ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 2

MWF – 12:00pm

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2062 ECE Building

Lecture 2 - Summary

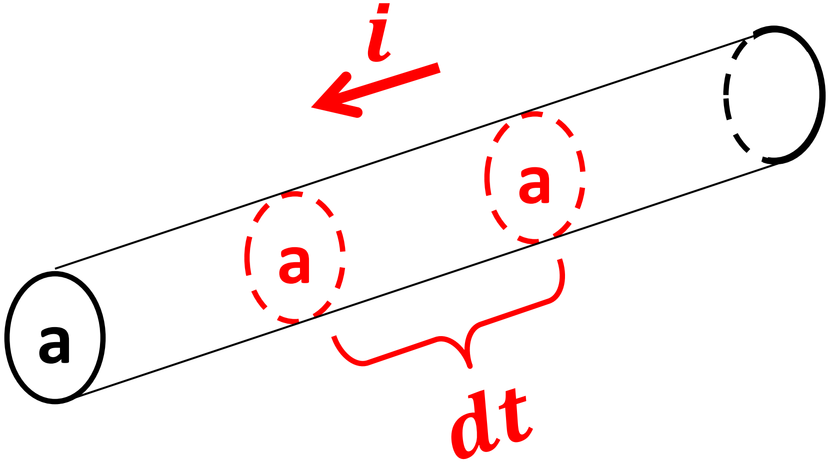
Learning Objectives

1. Define Voltage
2. Write difference between two points in symbolic form
3. Define “ground” and identify ground in a circuit
4. Compute Power absorbed/supplied by elements
5. Identify voltage and current sources
6. Define resistance, resistors
7. Define Ohm's Law
8. Use ohms law to compute voltage and current

Current

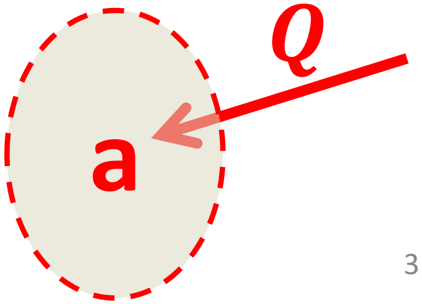
Charge in motion represents current which is defined as the rate of flow of charge through an area “a”

$$i = \frac{dq}{dt}$$



Alternatively we can say:

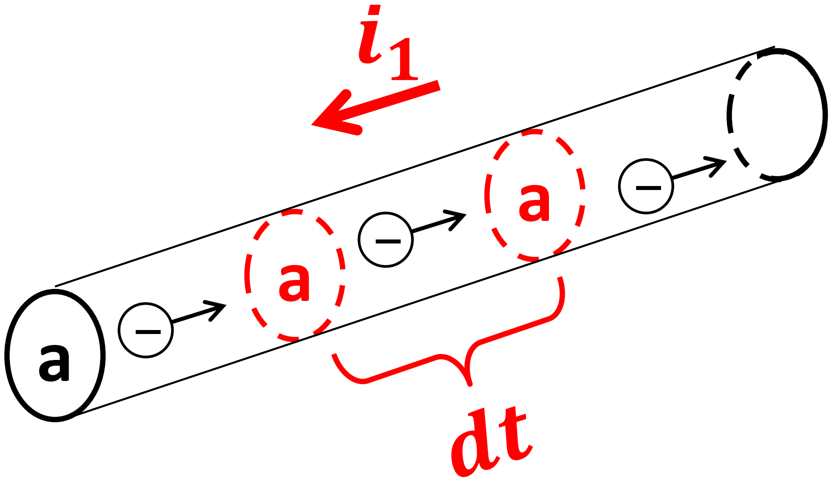
“Current is the (positive) charge that crosses an area “a” per second”



Current (can be confusing)

Current in metal wires was defined by Ampère in the 1820's when we did not know that it was due to electrons (discovered in 1897 by Thomson).

The positive direction of current, chosen at the time, turned out to be opposite to the flow of electrons. Therefore, electrons were assigned a negative charge.



Current units

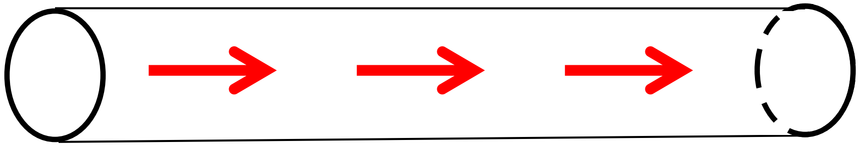
The unit of current is the Ampere with symbol [A]

A current of one ampere corresponds to a flow of one Coulomb of charge per second

$$\mathbf{1\ [A] = 1\ \left[\frac{C}{s}\right]}$$

Fluid analogy

PIPES



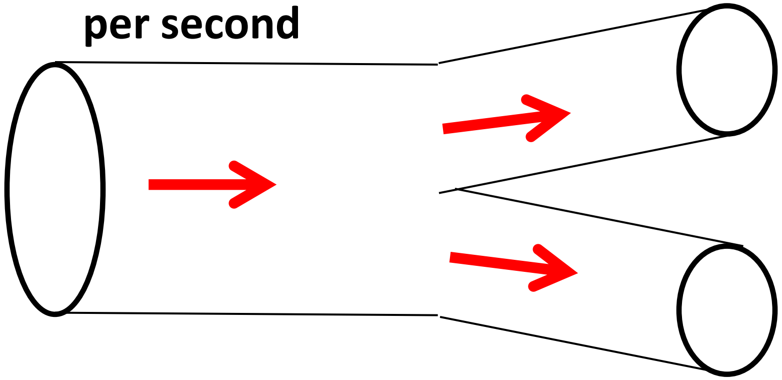
2 gallons per second

WIRES

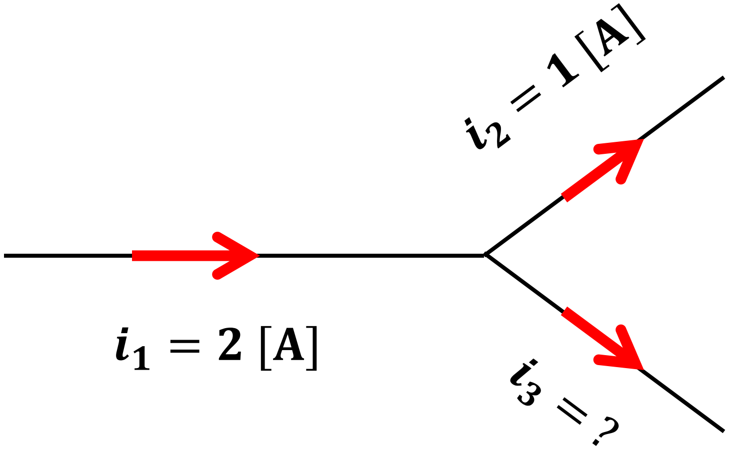
$2\ [\mathrm{A}]$ $2\ [\mathrm{A}]$ $2\ [\mathrm{A}]$

2 gallons per second

1 gallon per second



gallons per second?

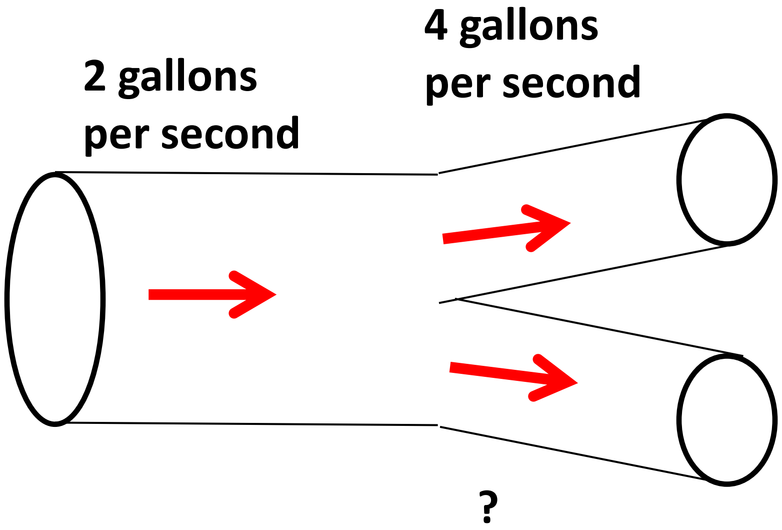


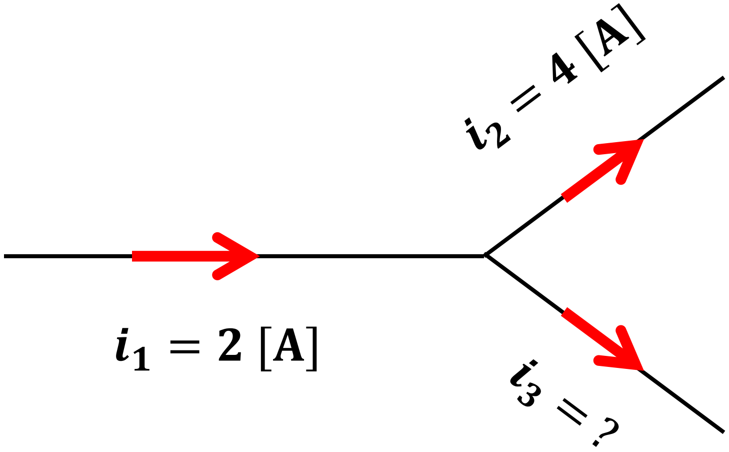
$i_2 = 1\ [\mathrm{A}]$

$i_1 = 2\ [\mathrm{A}]$

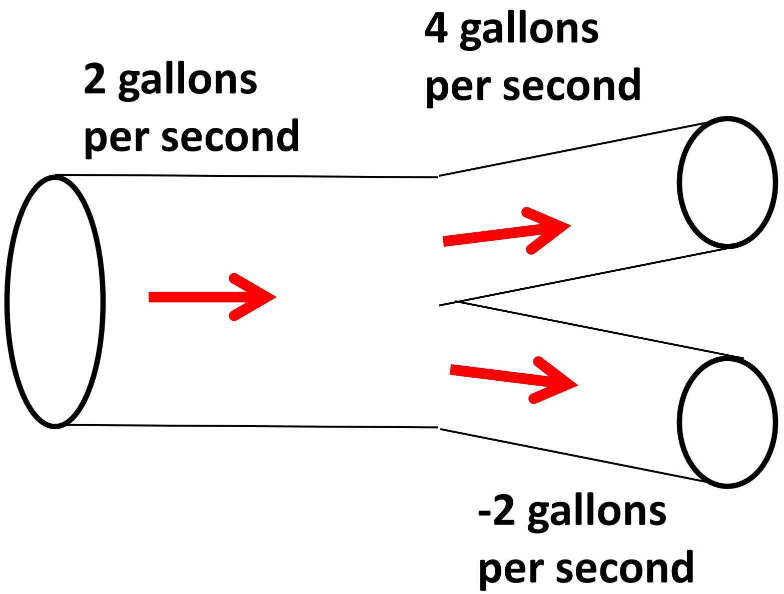
$i_3 = ?$

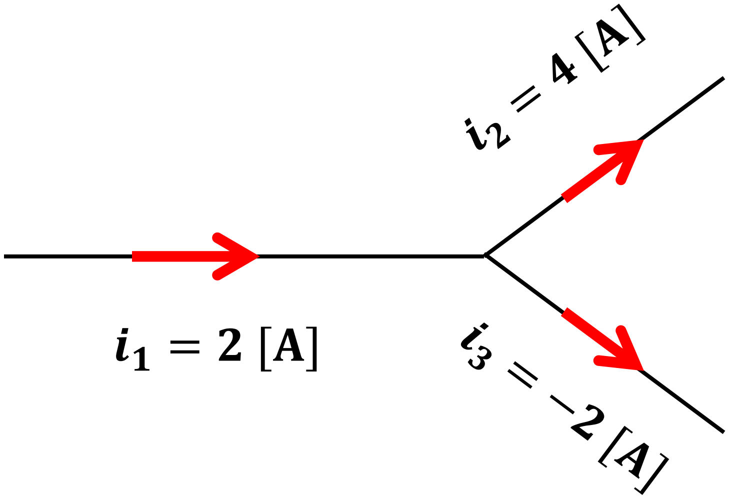
Fluid analogy

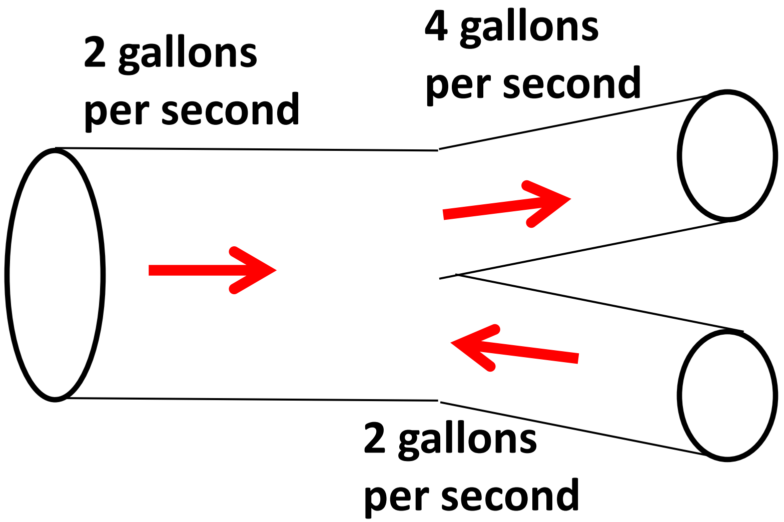


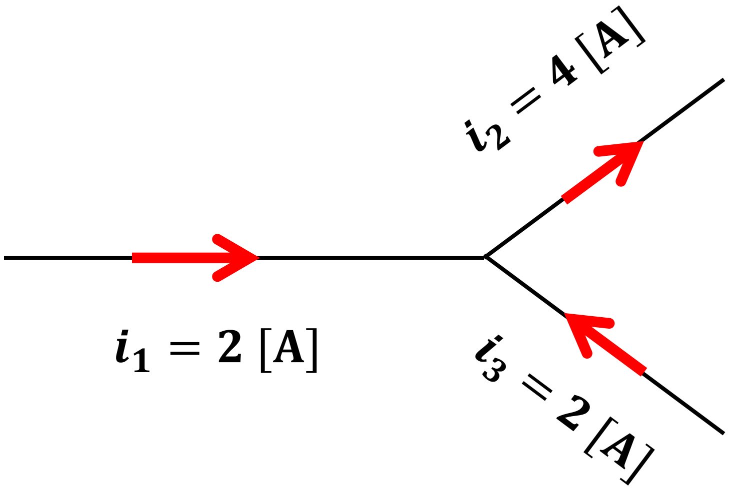


Fluid analogy



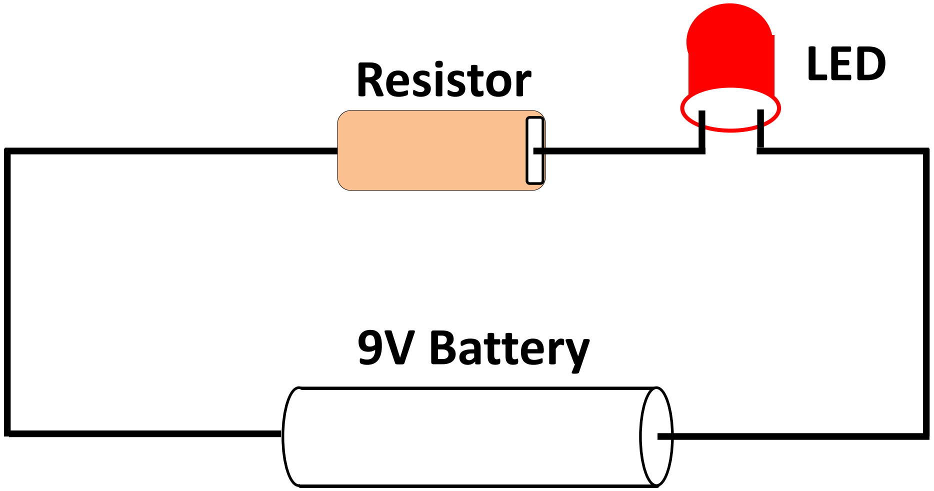






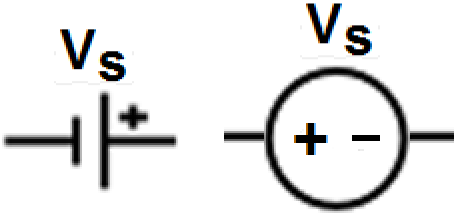
Electric circuits, elements, schematics

An electrical circuit is an interconnection of electrical elements through which current can flow. Electrical elements can be seen as a model or an abstraction of electrical devices. For example, let us say we would like to light up an LED using a battery. We may sketch a physical arrangement of this set-up as shown below.

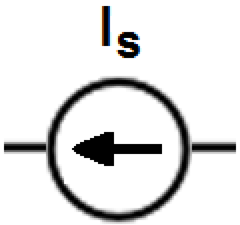


Symbols for common circuit elements

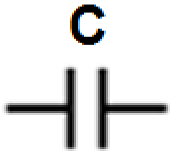
Resistor



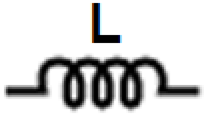
Voltage source



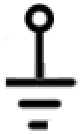
Current source



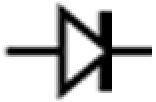
Capacitor



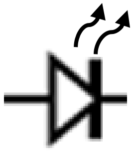
Inductor



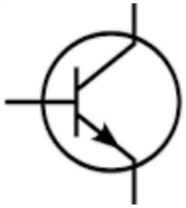
Ground node connection (zero reference voltage)



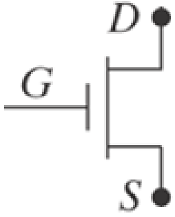
Diode



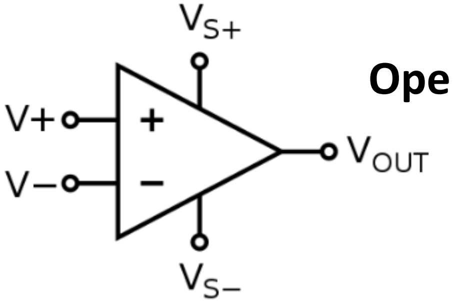
Light Emitting Diode (LED)



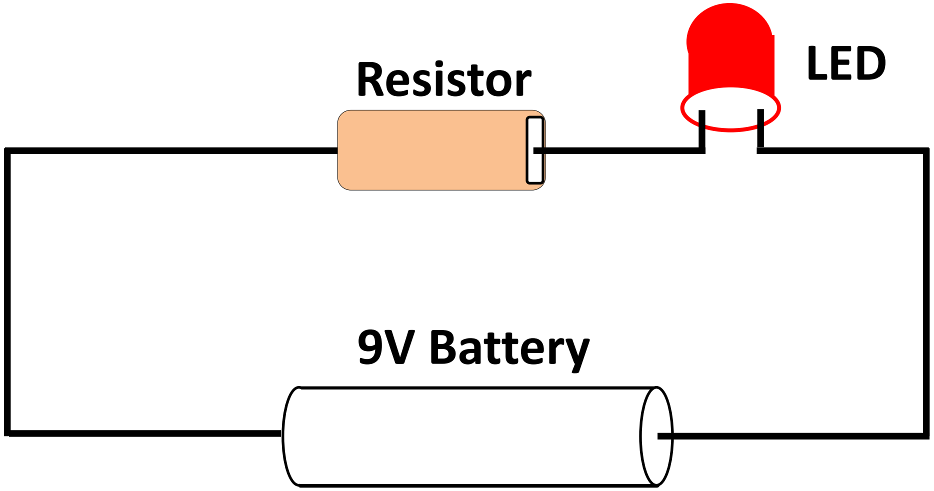
Bipolar Transistor



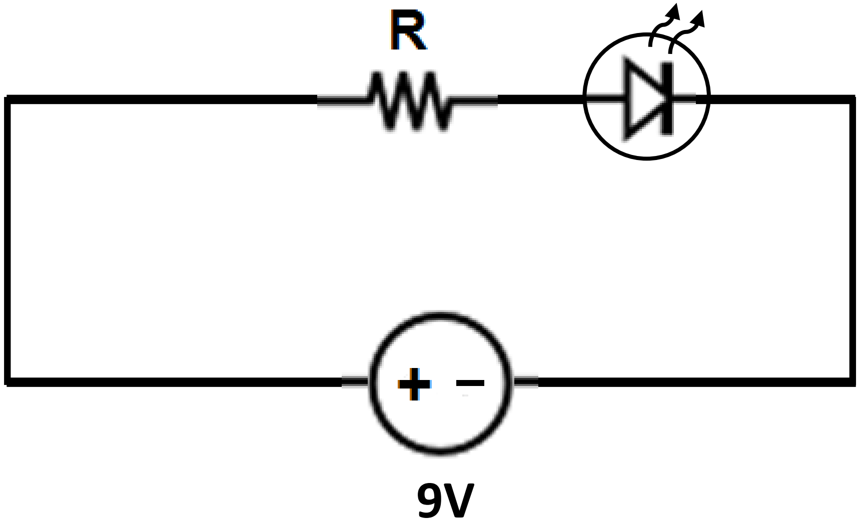
MOS Transistor



Operational Amplifier

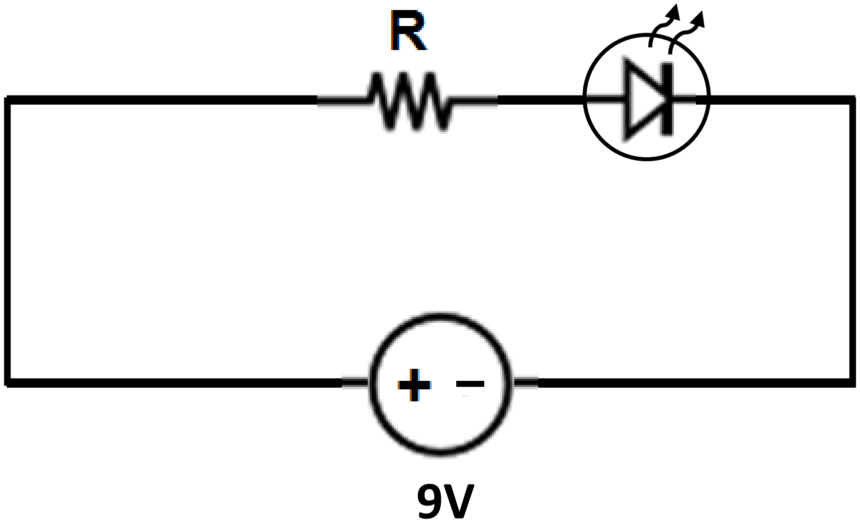


Circuit Schematic Diagram

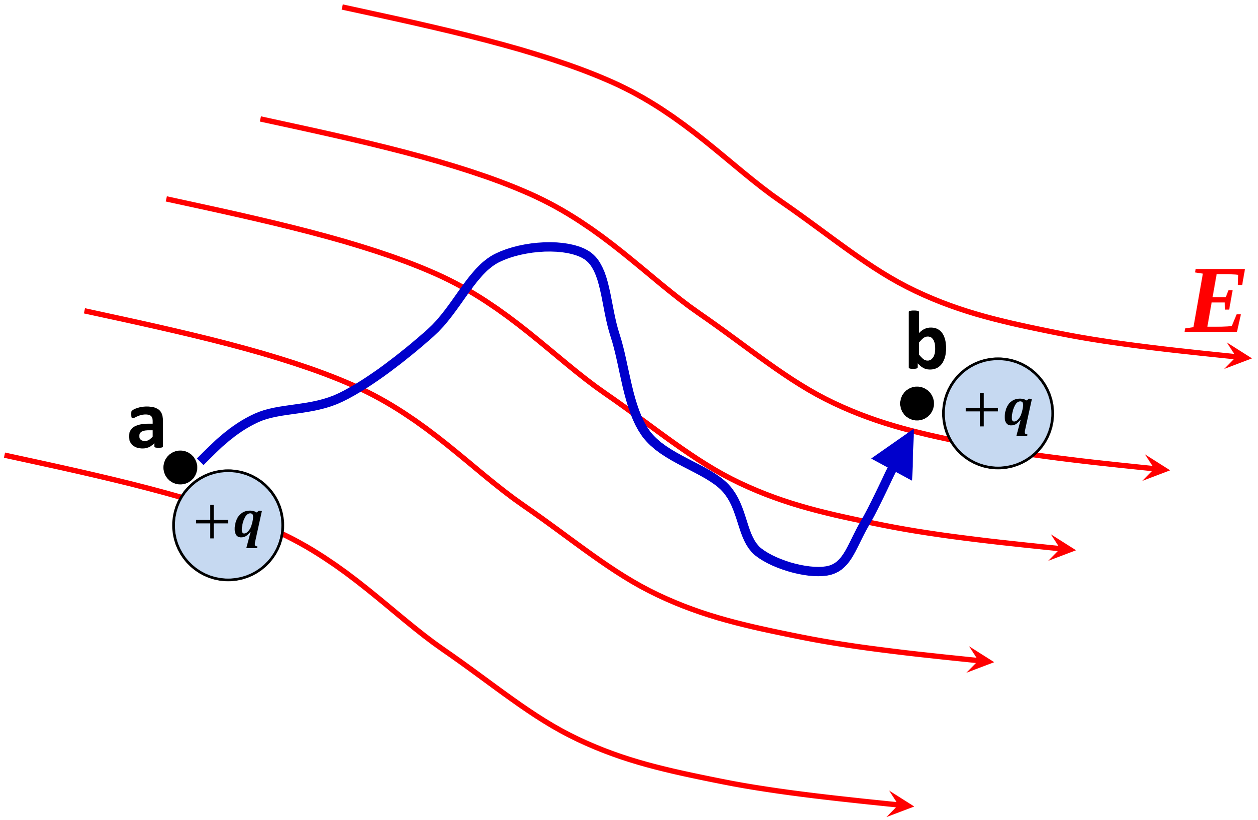


Voltage

The movement of charge in a system can transfer energy. For example, in the circuit shown below the chemical energy stored in the battery is converted to light in the LED.



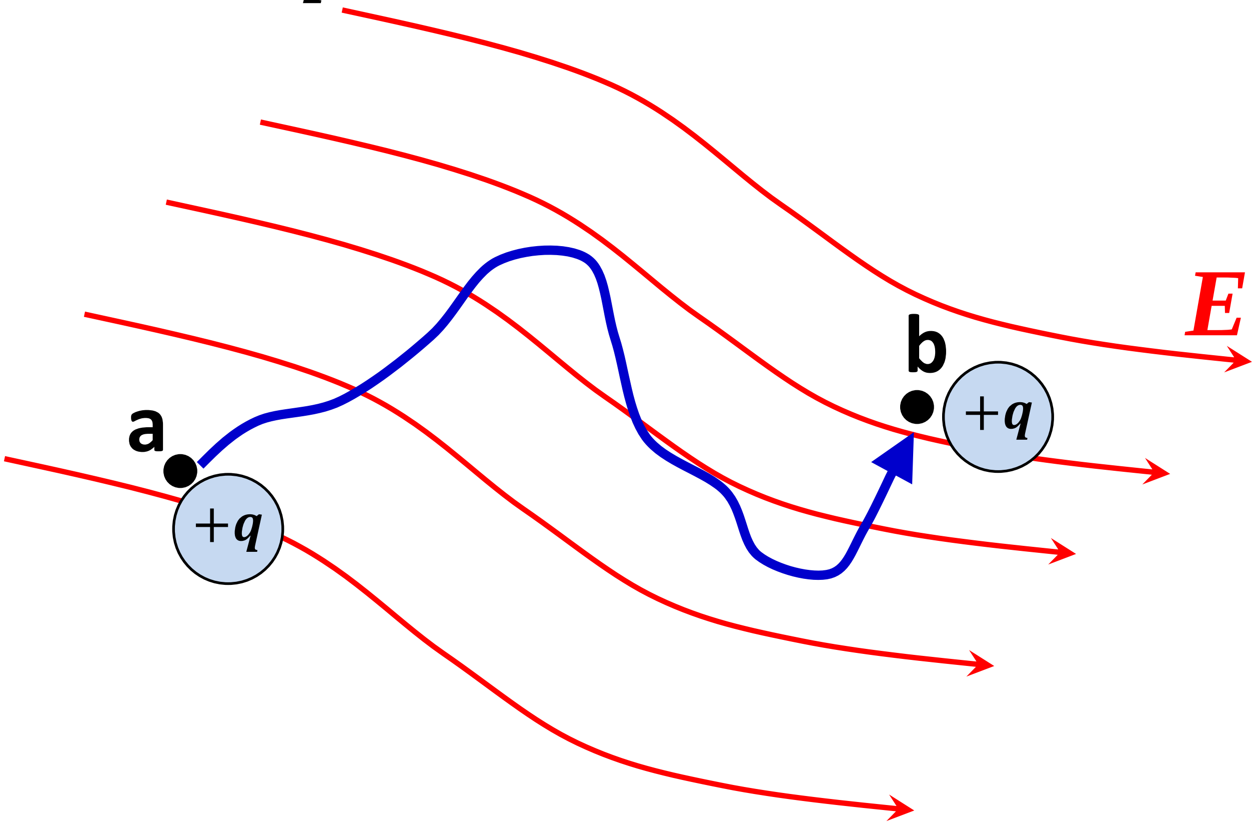
Voltage represents the work done in moving a unit charge from one point to another in an electric field E.



Voltage between point **a** and point **b**

$$V_{ab} = \frac{W_{ab}}{q}$$

work done in moving charge "q" from "a" to "b".



Unit of Voltage

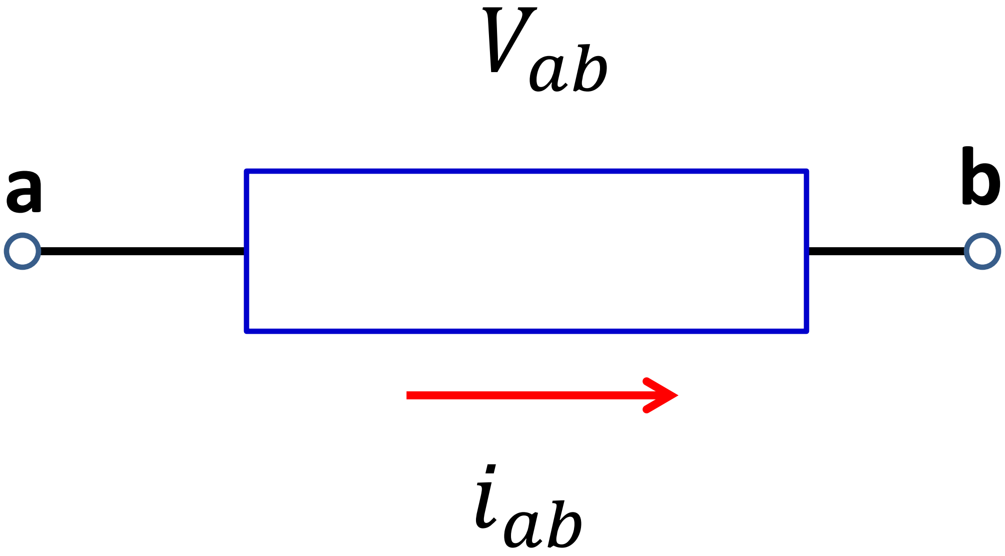
The unit of Voltage is the **Volt** with symbol [V]

$$1\ \mathrm{V} = \frac{1\ \mathrm{Joule}}{1\ \mathrm{Coulomb}}$$

Voltage is always defined between two points, as the difference in potential energy between those points.

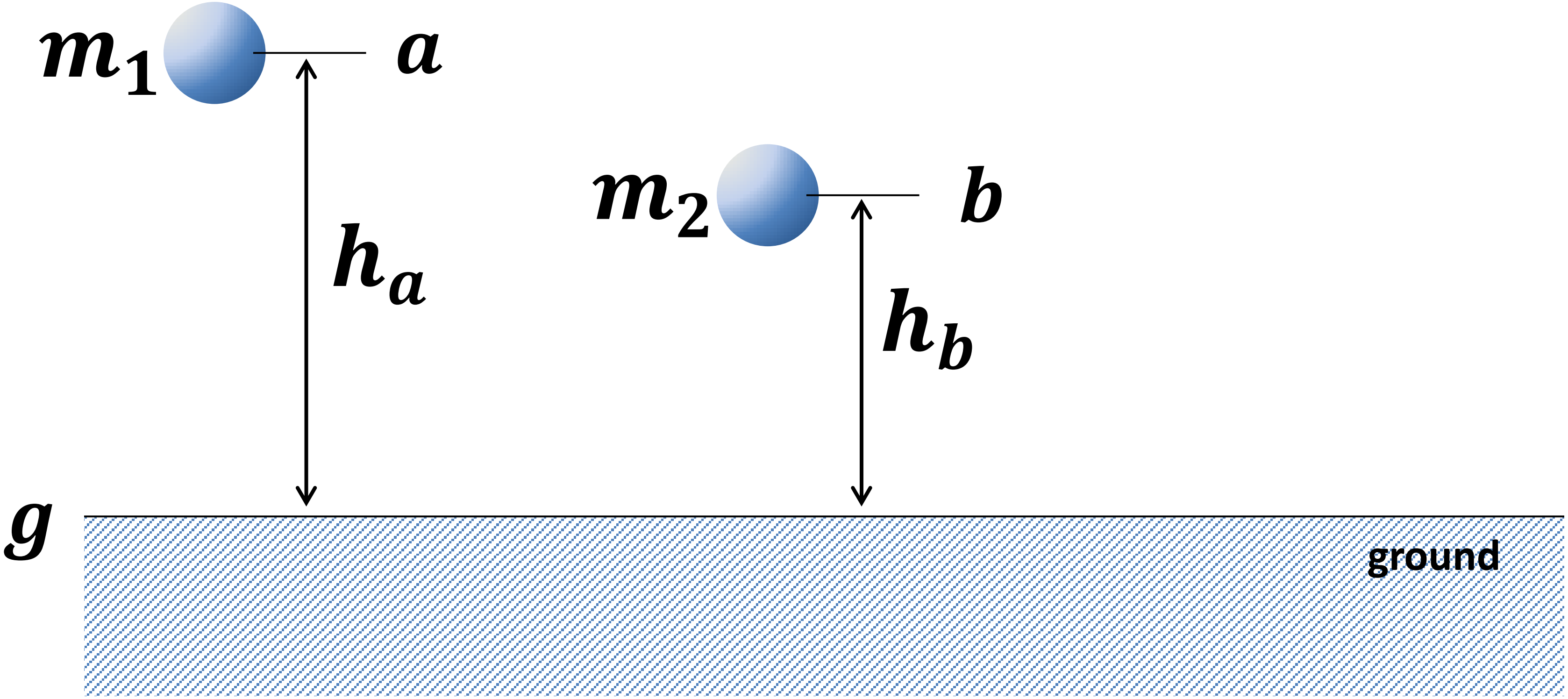
When one talks about a "**specific voltage**" at a "**specific point**" it is always assumed to be with respect to a **reference point** where the voltage is assigned to be zero.

The eletrical behavior of a circuit component is usually identified by the voltage difference at its terminals and the current flowing through it.



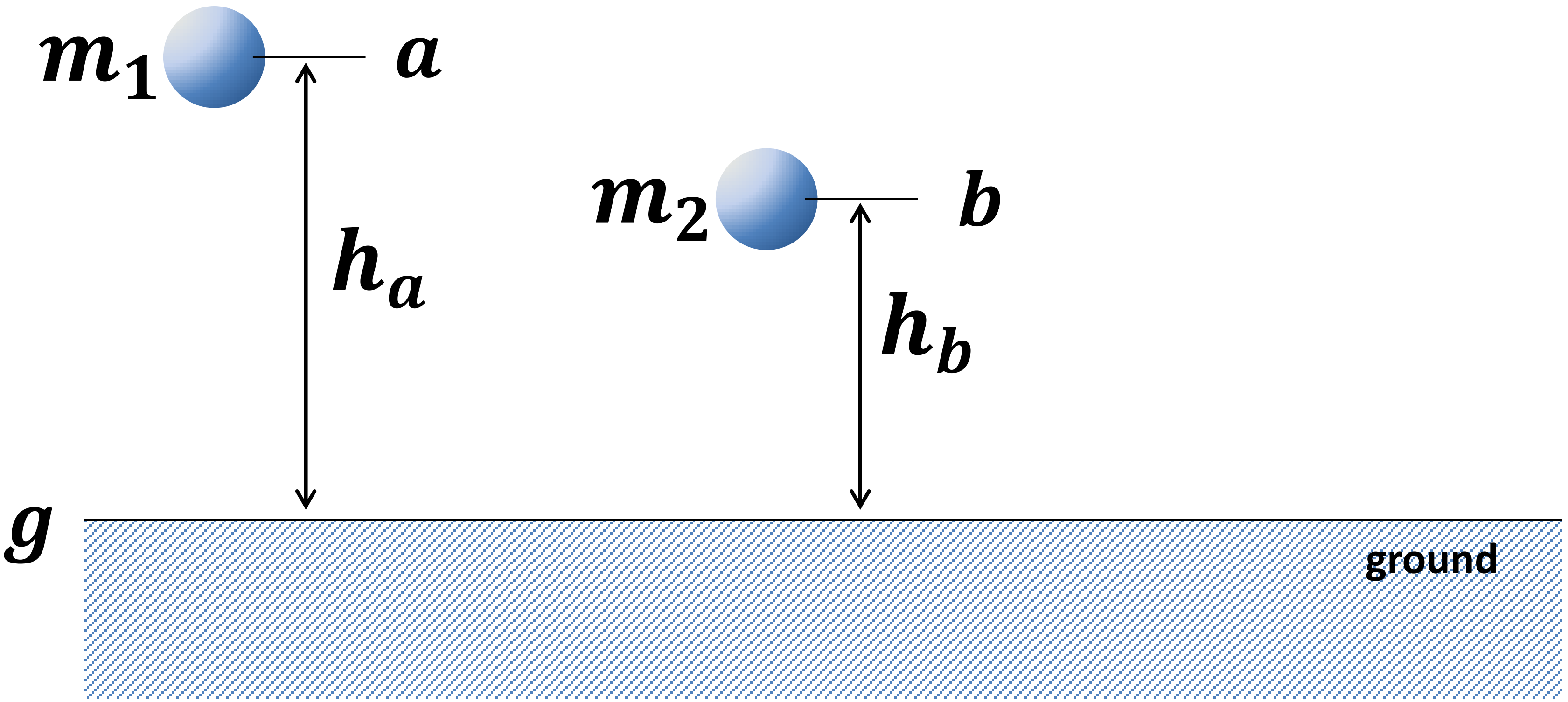
Ground

In circuit analysis it is useful to define a **reference point**. This is analogous to specifying the height of mass m with respect to a reference point called **ground**.



Ground

Let $\boldsymbol{h_{xy} = h_x - h_y}$ denote the difference in height between two points at heights $\boldsymbol{h_x}$ and $\boldsymbol{h_y}$ with respect to ground.

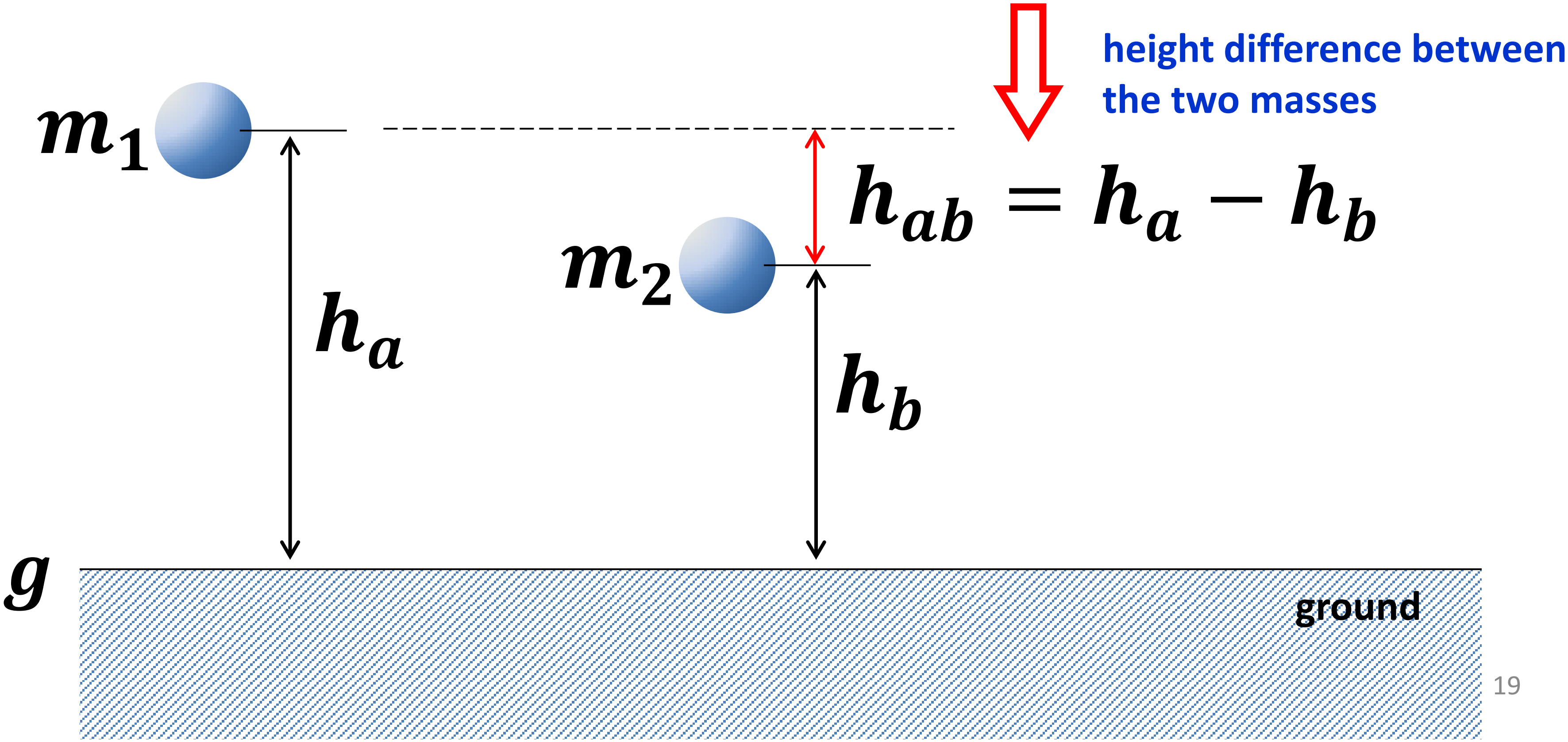


Ground

We have the following relationships for $\boldsymbol{a}$ and $\boldsymbol{b}$

$$h_{ag} = h_a - h_g = h_a$$

$$h_{bg} = h_b - h_g = h_b$$



height difference between the two masses

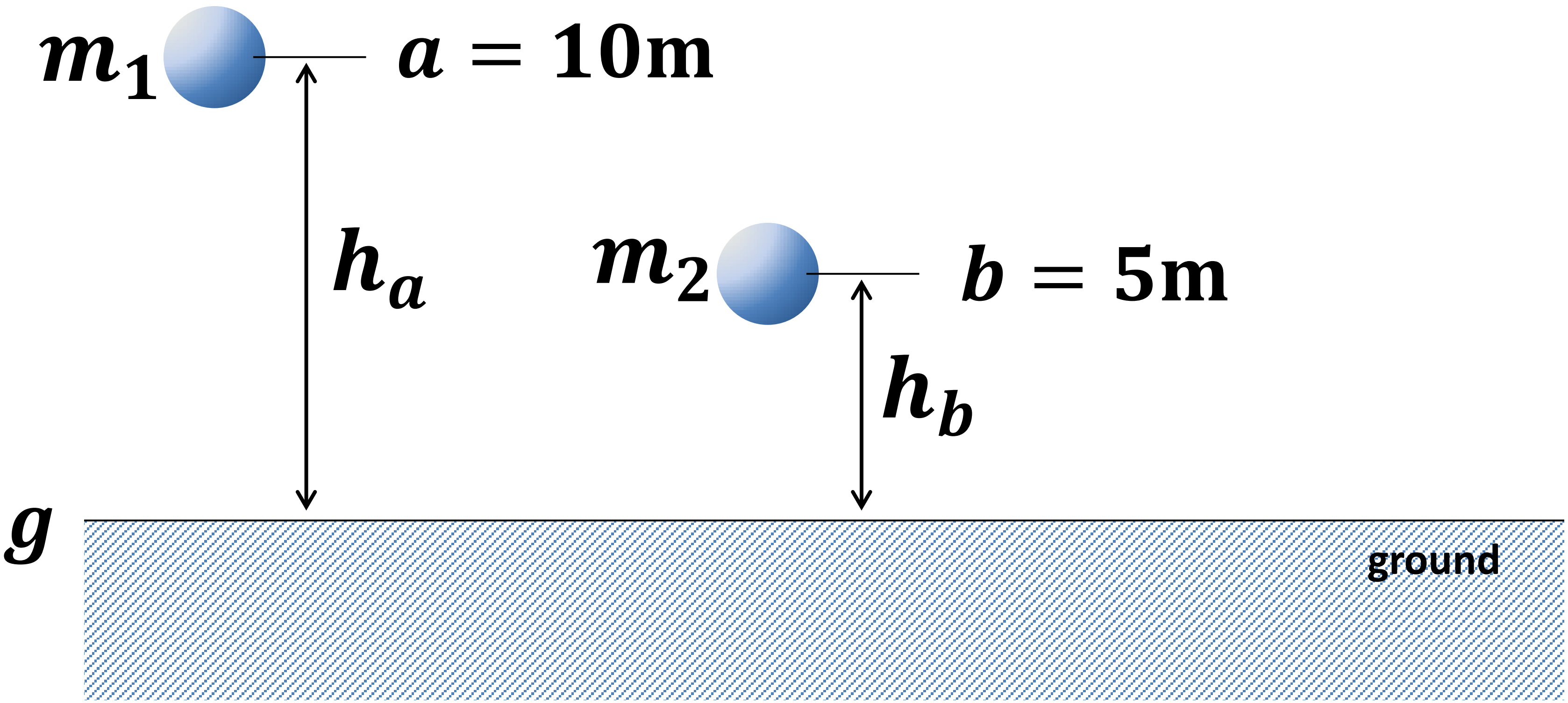
$$h_{ab} = h_a - h_b$$

Example

$$h_{ab} = h_a - h_b = 10 - 5 = 5\text{m}$$

Similarly:

$$h_{ba} = h_b - h_a = 5 - 10 = -5\text{m}$$

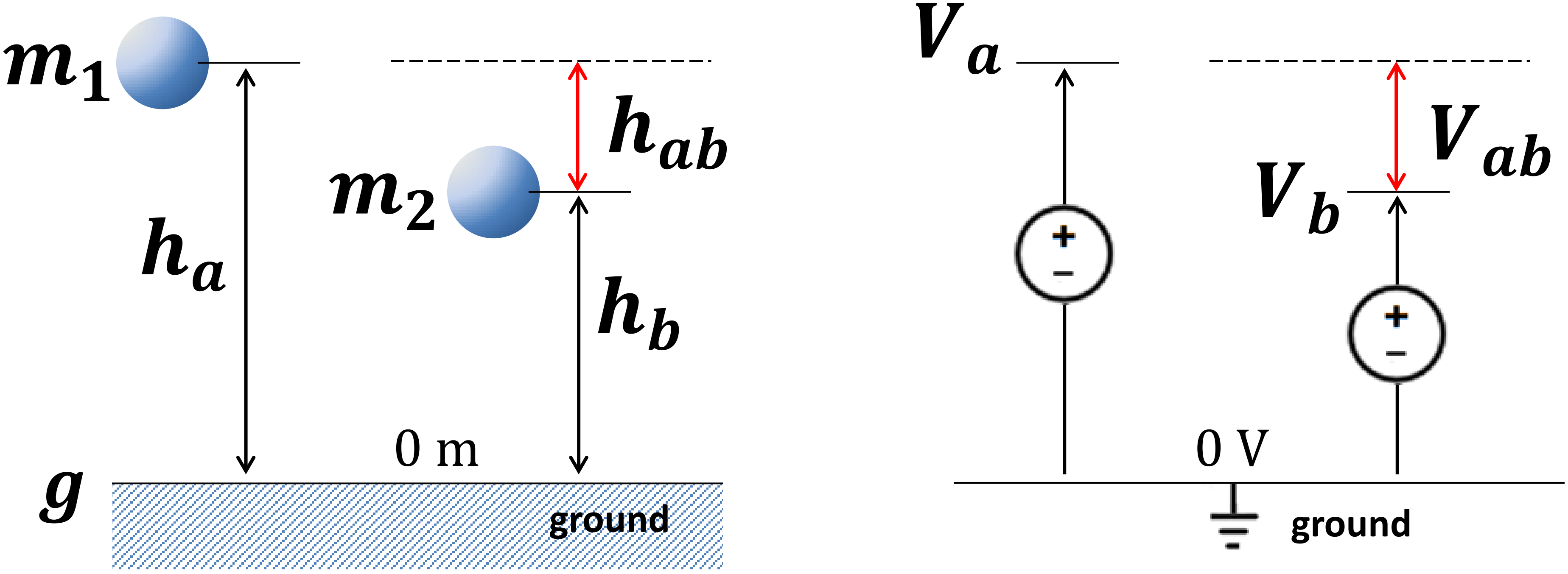


Equivalence between height & voltage

The reference for a circuit is also called "ground"

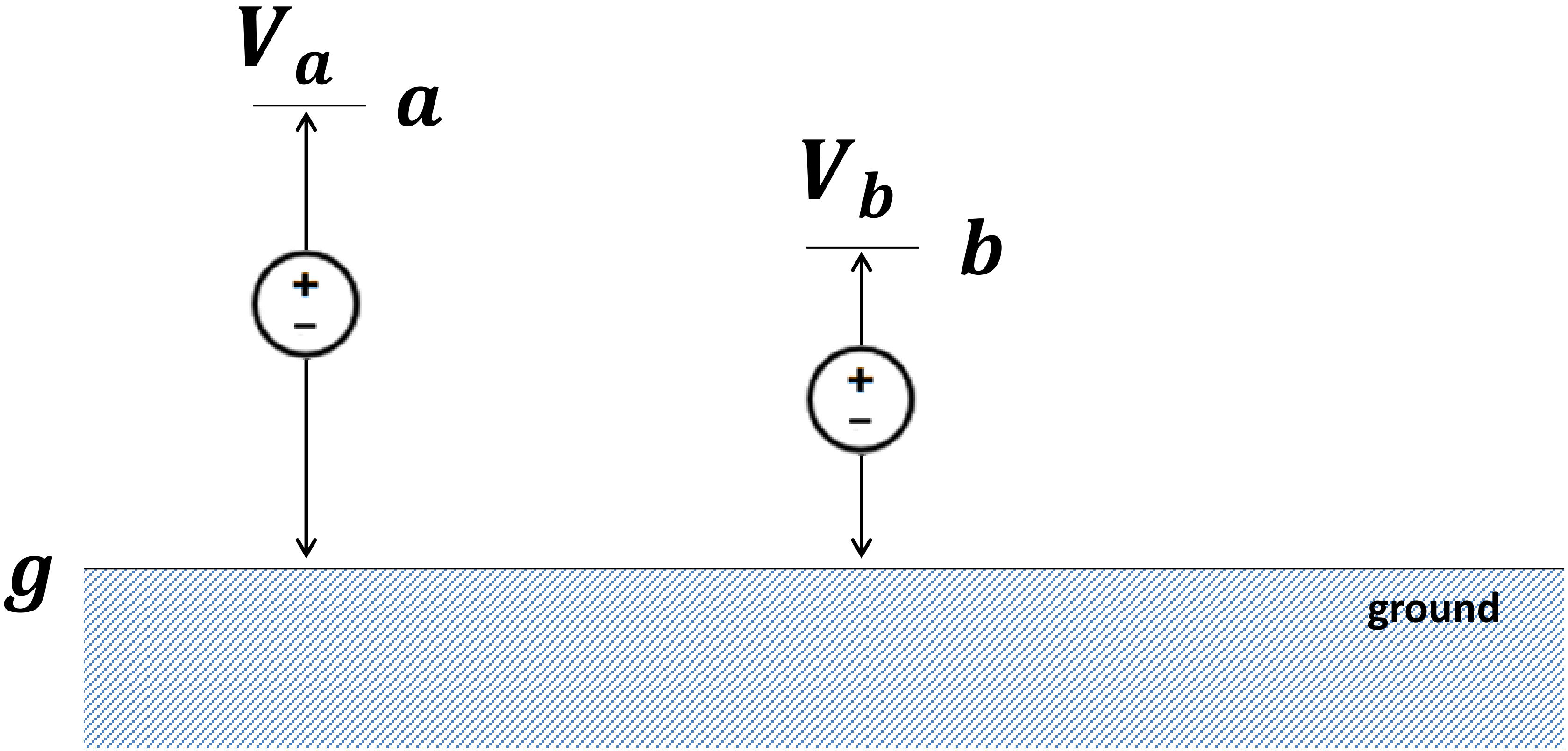
$$h_g = 0 \text{ m}$$

$$V_g = 0 \text{ V}$$



Voltage – detailed formulation

Let $\boldsymbol{V_{xy} = V_x - V_y}$ denote the difference in voltage between two points at heights $\boldsymbol{x}$ and $\boldsymbol{y}$, respectively.

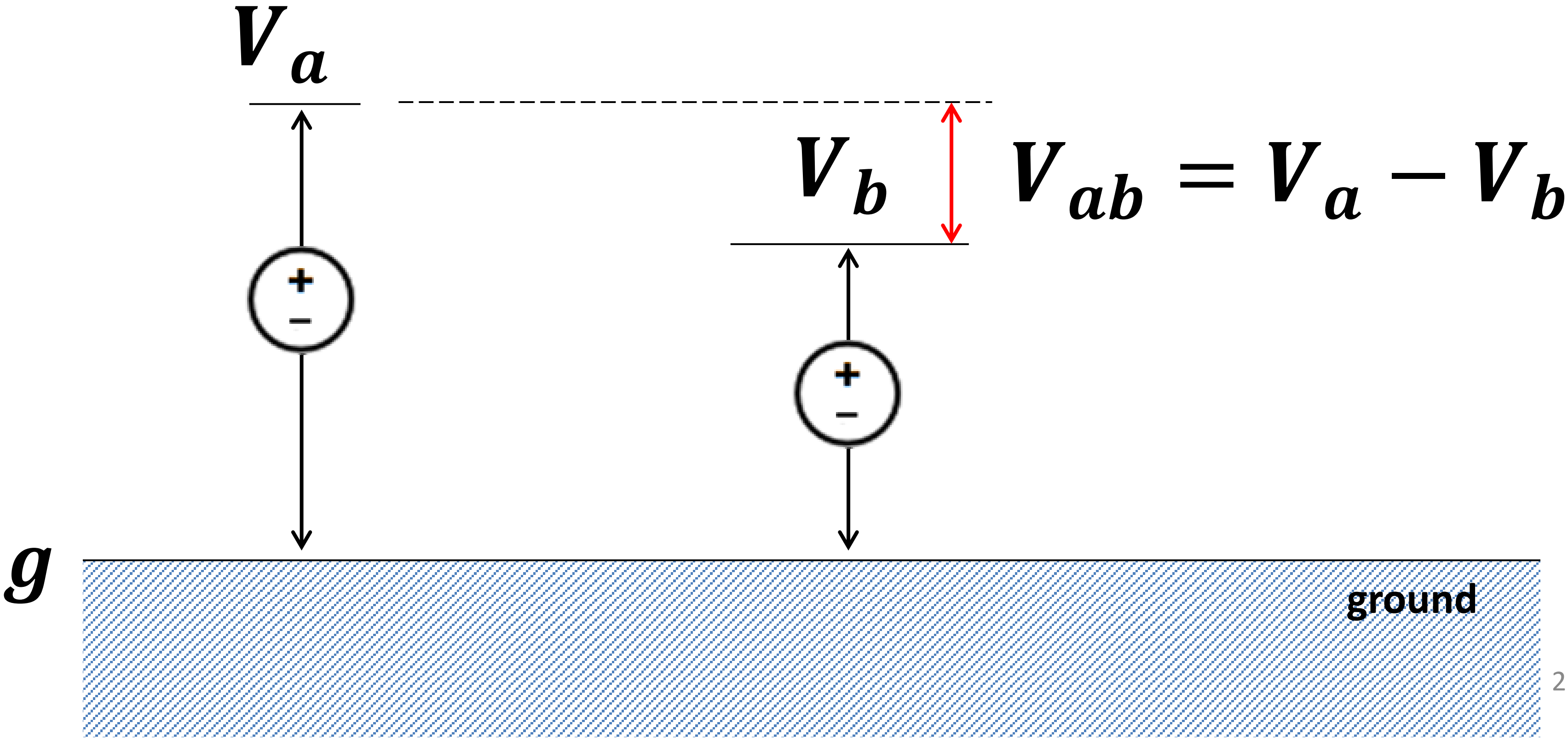


Voltages with ground reference

We have the following relationships

$$V_{ag} = V_a - V_g = V_a$$

$$V_{bg} = V_b - V_g = V_b$$



$$V_{ab} = V_a - V_b$$

Examples

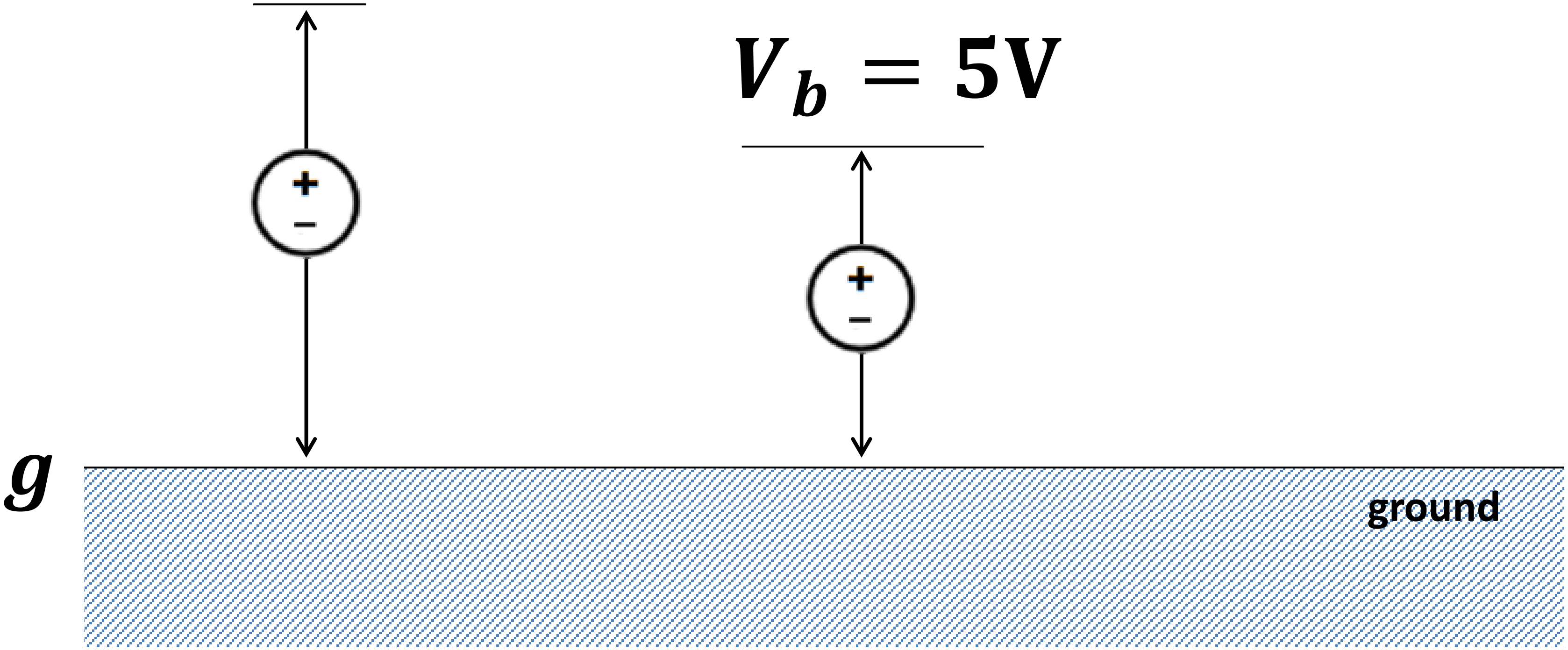
$$V_{ab} = V_a - V_b = 10 - 5 = 5\text{V}$$

Similarly:

$$V_{ba} = V_b - V_a = 5 - 10 = -5\text{V}$$

$$V_a = 10\text{V}$$

$$V_b = 5\text{V}$$



Remember the notation

$$V_{ab} \rightarrow Voltage\ between\ "a"\ and\ "b"$$

$$V_a \rightarrow Voltage\ between\ "a"\ and\ ground$$

Power

Power is the rate at which work is dissipated (consumed) or delivered (supplied)

$$P = \frac{dW}{dt} = \underbrace{\frac{dW}{dq}}_{V} \times \underbrace{\frac{dq}{dt}}_{I}$$

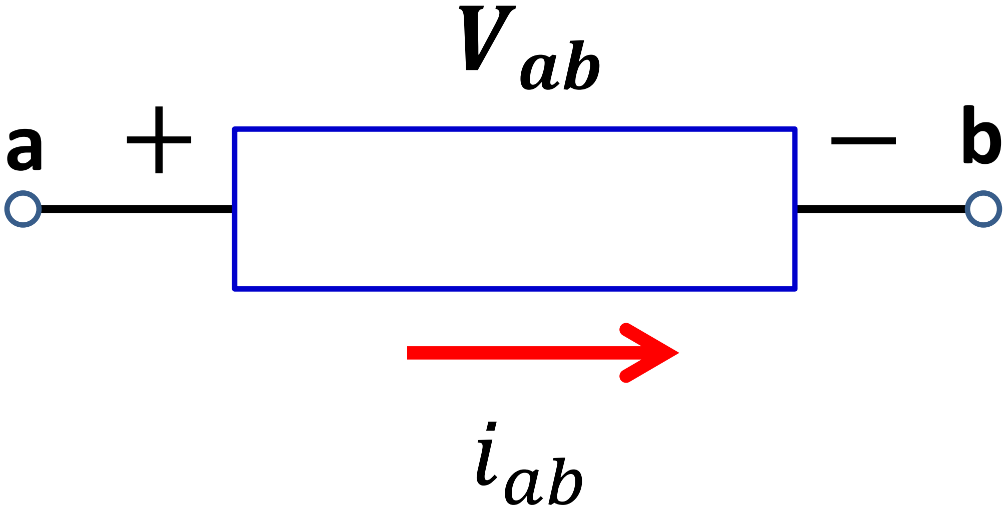
$$\boxed{P = V \times I}$$

$P > 0$ → **Power dissipated or consumed**

$P < 0$ → **Power delivered or supplied**

Power sign convention

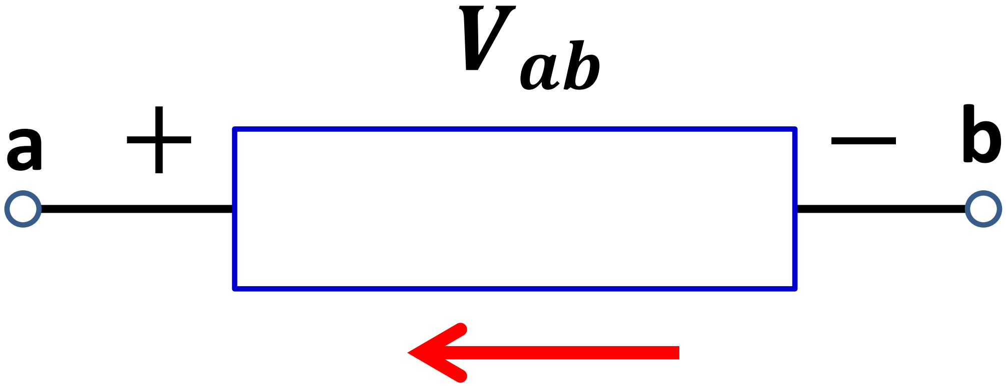
If current enters the positive end of an element and leaves the negative end, then the element is dissipating power. The power is positive in this case.



$$\boxed{P = V_{ab} \times i_{ab}}$$

Power sign convention

If current enters the negative end of an element and leaves the positive end the device, it is delivering (supplying) power. The power is negative in this case.



$$i_{ba} = -i_{ab}$$

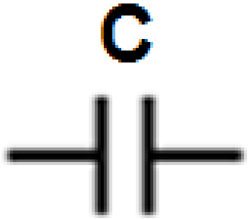
$$\boxed{P = V_{ab} \times i_{ba} = -V_{ab} \times i_{ab}}$$

Circuit elements and power

Passive elements can only consume power

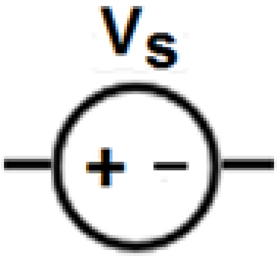
Resistor

Inductor

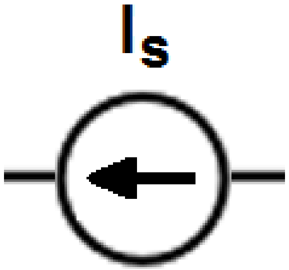


Capacitor

Sources can deliver or consume power

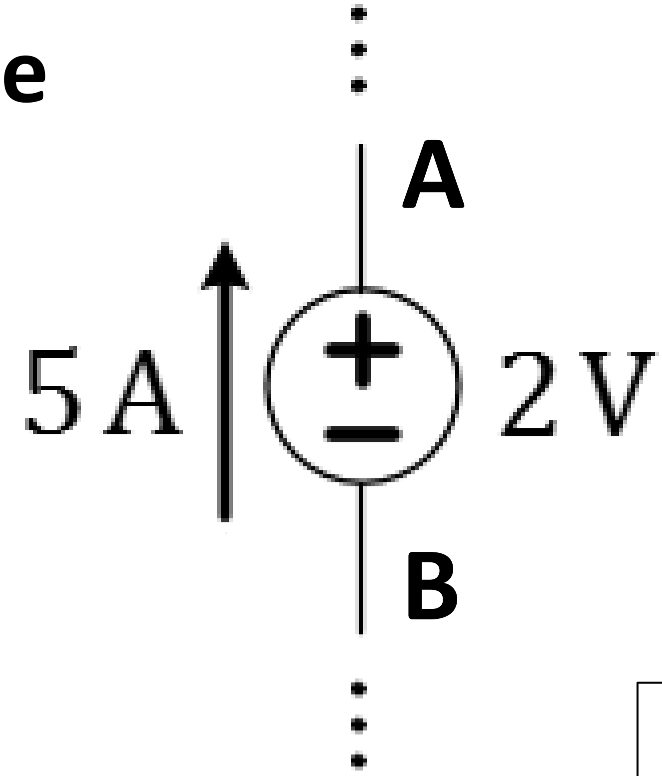


Voltage source



Current source

Voltage Source



$$\boxed{V_{AB} = 2\text{ V}}$$

$$\boxed{I_{AB} = -5\text{ A}}$$

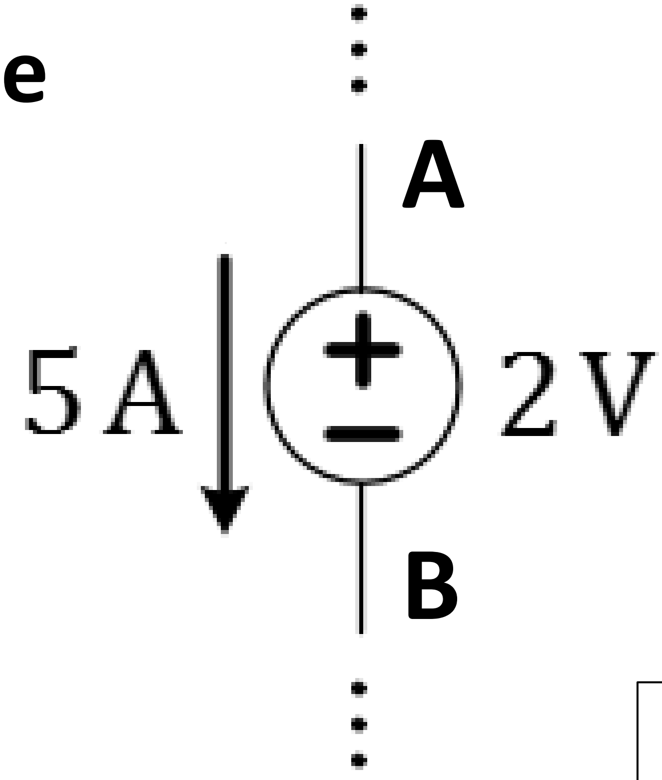
$$\boxed{P = -10\text{ W}}$$

DELIVERING POWER

VOLTAGE SOURCE ACTS LIKE A PUMP SENDING CURRENT “UPHILL”

GENERATES current that gives power to the rest of the circuit

Voltage Source



$$\boxed{V_{AB} = 2\text{ V}}$$

$$\boxed{I_{AB} = 5\text{ A}}$$

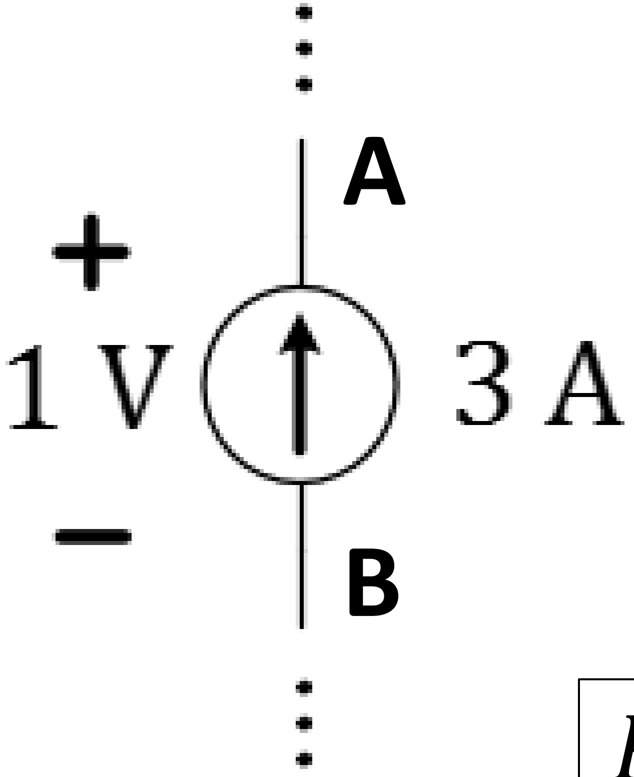
$$\boxed{P = 10\text{ W}}$$

ABSORBING POWER

CURRENT IN VOLTAGE SOURCE IS GOING “DOWNHILL” AS IN **RECHARGE** MODE

RECEIVES current and receives power from the rest of the circuit

Current Source



$$\boxed{I_{AB} = -3\ \mathrm{A}}$$

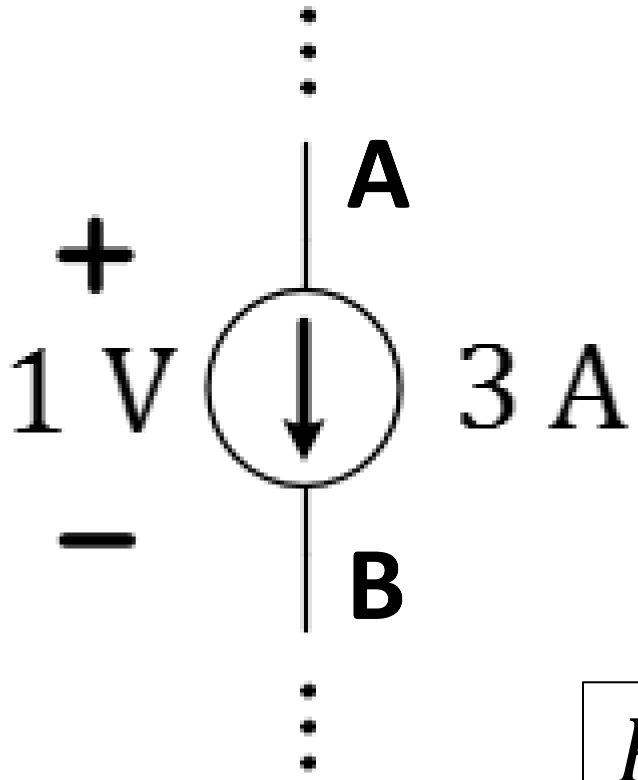
$$\boxed{V_{AB} = 1\ \mathrm{V}}$$

$$\boxed{P = -3\ \mathrm{W}}$$

DELIVERING POWER

CURRENT SOURCE ESTABLISHES “UPHILL” VOLTAGE PUMPING ENERGY INTO THE REST OF THE CIRCUIT

Current Source



$$\boxed{I_{AB} = 3\ \mathrm{A}}$$

$$\boxed{V_{AB} = 1\ \mathrm{V}}$$

$$\boxed{P = 3\ \mathrm{W}}$$

ABSORBING POWER

VOLTAGE DROP IS IMPOSED BY THE REST OF THE CIRCUIT WHICH MAKES CURRENT GO “DOWNHILL”

Ideal Sources

These are limit cases which simplify the study of circuits. In this course, we will assume that sources are ideal.

IDEAL VOLTAGE SOURCE

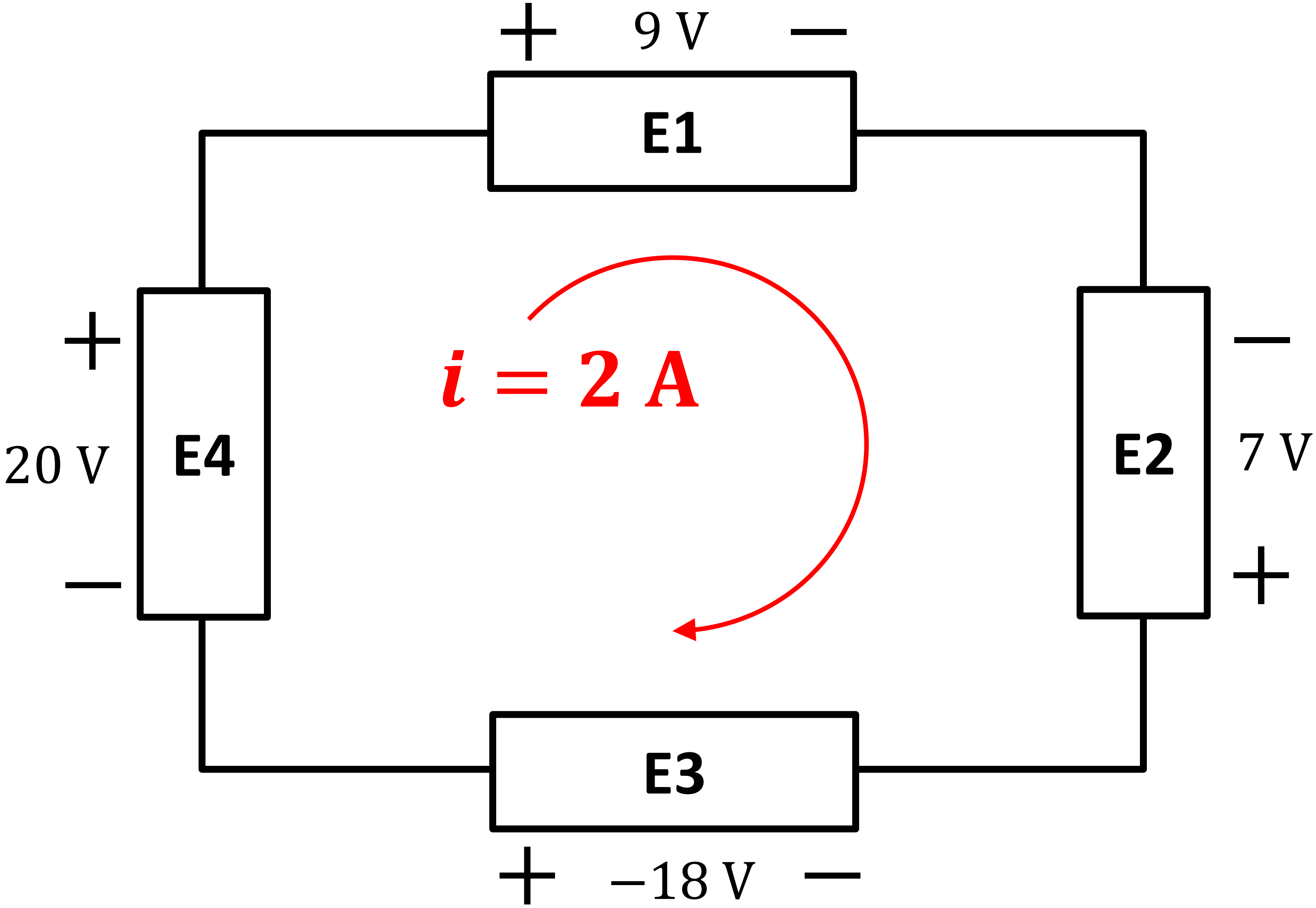
An ideal voltage source has zero internal resistance and can drive up to infinite current

IDEAL CURRENT SOURCE

An ideal curent source has infinite internal resistance and can have up to infinite voltage across its terminals

Example

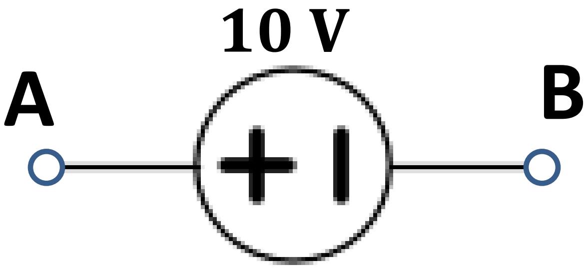
Find the power consumed or supplied by each element.



Electrical Circuit

An **electrical circuit** is made up of **electrical elements.** Initially, we will look at circuits with these elements:

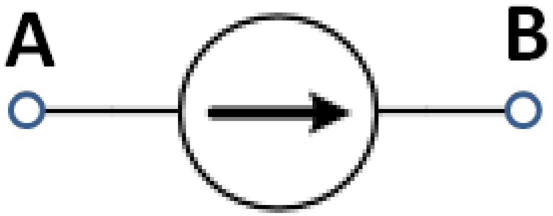
1)



Ideal Voltage Source

$$\boxed{V_{AB} = V_A - V_B}$$

2)



Ideal Current Source

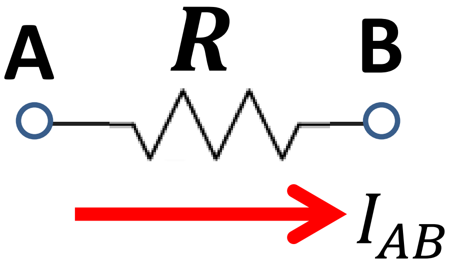
$$\boxed{I_{AB}}$$

3)

Wire (ideal conductor)

$$\boxed{V_{AB} = 0\ \text{V}}$$

4)

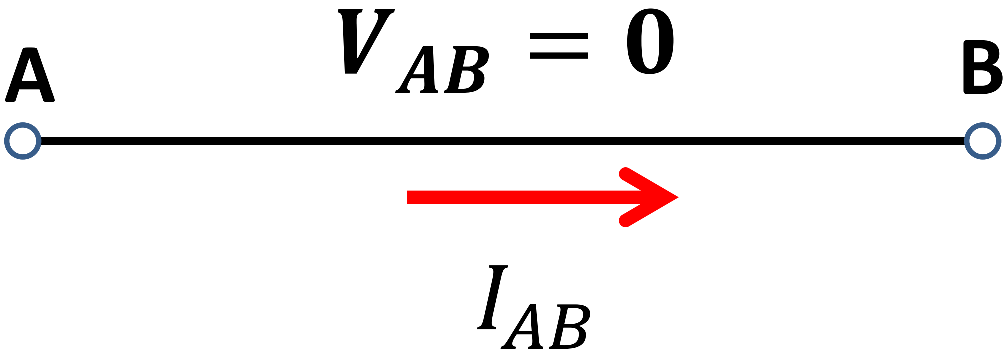


Resistor

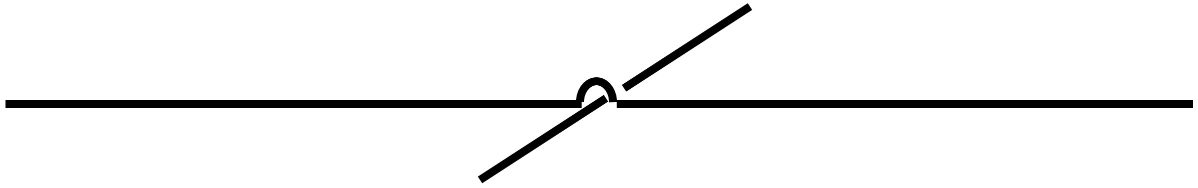
$$\boxed{V_{AB} = I_{AB}\, R}$$

Ideal Wire

Wires are represented by unbroken lines. Wires are assumed to be ideal conductors, i.e., the voltage difference between two point on a wire is zero (equipotential). Two points in a circuit that are connected by a wire are said to be **shorted** together.



If you have to draw two wires which cross but do not touch



Resistor

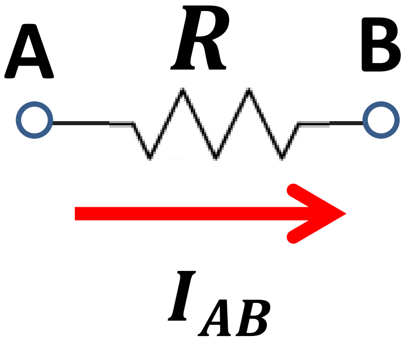
A resistor is an element which requires a certain effort on the part of the voltage to push a current through it. The resistance R is quantified by:

$$\boxed{R = \frac{\rho\,\ell}{A}}$$

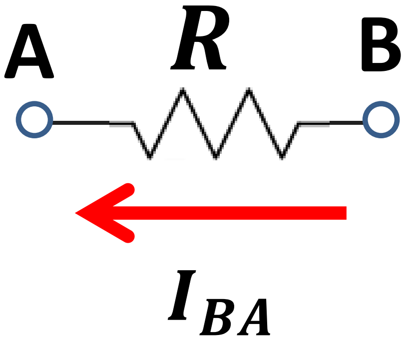
ρ	**Resistivity of the material**
ℓ	**Length**
A	**Cross-sectional area**

Ohm’s Law

Ohm's law captures the relationship between voltage across a resistor and current through it. Ohm's law can have the following two forms,



$$V_{AB} = I_{AB} R$$



$$V_{AB} = -I_{BA} R$$

Unit of Resistance

Unit of Resistance is the **Ohm** with symbol [Ω]

$$1\,\Omega = \frac{1\,\text{Volt}}{1\,\text{Ampere}}$$

In some applications is more convenient to use the “Conductance” which is the inverse of the resistance. The unit of conductance is the “Siemens” with symbol [S] but also [Ω^{-1}] or [mho] (which is ohm backwards)

$$1\,\text{S} = \frac{1\,\text{Ampere}}{1\,\text{Volt}}$$

Observations on Ohm’s Law

$$V = IR$$

For the same current, a higher resistance cause a higher voltage drop at the terminals

$$I = \frac{V}{R}$$

For the same voltage at the terminals, a higher resistance cause a smaller current

Observations on Ohm's Law

$$I = \frac{V}{R}$$

For low resolution resistance R, **a small voltage may cause a high current**.

$$V = IR$$

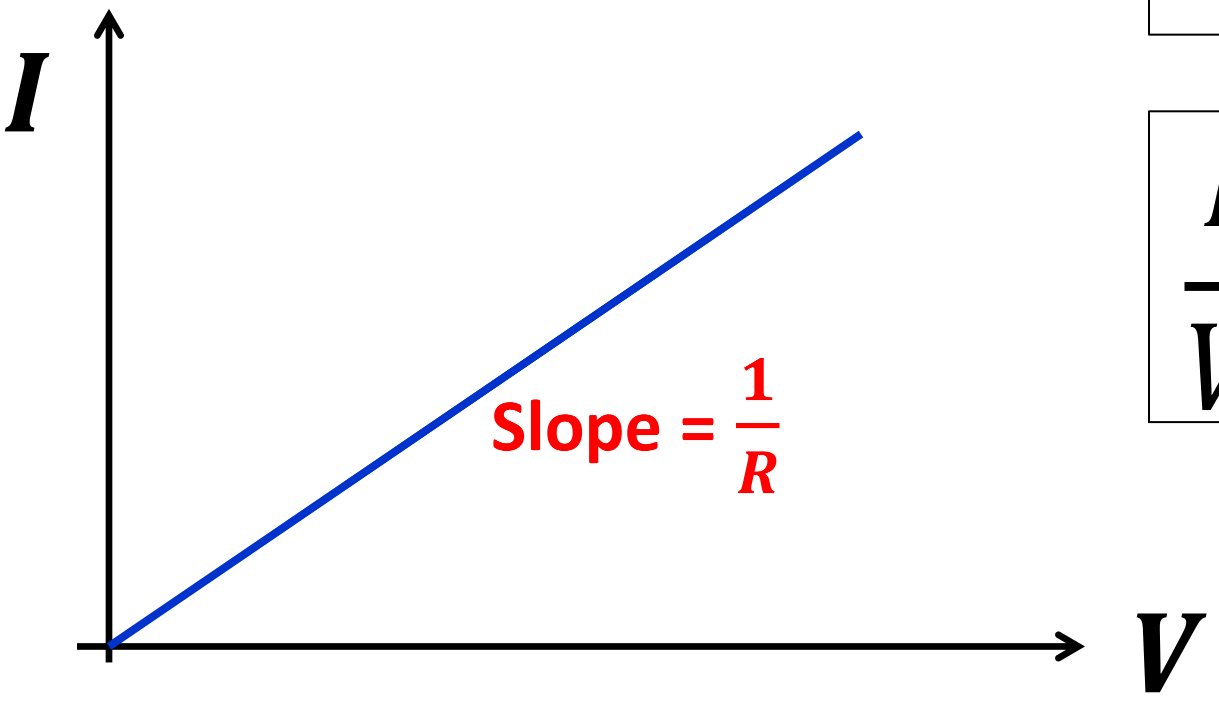
For large resistance R, **a small current may cause a high voltage**.

Current-Voltage or I-V Curves

I-V curves capture the relationship between current and voltage. For a **resistance**, the I-V is linear (straight line)

$$\boxed{V = IR}$$

$$\boxed{\frac{I}{V} = \frac{1}{R}}$$

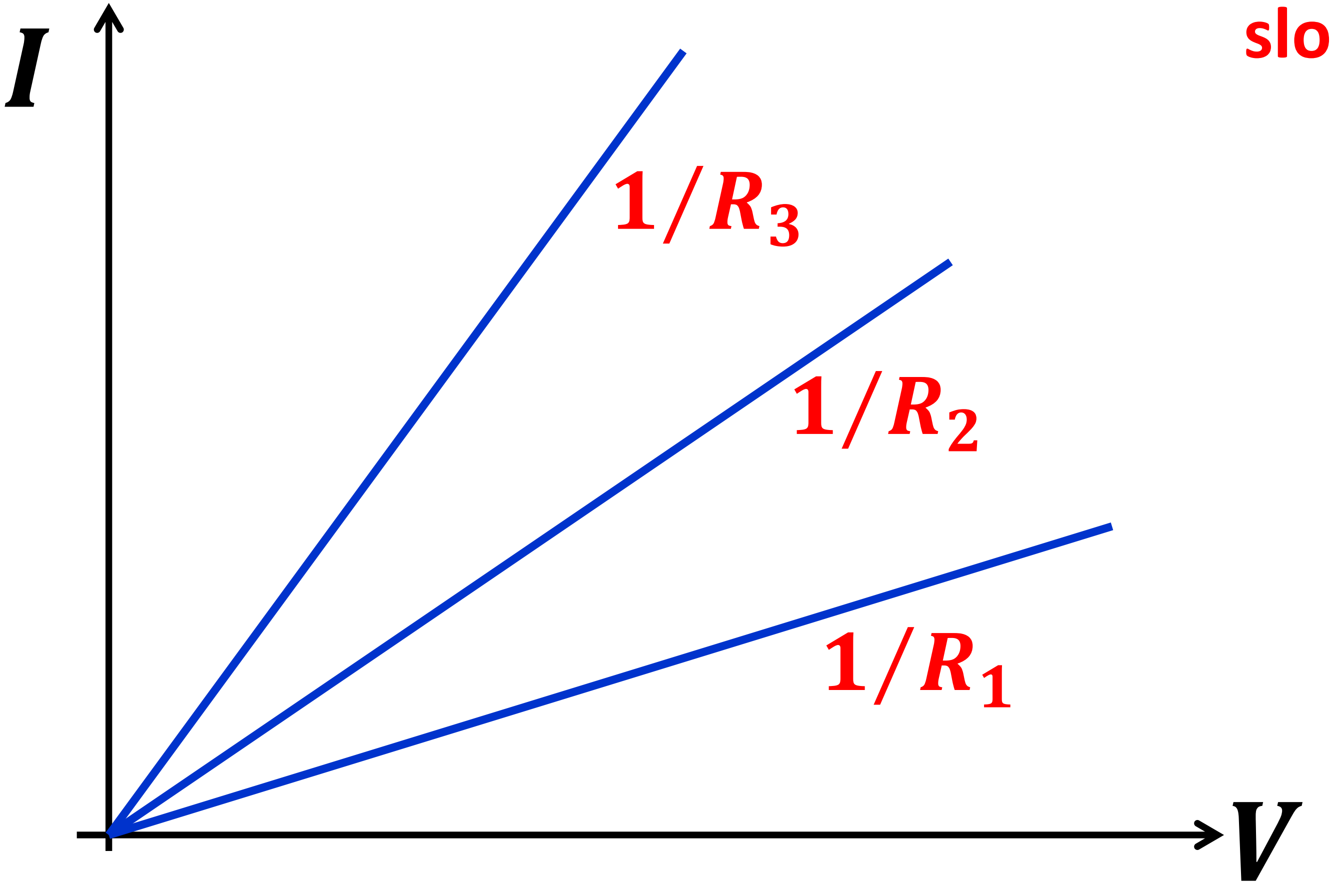


The inverse of the slope represents the resistance

$$\boxed{R_1 > R_2 > R_3}$$

$$\boxed{\frac{I}{V} = \frac{1}{R}}$$

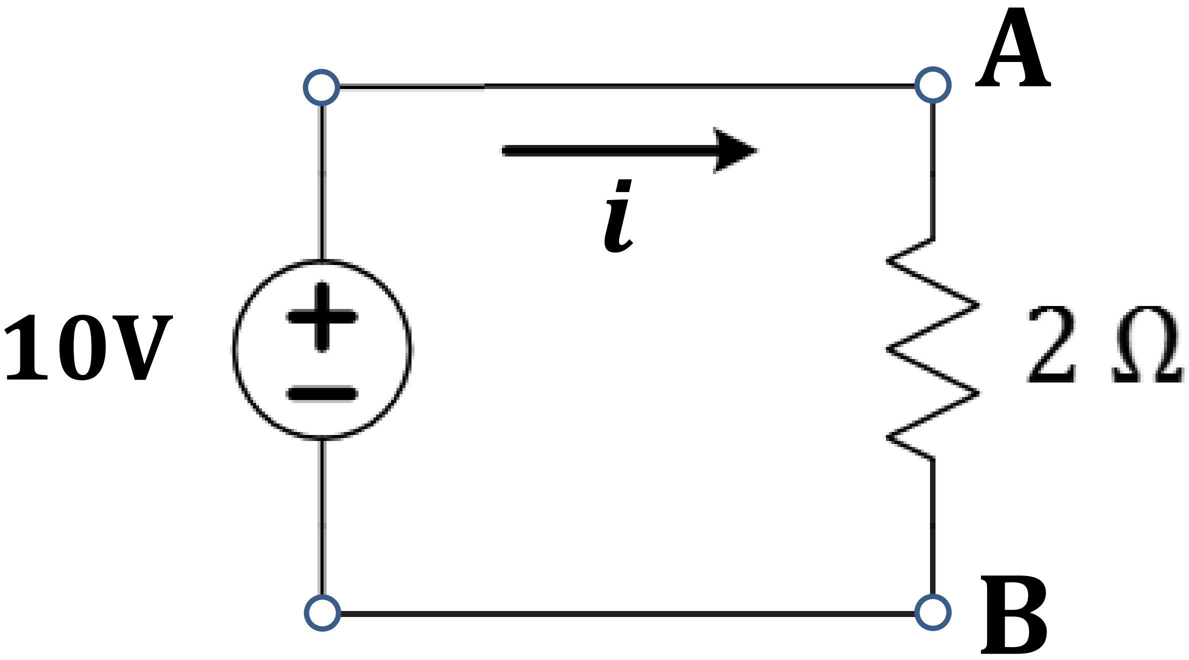
slope



The smaller the slope, the higher the resistance

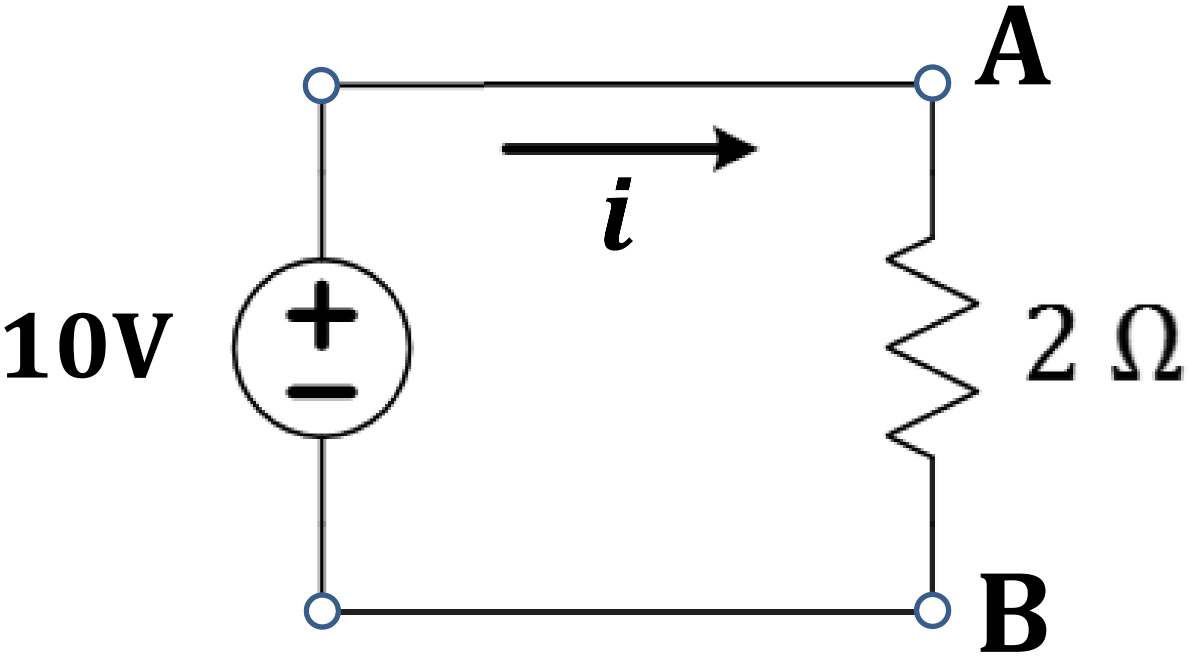
Example 1

Find the current i in the circuit below



Example 1

Find the current i in the circuit below

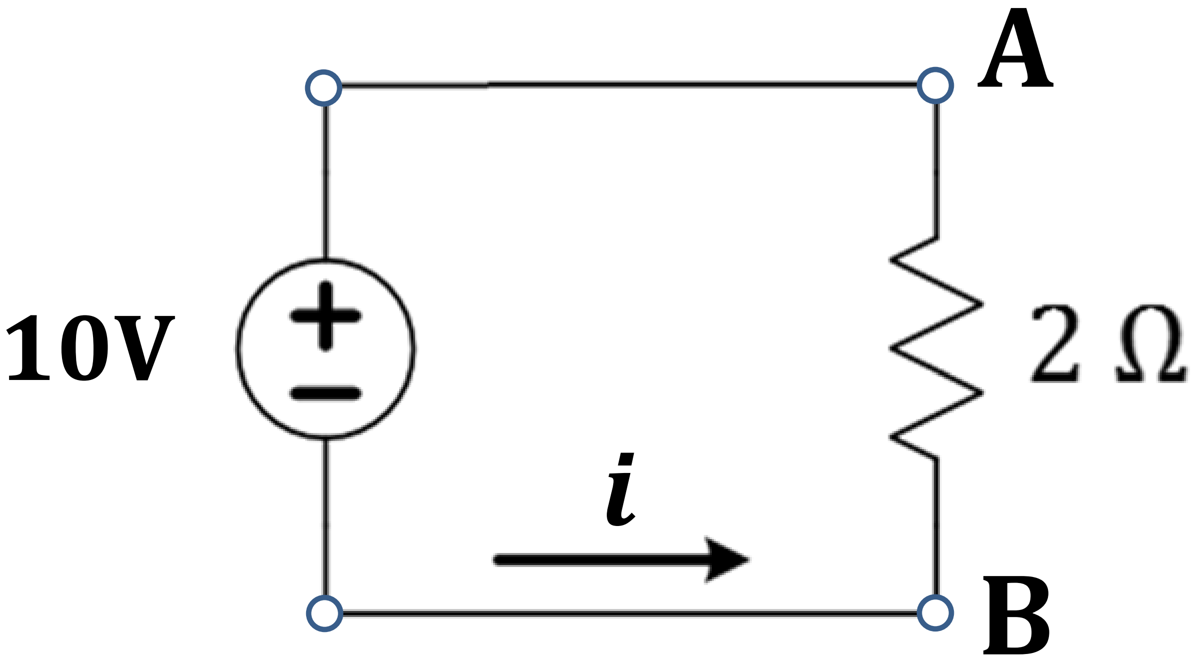


$$\boxed{V_{AB} = 10V = i_{AB} \times R = i_{AB} \times 2}$$

$$\boxed{i = i_{AB} = V_{AB}/R = 10/2 = 5\text{ A}}$$

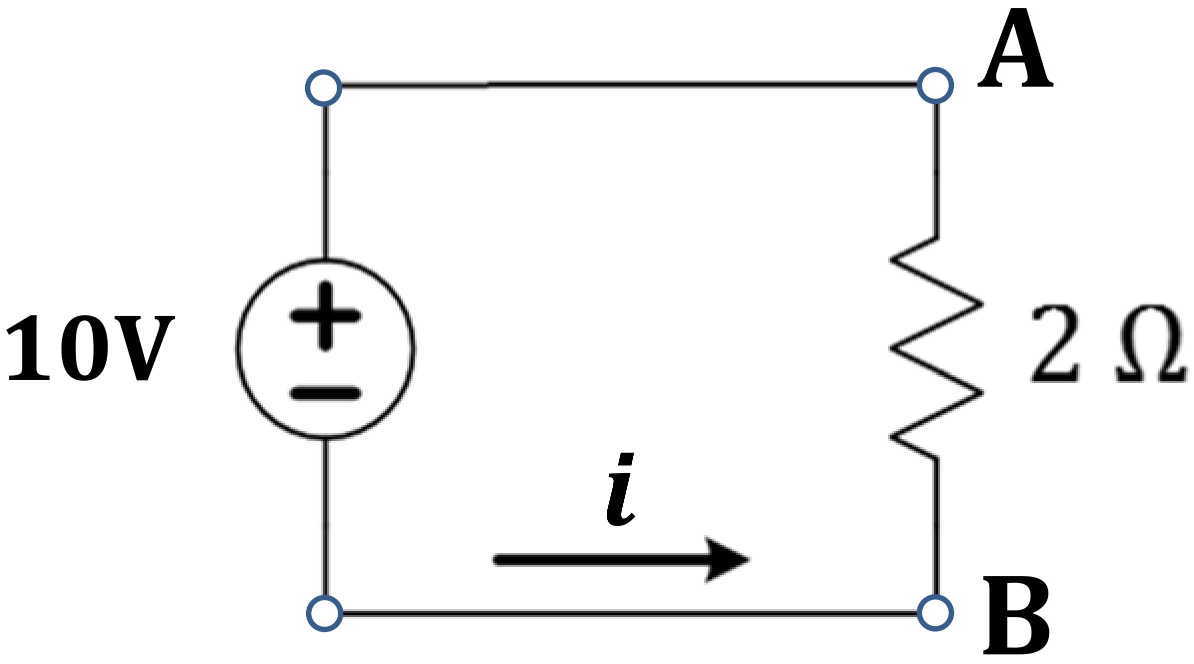
Example 2

Find the current i in the circuit below



Example 2

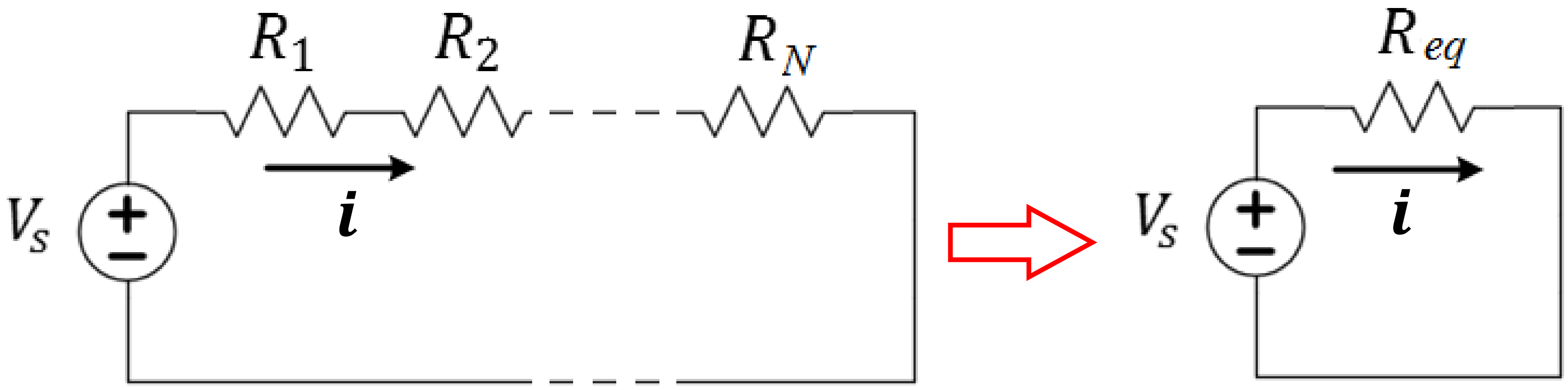
Find the current i in the circuit below



$$\boxed{V_{AB} = 10V = i_{AB} \times R = i_{AB} \times 2}$$

$$\boxed{i = i_{BA} = V_{BA}/R = -V_{AB}/2 = -5\text{ A}}$$

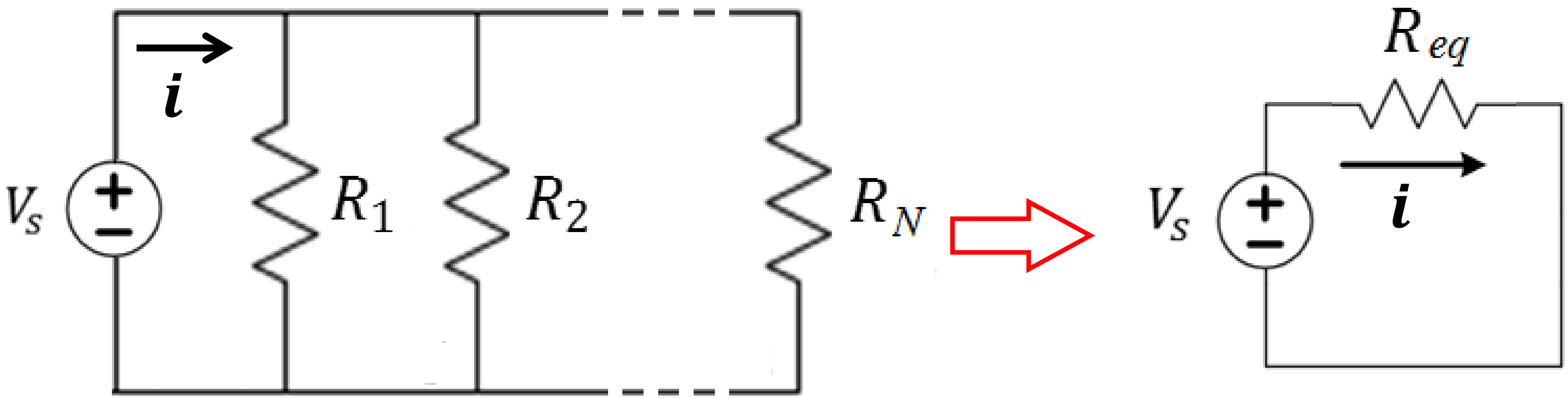
Series connected resistors



N **resistors connected in series can be replaced by an equivalent resistor** R_{eq}

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{k=1}^{N} R_k$$

Parallel connected resistors

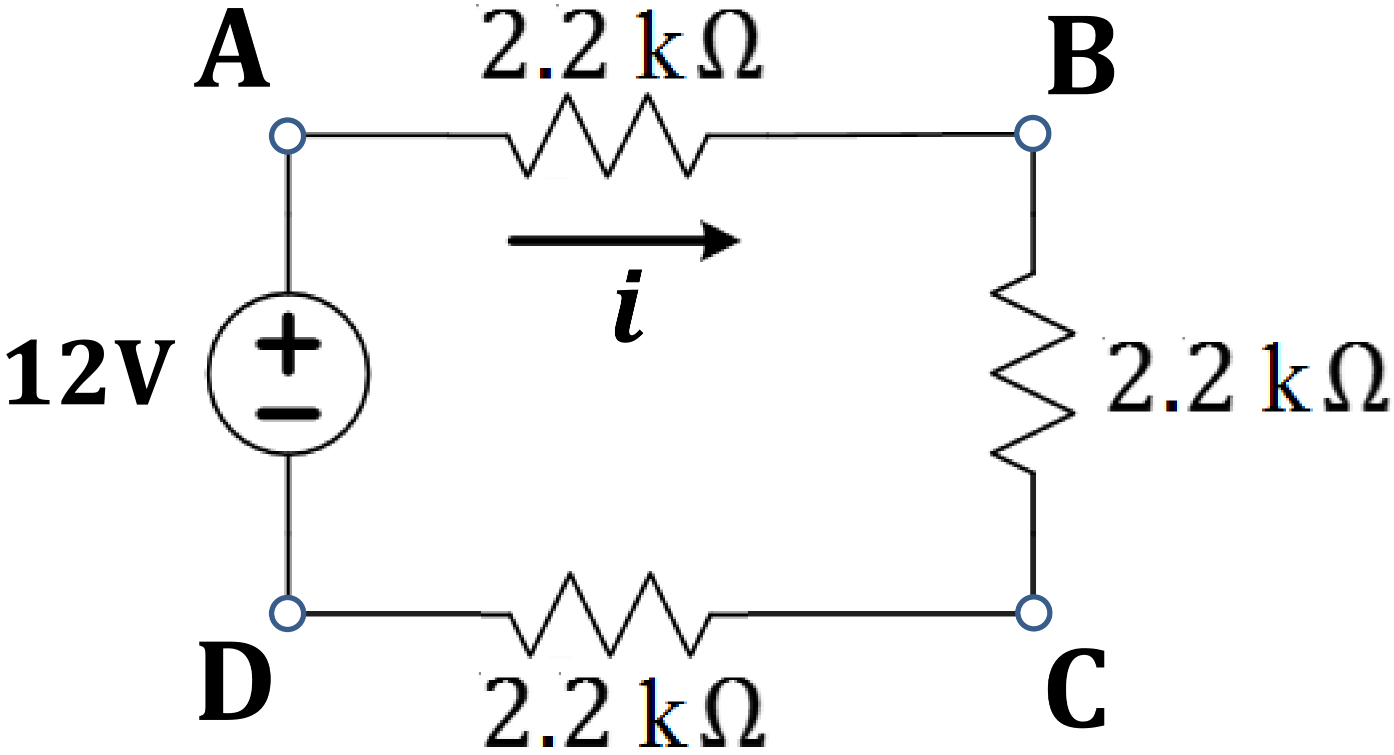


N **resistors connected in series can be replaced by an equivalent resistor** R_{eq} **given by**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{k=1}^{N} \frac{1}{R_k}$$

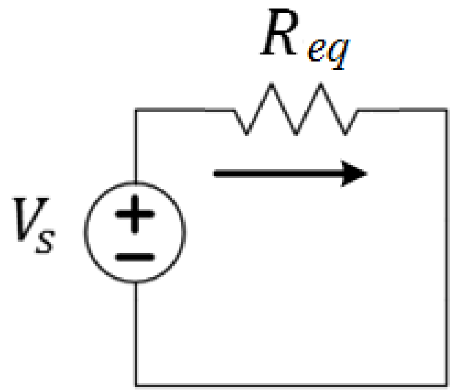
Example 3

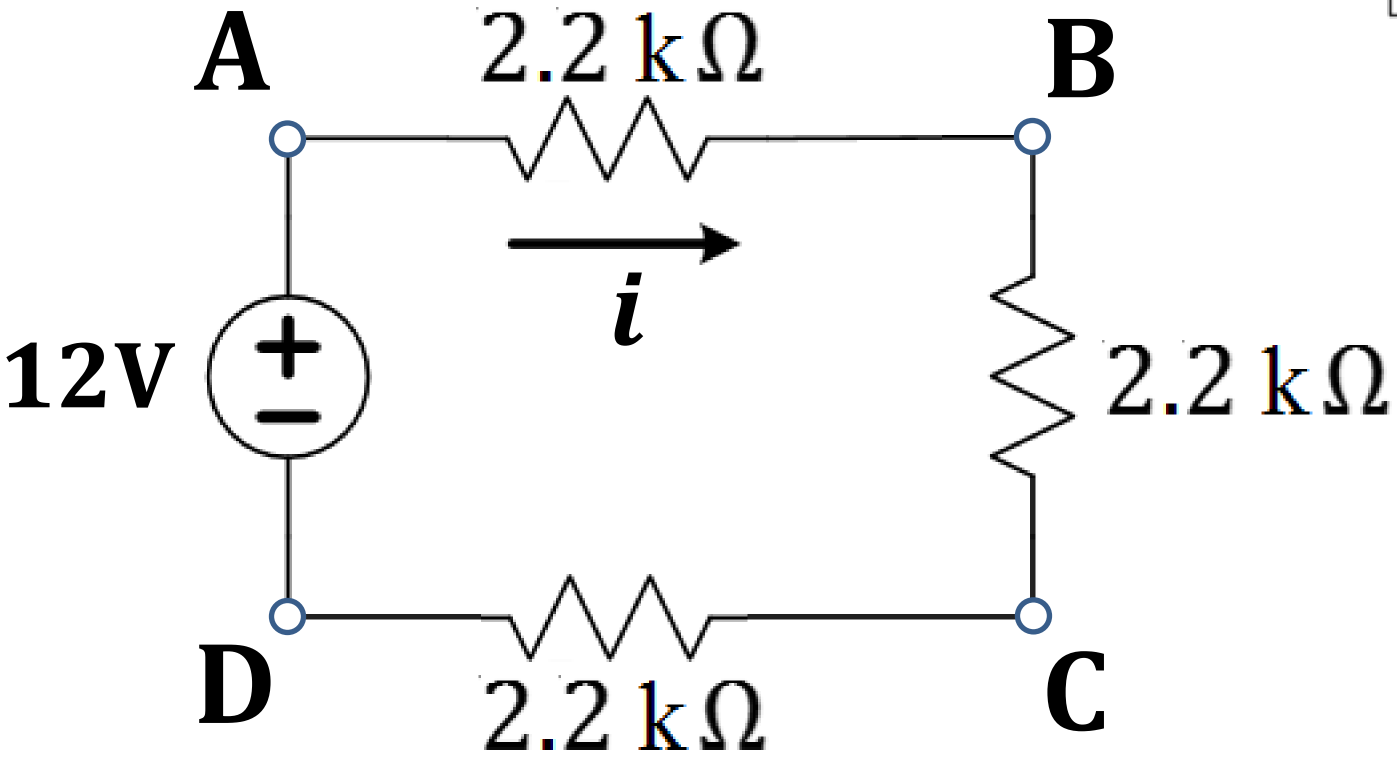
Find the current i in the circuit below



Example 3

Find the current i in the circuit below

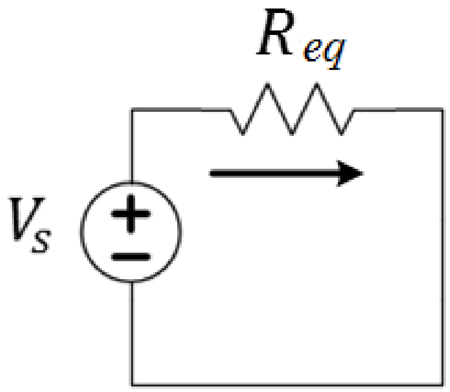


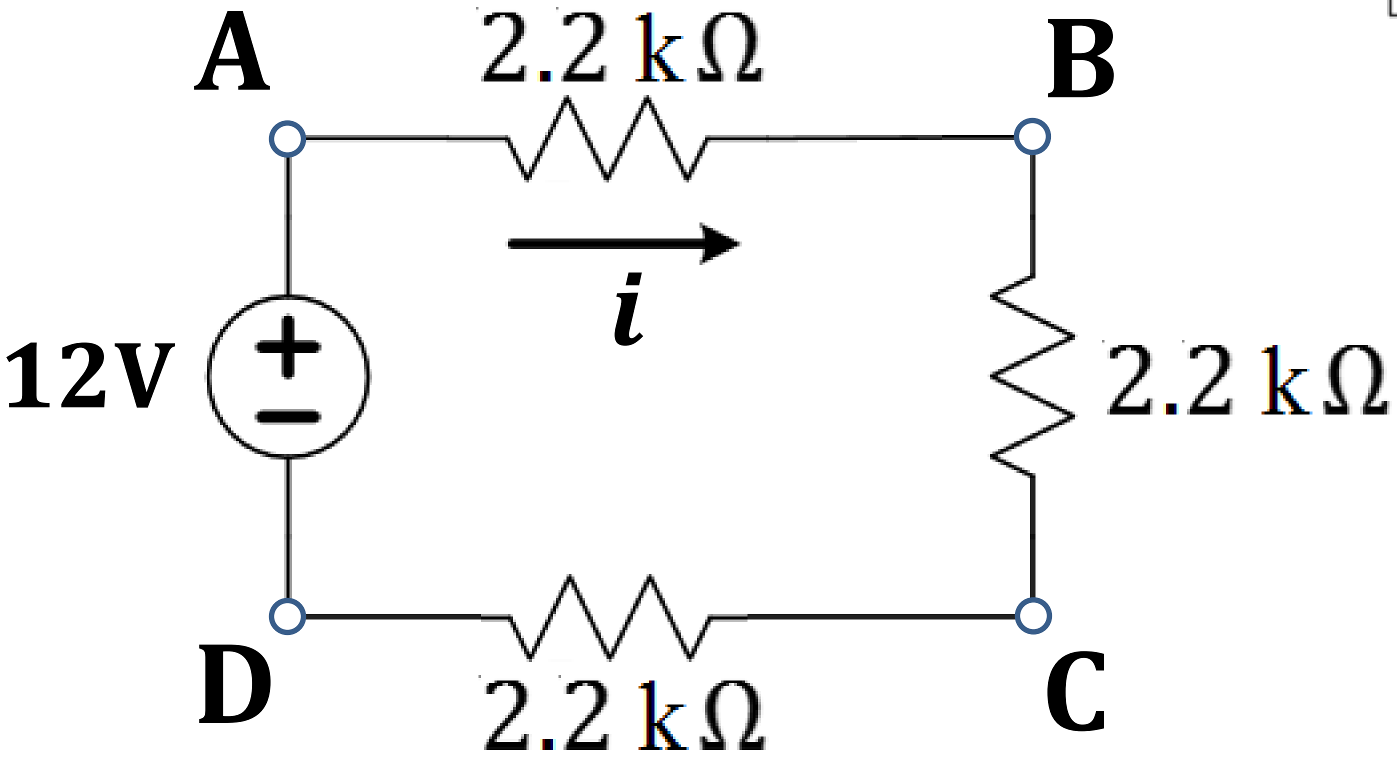


$$\boxed{R_{eq} = R_{AB} + R_{BC} + R_{CD} = 6.6\ \text{k}\Omega = 6{,}600\Omega}$$

Example 3

Find the current i in the circuit below

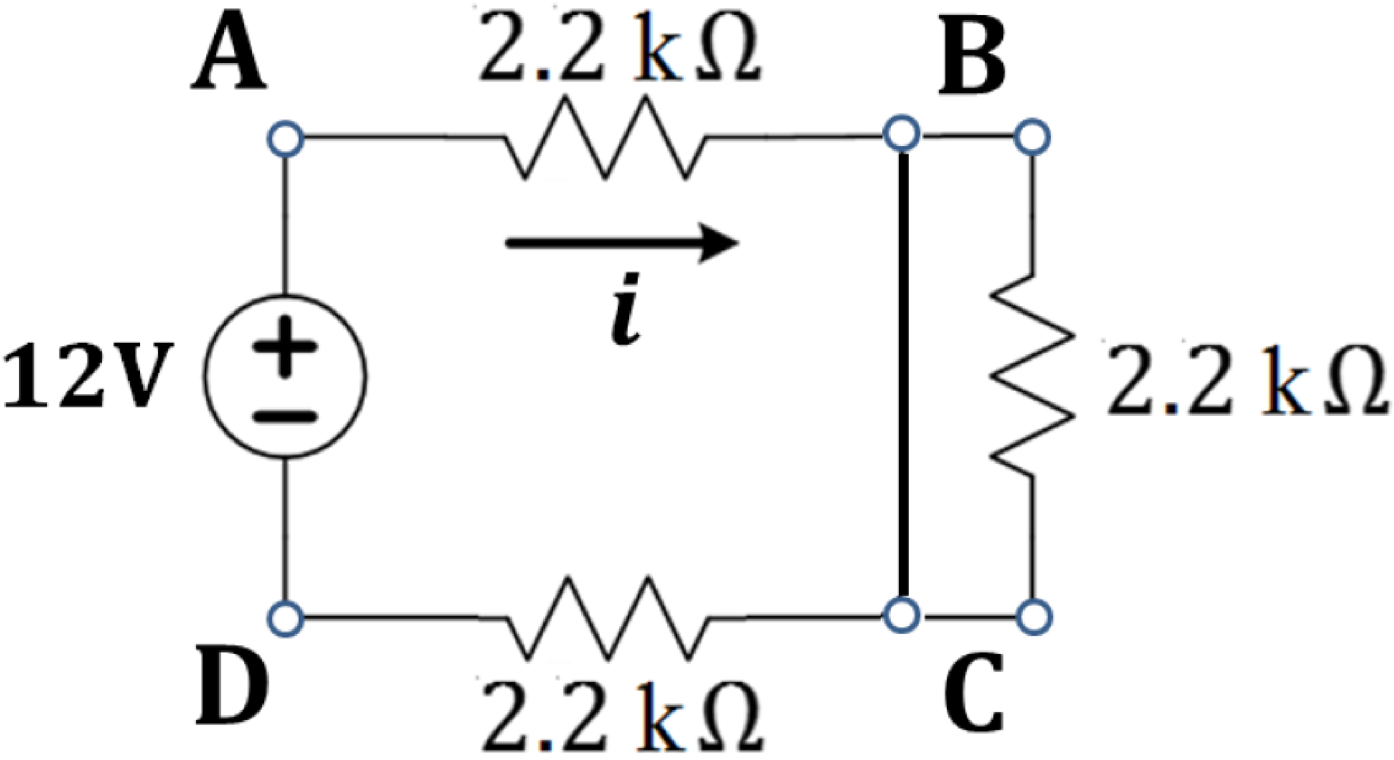




$$\boxed{i = 12/6{,}600 = 0.00\overline{18}\,\text{A} = 1.\overline{81}\text{mA}}$$

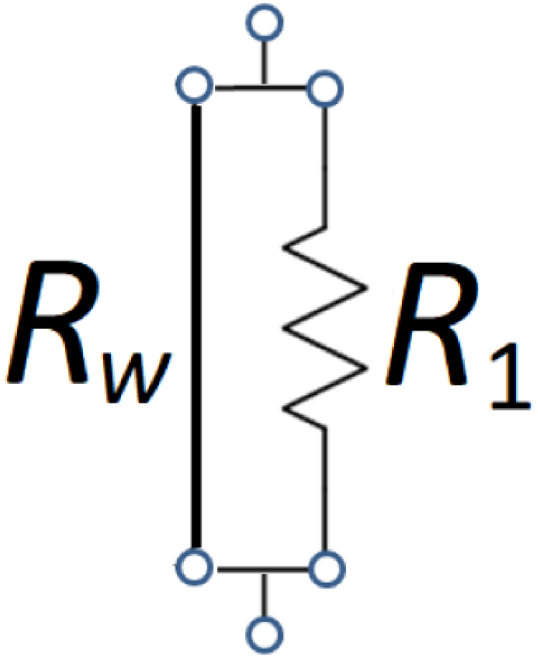
Example 3

If you "short-circuit" a resistor with a zero-resistance wire



$$\boxed{i = 12/4{,}400 = 0.00\overline{27}\ \mathrm{A} = 2.\overline{72}\mathrm{mA}}$$

Parallel between an ideal wire and a resistor



$$\boxed{R_{eq} = \left[\frac{1}{R_w} + \frac{1}{R_1}\right]^{-1}}$$

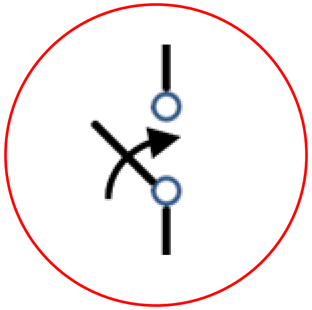
$$R_{eq} = \left[\frac{1}{0} + \frac{1}{R_1}\right]^{-1}$$

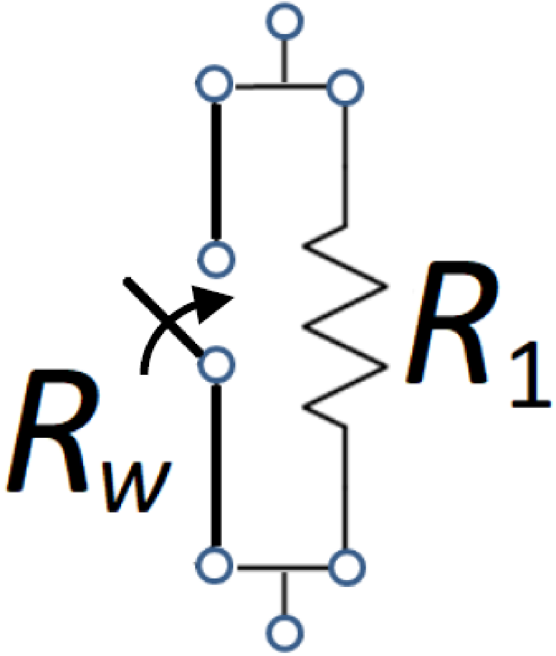
$$R_{eq} = \left[\infty + \frac{1}{R_1}\right]^{-1} = [\infty]^{-1} = 0$$

Current only flows in the wire regardless of R_1

Parallel between an ideal wire and a resistor

Symbol for a switch



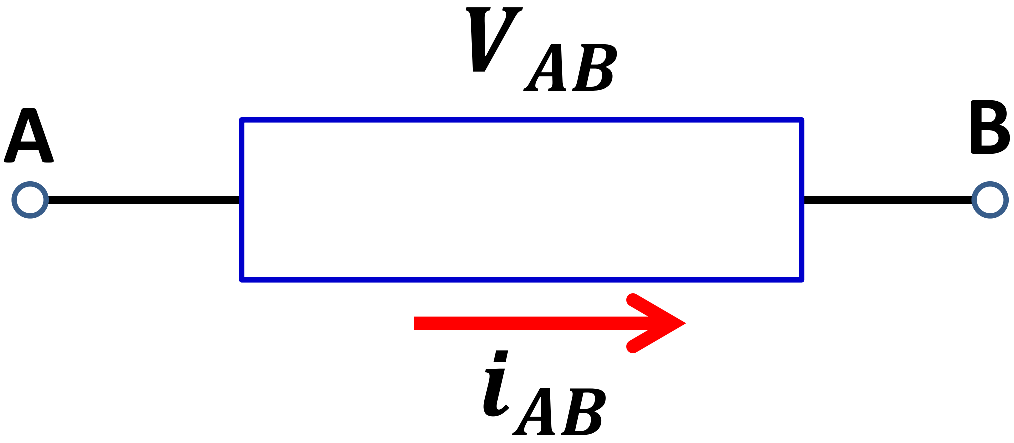


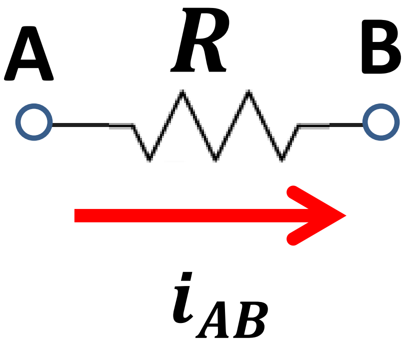
You can add a switch to turn on or off the effect of the shorting wire

Power

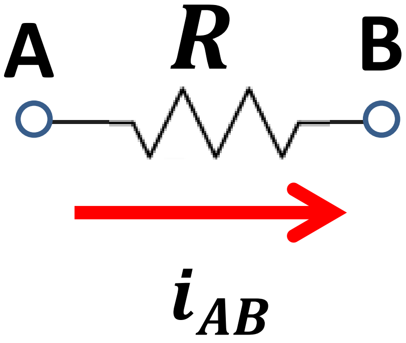
As discussed earlier, the power dissipated by an electrical element is given by

$$\boxed{P = V_{AB} \times i_{AB}}$$





Power



The voltage V_{AB} across a resistor is

$$\boxed{V_{AB} = i_{AB} \times R}$$

which gives the power

$$\boxed{P = V_{AB} \times i_{AB} = i_{AB} \times R \times i_{AB}}$$

$$\boxed{P = i_{AB}^2 R} \quad \text{or} \quad \boxed{P = \frac{V_{AB}^2}{R}} \quad \text{[Watts]}$$