ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 3

MWF – 12:00pm

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2062 ECE Building

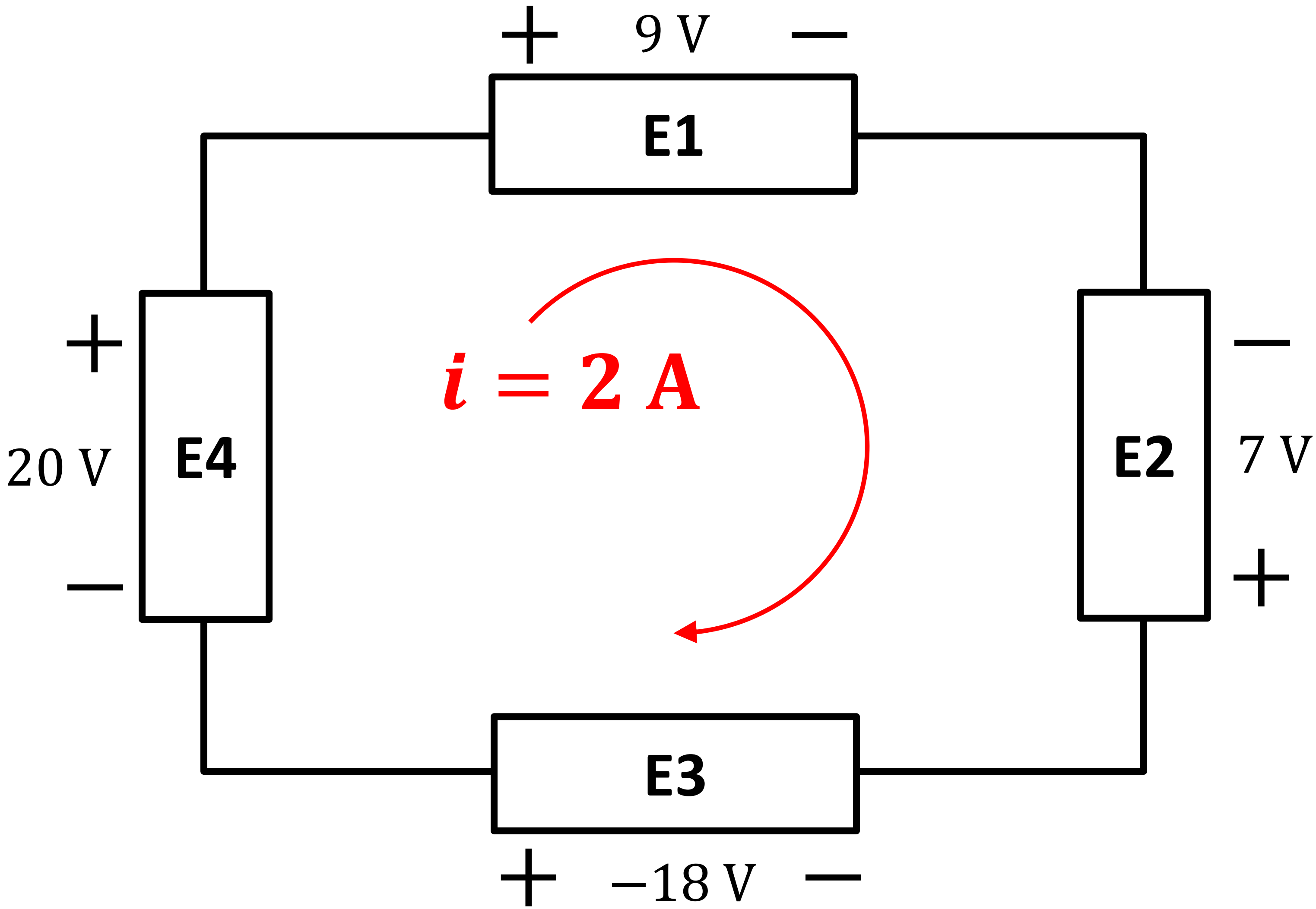
Lecture 3 - Summary

Learning Objectives

1. Define Ohm's Law
2. Use ohms law to compute voltage and current
3. Combine basic elements to sketch a complete circuit
4. Identify series and parallel combination of resistors
5. Compute equivalent resistance between two terminals

Example

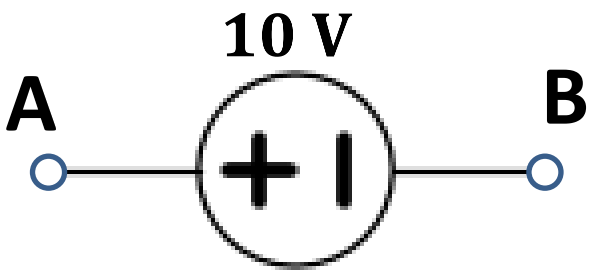
Find the power consumed or supplied by each element.



Electrical Circuit

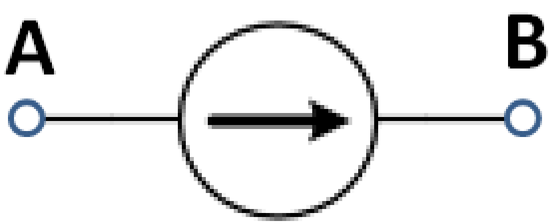
An **electrical circuit** is made up of **electrical elements.**

Initially, we will look at circuits with these elements:

1) 

Ideal Voltage Source

$$V_{AB} = V_A - V_B$$

2) 

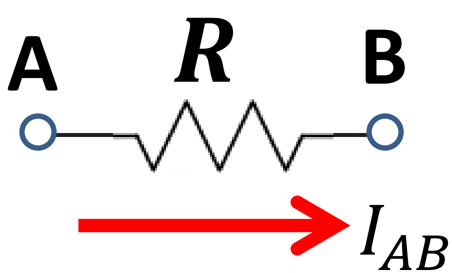
Ideal Current Source

$$I_{AB}$$

3) 

Wire (ideal conductor)

$$V_{AB} = 0\ \mathrm{V}$$

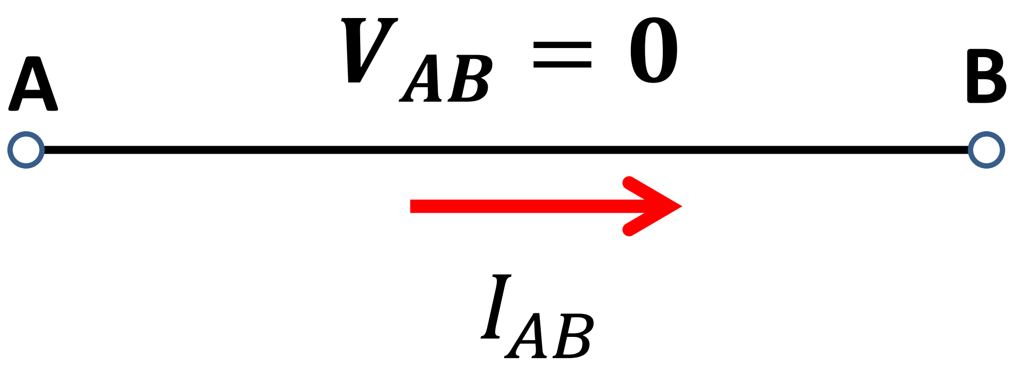
4) 

Resistor

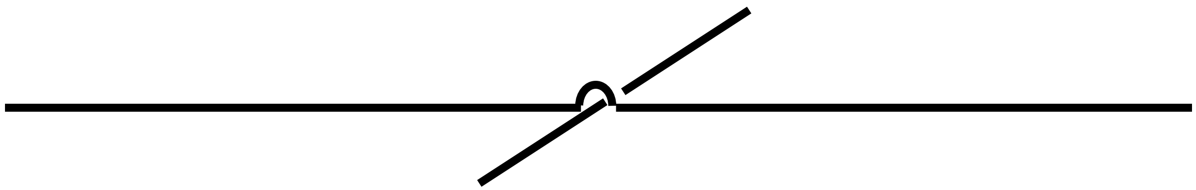
$$V_{AB} = I_{AB}\, R$$

Ideal Wire

Wires are represented by unbroken lines. Wires are assumed to be ideal conductors, i.e., the voltage difference between two point on a wire is zero (equipotential). Two points in a circuit that are connected by a wire are said to be **shorted** together.



If you have to draw two wires which cross but do not touch



Resistor

A resistor is an element which requires a certain effort on the part of the voltage to push a current through it. The resistance R is quantified by:

$$R = \frac{\rho \, \ell}{A}$$

ρ **Resistivity of the material**

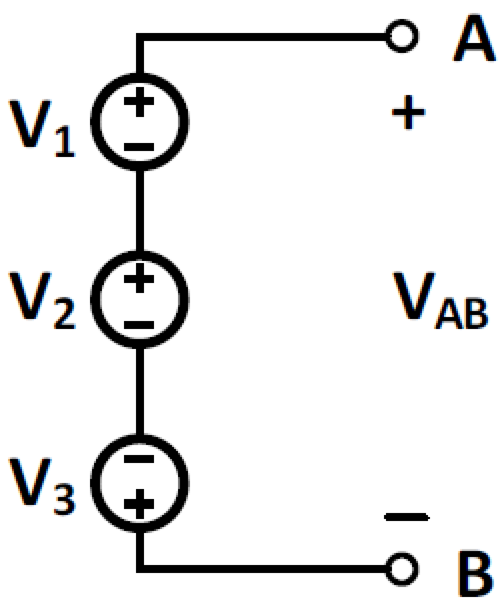
ℓ **Length**

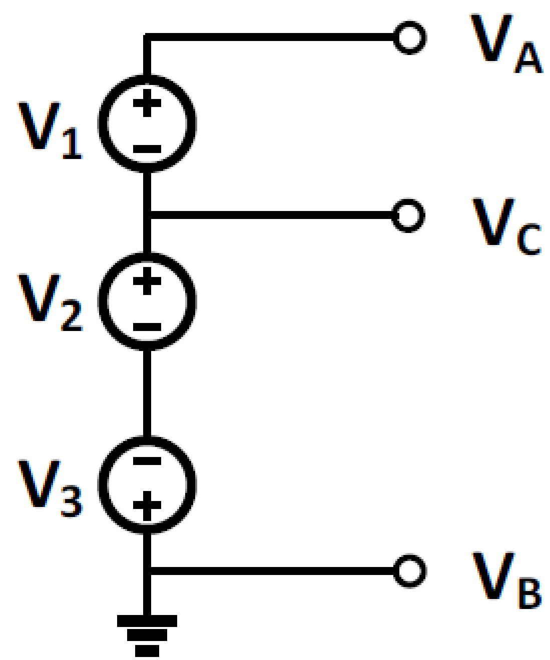
A **Cross-sectional area**

Worksheet 1

1. Compute voltage $\mathbf{V_{AB}}$, $\mathbf{V_A}$, $\mathbf{V_B}$, and $\mathbf{V_C}$ in the circuits shown below.

   Assume $\mathbf{V_1 = 10V}$, $\mathbf{V_2 = 5V}$, and $\mathbf{V_3 = 6V}$.

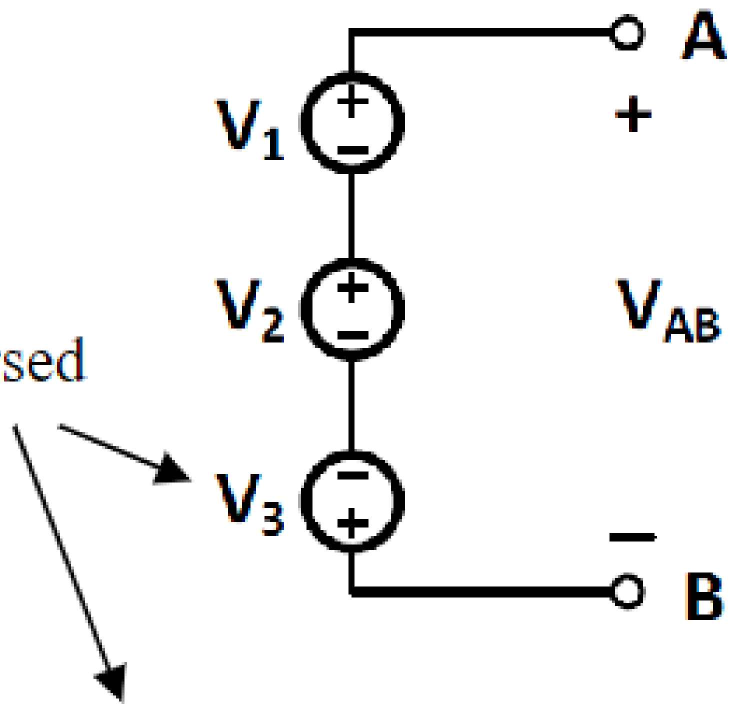




Worksheet 1

1. Compute voltage $\mathbf{V_{AB}}$, $\mathbf{V_{A}}$, $\mathbf{V_{B}}$, and $\mathbf{V_{C}}$ in the circuits shown below. Assume $\mathbf{V_1 = 10V}$, $\mathbf{V_2 = 5V}$, and $\mathbf{V_3 = 6V}$.

Polarity is reversed

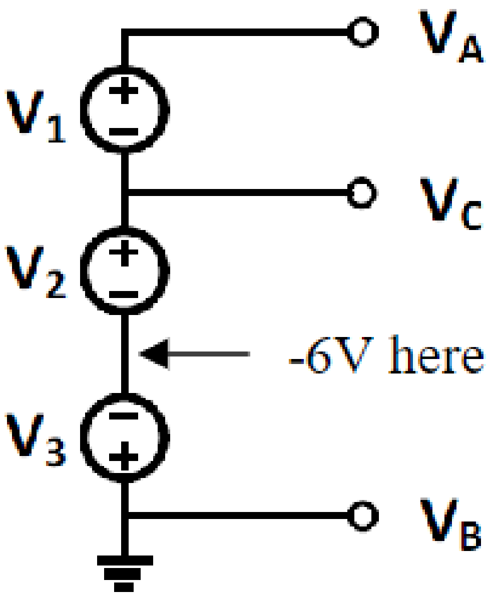


$$V_{AB} = V_1 + V_2 - V_3 = 10 + 5 - 6 = 9V$$

Worksheet 1

1. Compute voltage $\mathbf{V_{AB}}$, $\mathbf{V_A}$, $\mathbf{V_B}$, and $\mathbf{V_C}$ in the circuits shown below.

   Assume $\mathbf{V_1 = 10V}$, $\mathbf{V_2 = 5V}$, and $\mathbf{V_3 = 6V}$.

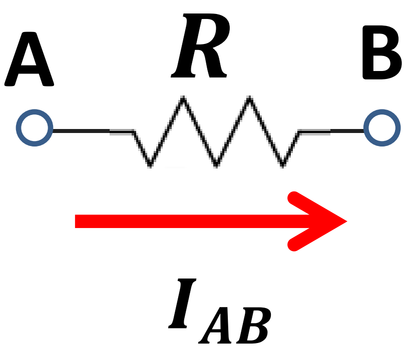


$$V_A = V_1 + V_2 - V_3 = 9V$$
$$V_C = V_2 - V_3 = -1V$$
$$V_B = V_g = 0V$$

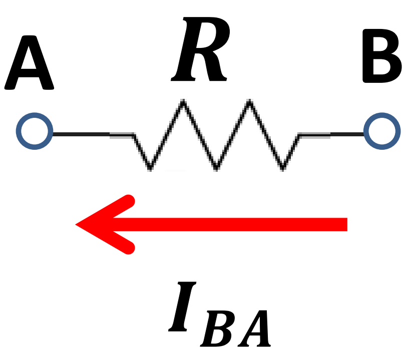
Ground is at 0 potential (V_B). The battery V_3 lowers the potential to -6V at its top terminal because it is reversed. Batteries V2 and V1 then raise the potential higher.

Ohm’s Law

Ohm's law captures the relationship between voltage across a resistor and current through it. Ohm's law can have the following two forms,



$$V_{AB} = I_{AB}R$$



$$V_{AB} = -I_{BA}R$$

Observations on Ohm's Law

$$V = IR$$

For the same current, a higher resistance cause a higher voltage drop at the terminals

$$I = \frac{V}{R}$$

For the same voltage at the terminals, a higher resistance cause a smaller current

Observations on Ohm's Law

$$I = \frac{V}{R}$$

For **low** resistance R, **a small voltage may cause a high current**.

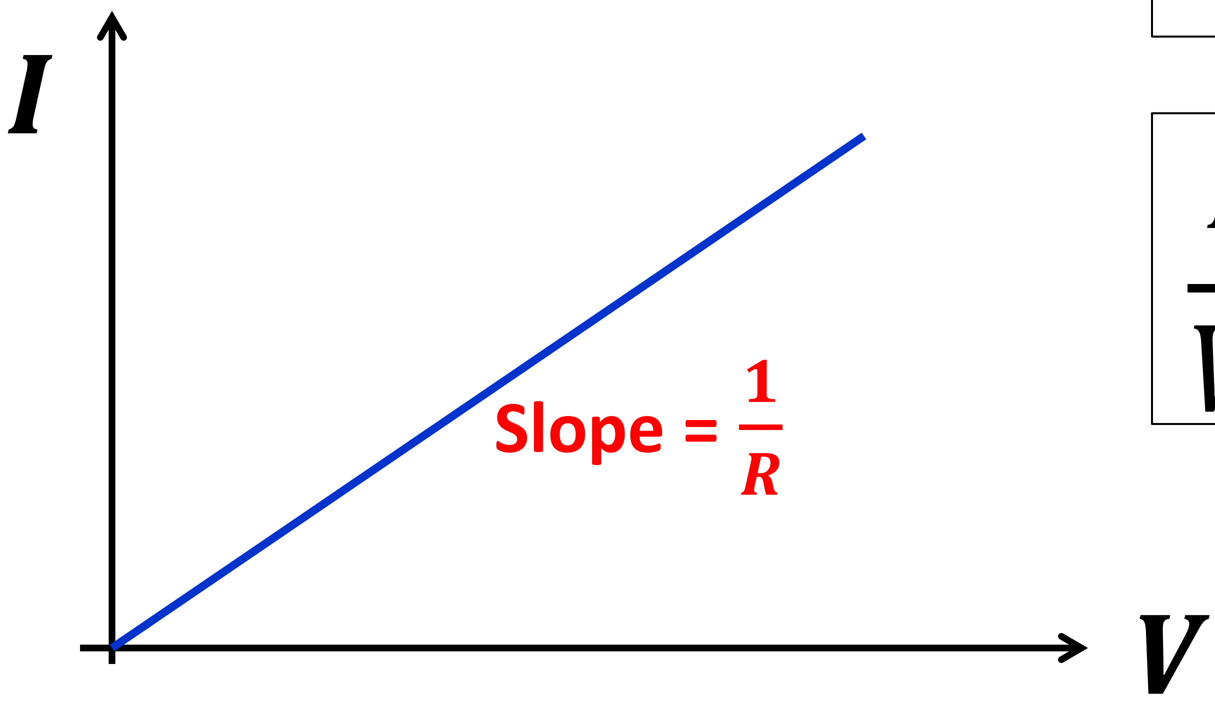
$$V = IR$$

For **large** resistance R, **a small current may cause a high voltage**.

Current-Voltage or I-V Curves

I-V curves capture the relationship between current and voltage. For a **resistance**, the I-V is linear (straight line)

$$V = IR$$



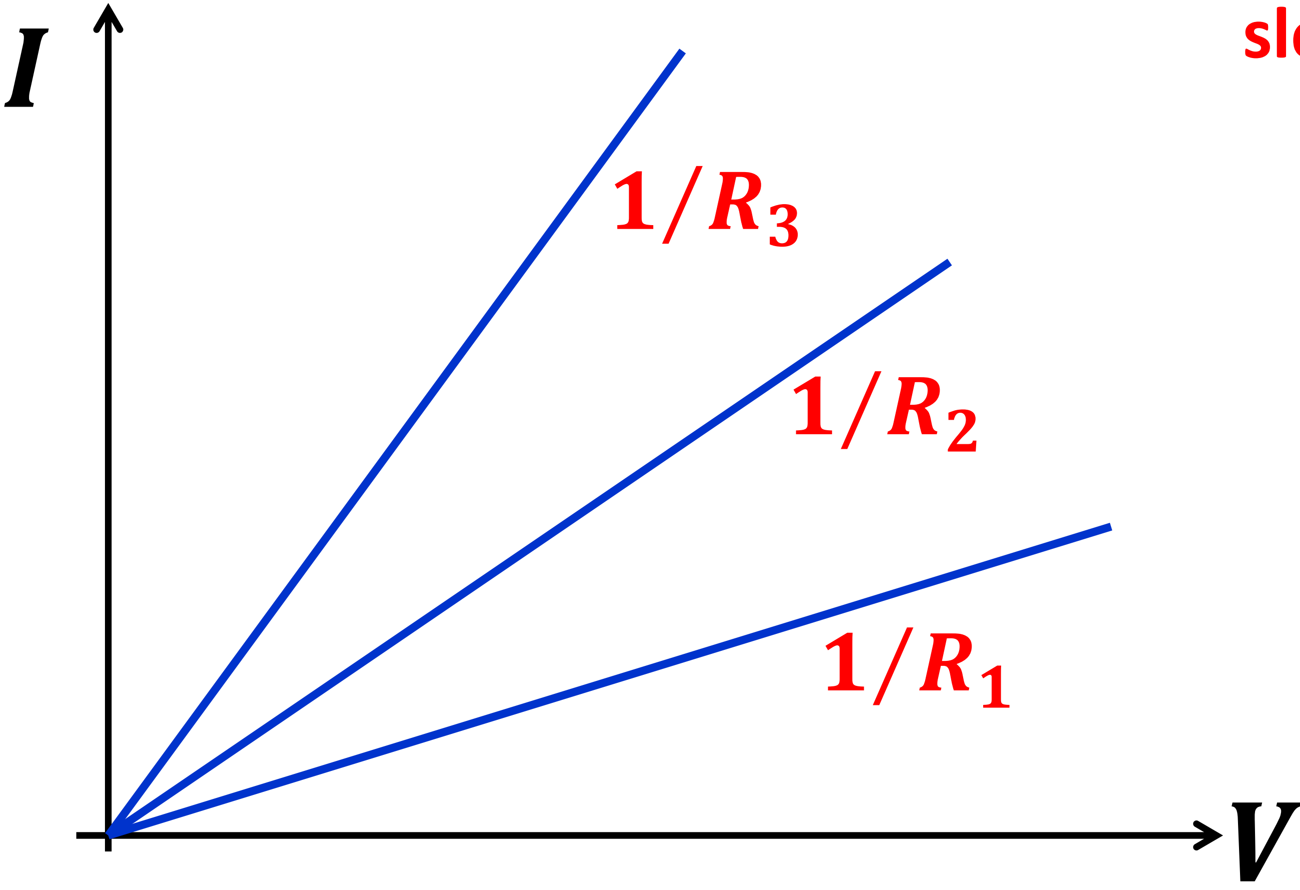
$$\frac{I}{V} = \frac{1}{R}$$

The inverse of the slope represents the resistance

$$R_1 > R_2 > R_3$$

$$\frac{I}{V} = \frac{1}{R}$$

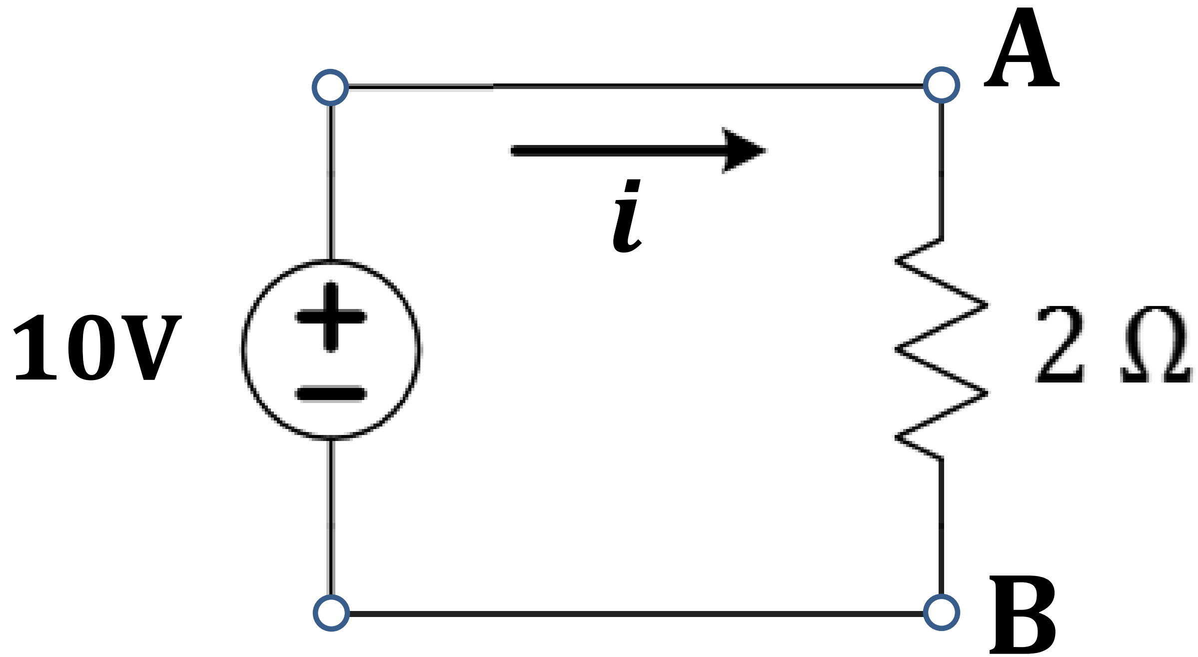
slope



The smaller the slope, the higher the resistance

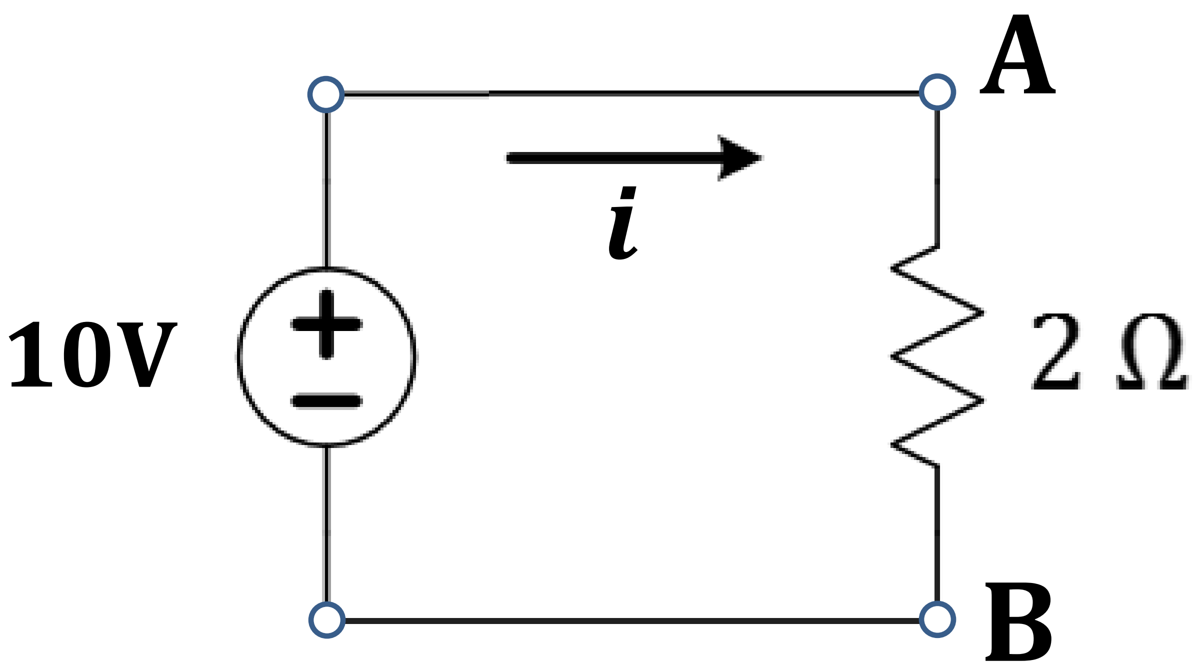
Example 1

Find the current i in the circuit below



Example 1

Find the current i in the circuit below

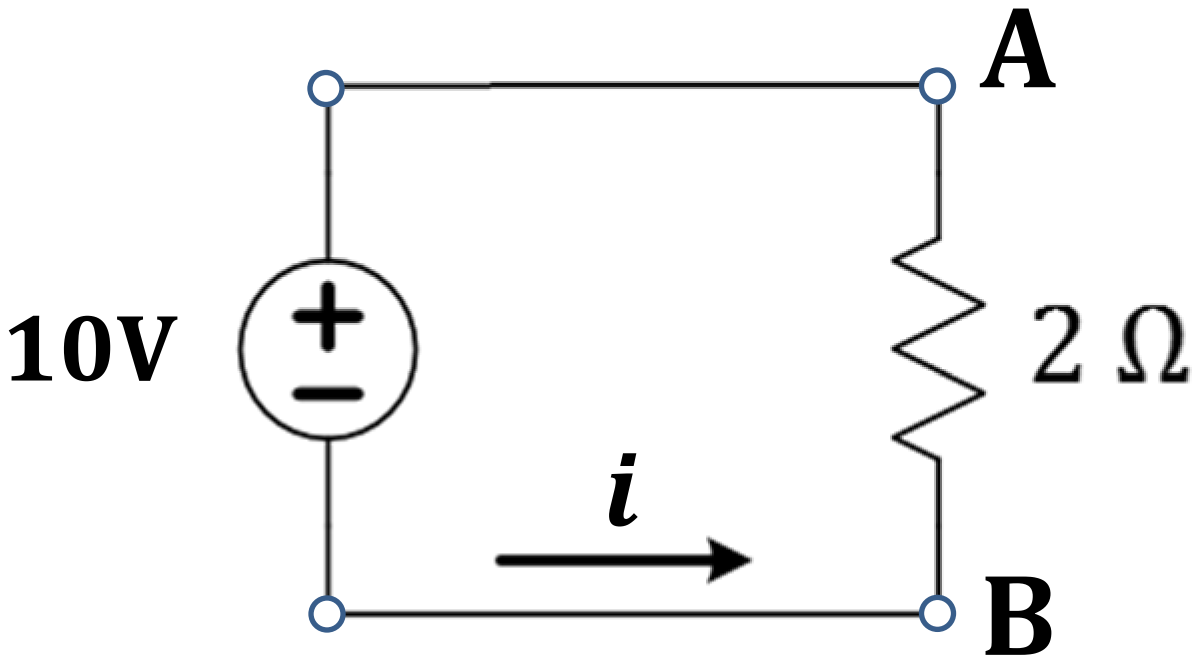


$$V_{AB} = 10V = i_{AB} \times R = i_{AB} \times 2$$

$$i = i_{AB} = V_{AB}/R = 10/2 = 5\ \text{A}$$

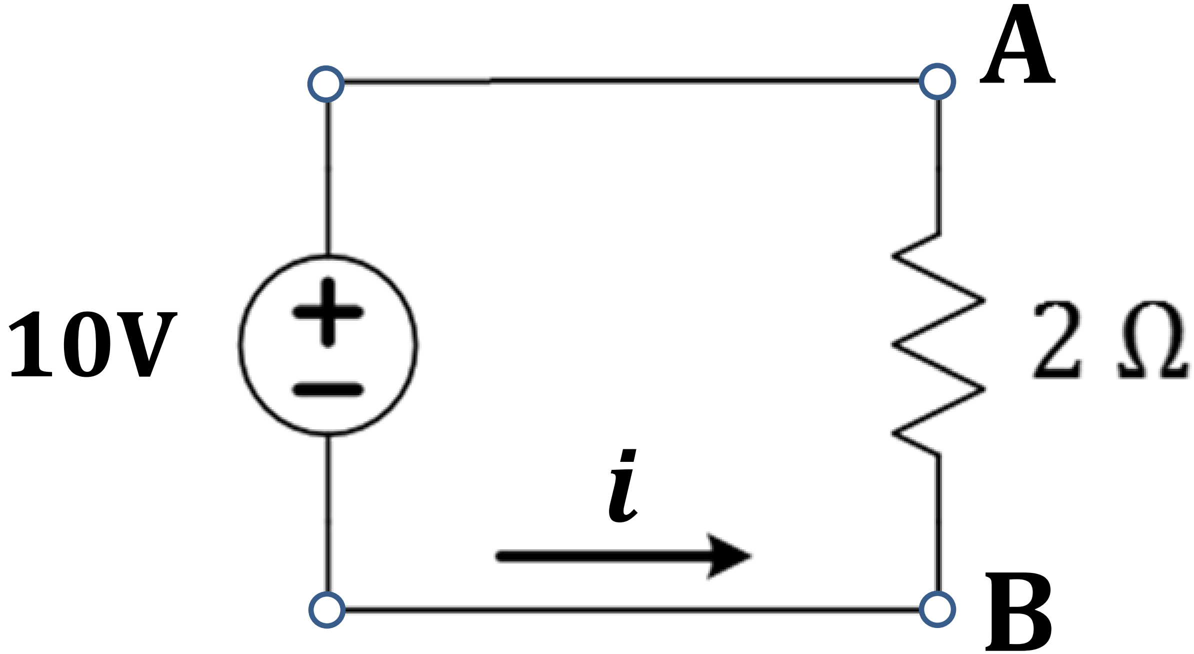
Example 2

Find the current i in the circuit below



Example 2

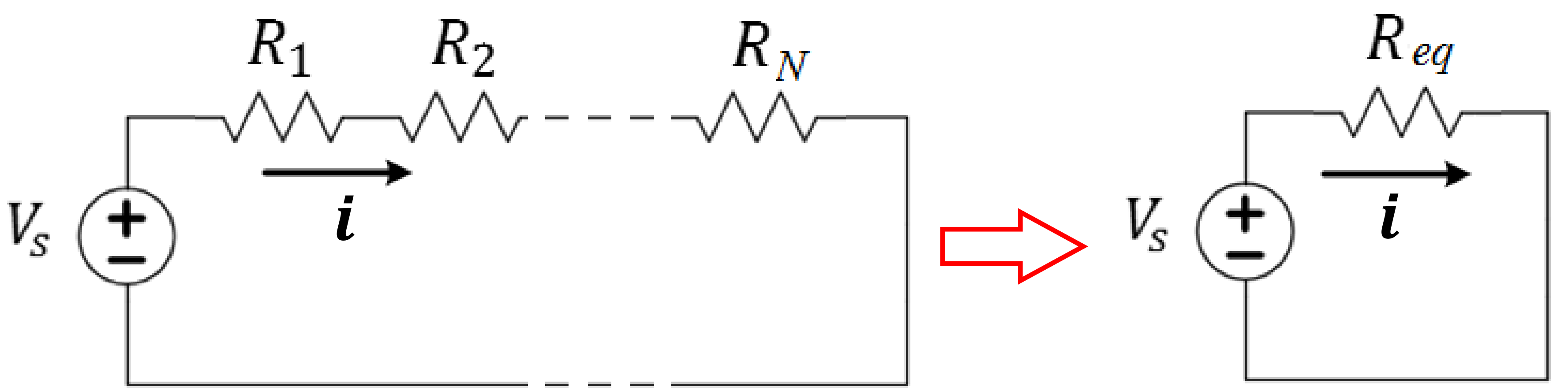
Find the current i in the circuit below



$$V_{AB} = 10V = i_{AB} \times R = i_{AB} \times 2$$

$$i = i_{BA} = V_{BA}/R = -V_{AB}/2 = -5\ \text{A}$$

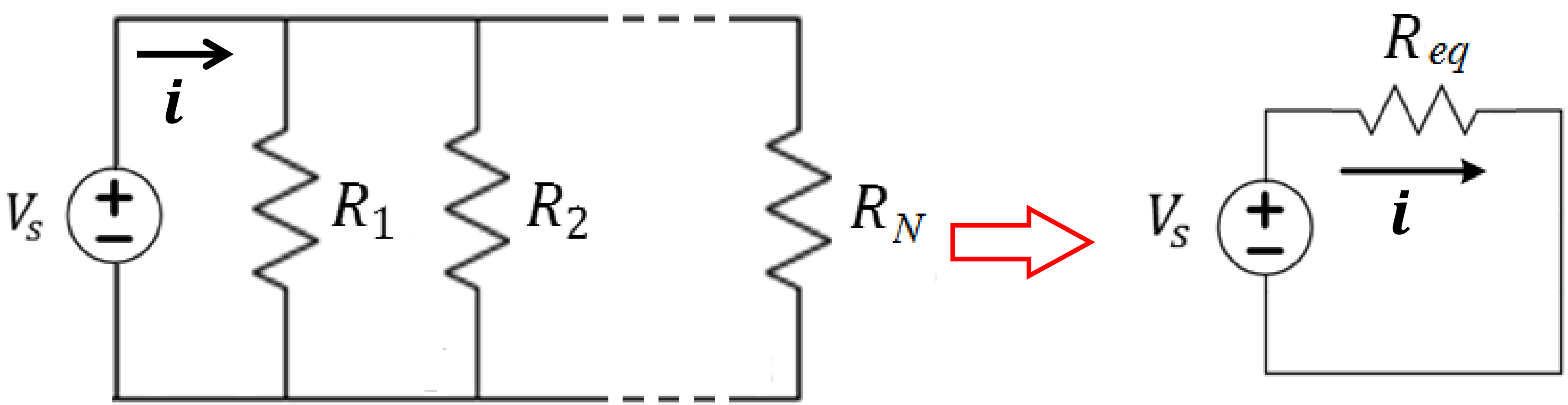
Series connected resistors



N **resistors connected in series can be replaced by an equivalent resistor** R_{eq}

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{k=1}^{N} R_k$$

Parallel connected resistors

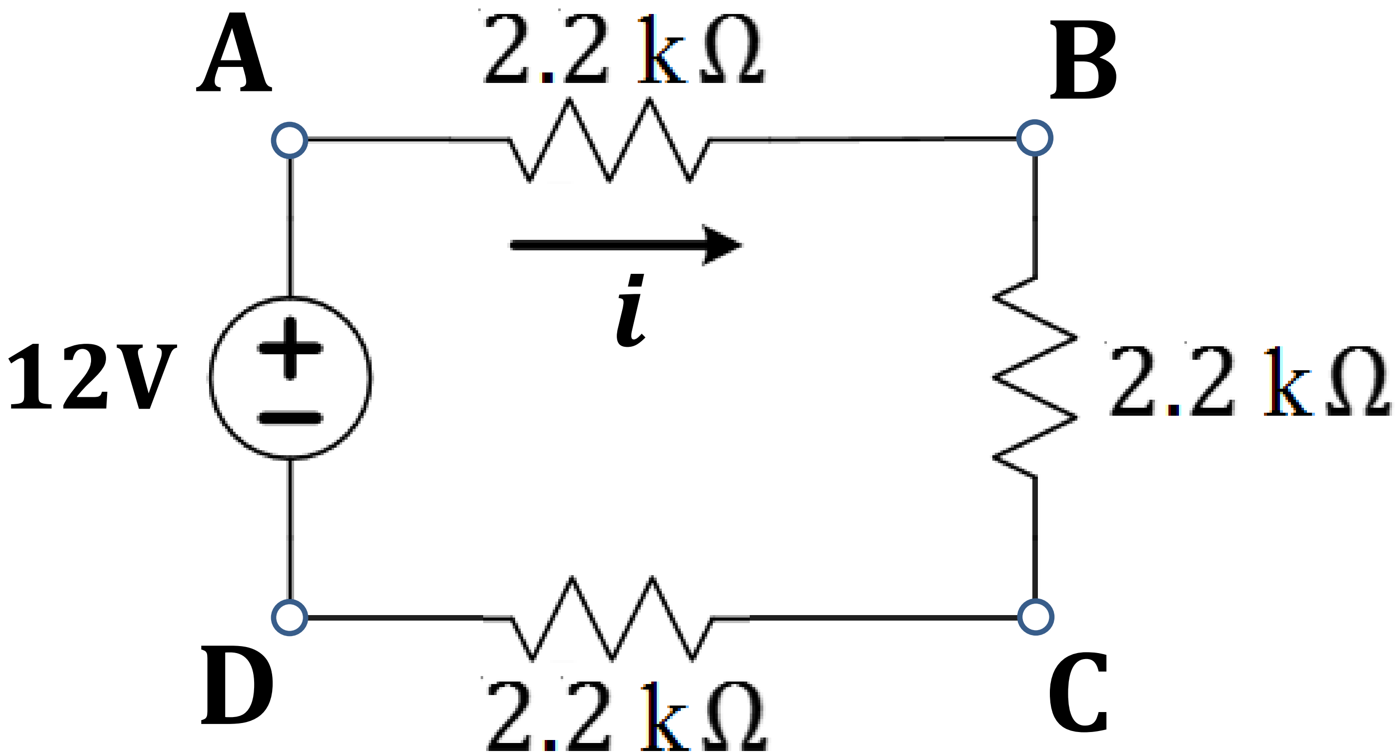


N resistors connected in series can be replaced by an equivalent resistor R_{eq} given by

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{k=1}^{N} \frac{1}{R_k}$$

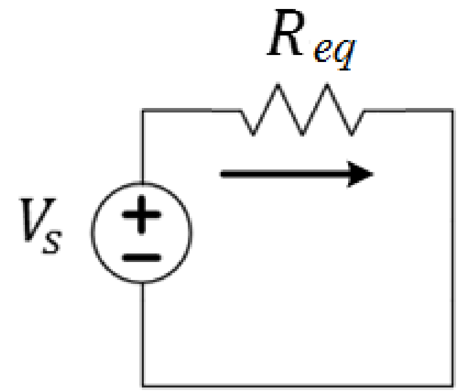
Example 3

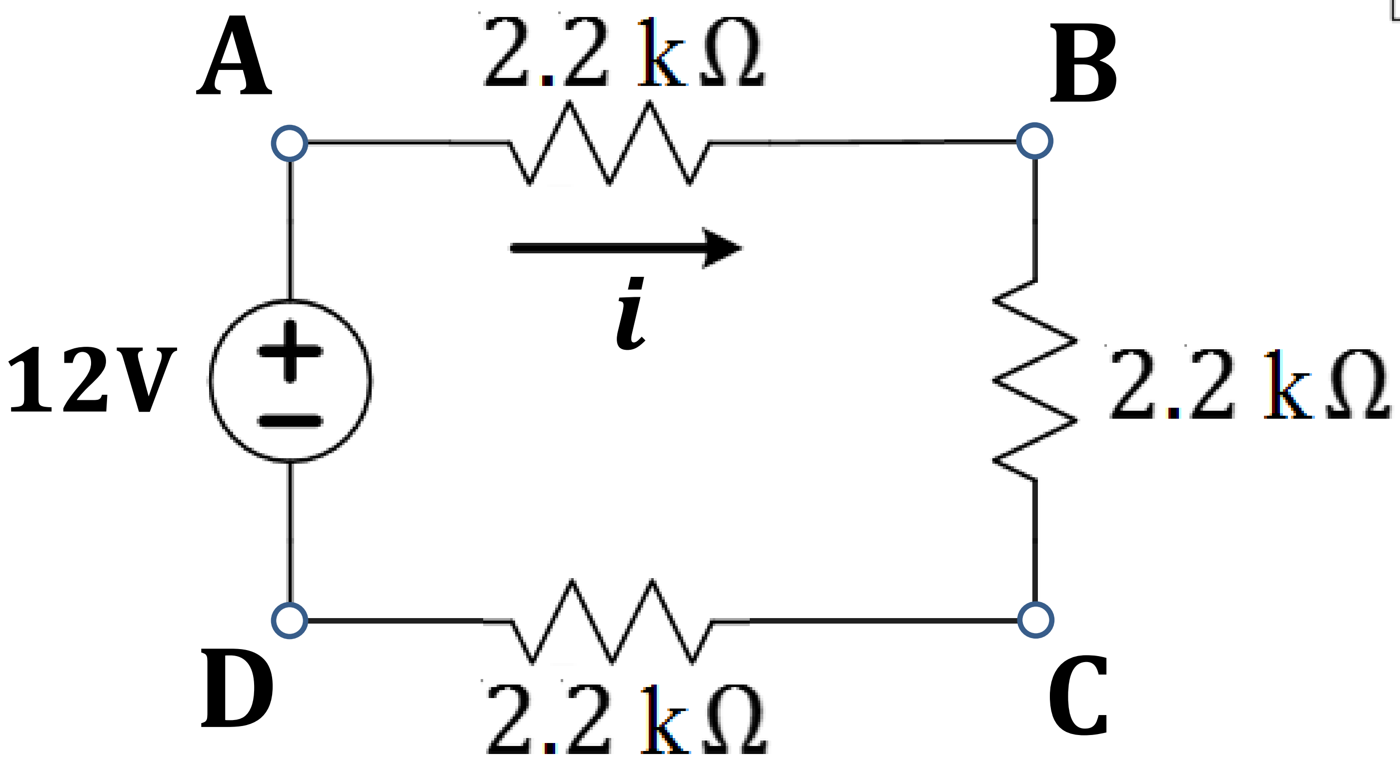
Find the current i in the circuit below



Example 3

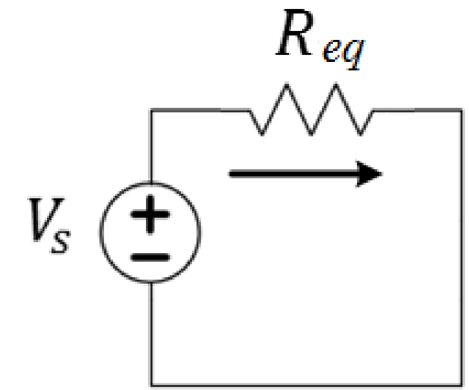
Find the current *i* in the circuit below



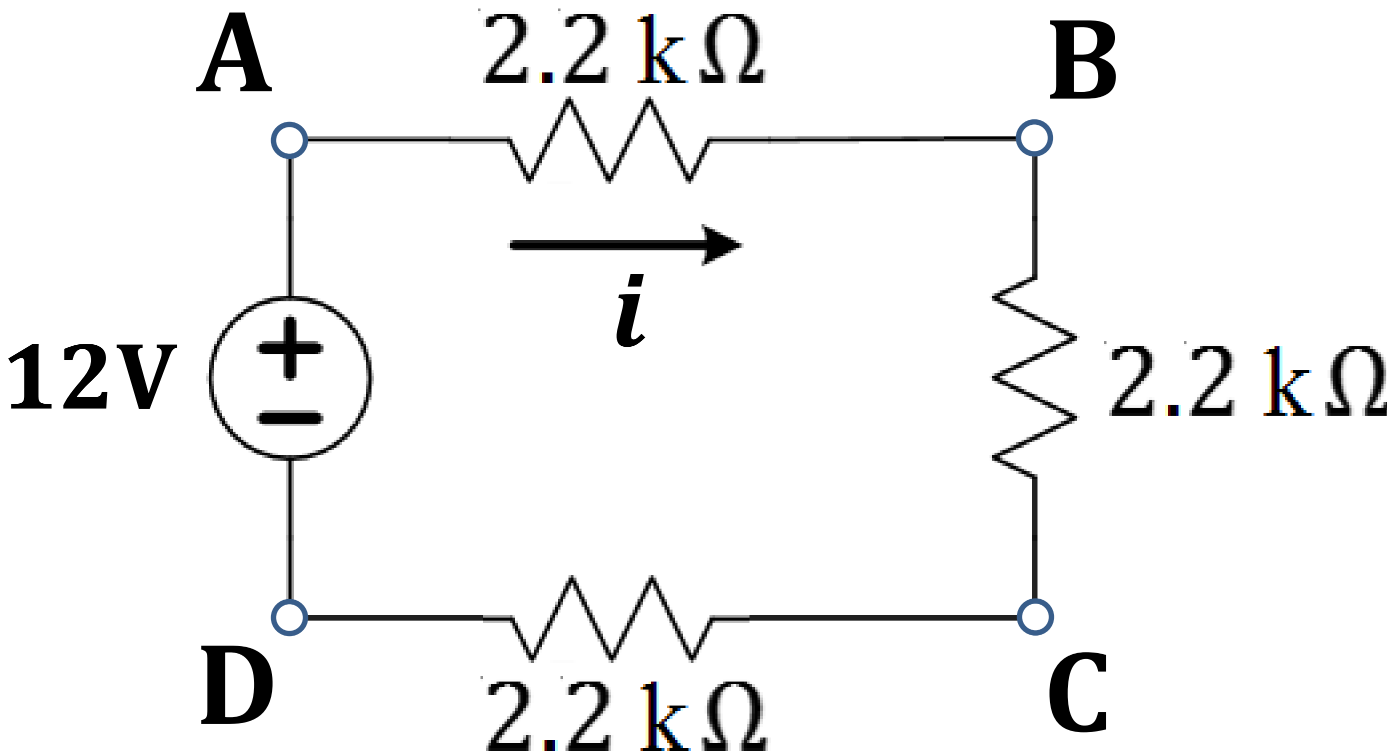


$$R_{eq} = R_{AB} + R_{BC} + R_{CD} = 6.6\text{ k}\Omega = 6{,}600\Omega$$

Example 3



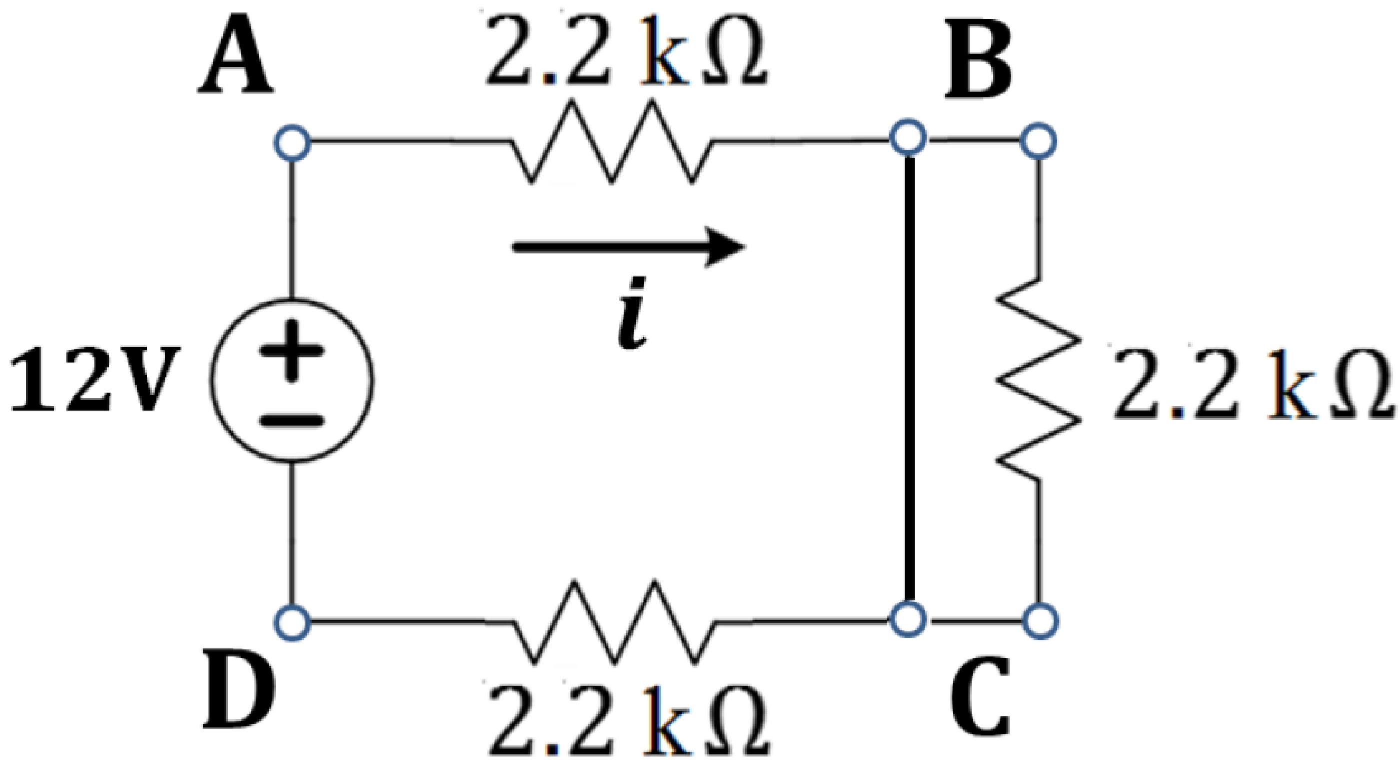
Find the current i in the circuit below



$$i = 12/6{,}600 = 0.00\overline{18}\ \text{A} = 1.\overline{81}\text{mA}$$

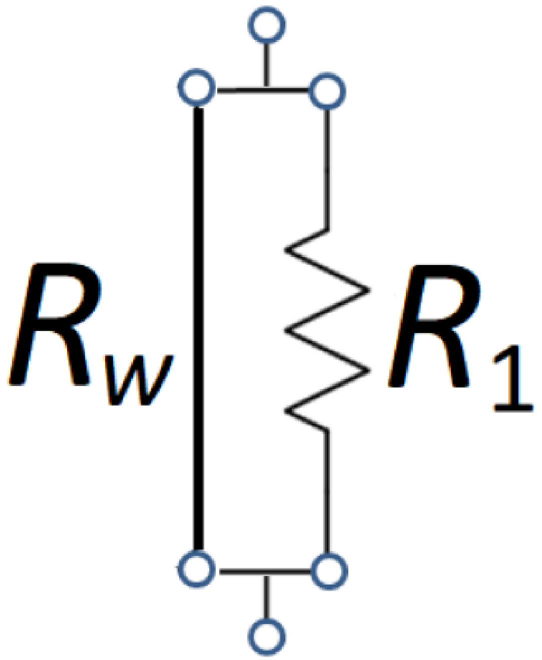
Example 3

If you "short-circuit" a resistor with a zero-resistance wire



$$i = 12/4{,}400 = 0.00\overline{27}\ \text{A} = 2.\overline{72}\text{mA}$$

Parallel between an ideal wire and a resistor



$$R_{eq} = \left[\frac{1}{R_w} + \frac{1}{R_1}\right]^{-1}$$

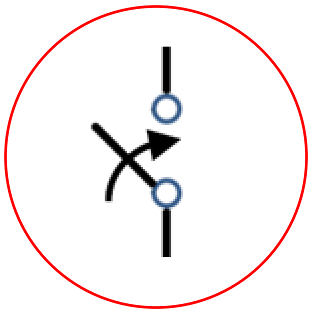
$$R_{eq} = \left[\frac{1}{0} + \frac{1}{R_1}\right]^{-1}$$

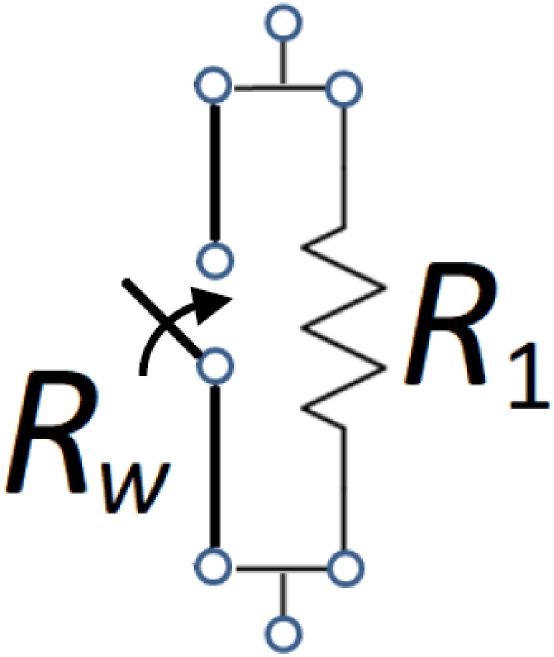
$$R_{eq} = \left[\infty + \frac{1}{R_1}\right]^{-1} = [\infty]^{-1} = 0$$

Current only flows in the wire regardless of R_1

Parallel between an ideal wire and a resistor

Symbol for a switch

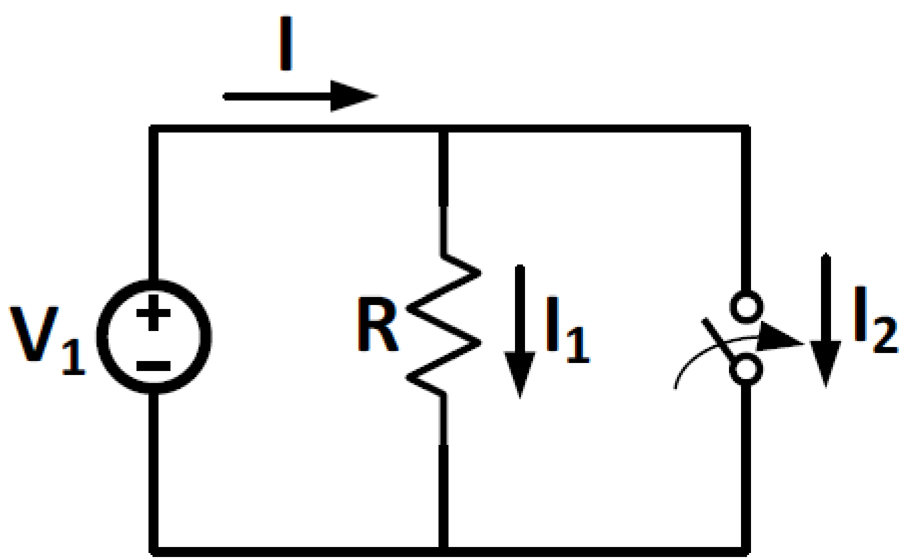




You can add a switch to turn on or off the effect of the shorting wire

Worksheet 1

2. In the circuit shown below, current $\mathbf{I = 5mA}$. Compute currents $\mathbf{I_1}$ and $\mathbf{I_2}$ when the switch is (a) open and (b) closed.



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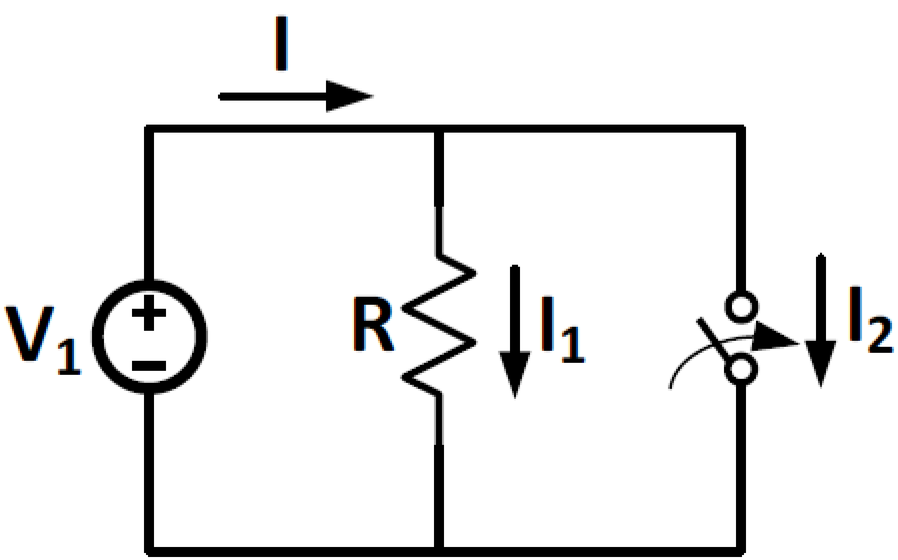


Open switch
$I_1 = I = 5\text{mA}$
$I_2 = 0$

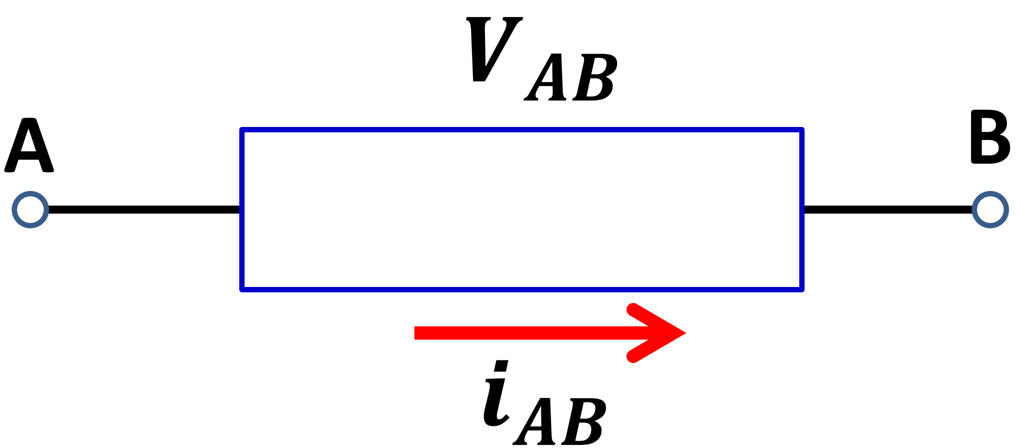
Closed switch (short circuit)
$I_1 = 0$
$I_2 \rightarrow \infty$

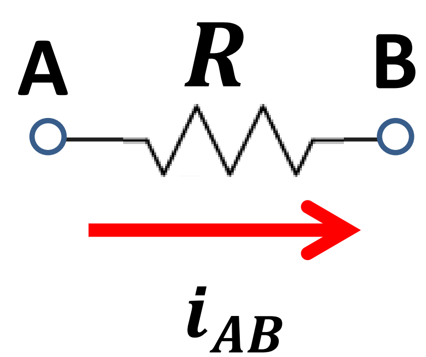
The battery only sees the zero resistance in the shorting bypass wire when the switch is closed.

Power

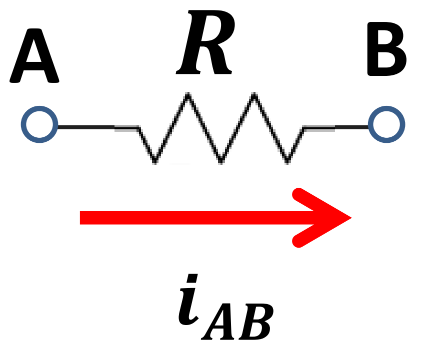
As discussed earlier, the power dissipated by an electrical element is given by

$$P = V_{AB} \times i_{AB}$$





Power



The voltage V_{AB} across a resistor is

$$V_{AB} = i_{AB} \times R$$

which gives the power

$$P = V_{AB} \times i_{AB} = i_{AB} \times R \times i_{AB}$$

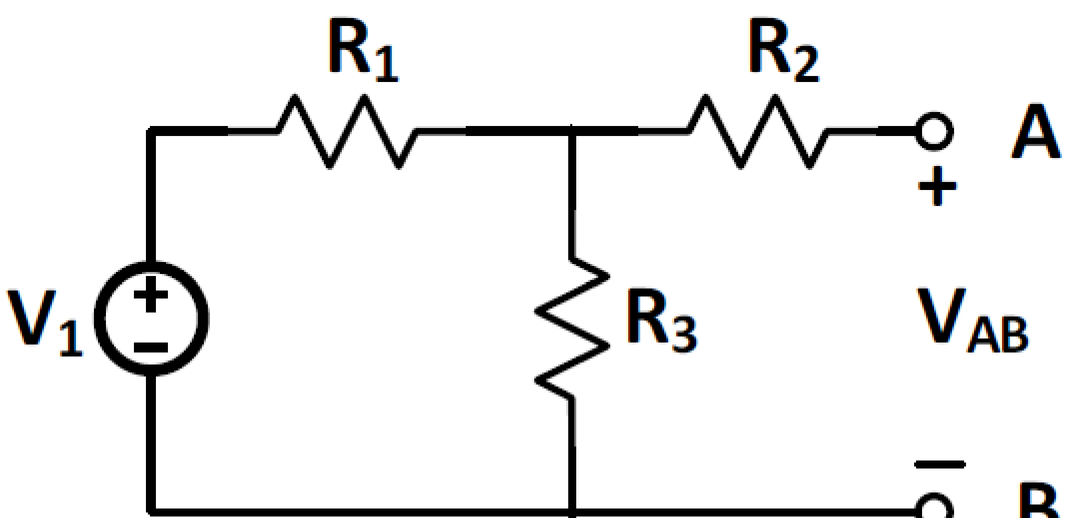
$$P = i_{AB}^2 R$$ or $$P = \frac{V_{AB}^2}{R}$$ [Watts]

Worksheet 1

3. In the circuit shown below,

   $\mathbf{V_1 = 5V,\ R_1 = 2k\Omega,\ R_2 = 5k\Omega,\ R_3 = 3k\Omega.}$

   Compute voltage $\mathbf{V_{AB}}$.



Worksheet 1

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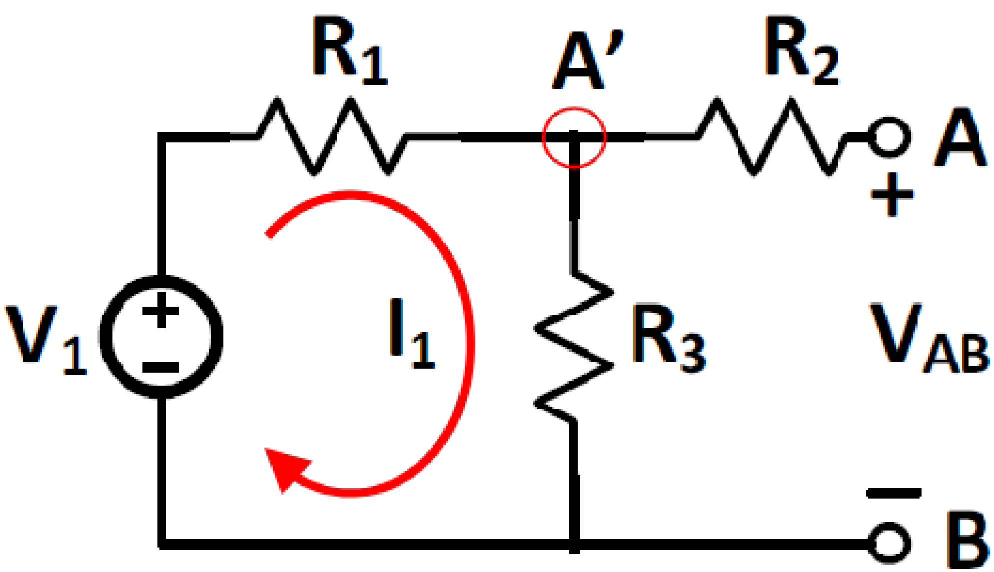
Compute voltage $\mathbf{V_{AB}}$.



All points in the bottom wire are at the same potential as B. The circled node A' has the same potential as A, because no current flows in R_2. The potential across R_3 is V_{AB}. The only current flowing is I_1 through the series of R_1+R_3:

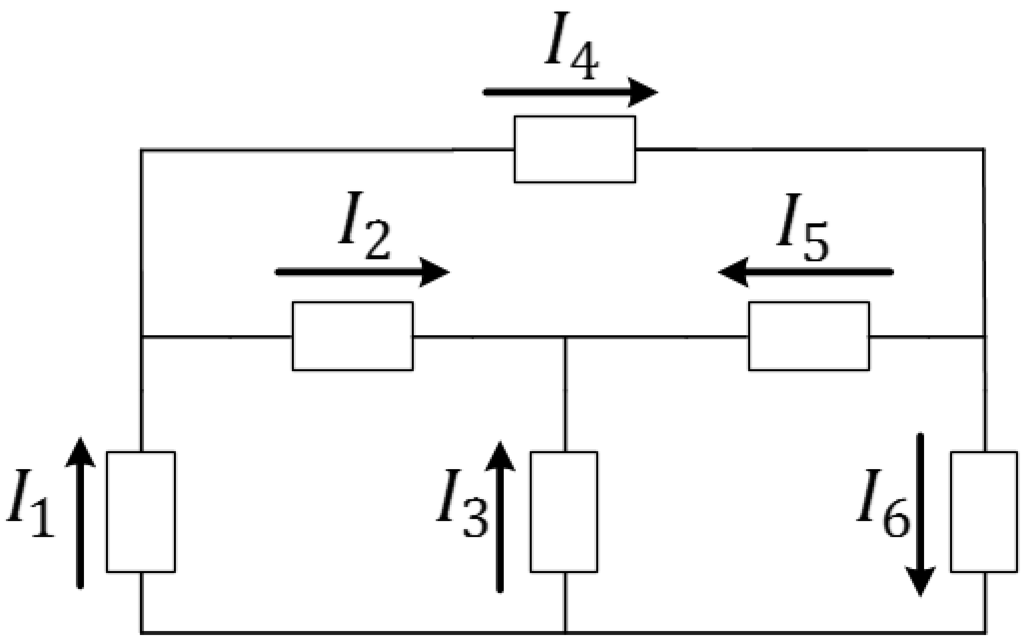
$$I_1 = \frac{V_1}{R_1 + R_3} = \frac{5}{5k} = 1\text{mA}$$

Ohm's Law:

$$V_{AB} = R_3 I_1 = 3k\Omega \times 1\text{mA} = 3\text{V}$$

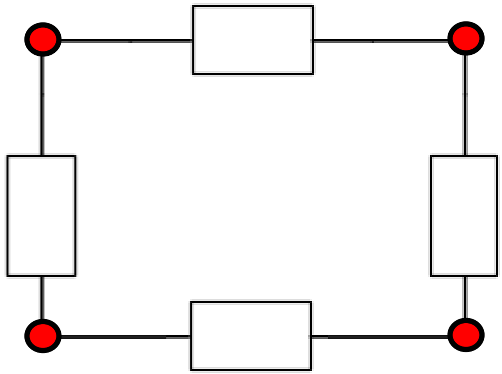
Electrical Circuit

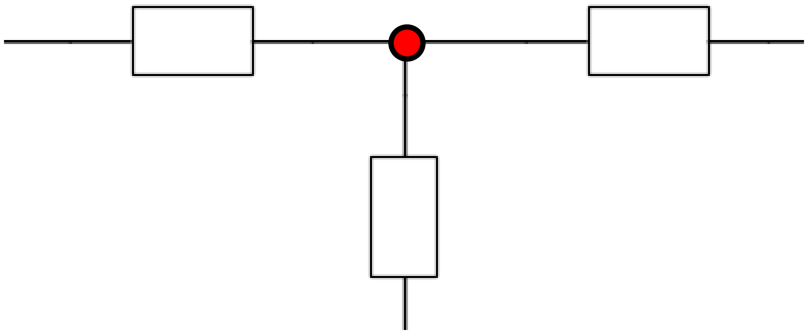
An electrical circuit is a network of electrical elements interconnected in a closed path such that currents can continuously flow. Example:



Circuit Node

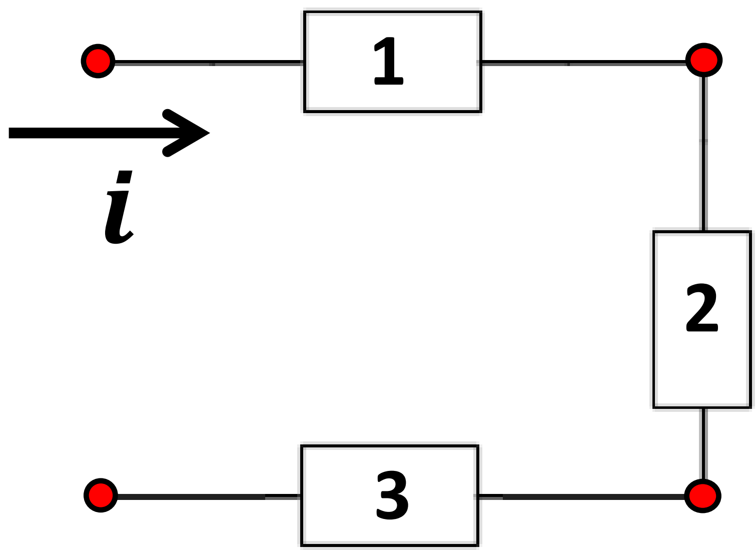
Node is a point at which two or more elements are connected. Examples:

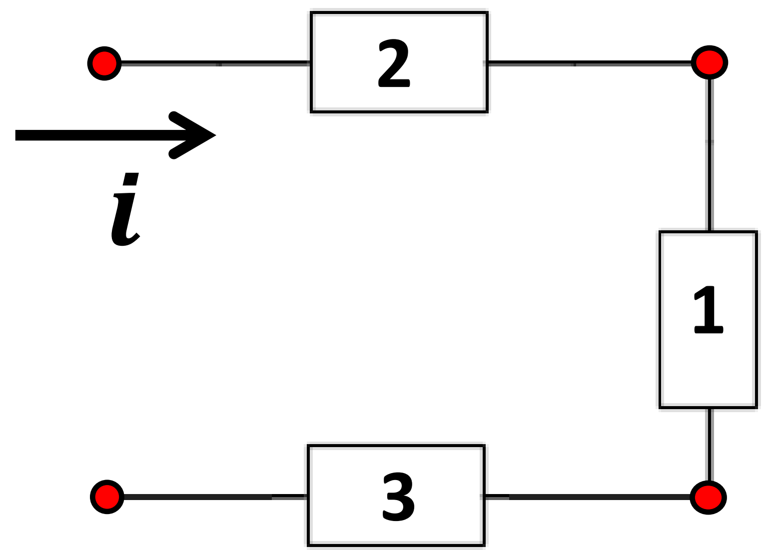




Series connected elements

Elements are said to be connected in series if: 1) they share only one common node with other elements in the series and 2) they all carry the same current.

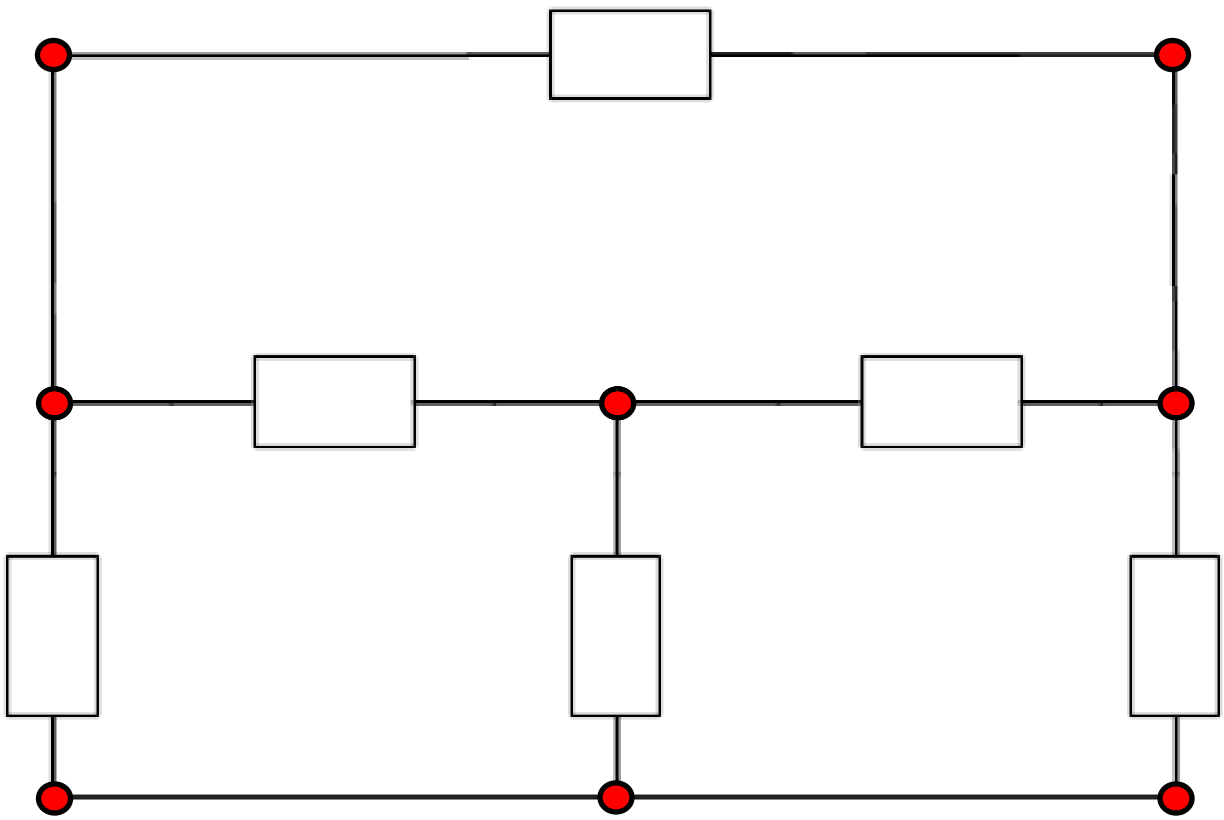




It does not matter if the order of series elements is changed

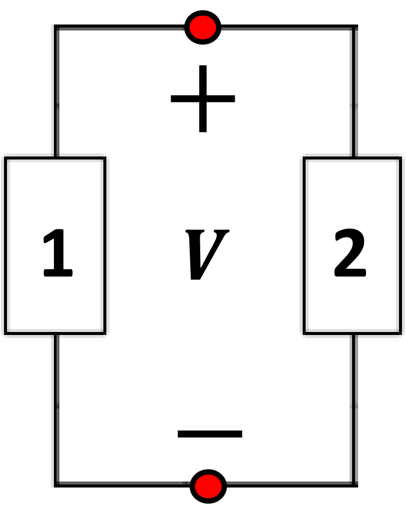
Question

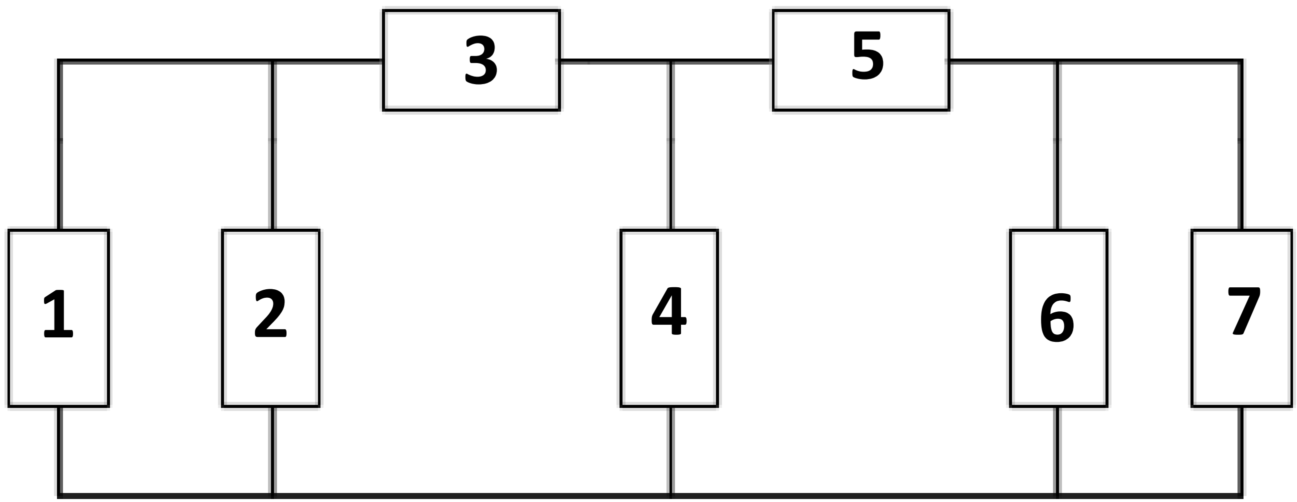
Which elements in the circuit below are in series?



Parallel connected elements

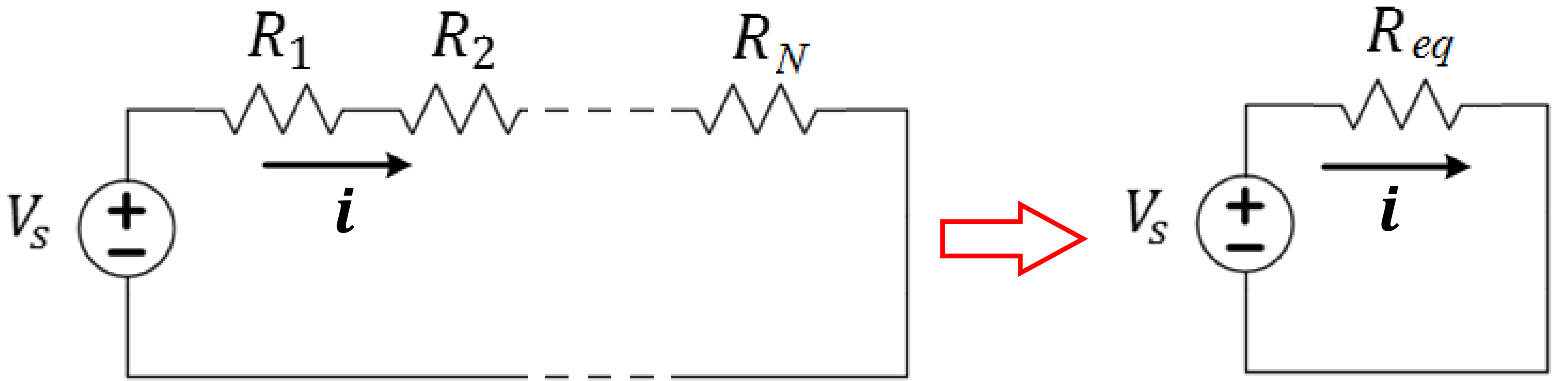
Elements are said to be connected in parallel if: 1) they all share both terminal nodes and 2) they have the same voltage across them.





Which elements in the circuits above are connected in parallel?

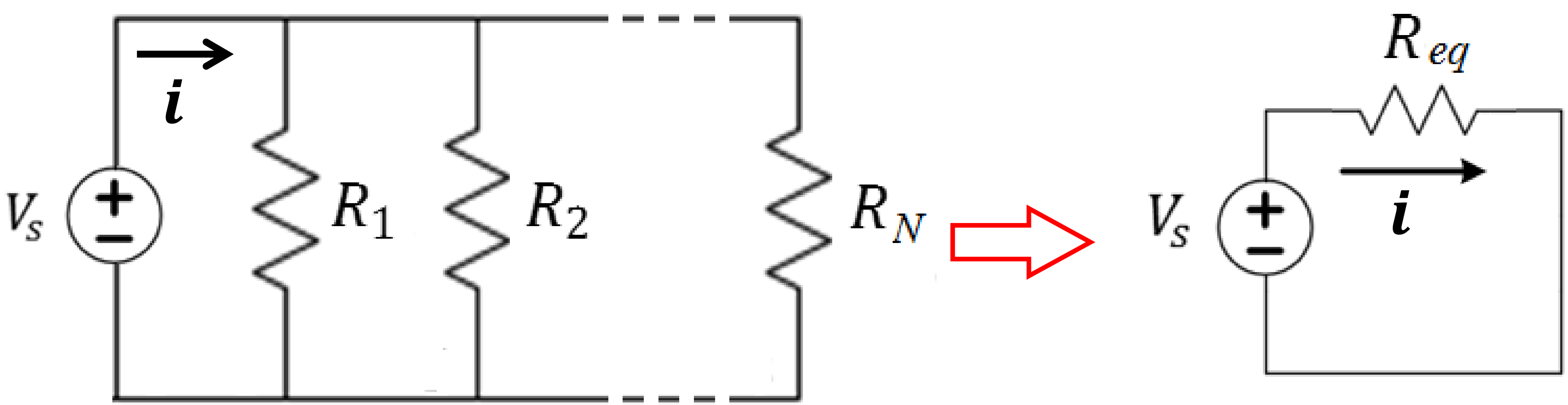
Series connected resistors



N **resistors connected in series can be replaced by an equivalent resistor** $\boldsymbol{R_{eq}}$

$$R_{eq} = R_1 + R_2 + \cdots + R_N = \sum_{k=1}^{N} R_k$$

Parallel connected resistors

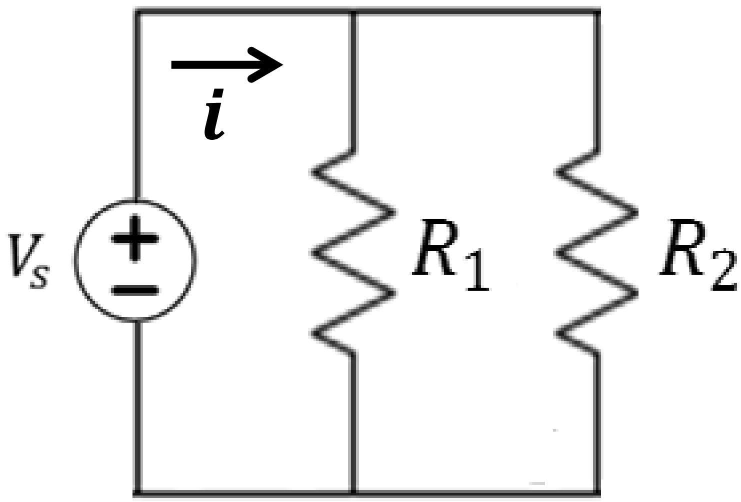


N **resistors connected in series can be replaced by an equivalent resistor** R_{eq} **given by**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{k=1}^{N} \frac{1}{R_k}$$

Special case: Two parallel resistors

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$



If the resistors are identical

$$\boldsymbol{R_1 = R_2 = R}$$

$$R_{eq} = \frac{RR}{R+R} = \frac{R}{2}$$

Corollary: *N* identical parallel resistors

N identical resistors in parallel have an equivalent resistance

$$R_{eq} = \frac{R}{N}$$