ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 4

MWF – 12:00pm

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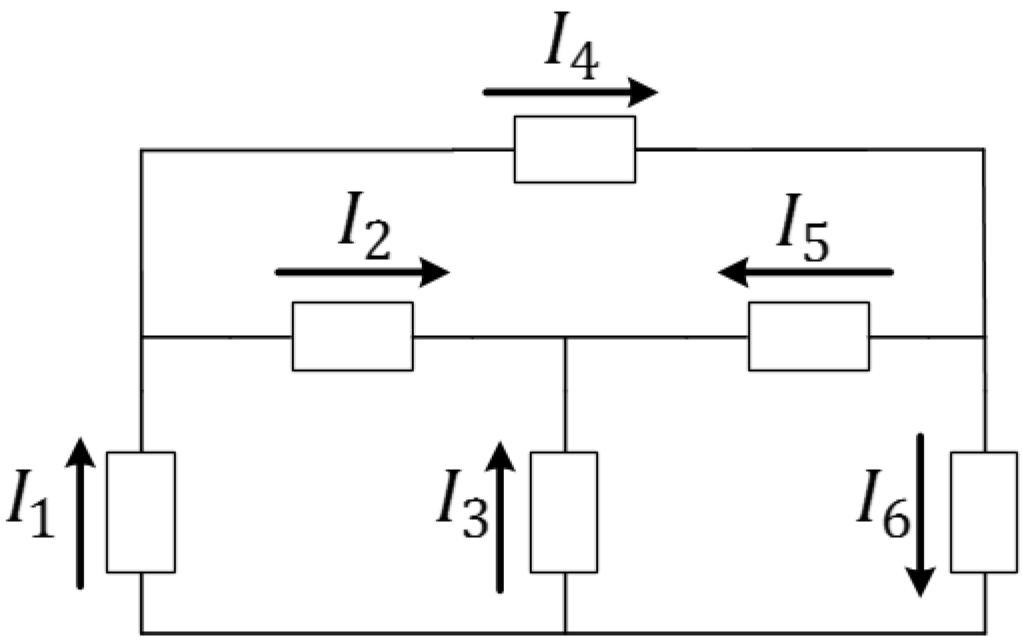
Lecture 4 - Summary

Learning Objectives

1. Basics of electrical circuits
2. Define Kirchhoff's voltage law (KVL)
3. Compute currents in simple circuits using KVL and Ohm's law
4. Use loop analysis method to compute loop currents

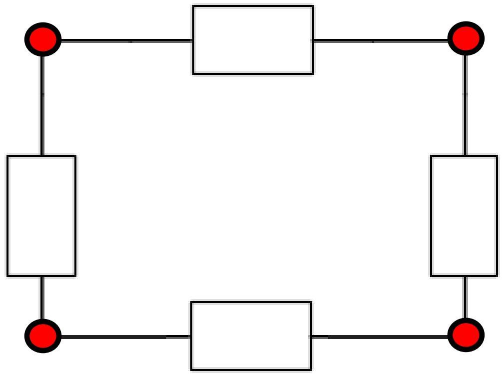
Electrical Circuit

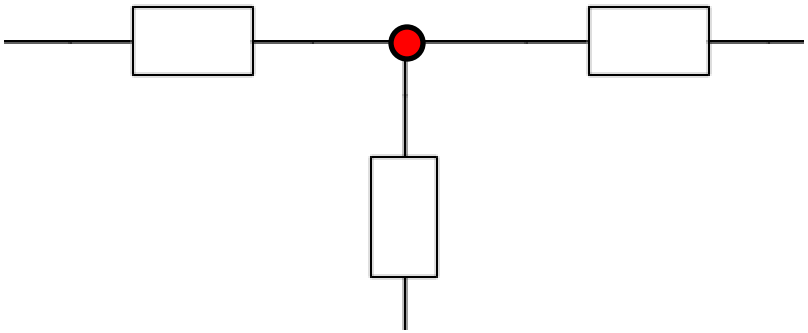
An electrical circuit is a network of electrical elements interconnected in a closed path such that currents can continuously flow. Example:



Circuit Node

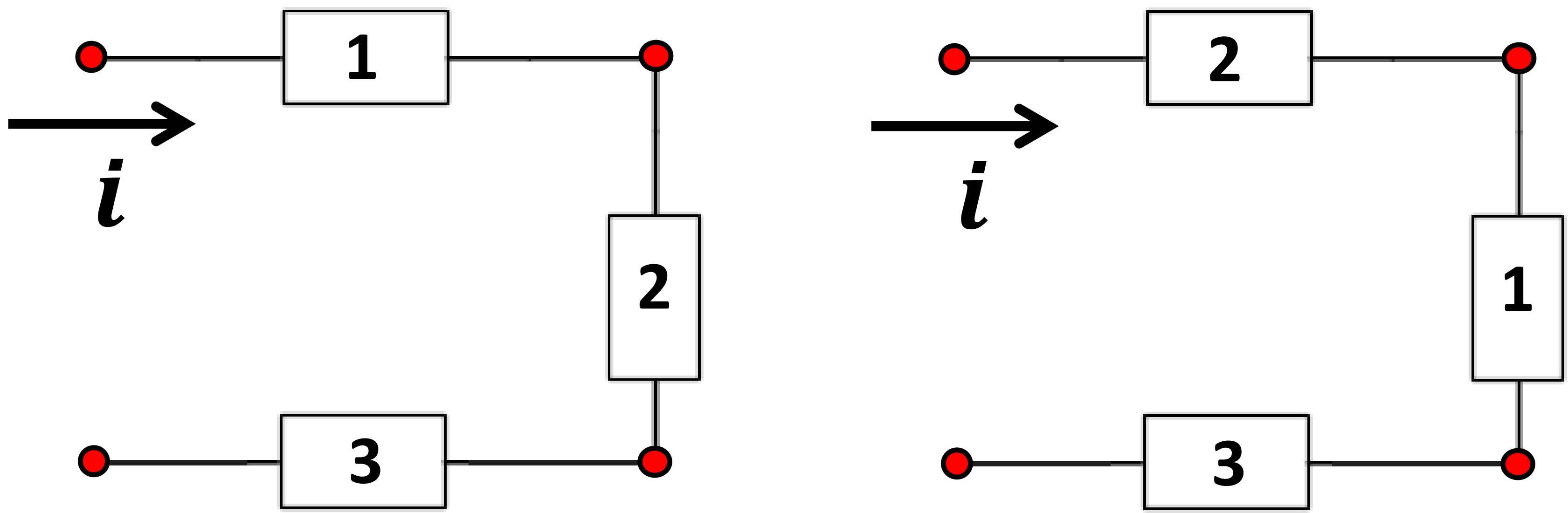
Node is a point at which two or more elements are connected. Examples:





Series connected elements

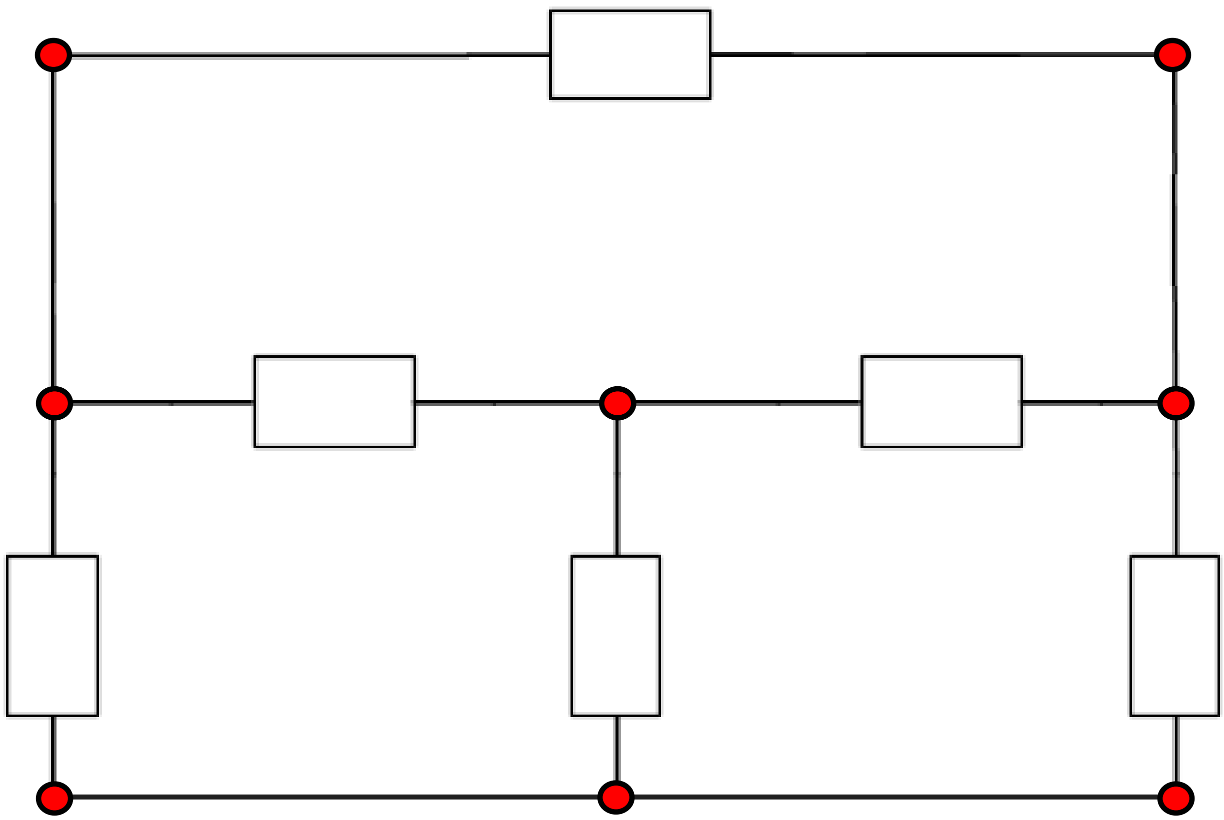
Elements are said to be connected in series if: 1) they share only one common node with other elements in the series and 2) they all carry the same current.



It does not matter if the order of series elements is changed

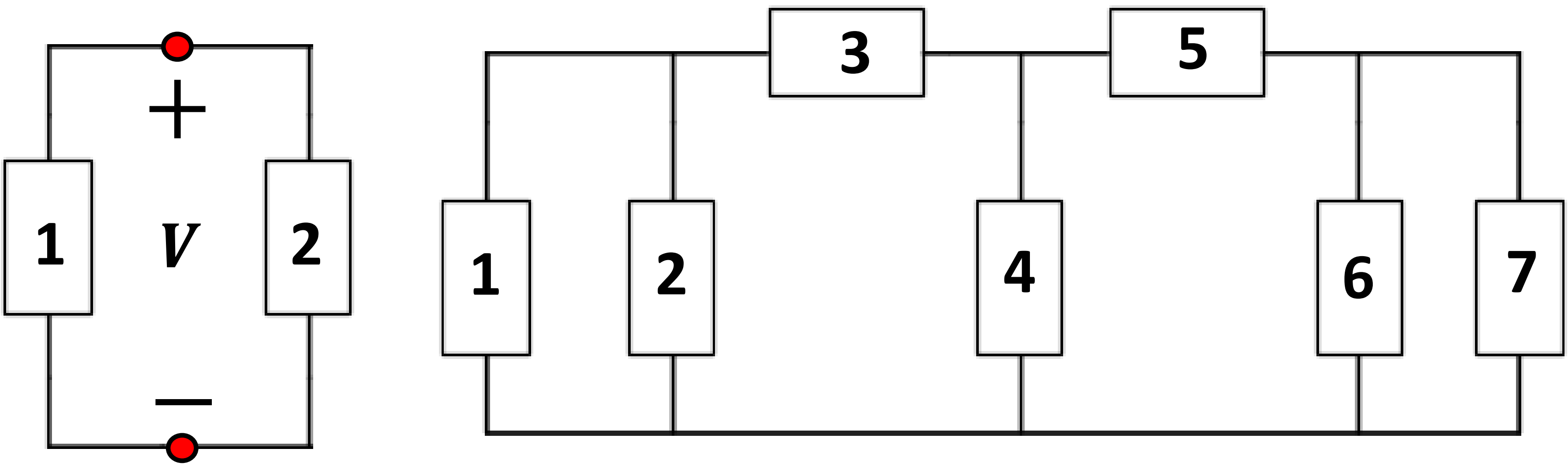
Question

Which elements in the circuit below are in series?



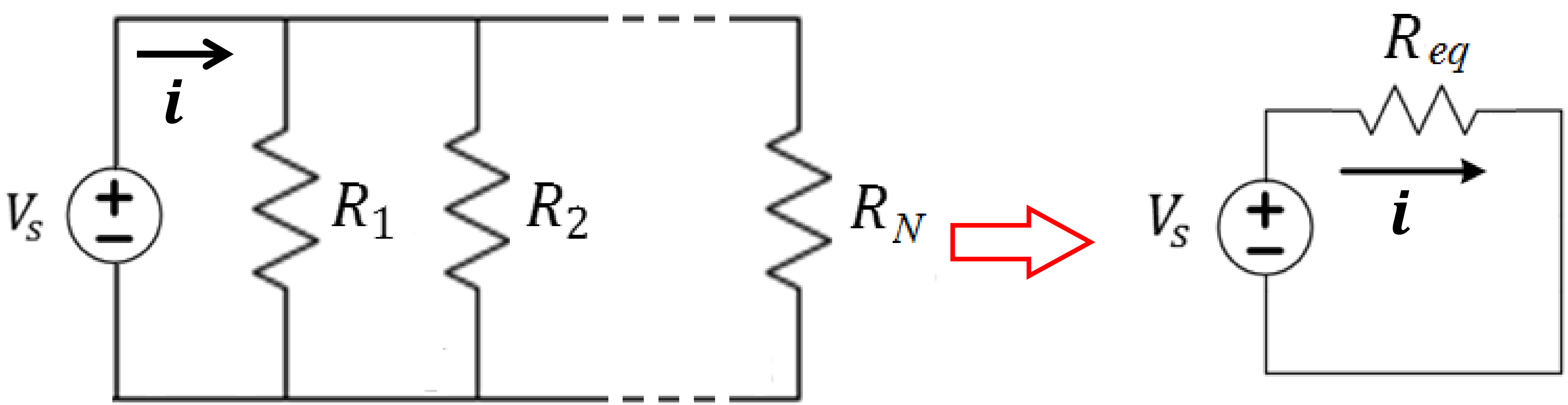
Parallel connected elements

Elements are said to be connected in parallel if: 1) they all share both terminal nodes and 2) they have the same voltage across them.



Which elements in the circuits above are connected in parallel?

RECALL: Parallel connected resistors

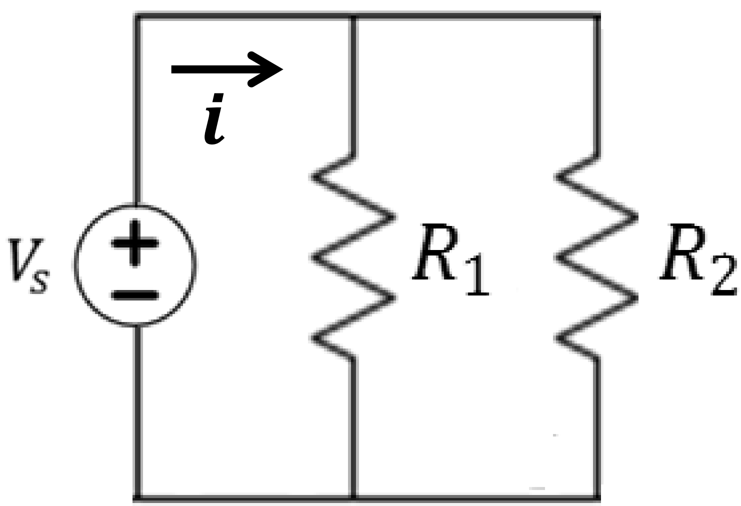


N **resistors connected in series can be replaced by an equivalent resistor R_{eq} given by**

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N} = \sum_{k=1}^{N} \frac{1}{R_k}$$

Special case: Two parallel resistors

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \frac{R_1 R_2}{R_1 + R_2}$$



If the resistors are identical

$$\boldsymbol{R_1 = R_2 = R}$$

$$R_{eq} = \frac{RR}{R + R} = \frac{R}{2}$$

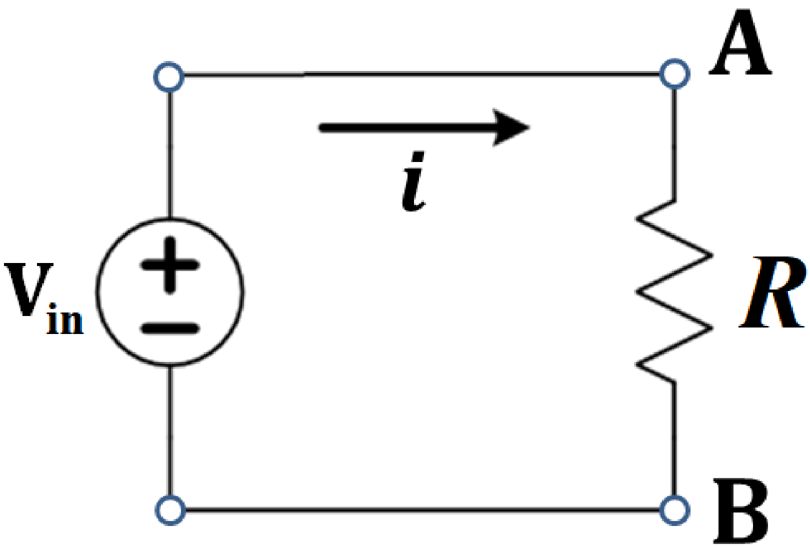
Corollary: *N* identical parallel resistors

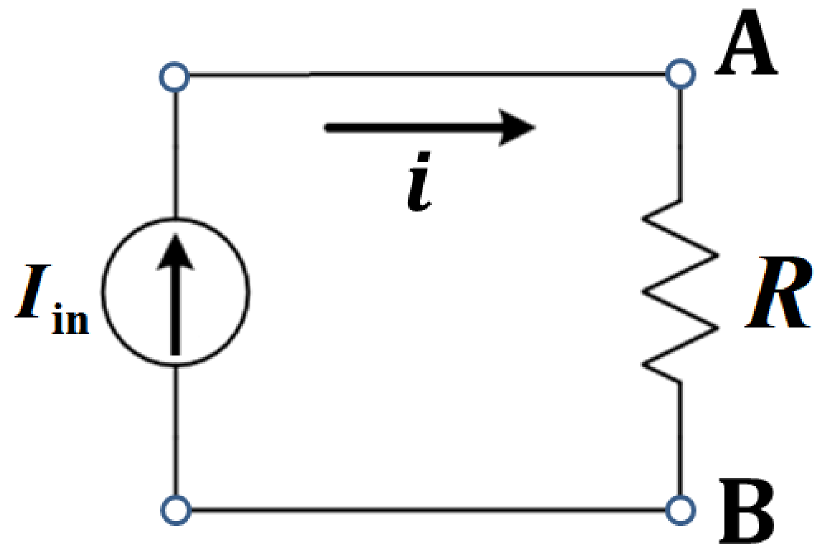
N identical resistors in parallel have an equivalent resistance

$$R_{eq} = \frac{R}{N}$$

Solution of circuits

Ohm's law describes the current-voltage relationship across a resistor element excited by a voltage source or by a current source.





$$V_{AB} = V_{in}$$

$$i = V_{AB}/R$$

$$i = I_{in}$$

$$V_{AB} = R\,i$$

Solution of circuits

We examine now methods to solve more complex circuits.

The goal is to find the voltage at each node and the current through each element of the circuit, to characterize completely the electrical behavior.

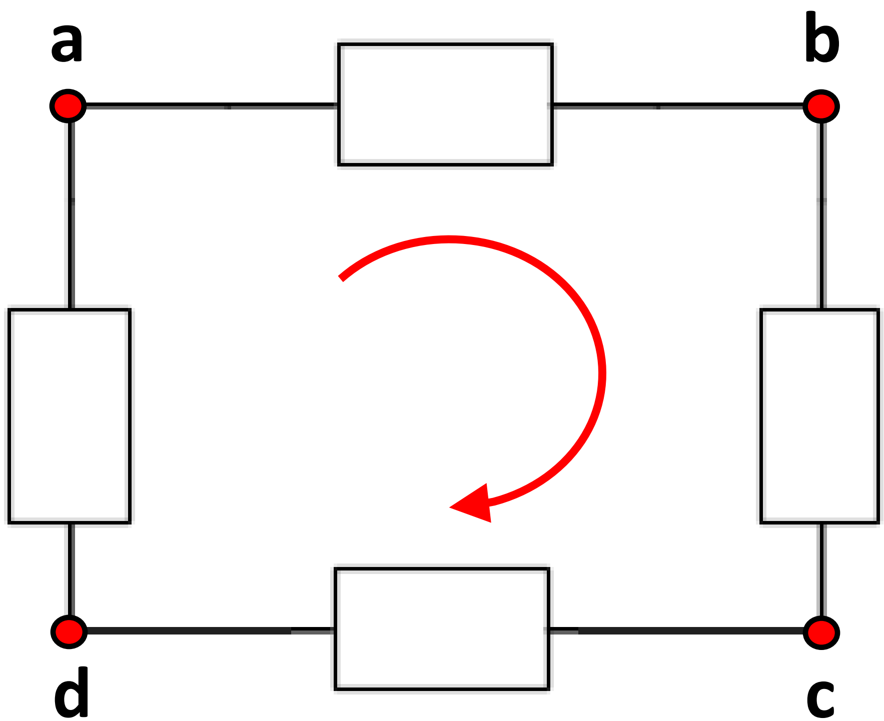
The most important equations to solve a circuit are:

- **Kirchhoff Voltage Law (KVL)**
- **Kirchhoff Current Law (KCL)**

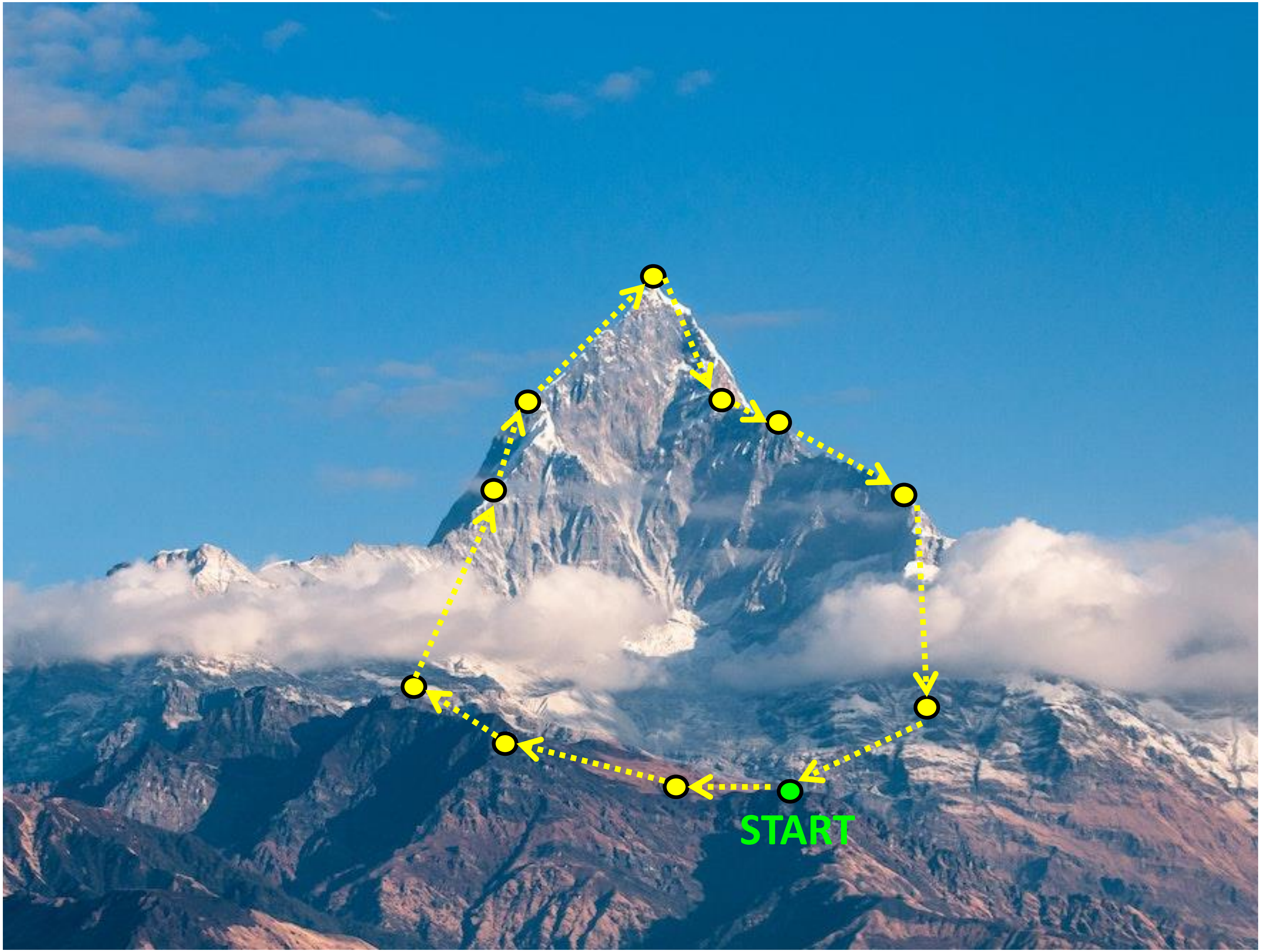
Kirchhoff Voltage Law (KVL)

KVL states that the algebraic sum of voltage around a circuit loop (a closed path) is zero.

KVL) $$V_{ab} + V_{bc} + V_{cd} + V_{da} = 0$$



Direction of rotation is arbitrary, but we will normally consider a clockwise path.

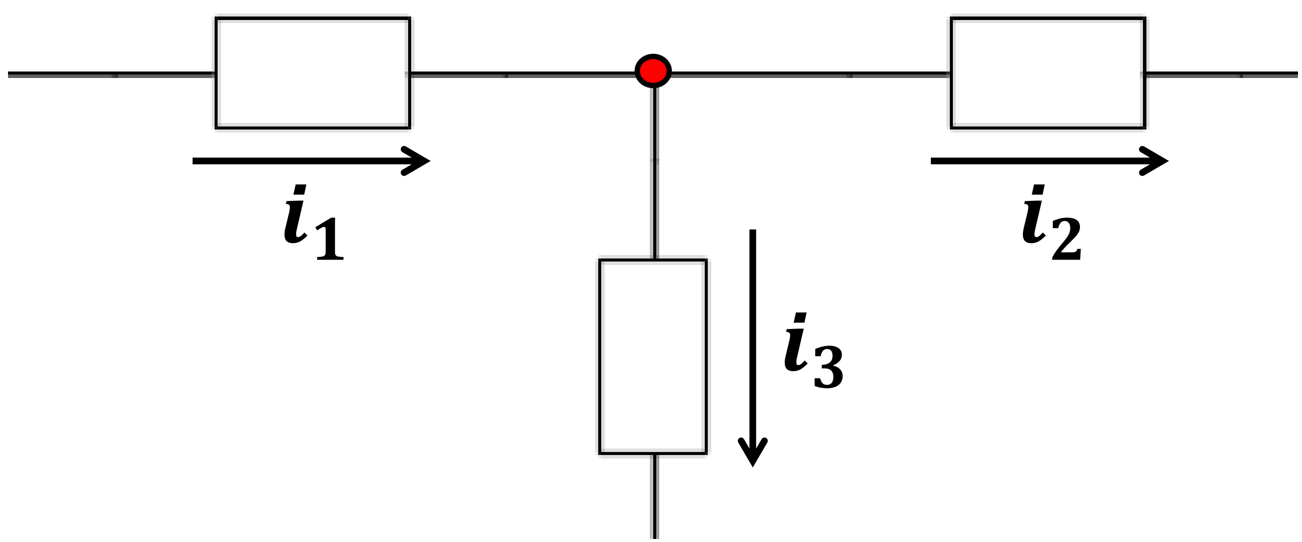

START

Kirchhoff Current Law (KCL)

KVL states that the algebraic sum of current entering or leaving a node is zero (conservation of charge).

$$i_1 = i_2 + i_3$$ current in = current out

KCL) $-i_1 + i_2 + i_3 = 0$



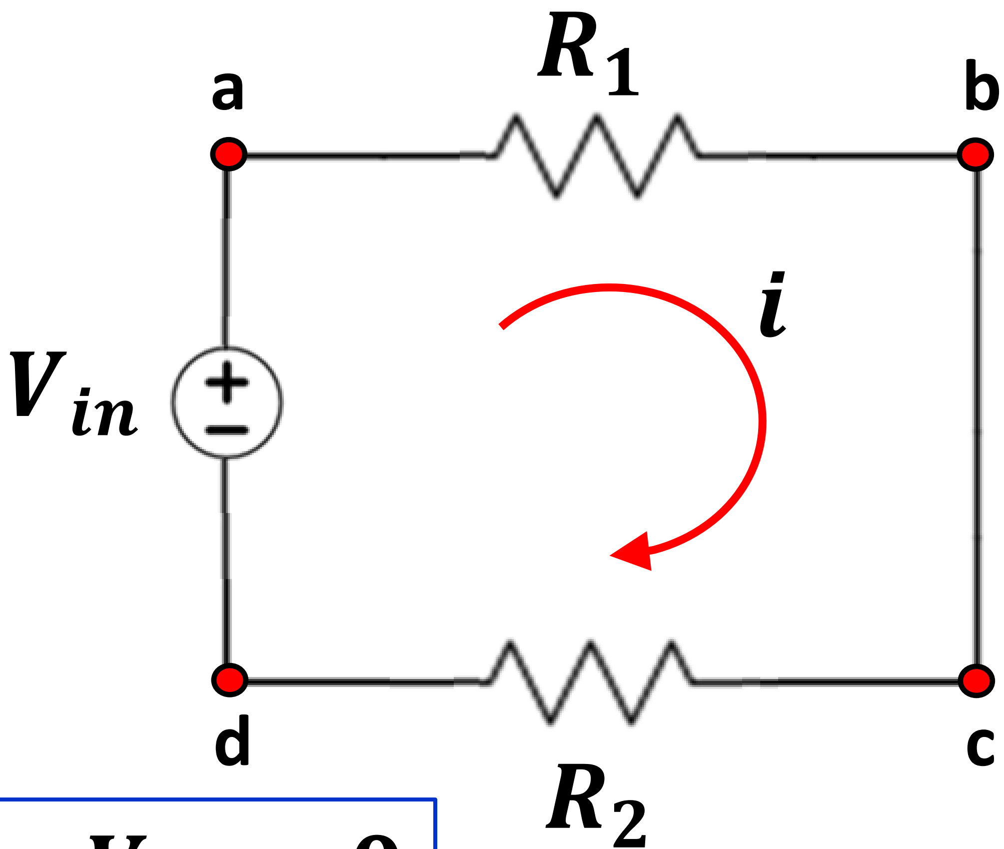
Example 1

KVL) $V_{\mathrm{ab}} + V_{\mathrm{bc}} + V_{\mathrm{cd}} + V_{\mathrm{da}} = 0$

Then, express voltage in each segment using Ohm's Law

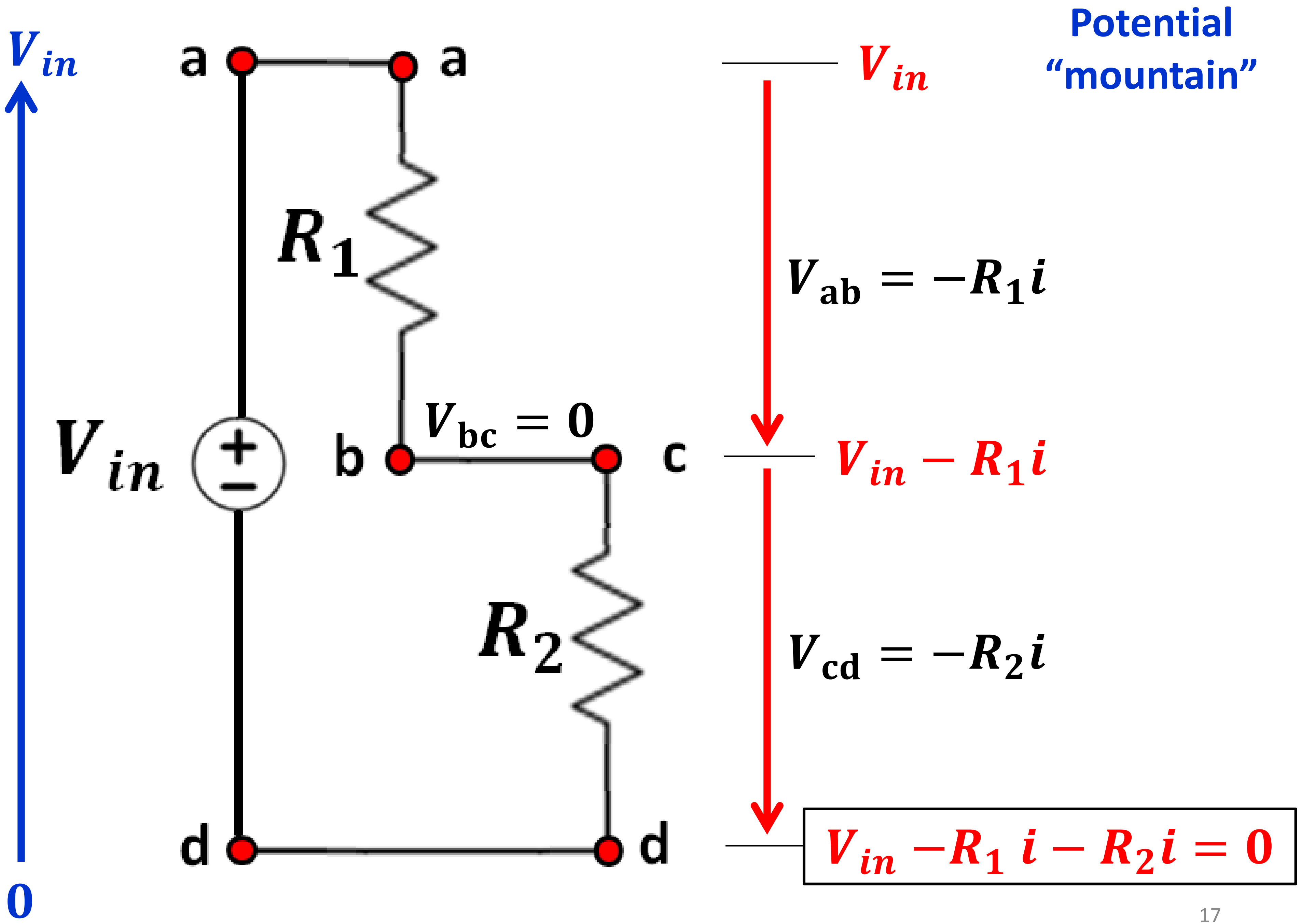
Ohm's Law for each segment

$$V_{\mathrm{ab}} = R_1 i$$
$$V_{\mathrm{bc}} = 0$$
$$V_{\mathrm{cd}} = R_2 i$$
$$V_{\mathrm{da}} = -V_{in}$$



KVL) $R_1 i + R_2 i - V_{in} = 0$

$$V_{in}/i = \mathrm{R}_1 + R_2$$



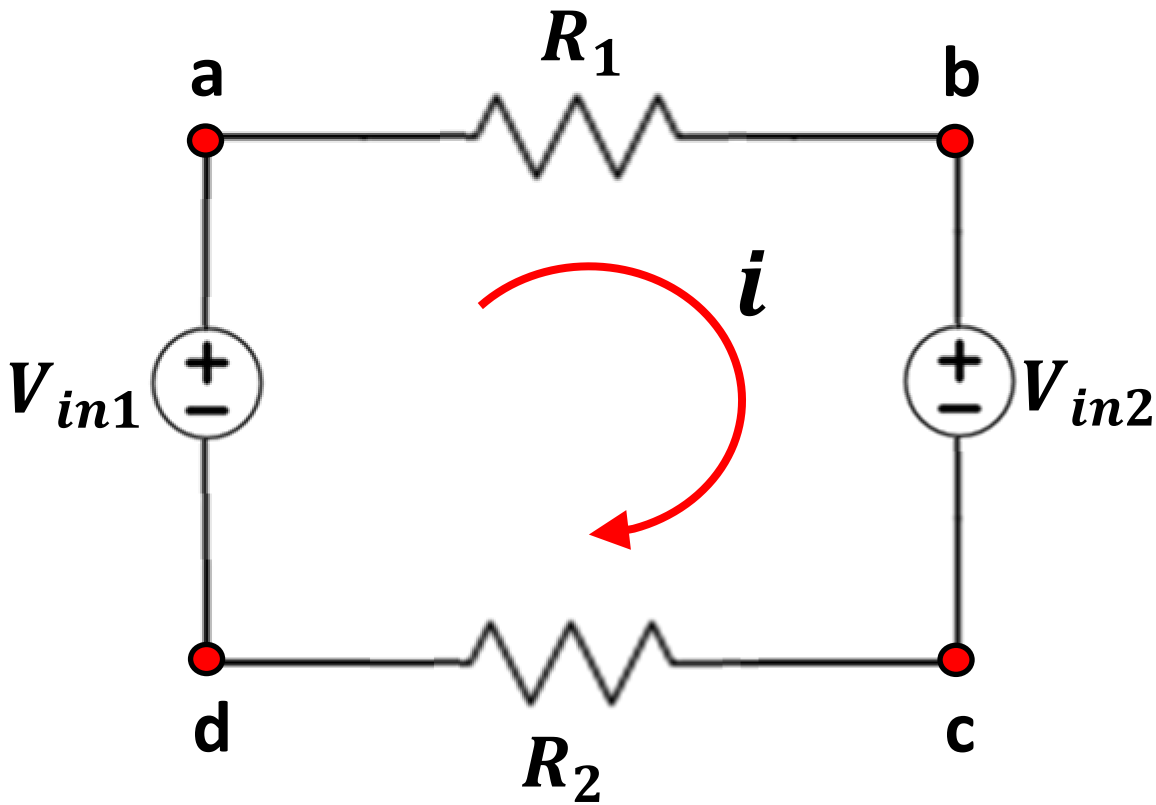
Potential
"mountain"
V_{in}
0
a
a
R_1
V_{in}
b
$V_{bc} = 0$
c
R_2
d
d
V_{in}
$V_{ab} = -R_1 i$
$V_{in} - R_1 i$
$V_{cd} = -R_2 i$
$V_{in} - R_1 i - R_2 i = 0$

Example 2

KVL) $V_{\mathrm{ab}} + V_{\mathrm{bc}} + V_{\mathrm{cd}} + V_{\mathrm{da}} = 0$

Ohm's Law for each segment

$$V_{\mathrm{ab}} = R_1 i$$
$$V_{\mathrm{bc}} = V_{in2}$$
$$V_{\mathrm{cd}} = R_2 i$$
$$V_{\mathrm{da}} = -V_{in}$$

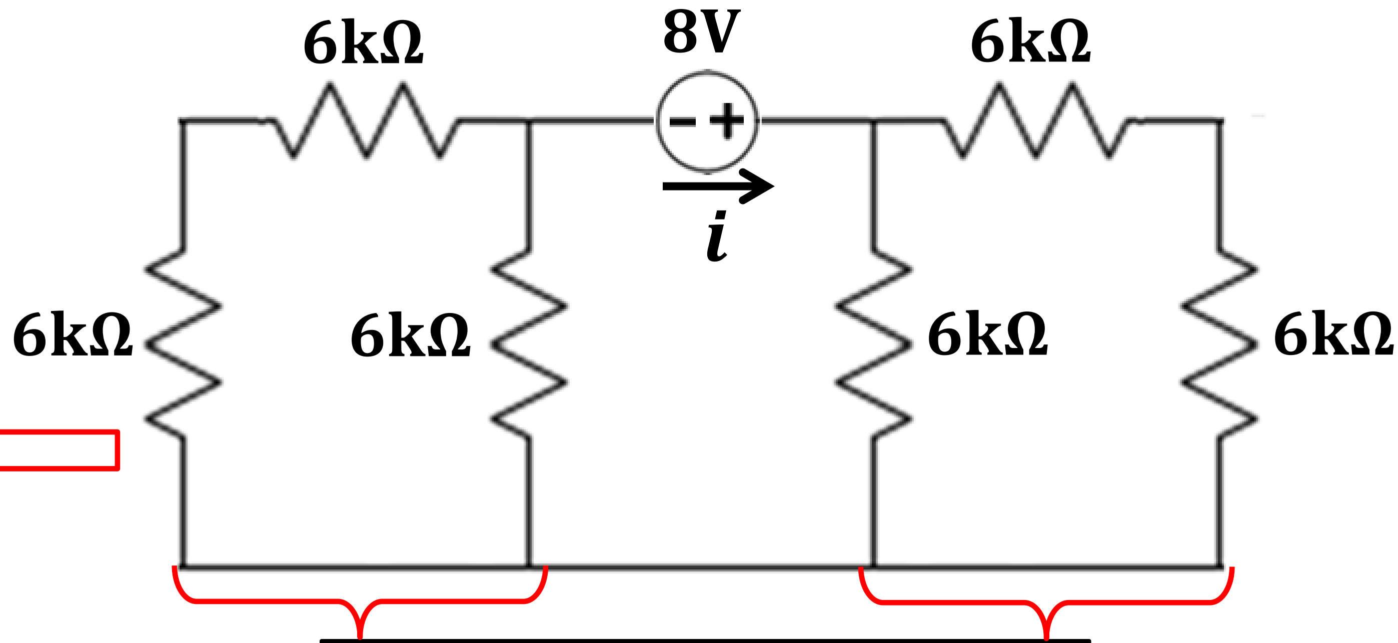


KVL) $R_1 i + R_2 i = V_{in1} - V_{in2}$

$$\frac{V_{in1} - V_{in2}}{i} = \mathrm{R}_1 + R_2$$

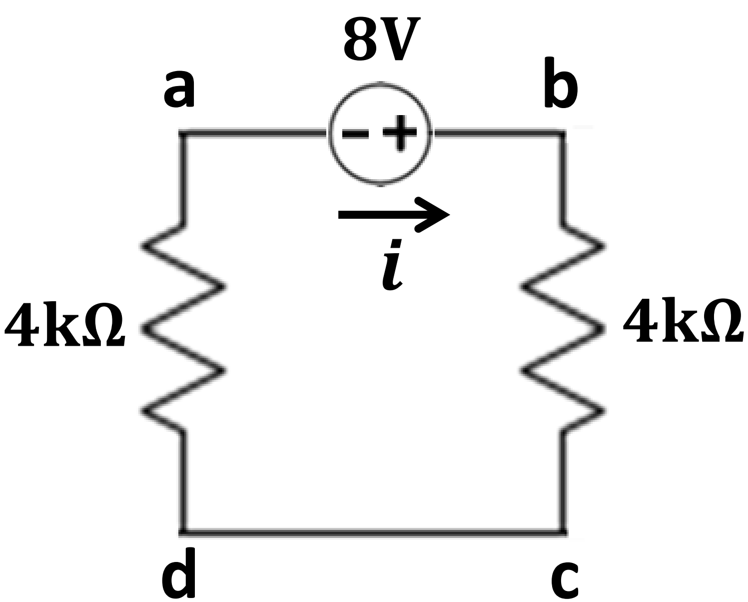
Example 3

Find i



$$(6+6)\text{k}\Omega \,//\, 6\text{k}\Omega = 4\text{k}\Omega$$

Equivalent Circuit



KVL) $V_{\text{ab}} + V_{\text{bc}} + V_{\text{cd}} + V_{\text{da}} = 0$

$$-8 + 4\text{k}i_{bc} + 0 + 4\text{k}i_{da} = 0$$

$$8 = 8\text{k}i$$

$$i = 1\text{mA}$$

Loop Analysis

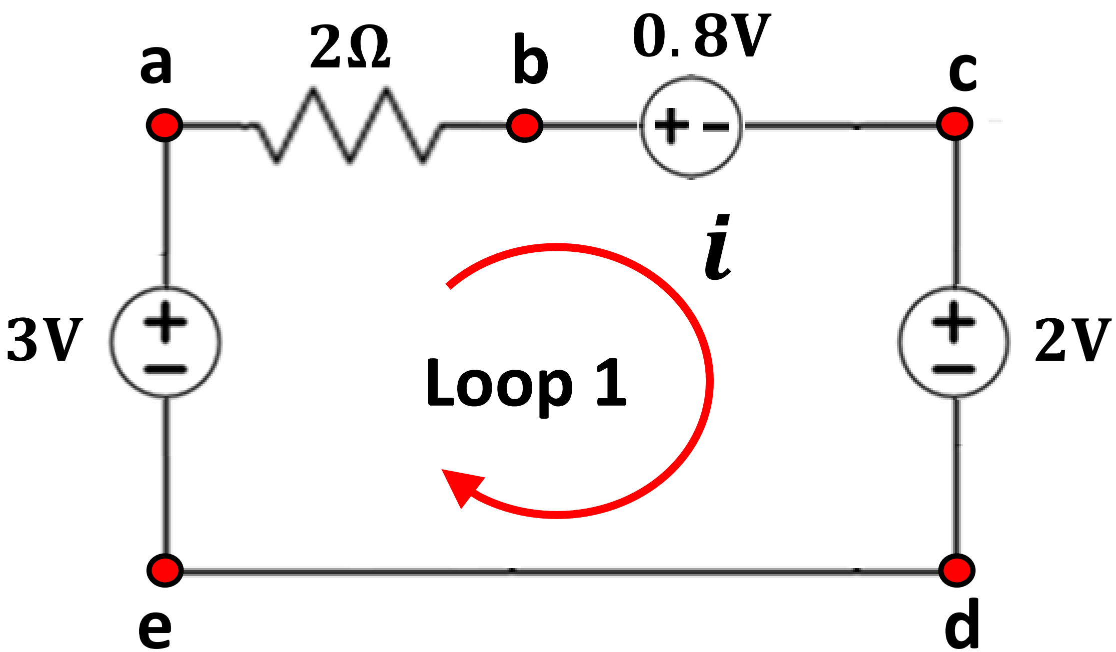
Loop analysis is a systematic procedure based on KVL to solve for currents in more complex circuits.

STEPS

- **Identify loops in a circuit.**
- **Pick currents in clockwise direction**
- **Set up loop equations**
- **Solve system of linear equations to obtain unknown currents**

Example 1 – Single loop circuit

Obtain the unknown current i



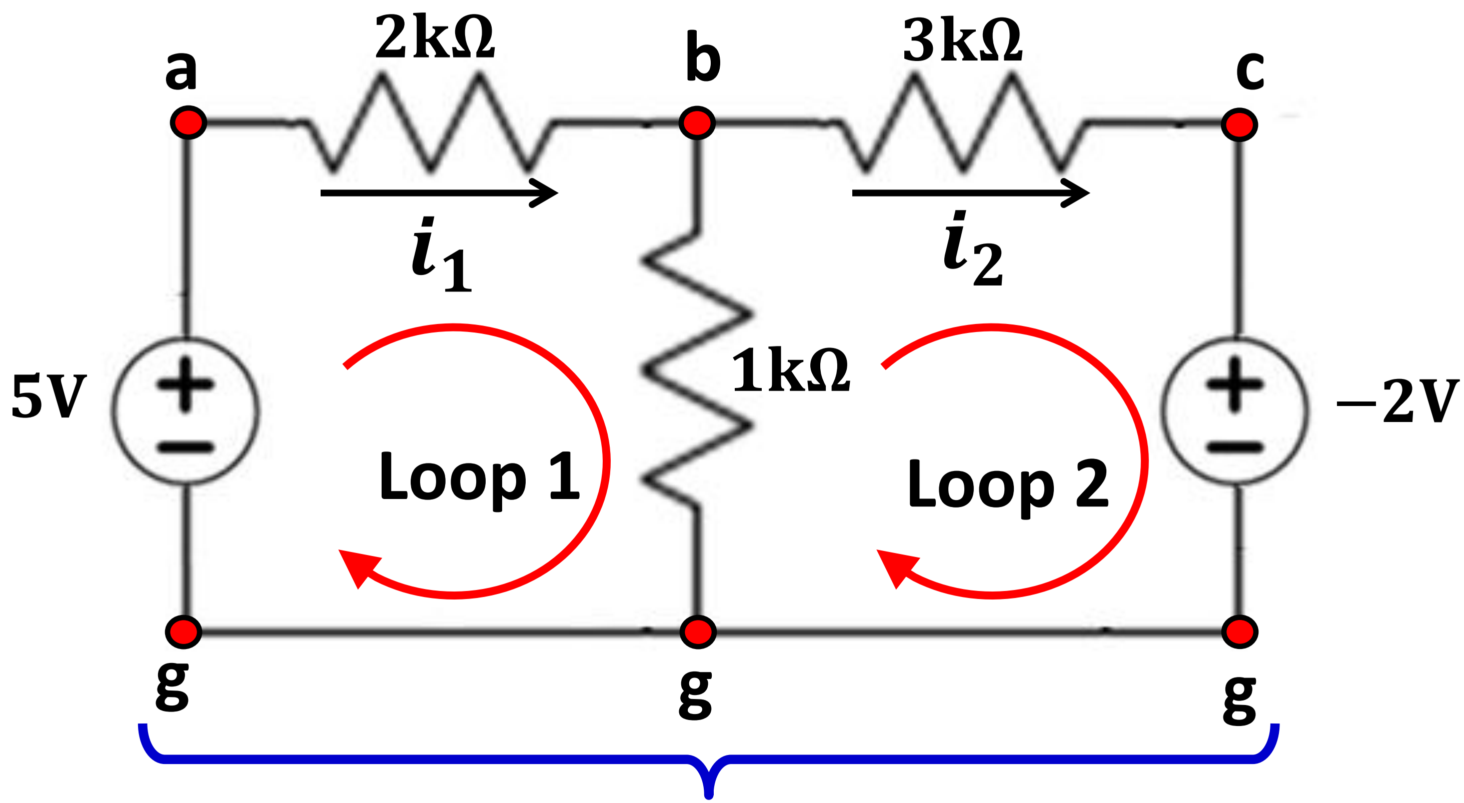
KVL) $V_{ab} + V_{bc} + V_{cd} + V_{de} + V_{ea} = 0$

Ohm's Law: $$2i + 0.8 + 2 + 0 - 3 = 0$$

$$\boxed{i = 0.1\text{A}}$$

Example 2 – Two loops

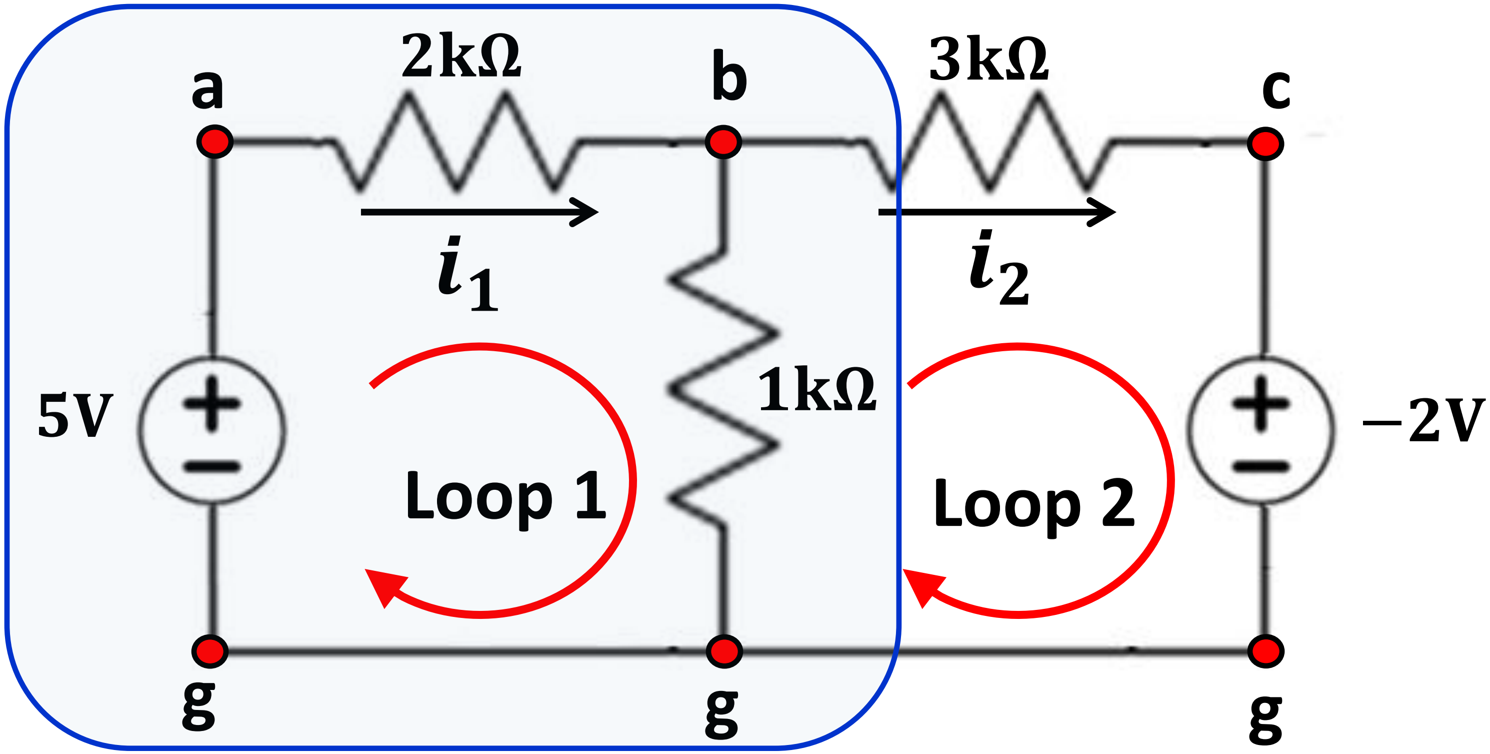
Obtain the unknown currents i_1 and i_2



Let's designate these nodes as "ground" g since they are at the same potential. We will not include in equations the potential between these nodes, because = 0.

Example 2 – Two loops

Obtain the unknown currents i_1 and i_2



LOOP #1

KVL) $V_{ab} + V_{bg} + V_{ga} = 0$

$i_{ab} = i_1$

$i_{bg} = i_1 - i_2$

Ohm's Law:

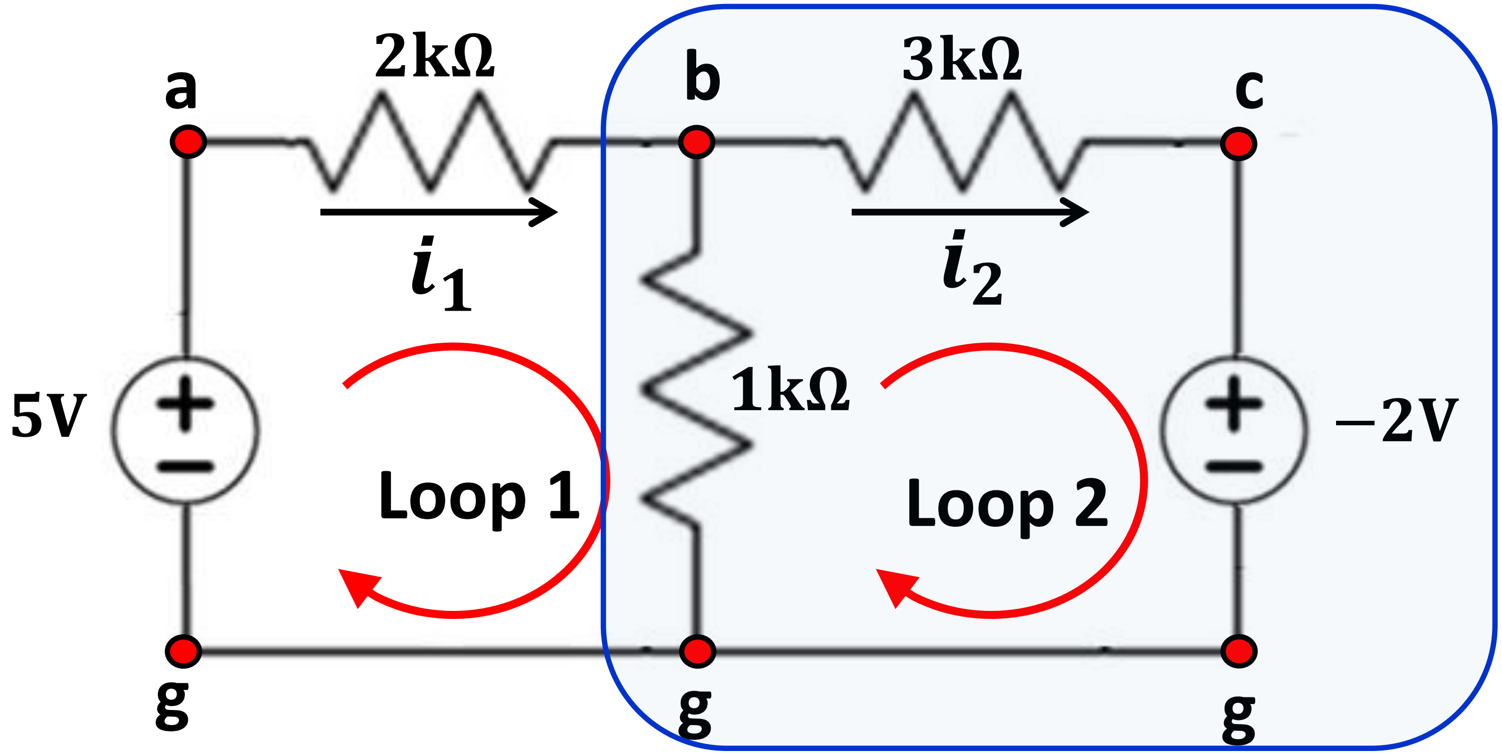
$$2k\, i_{ab} + 1k\, i_{bg} - 5 = 0$$

$$2k\, i_1 + 1k(i_1 - i_2) - 5 = 0$$

$$3k\, i_1 - 1k\, i_2 = 5$$ **Eq. (1)**

Example 2 – Two loops

Obtain the unknown currents i_1 and i_2



LOOP #2

KVL) $V_{bc} + V_{cg} + V_{gb} = 0$

$i_{ab} = i_1$

$i_{bg} = i_1 - i_2$

Ohm's Law:

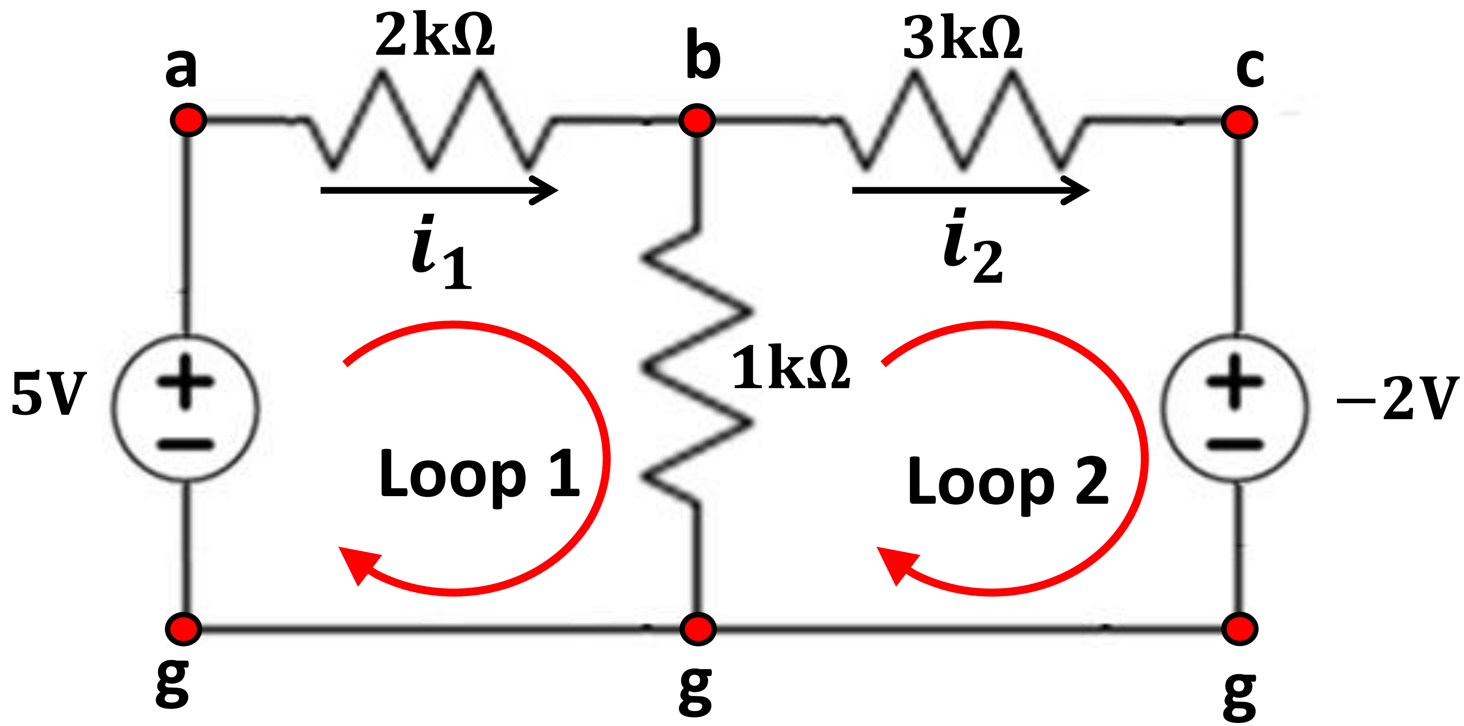
$$3\text{k}\, i_{bc} - 2 + 1\text{k}\, i_{gb} = 0$$

$$3\text{k}\, i_2 - 2 + 1\text{k}\,(i_2 - i_1) = 0$$

$$-1\text{k}\, i_1 + 4\text{k}\, i_2 = 2$$ **Eq. (2)**

Example 2 – Two loops

Obtain the unknown currents i_1 and i_2



Solve system of equations

$$3\text{k}i_1 - \text{k}i_2 = 5$$

$$-1\text{k}\, i_1 + 4\text{k}\, i_2 = 2$$

$$\begin{bmatrix} 3\text{k} & -1\text{k} \\ -1\text{k} & 4\text{k} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 3\text{k} & -1\text{k} \\ -1\text{k} & 4\text{k} \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{mA}$$

Inverse of 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

- A^{-1}: Inverse of A
- $ad - bc$: Determinant of A
- $\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$: Adjoint of A

Example 2 – Two loops

The simple system of equations can be solved by substitution

$$\boxed{3\mathrm{k}\, i_1 - 1\mathrm{k}\, i_2 = 5}$$

$$\boxed{-1\mathrm{k}\, i_1 + 4\mathrm{k}\, i_2 = 2}$$

$$i_2 = 3i_1 - 5/\mathrm{k}$$

$$-i_1 + 4(3i_1 - 5/\mathrm{k}) = 2/\mathrm{k}$$

$$11i_1 - 20/\mathrm{k} = 2/\mathrm{k}$$

$$11i_1 = 22/\mathrm{k}$$

$$\boxed{i_1 = 2\ \mathrm{mA}}$$

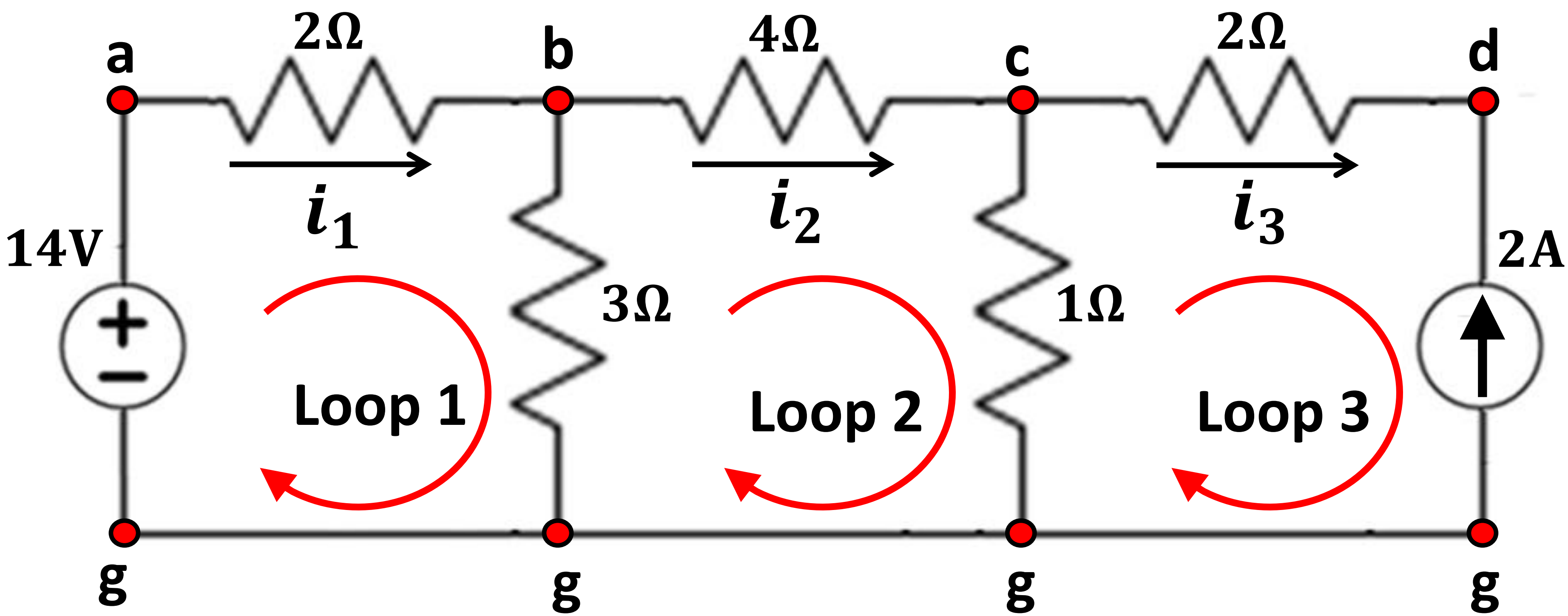
$$i_2 = 3 \times 2/\mathrm{k} - 5/\mathrm{k}$$

$$i_2 = 6/\mathrm{k} - 5/\mathrm{k}$$

$$\boxed{i_2 = 1\ \mathrm{mA}}$$

Example 3 – Three loops

Obtain the unknown currents i_1, i_2 and i_3



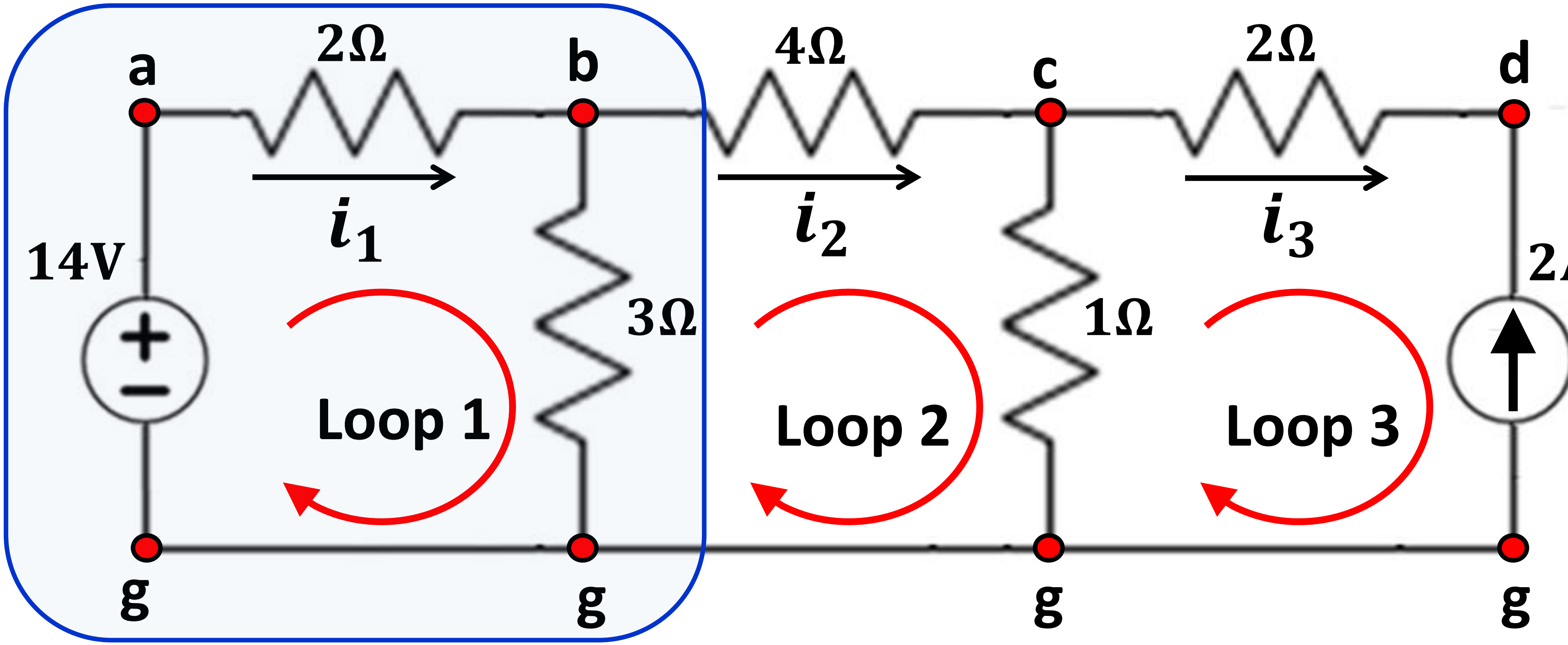
By inspection of Loop 3: $\boxed{i_3 = -2\ \mathrm{A}}$

NOTE: Loop 3 has a current source. It is not possible to write a loop equation for it because the voltage V_{dg} depends on the rest of the circuit.

Example 3 – Three loops

$i_3 = -2\ \text{A}$

Obtain the unknown currents i_1, i_2 and i_3



LOOP #1

KVL) $V_{ab} + V_{bg} + V_{ga} = 0$

$i_{ab} = i_1$

$i_{bg} = i_1 - i_2$

Ohm's Law:

$$2\ i_{ab} + 3\ i_{bg} - 14 = 0$$

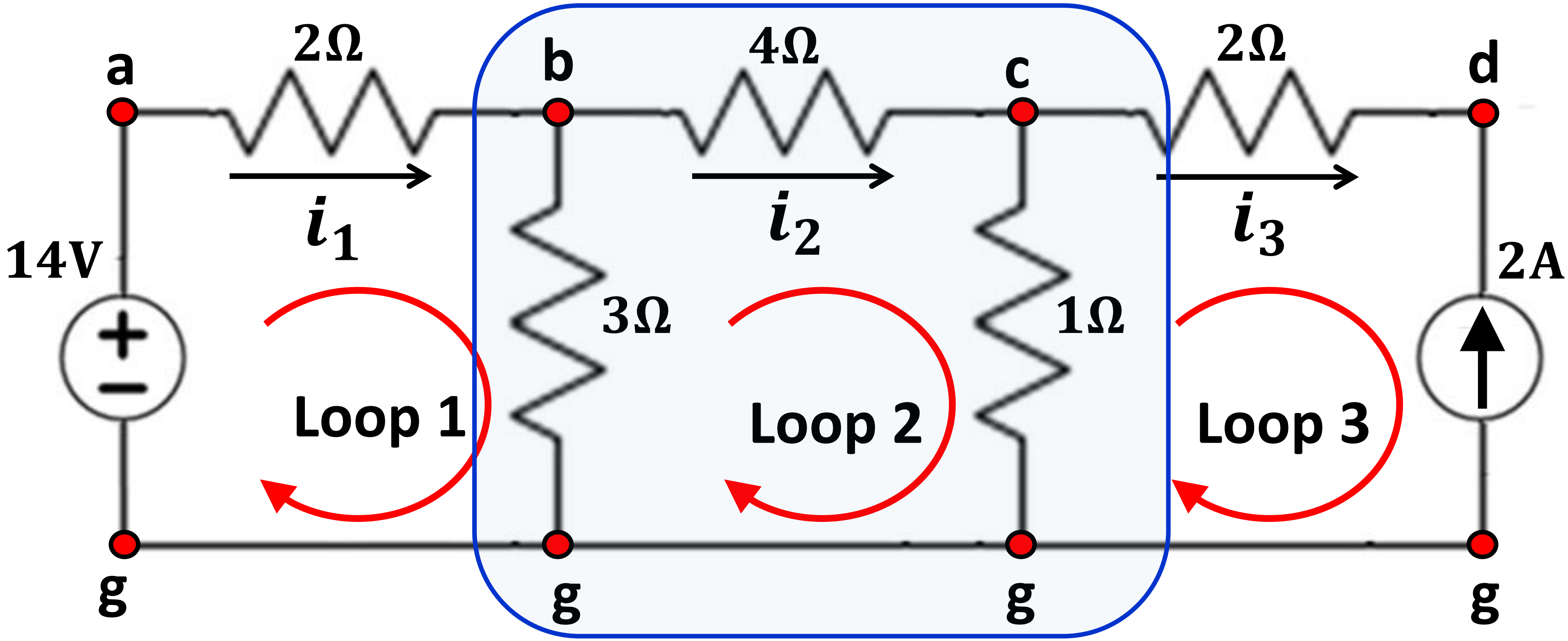
$$2\ i_1 + 3(i_1 - i_2) - 14 = 0$$

$$5\ i_1 - 3\ i_2 = 14 \quad \text{Eq. (1)}$$

Example 3 – Three loops

$$i_3 = -2 \text{ A}$$

Obtain the unknown currents i_1, i_2 and i_3



LOOP #2

KVL) $V_{bc} + V_{cg} + V_{gb} = 0$

Ohm's Law:

$$4\, i_{bc} + 1\, i_{cg} + 3\, i_{gb} = 0$$

$$i_{cg} = i_2 - i_3$$

$$4\, i_2 + 1(i_2 - i_3) + 3(i_2 - i_1) = 0$$

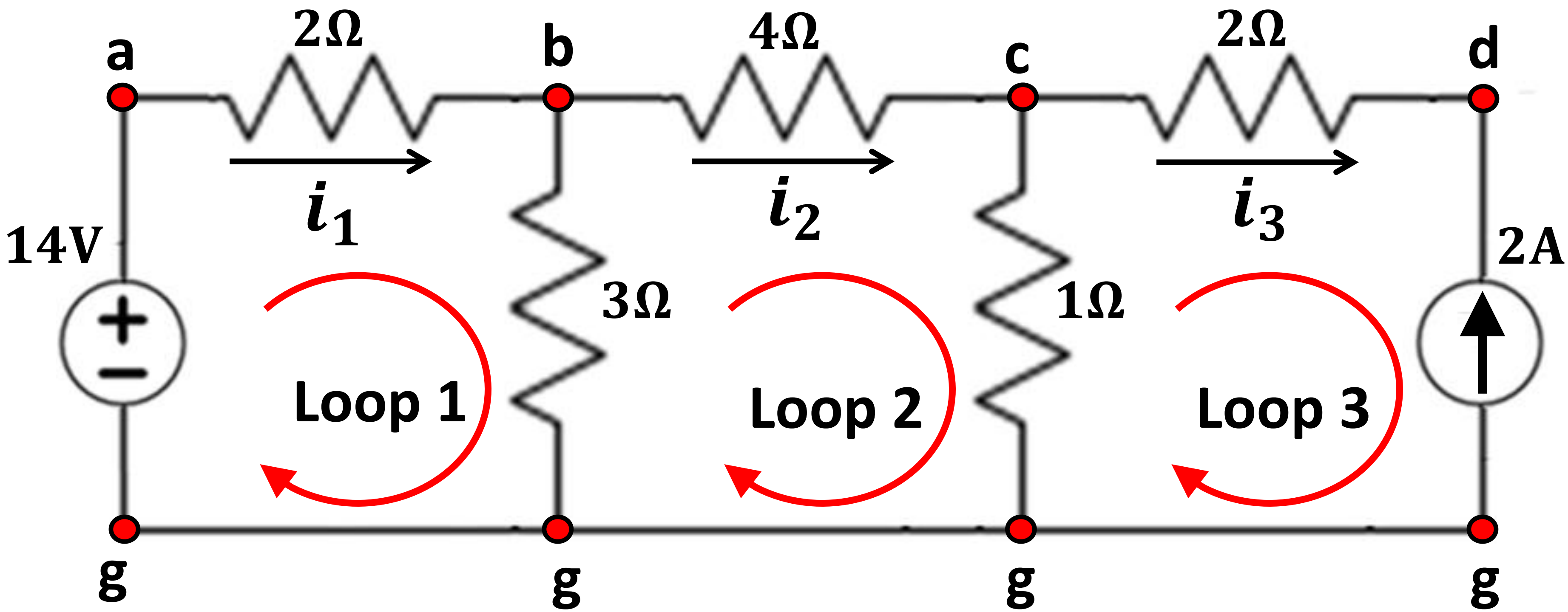
$$i_{bg} = i_1 - i_2$$

$$3\, i_1 - 8\, i_2 = 2$$ Eq. (2)

Example 3 – Three loops

$i_3 = -2\ \text{A}$

Obtain the unknown currents i_1, i_2 and i_3



Solve system of equations

$5\, i_1 - 3\, i_2 = 14$

$3\, i_1 - 8\, i_2 = 2$

$$\begin{bmatrix} 5 & -3 \\ 3 & -8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 3 & -8 \end{bmatrix}^{-1} \begin{bmatrix} 14 \\ 2 \end{bmatrix} = \begin{bmatrix} 3.42 \\ 1.03 \end{bmatrix} \text{A}$$

Example 3 – Three loops

The simple system of equations can be solved by substitution

$$\boxed{5\,i_1 - 3\,i_2 = 14}$$

$$\boxed{3\,i_1 - 8\,i_2 = 2}$$

$$i_2 = (5i_1 - 14)/3$$

$$3i_1 - 8\,(5i_1 - 14)/3 = 2$$

$$i_1 - (40i_1 - 112)/9 = 2/3$$

$$3.\overline{4}\,i_1 = 11.\overline{7}$$

$$\boxed{i_1 = 3.419\text{ A}}$$

$$i_2 = (5i_1 - 14)/3$$

$$i_2 = 5.6989 - 4.\overline{6}$$

$$\boxed{i_2 = 1.032\text{ A}}$$