ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 5

MWF – 12:00pm

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2062 ECE Building

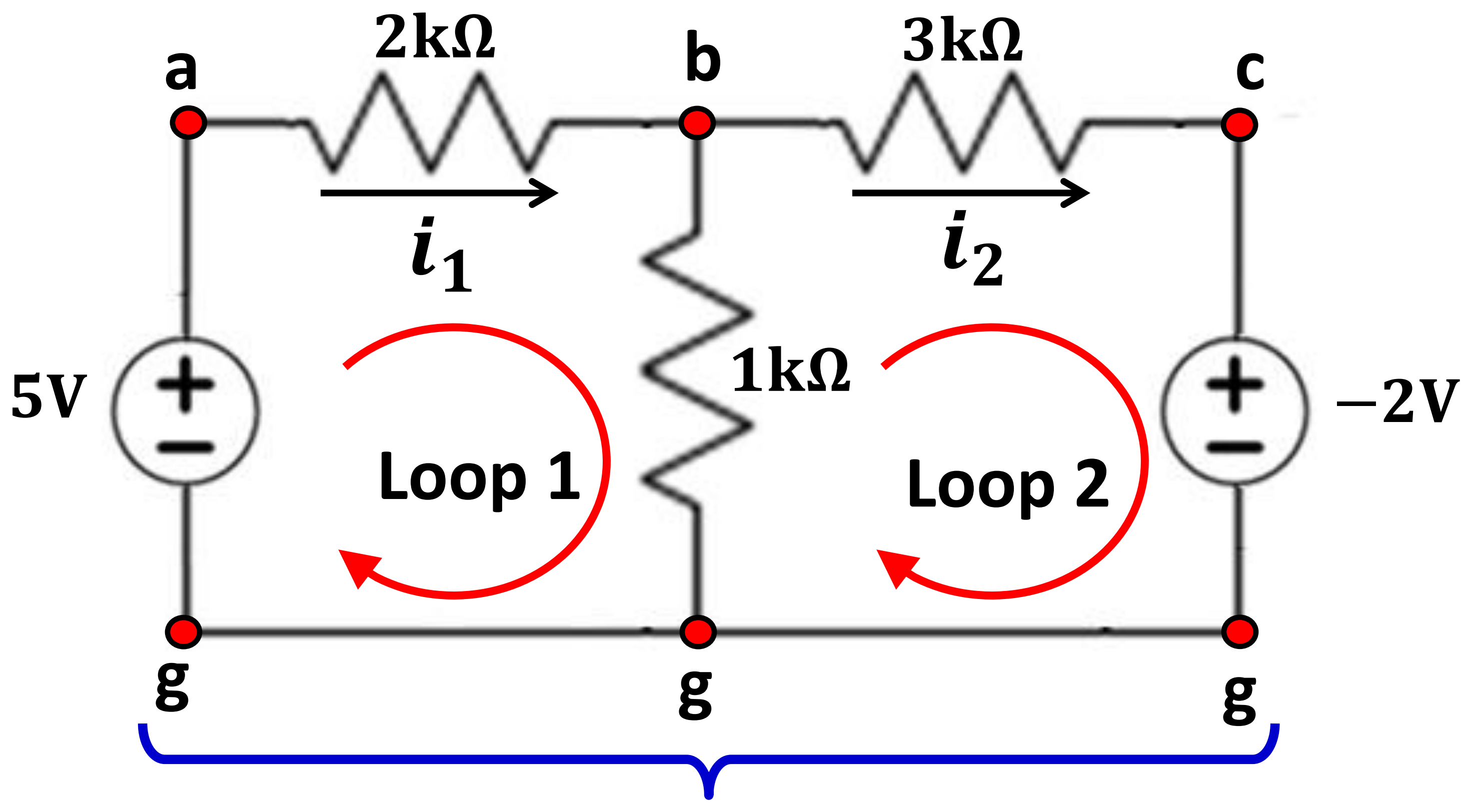
Lecture 5 - Summary

Learning Objectives

1. Use loop analysis method to compute loop currents
2. Understand “Superloops”
3. Solve circuits with current sources
4. Derive voltage division formula and analyze the limitations of voltage divider

Example 2 – Two loops

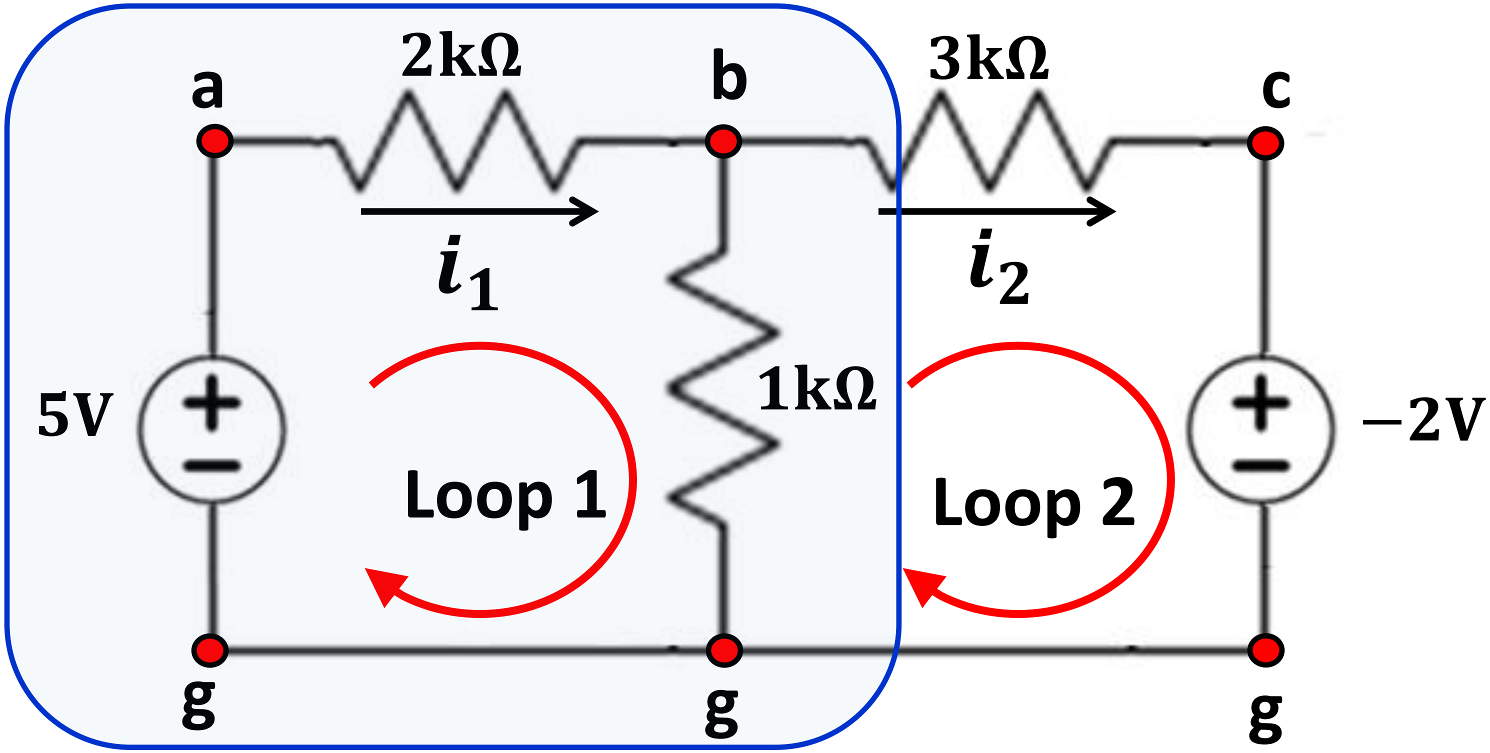
Obtain the unknown currents i_1 and i_2



Let's designate these nodes as "ground" g since they are at the same potential. We will not include in equations the potential between these nodes, because = 0.

Example 2 – Two loops

Obtain the unknown currents i_1 and i_2



LOOP #1

KVL) $V_{ab} + V_{bg} + V_{ga} = 0$

$i_{ab} = i_1$

$i_{bg} = i_1 - i_2$

Ohm's Law:

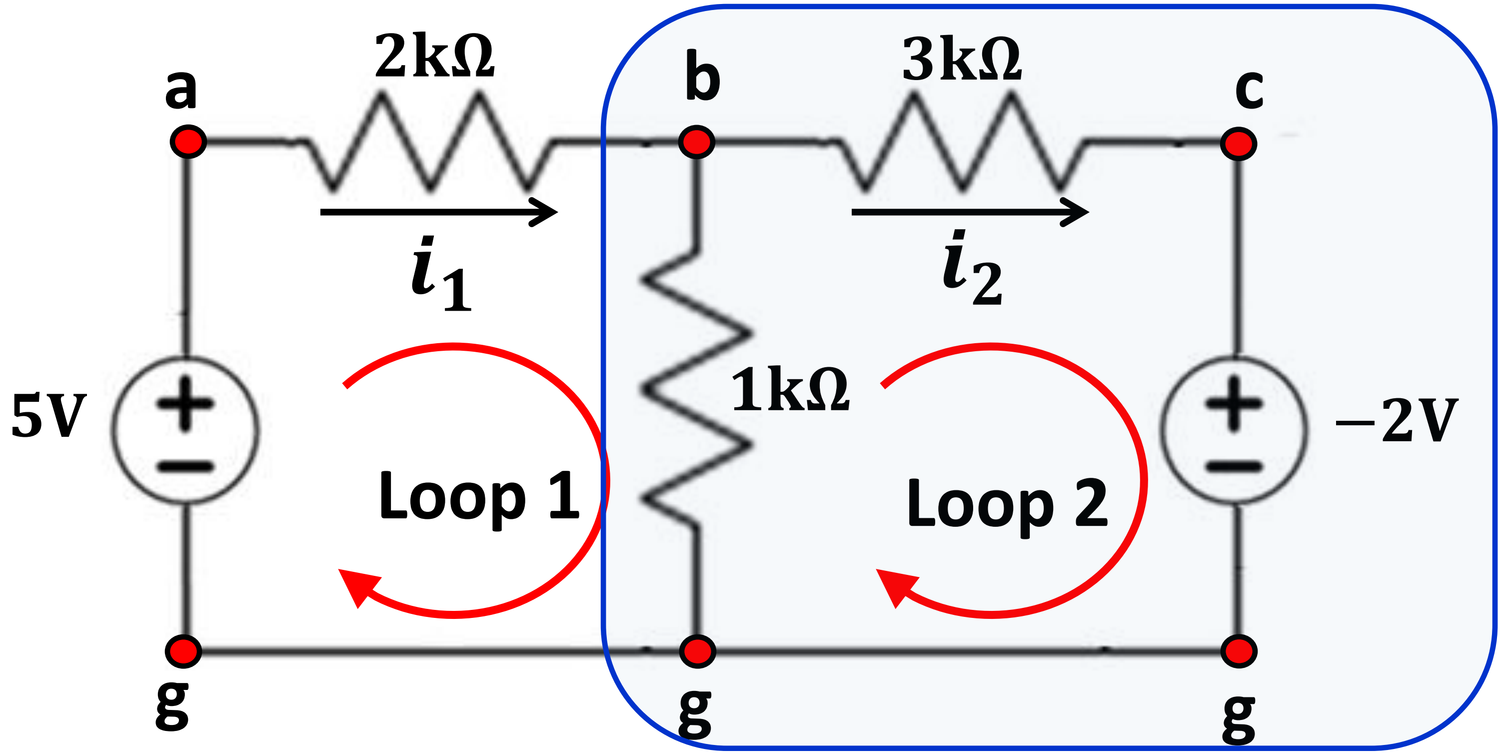
$$2\text{k}\, i_{ab} + 1\text{k}\, i_{bg} - 5 = 0$$

$$2\text{k}\, i_1 + 1\text{k}(i_1 - i_2) - 5 = 0$$

$$3\text{k}\, i_1 - 1\text{k}\, i_2 = 5$$ Eq. (1)

Example 2 – Two loops

Obtain the unknown currents i_1 and i_2



LOOP #2

KVL) $V_{bc} + V_{cg} + V_{gb} = 0$

$i_{ab} = i_1$

$i_{bg} = i_1 - i_2$

Ohm's Law:

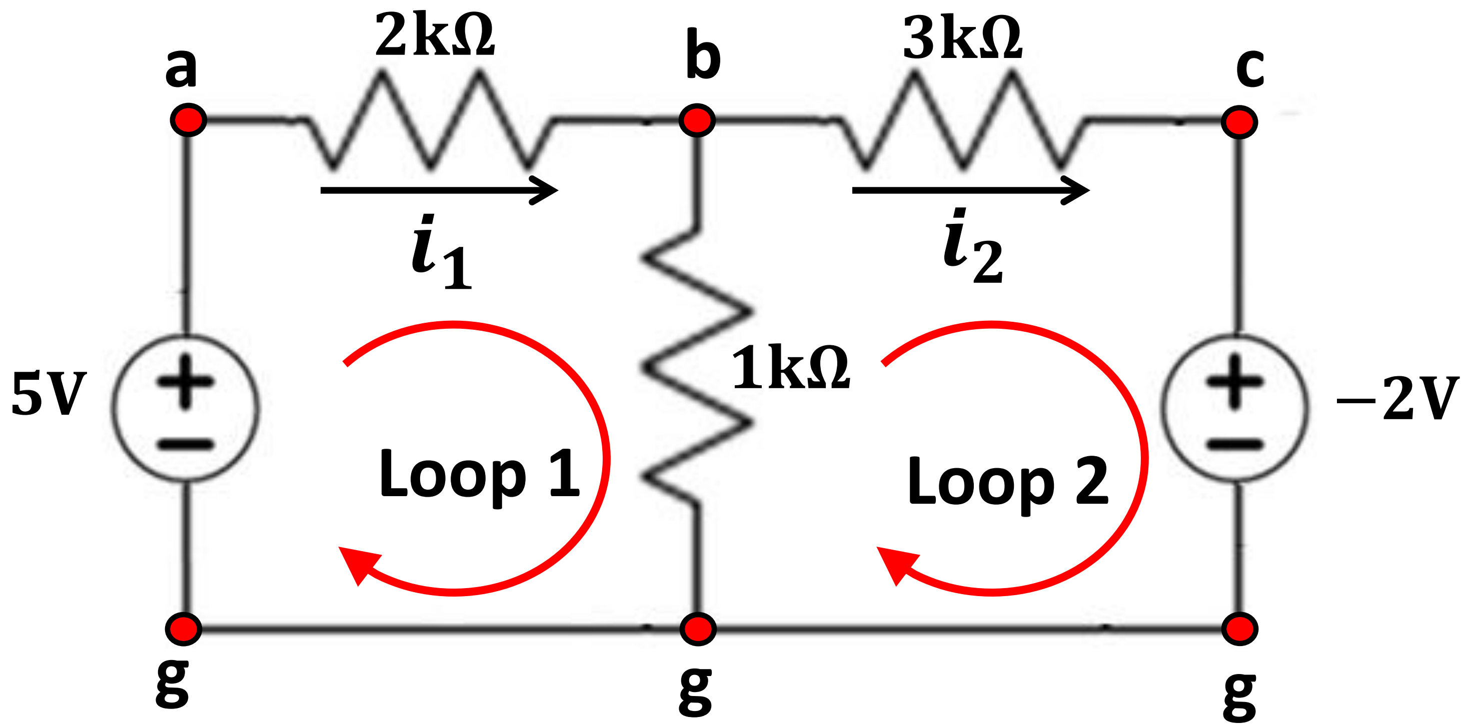
$$3\text{k}\, i_{bc} - 2 + 1\text{k}\, i_{gb} = 0$$

$$3\text{k}\, i_2 - 2 + 1\text{k}\, (i_2 - i_1) = 0$$

$$-1\text{k}\, i_1 + 4\text{k}\, i_2 = 2 \qquad \textbf{Eq. (2)}$$

Example 2 – Two loops

Obtain the unknown currents i_1 and i_2



Solve system of equations

$$3\text{k}i_1 - \text{k}i_2 = 5$$

$$-1\text{k}\, i_1 + 4\text{k}\, i_2 = 2$$

$$\begin{bmatrix} 3\text{k} & -1\text{k} \\ -1\text{k} & 4\text{k} \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 3\text{k} & -1\text{k} \\ -1\text{k} & 4\text{k} \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix} \text{mA}$$

Inverse of 2×2 matrix

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

$$A^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

Inverse of A

Determinant of A

Adjoint of A

Example 2 – Two loops

The simple system of equations can be solved by substitution

$$\boxed{3\text{k}\, i_1 - 1\text{k}\, i_2 = 5}$$

$$\boxed{-1\text{k}\, i_1 + 4\text{k}\, i_2 = 2}$$

$$i_2 = 3i_1 - 5/\text{k}$$

$$-i_1 + 4(3i_1 - 5/\text{k}) = 2/\text{k}$$

$$11i_1 - 20/\text{k} = 2/\text{k}$$

$$11i_1 = 22/\text{k}$$

$$\boxed{i_1 = 2\ \text{mA}}$$

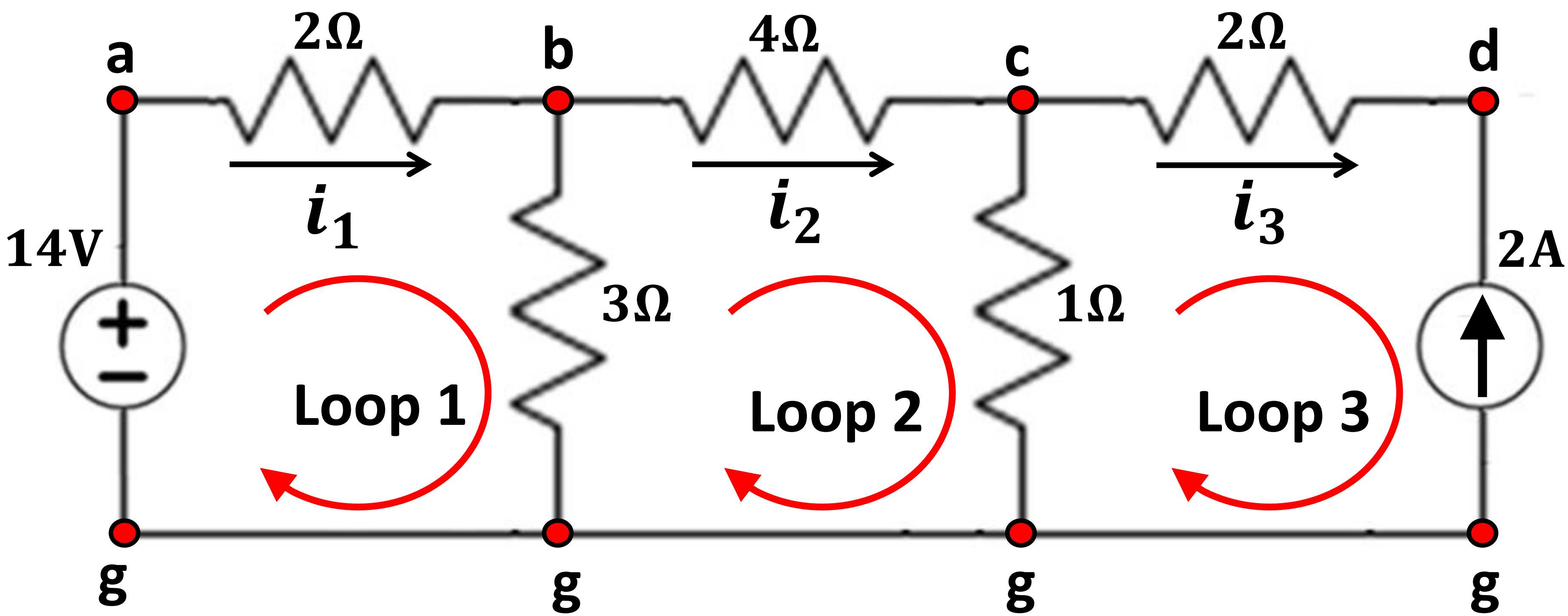
$$i_2 = 3 \times 2/\text{k} - 5/\text{k}$$

$$i_2 = 6/\text{k} - 5/\text{k}$$

$$\boxed{i_2 = 1\ \text{mA}}$$

Example 3 – Three loops

Obtain the unknown currents i_1, i_2 and i_3



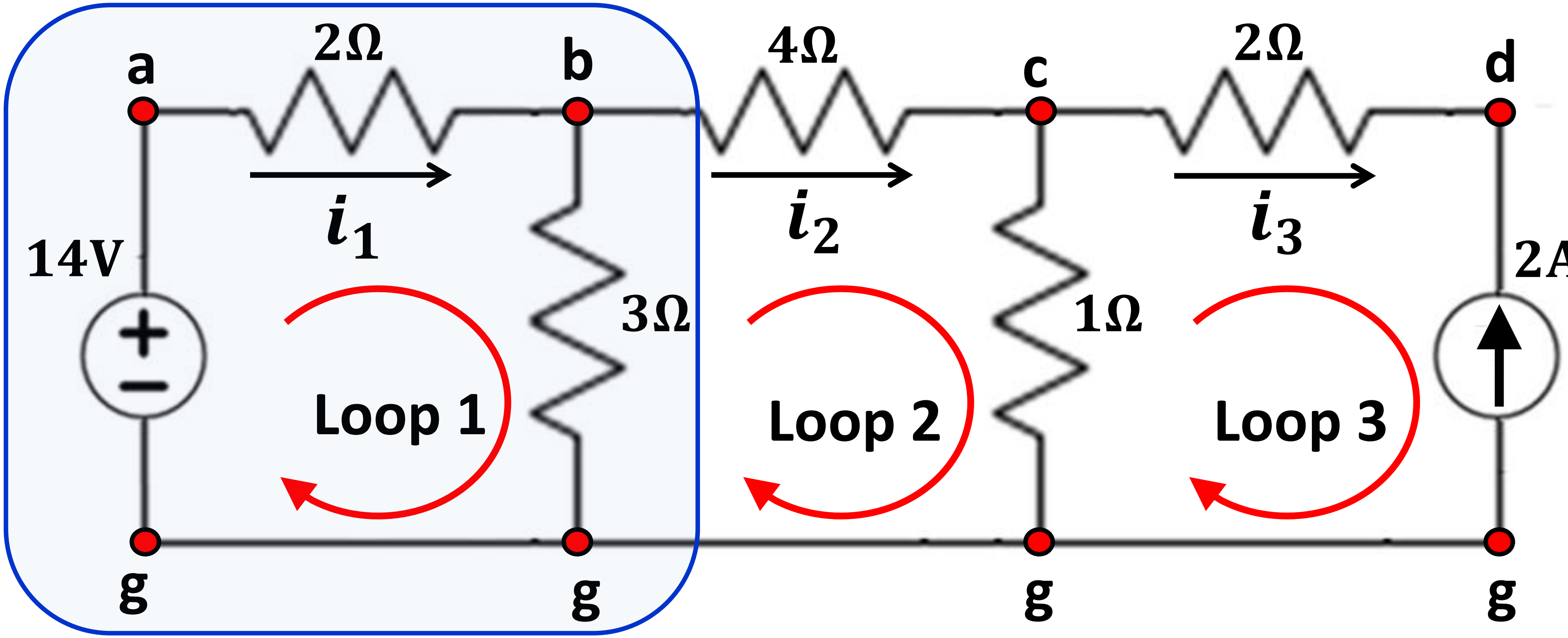
By inspection of Loop 3: $\boxed{i_3 = -2\text{ A}}$

NOTE: Loop 3 has a current source. It is not possible to write a loop equation for it because the voltage V_{dg} depends on the rest of the circuit.

Example 3 – Three loops

$i_3 = -2\ \text{A}$

Obtain the unknown currents i_1, i_2 and i_3



LOOP #1

KVL) $V_{ab} + V_{bg} + V_{ga} = 0$

$i_{ab} = i_1$

$i_{bg} = i_1 - i_2$

Ohm's Law:

$$2\ i_{ab} + 3\ i_{bg} - 14 = 0$$

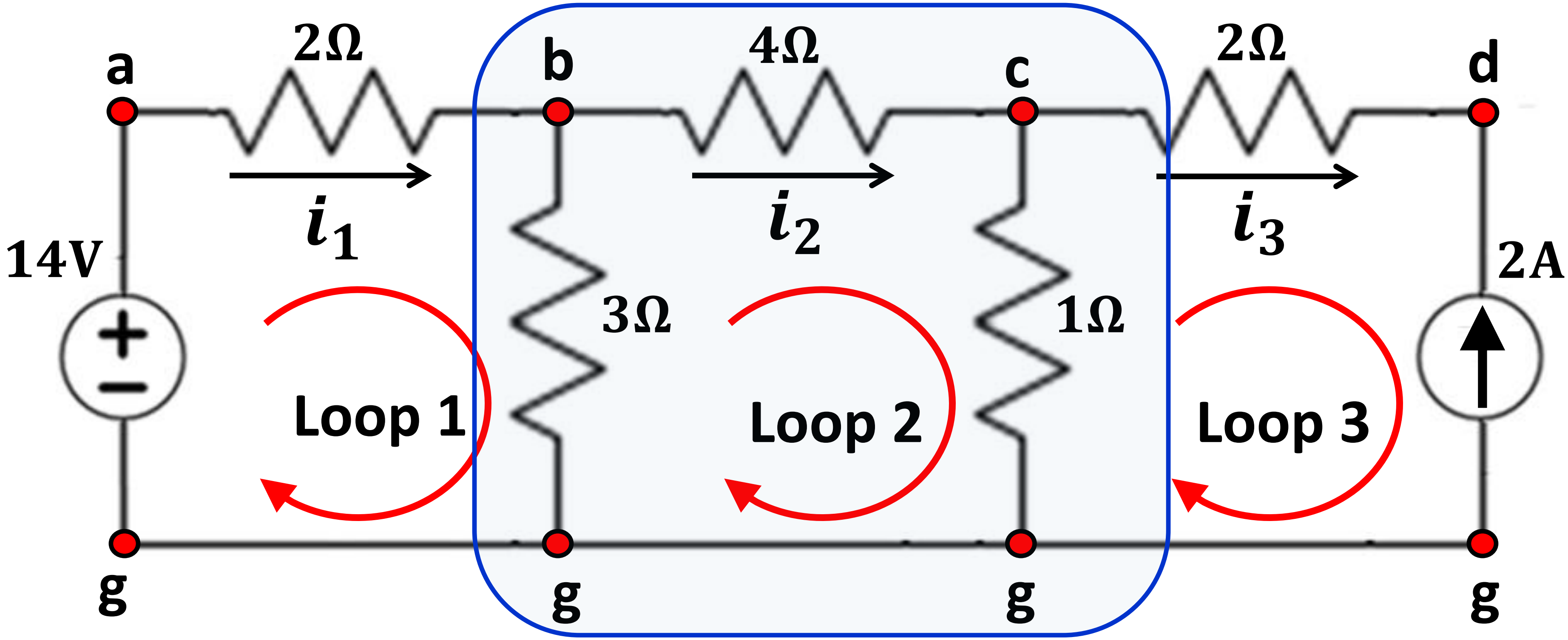
$$2\ i_1 + 3(i_1 - i_2) - 14 = 0$$

$$5\ i_1 - 3\ i_2 = 14$$ Eq. (1)

Example 3 – Three loops

$i_3 = -2\text{ A}$

Obtain the unknown currents i_1, i_2 and i_3



LOOP #2

KVL) $V_{bc} + V_{cg} + V_{gb} = 0$

Ohm's Law:

$$4\, i_{bc} + 1\, i_{cg} + 3\, i_{gb} = 0$$

$$i_{cg} = i_2 - i_3$$

$$4\, i_2 + 1(i_2 - i_3) + 3(i_2 - i_1) = 0$$

$$i_{bg} = i_1 - i_2$$

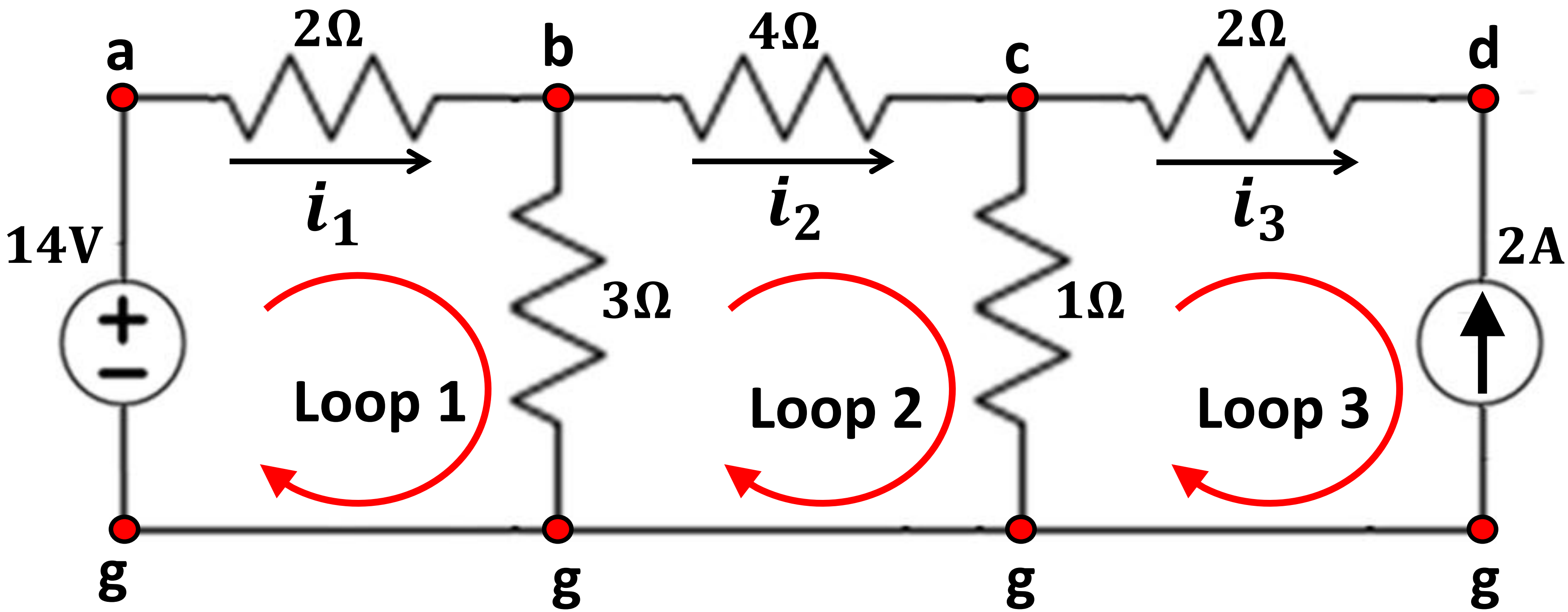
$$3\, i_1 - 8\, i_2 = 2$$

Eq. (2)

Example 3 – Three loops

$$i_3 = -2\ \text{A}$$

Obtain the unknown currents i_1, i_2 and i_3



Solve system of equations

$$5\, i_1 - 3\, i_2 = 14$$

$$3\, i_1 - 8\, i_2 = 2$$

$$\begin{bmatrix} 5 & -3 \\ 3 & -8 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 14 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 5 & -3 \\ 3 & -8 \end{bmatrix}^{-1} \begin{bmatrix} 14 \\ 2 \end{bmatrix} = \begin{bmatrix} 3.42 \\ 1.03 \end{bmatrix} \text{A}$$

Example 3 – Three loops

The simple system of equations can be solved by substitution

$$\boxed{5\,i_1 - 3\,i_2 = 14}$$

$$\boxed{3\,i_1 - 8\,i_2 = 2}$$

$$i_2 = (5i_1 - 14)/3$$

$$3i_1 - 8\,(5i_1 - 14)/3 = 2$$

$$i_1 - (40i_1 - 112)/9 = 2/3$$

$$3.\overline{4}\,i_1 = 11.\overline{7}$$

$$\boxed{i_1 = 3.419\text{ A}}$$

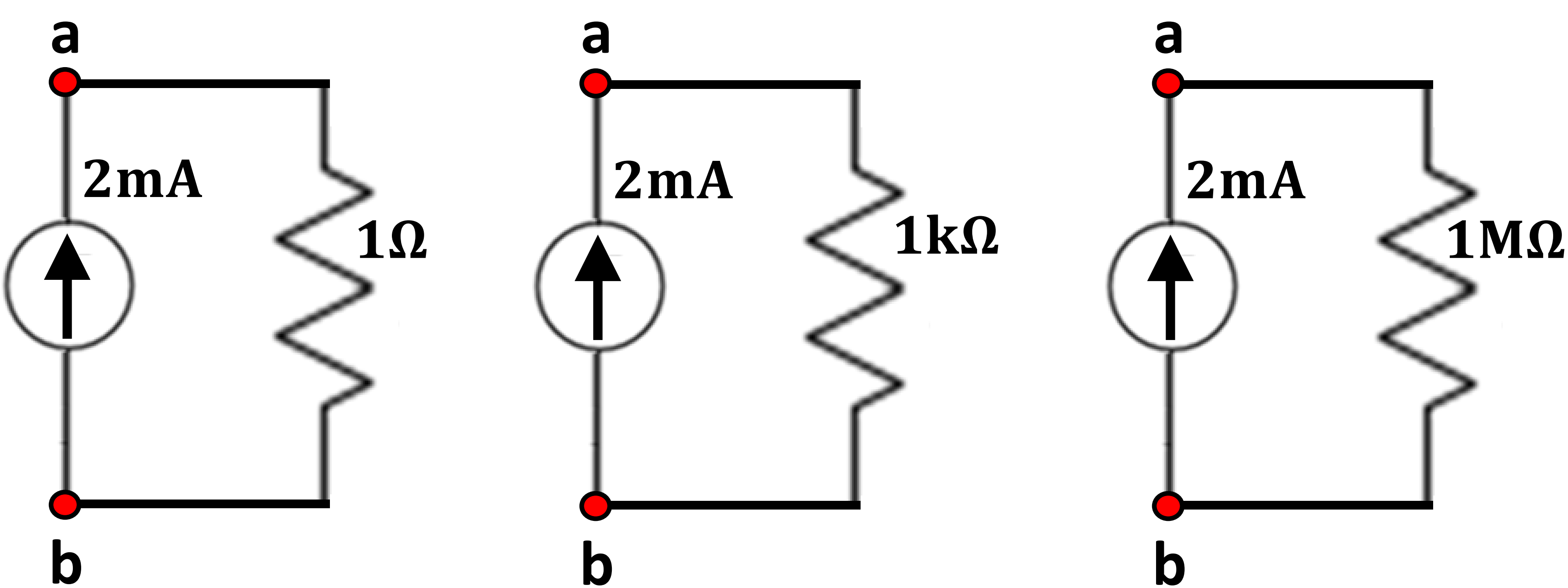
$$i_2 = (5i_1 - 14)/3$$

$$i_2 = 5.6989 - 4.\overline{6}$$

$$\boxed{i_2 = 1.032\text{ A}}$$

Voltage across a current source

The following elementary examples show how the voltage across a current source depends on the circuit connected to it.



$V_{ab} = 2\ \mathrm{mV}$ $V_{ab} = 2\ \mathrm{V}$ $V_{ab} = 2\ \mathrm{kV}$

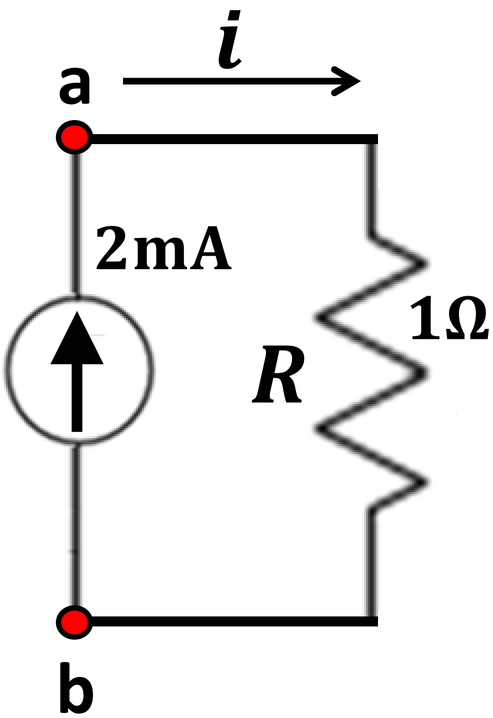
Power absorbed by resistor

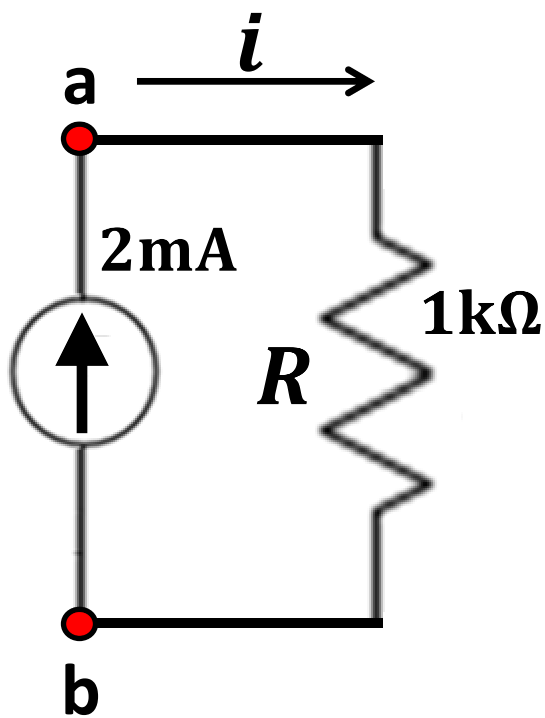
$$P_R = V_{ab} \times i = R \times i^2$$

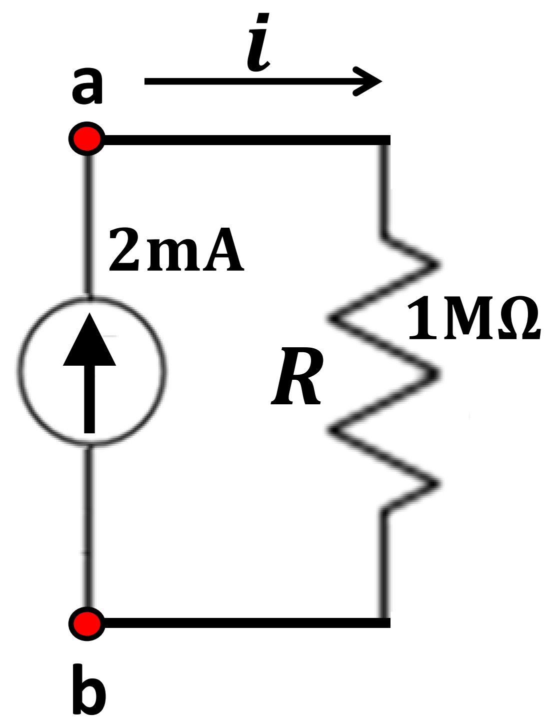
$P_R = 4\mu W$

$P_R = 4 mW$

$P_R = 4 W$







$V_{ab} = 2\ mV$

$V_{ab} = 2\ V$

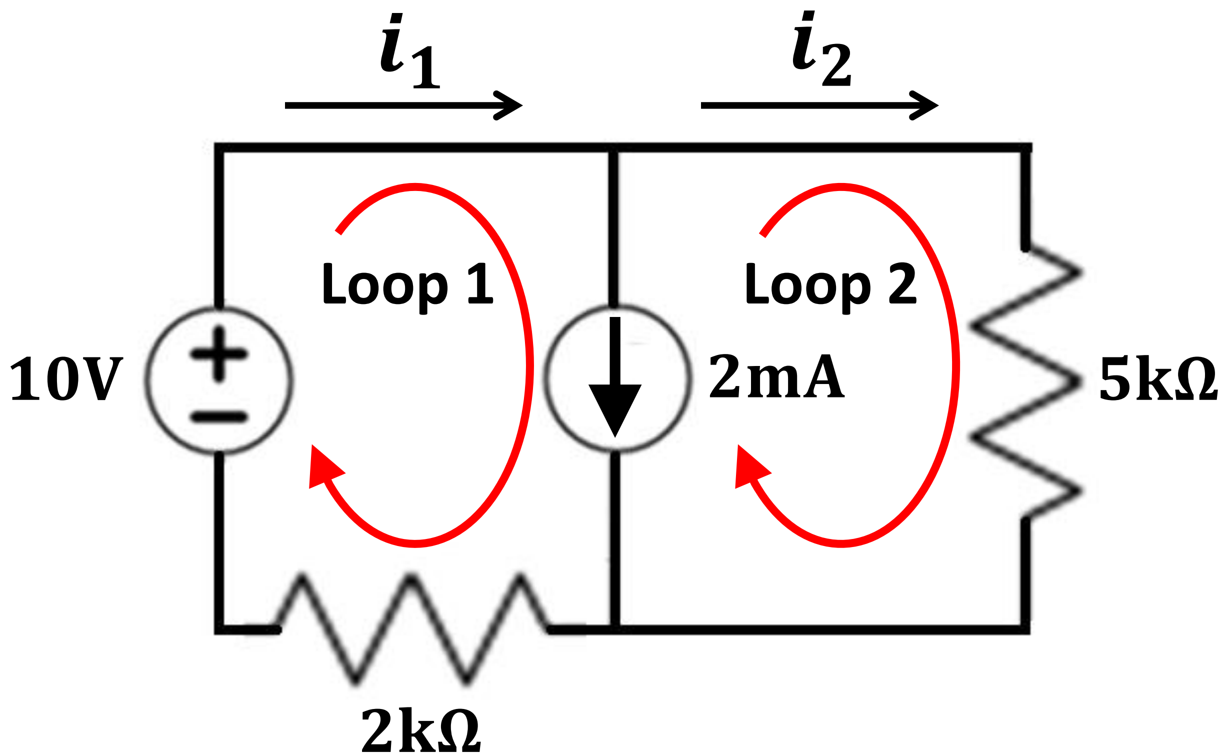
$V_{ab} = 2\ kV$

Superloops

It is not possible to write an equation for a loop with a current source in a branch. The voltage across the current source is not as easily determined as for a resistor or a voltage source.

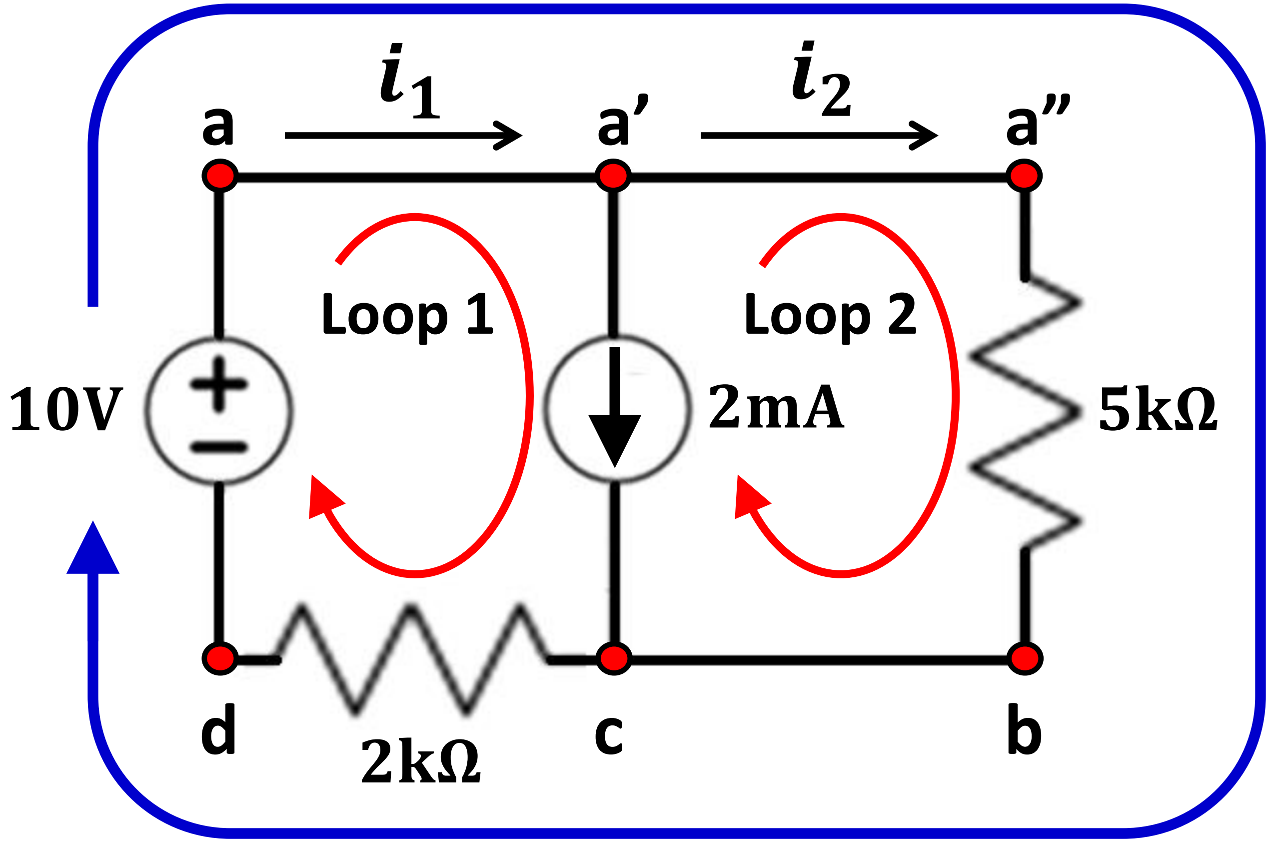
A way to get around this is to use a bigger loop or "superloop" in which the KVL can still be used.

Example – Obtain the unknown currents i_1 and i_2



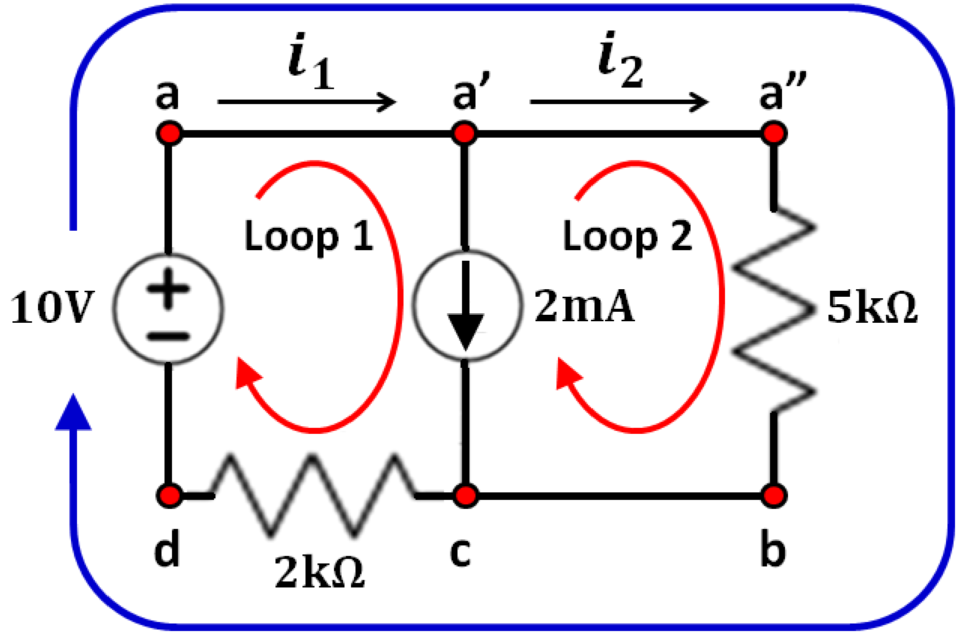
The two loops have a current source in the common branch and loop equations cannot be formulated.

Example – Obtain the unknown currents i_1 and i_2



Define a "superloop". Note that nodes a, a', a" have same potential. Also b and c have same potential.

Example – Obtain the unknown currents i_1 and i_2



Superloop equation — KVL) $V_{ab} + V_{bc} + V_{cd} + V_{da} = 0$

Ohm's Law

$$5\text{k}\, i_2 + 2\text{k}\, i_1 = 10$$

$$\boxed{2\text{k}\, i_1 + 5\text{k}\, i_2 = 10} \quad \text{Eq. (1)}$$

Currents at node a'

$$\boxed{i_1 - i_2 = 2\text{ mA}} \quad \text{Eq. (2)}$$

Example – Obtain the unknown currents i_1 and i_2

Solve the equations

$$\boxed{2\text{k}\, i_1 + 5\text{k}\, i_2 = 10}$$

$$\boxed{i_1 - i_2 = 2\ \text{mA}}$$

$$\begin{bmatrix} 2\text{k} & 5\text{k} \\ 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 2\text{m} \end{bmatrix}$$

Simplify to

$$\begin{bmatrix} 2 & 5 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 10\text{m} \\ 2\text{m} \end{bmatrix}$$

Final result

$$\begin{bmatrix} i_1 \\ i_2 \end{bmatrix} = \begin{bmatrix} 2.857 \\ 0.857 \end{bmatrix} \text{mA}$$

Example – Obtain the unknown currents i_1 and i_2

Solve the equations by substitution

$$2\text{k}\, i_1 + 5\text{k}\, i_2 = 10$$

$$i_1 - i_2 = 2\text{ mA}$$

$$i_2 = i_1 - 2\text{ mA}$$

$$2\text{k}\, i_1 + 5\text{k}\,(i_1 - 2\text{m}) = 10$$

$$2\text{k}\, i_1 + 5\text{k} i_1 - 5\text{k}\, 2\text{m} = 10$$

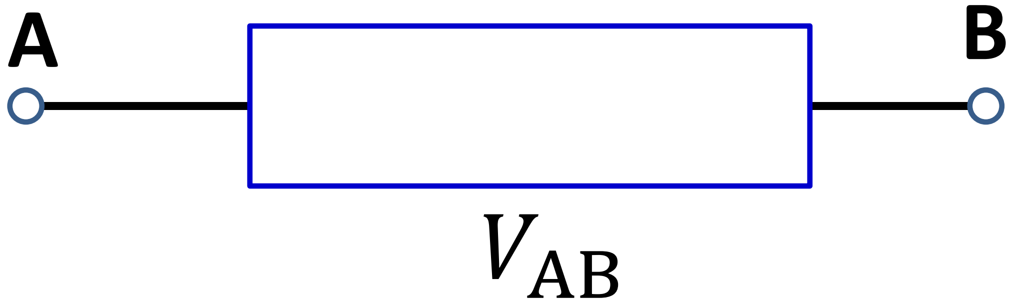
$$7\text{k}\, i_1 = 20$$

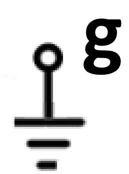
$$i_1 = \frac{20}{7}\text{mA} = 2.857\text{ mA}$$

$$i_2 = (2.857 - 2)\text{mA} = 0.857\text{ mA}$$

Alternative notation for Voltage

So far, we have used the notation below to represent the voltage between two points A and B.

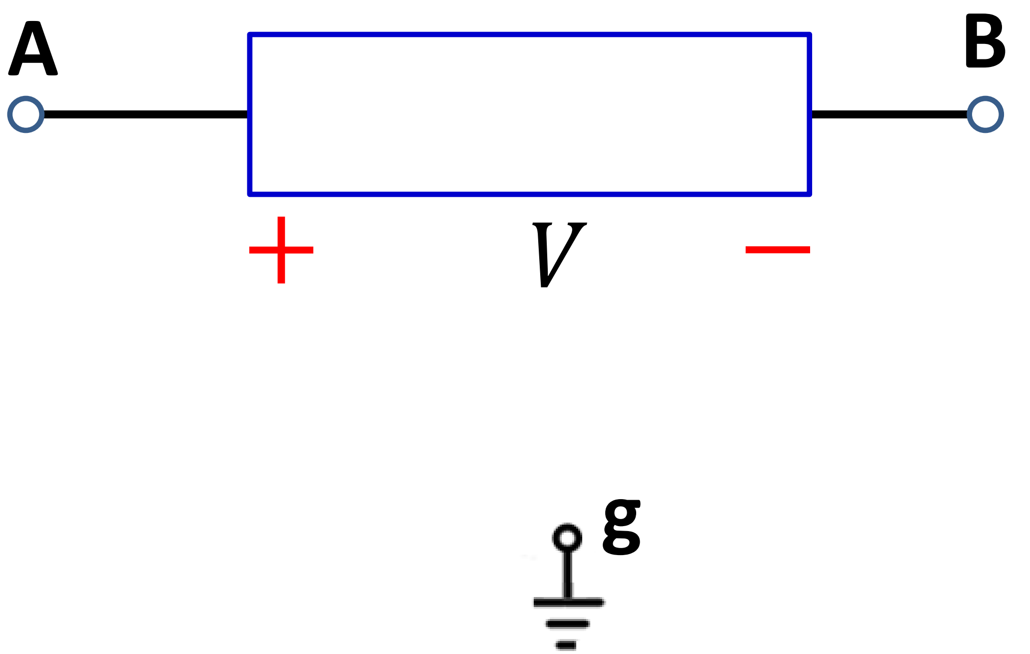




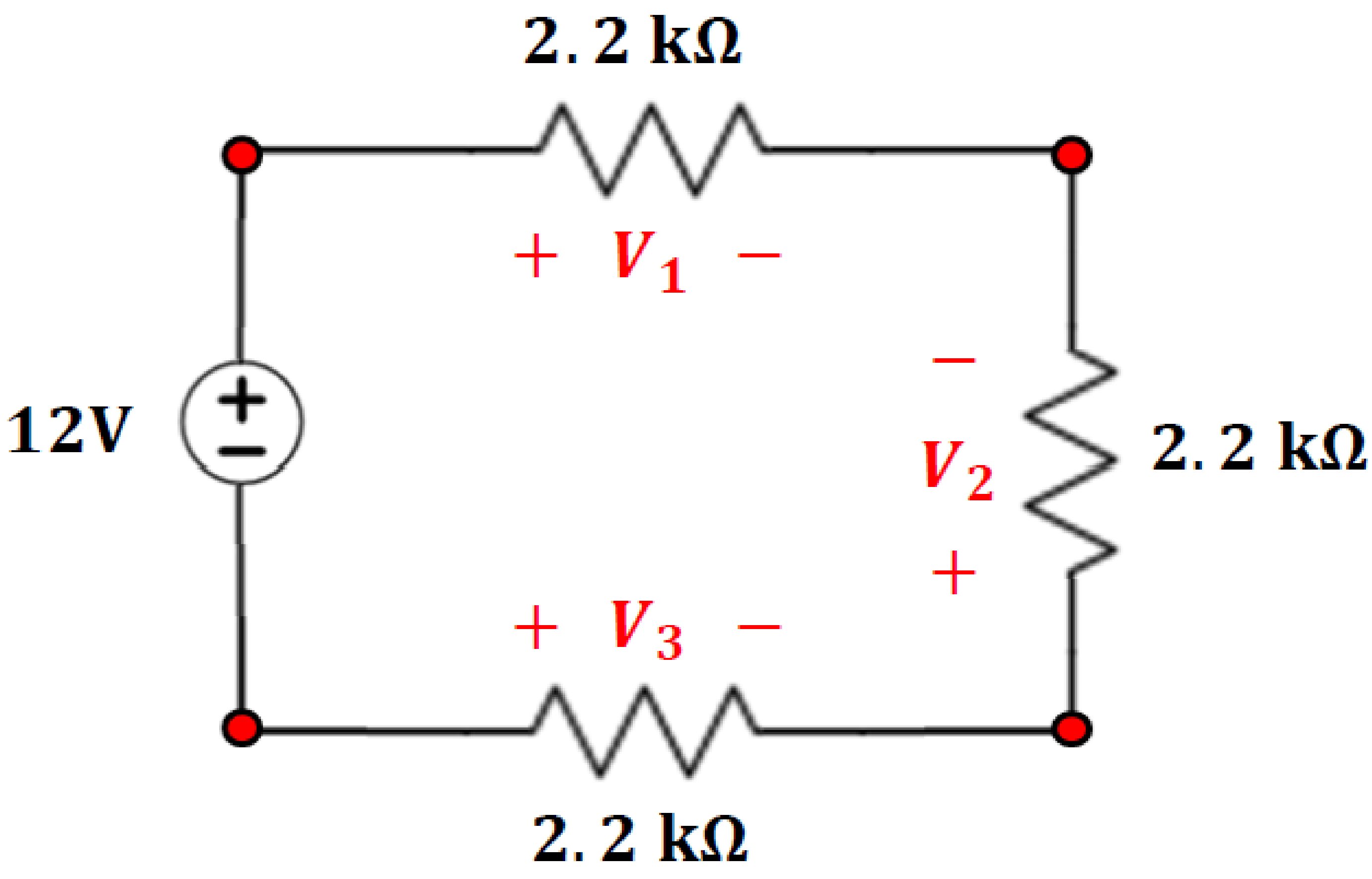
Alternative notation for Voltage

Another very common representation indicates the reference positive and negative potential location.

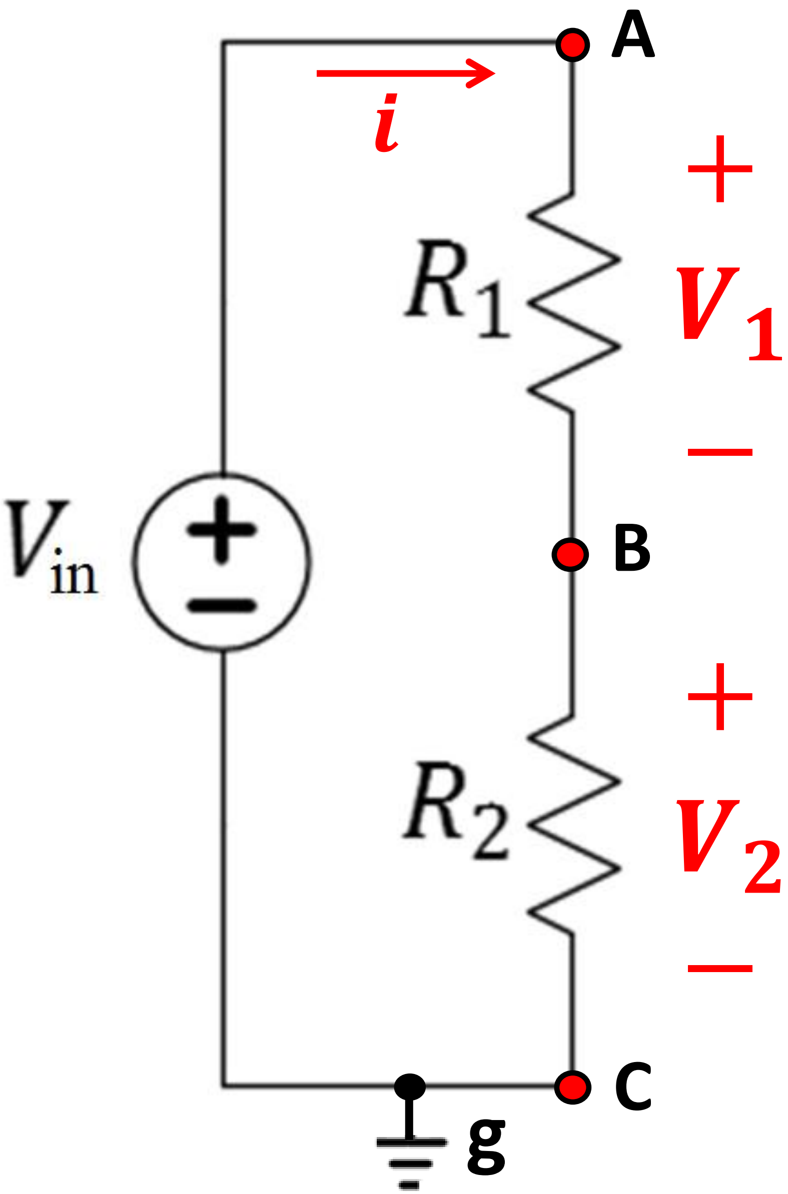
$$V = V_{\mathrm{AB}} = V_{\mathrm{A}} - V_{\mathrm{B}}$$



Example – Find Voltages V_1, V_2, V_3



Voltage Divider



$$V_1 = V_{AB} \quad \& \quad V_2 = V_{BC}$$

$$i = \frac{V_{in}}{R_1 + R_2}$$

$$V_{AB} = i_{AB} R_1 = i R_1$$

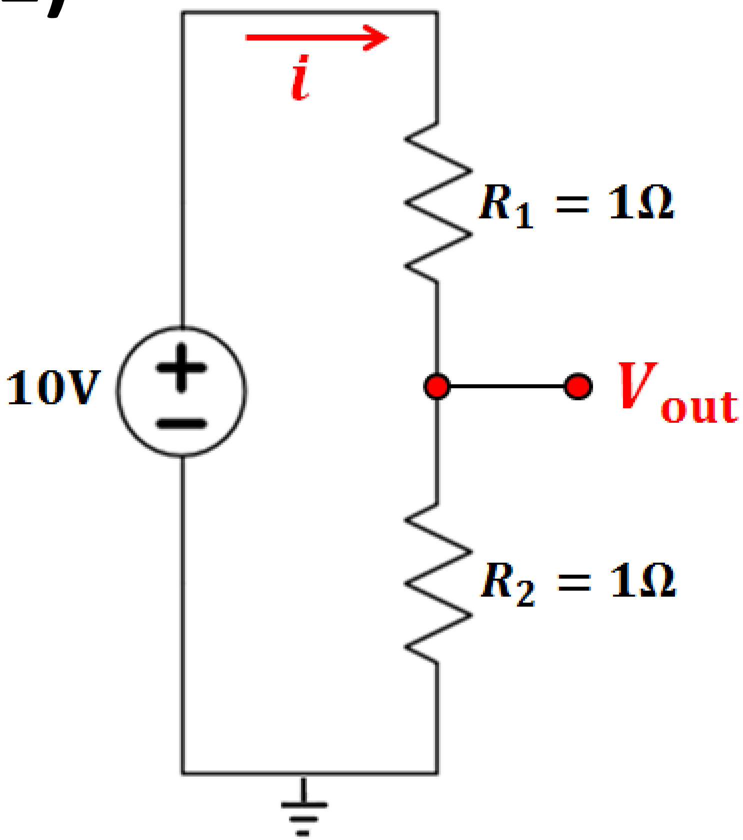
$$V_1 = \frac{V_{in} R_1}{R_1 + R_2}$$

$$V_{BC} = i_{BC} R_2 = i R_2$$

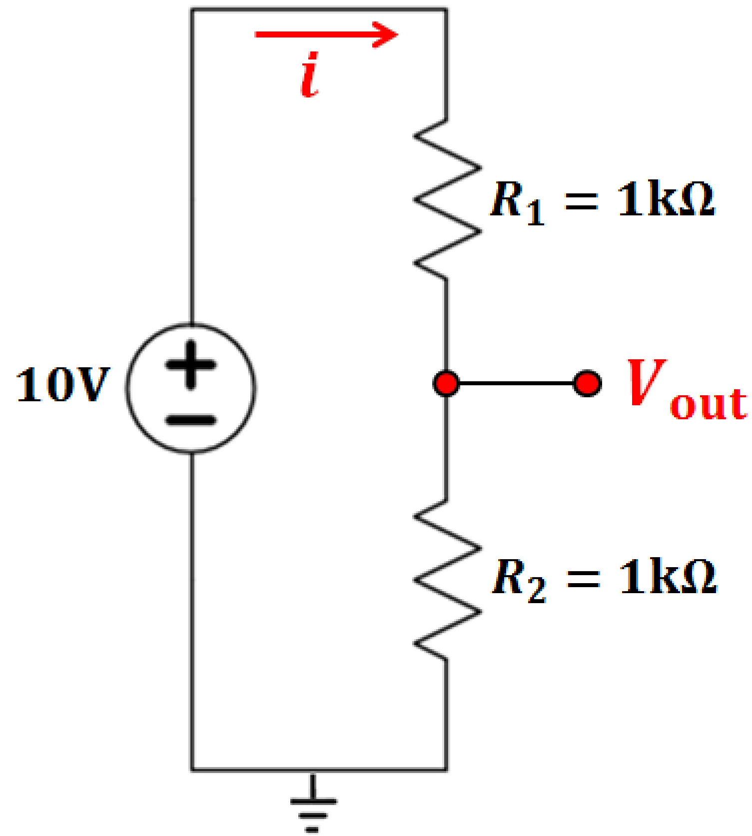
$$V_2 = \frac{V_{in} R_2}{R_1 + R_2}$$

Example – Compute V_{out} for these two cases

(1)

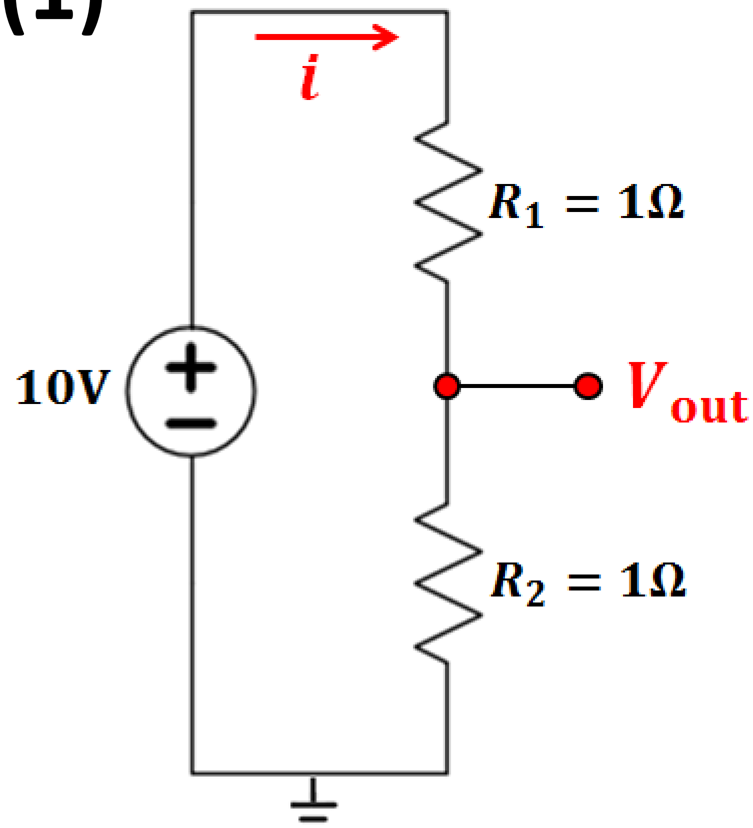


(2)



Example – Compute Voltage V_{out}

(1)



$$V_{out} = \frac{V_{in} R_2}{R_1 + R_2}$$

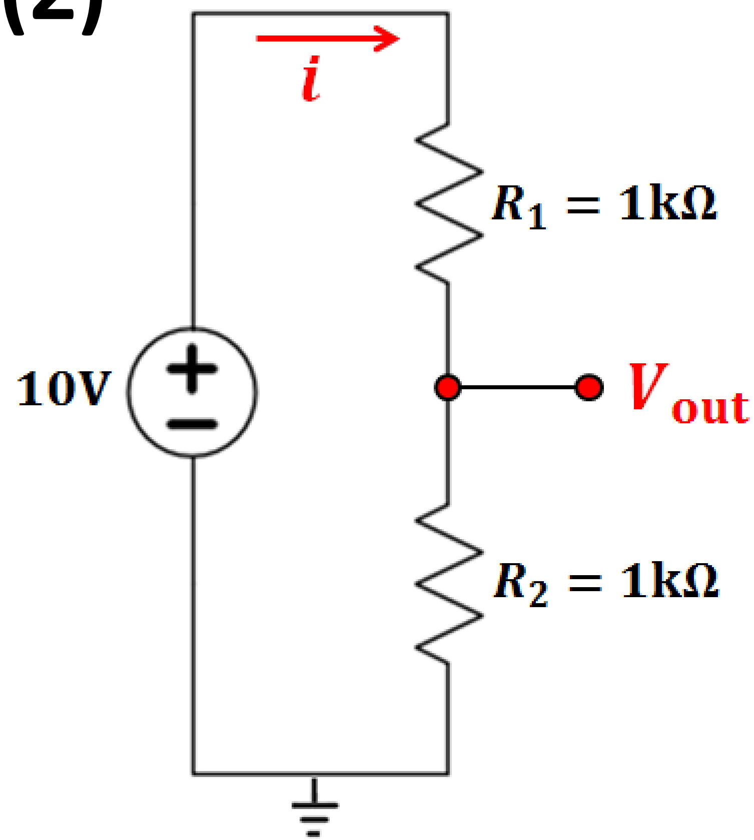
$$V_{out} = \frac{10 \times 1}{1 + 1} = 5\text{V}$$

$$i = \frac{V_{in}}{R_1 + R_2}$$

$$i = \frac{10}{1 + 1} = 5\text{ A}$$

Example – Compute Voltage V_{out}

(2)



$$V_{out} = \frac{V_{in} R_2}{R_1 + R_2}$$

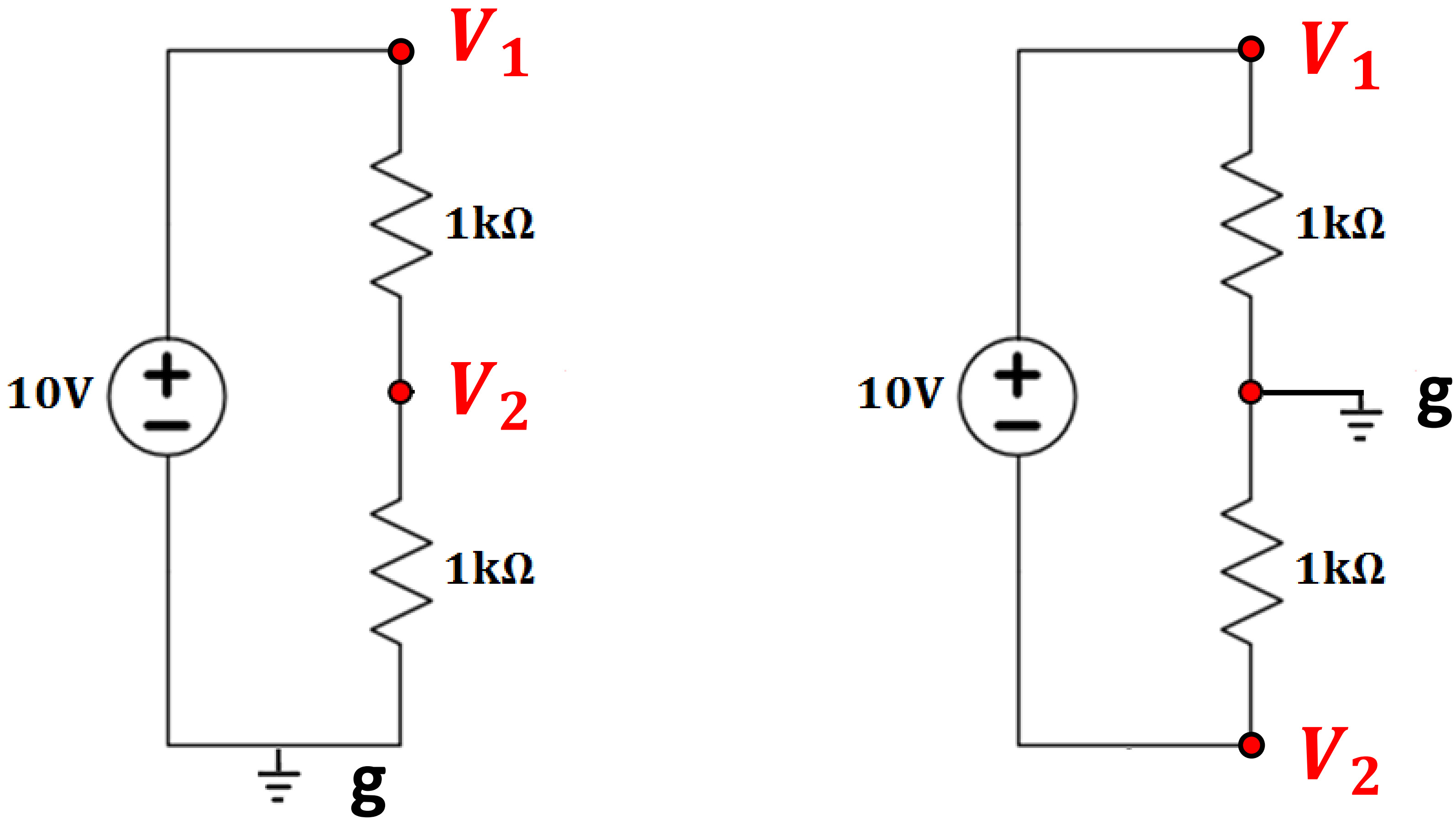
$$V_{out} = \frac{10 \times 1\text{k}}{1\text{k} + 1\text{k}} = 5\text{V}$$

$$i = \frac{V_{in}}{R_1 + R_2}$$

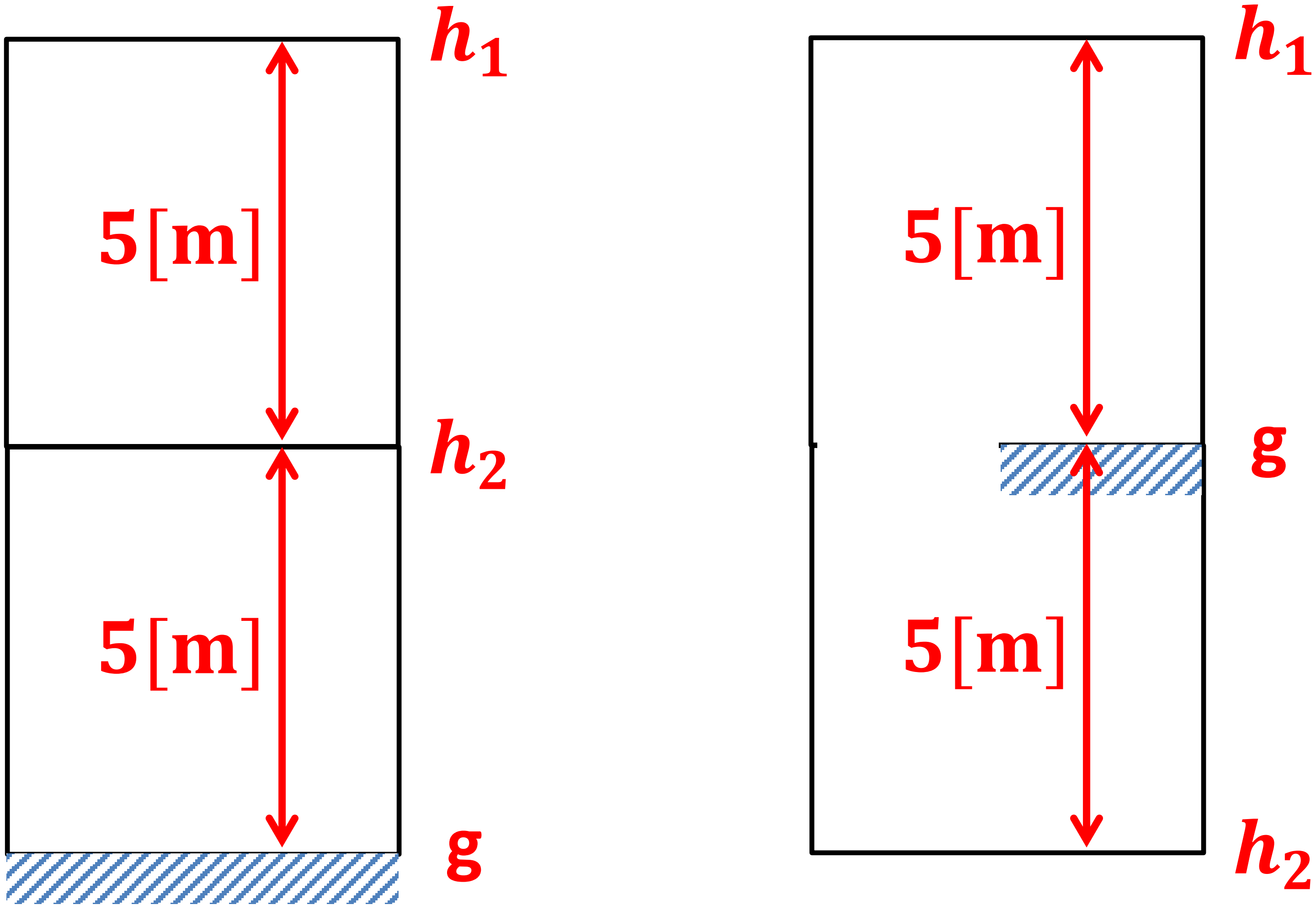
$$i = \frac{10}{1\text{k} + 1\text{k}} = 5\text{ mA}$$

Question: Voltage results are identical. Which of the two realizations do you prefer?

Example – Find V_1 and V_2 for these two cases



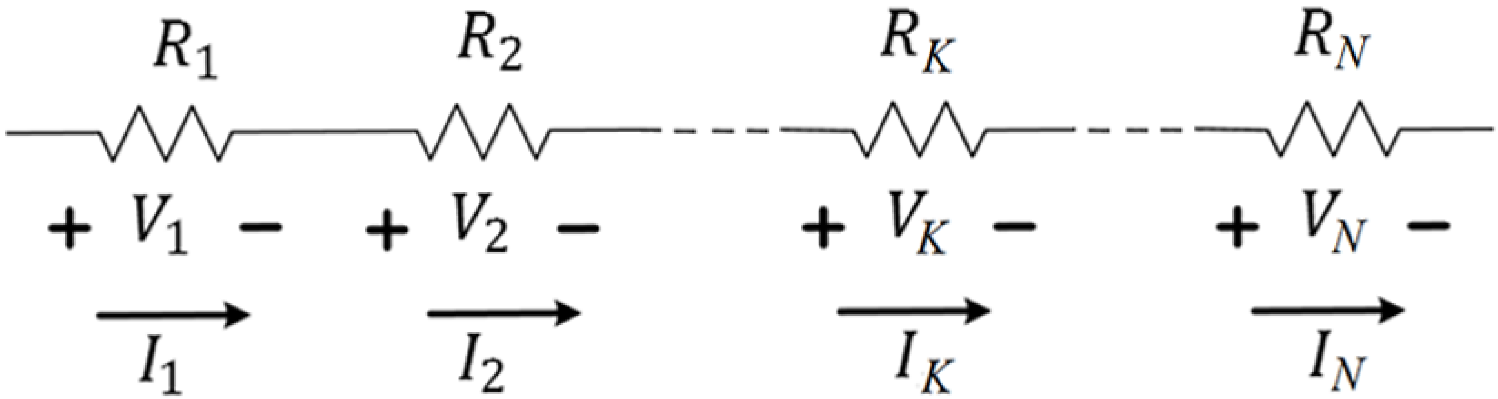
Mechanical analogy



Multiple resistors

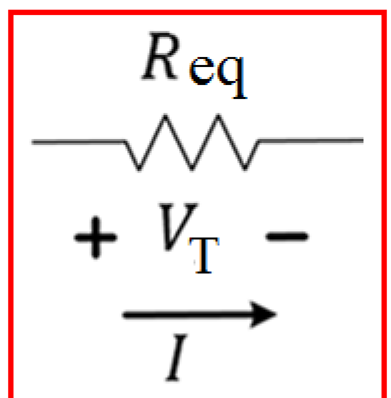
$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

$$I_K = I$$



$$V_T = \sum_K V_K$$

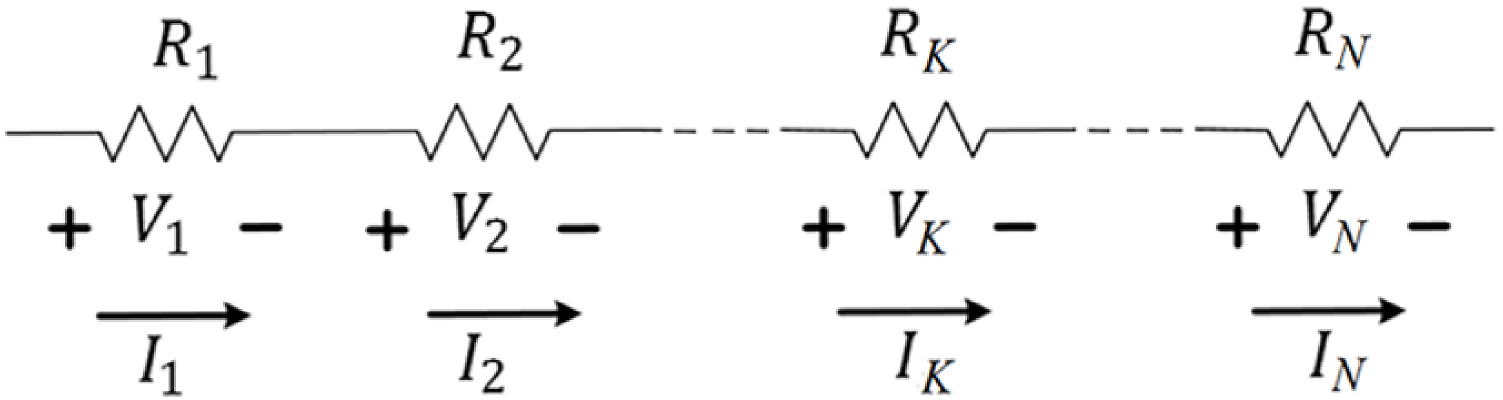
Equivalent to =



Multiple resistors

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

$$I_K = I$$



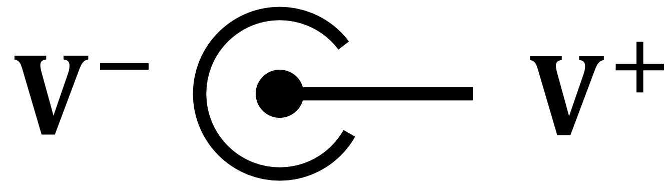
Across each resistor there is a voltage drop

$$V_k = \frac{R_k}{R_{eq}} \cdot V_T$$

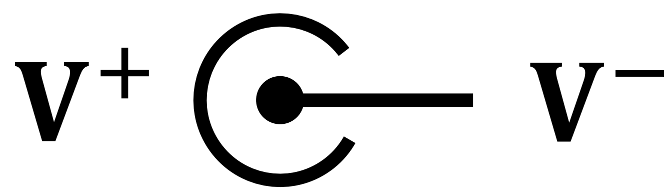
The larger the resistor the larger the drop.

Have you ever noticed the polarity markings on d.c. power supplies?

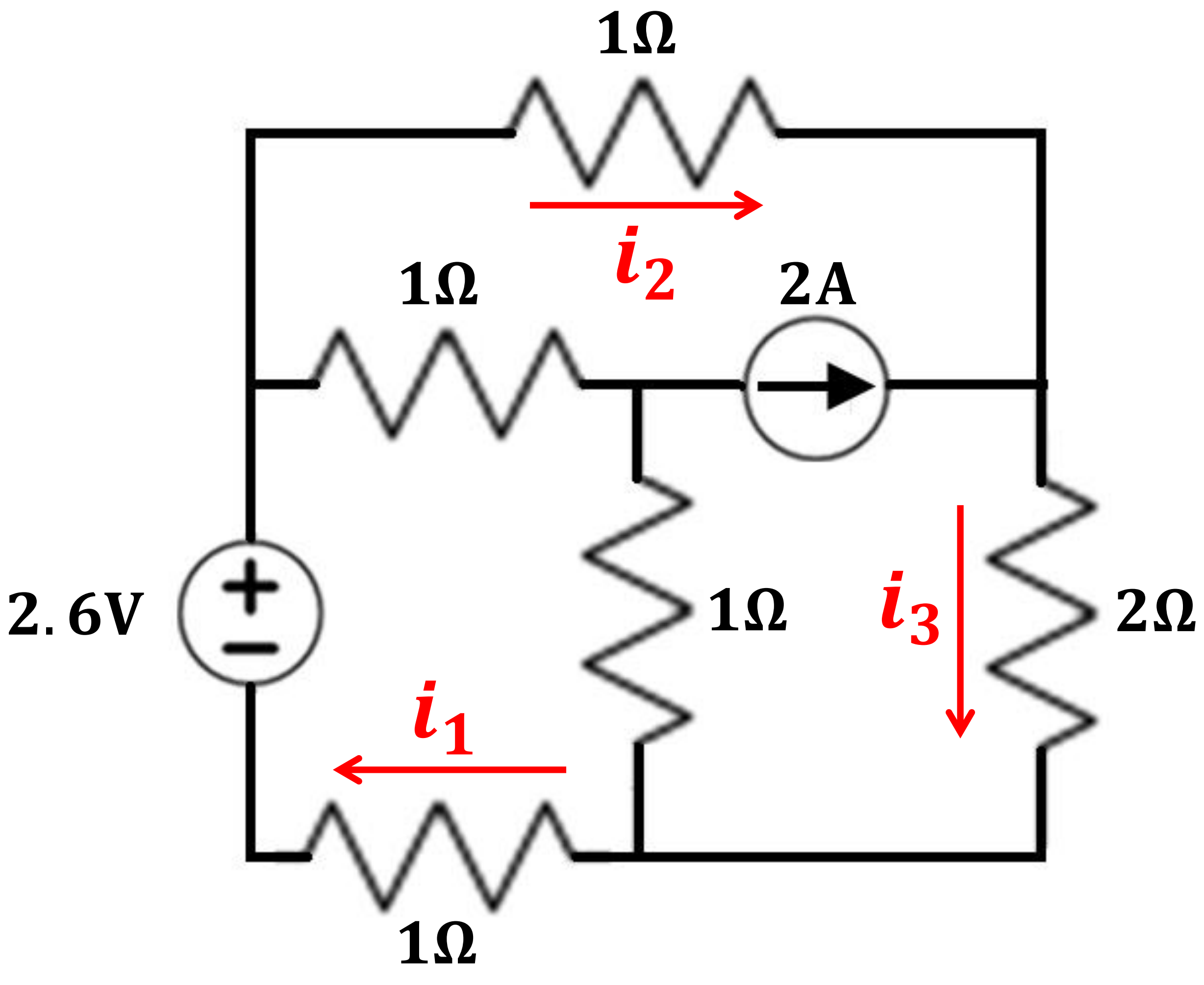
Ground reference
is negative terminal



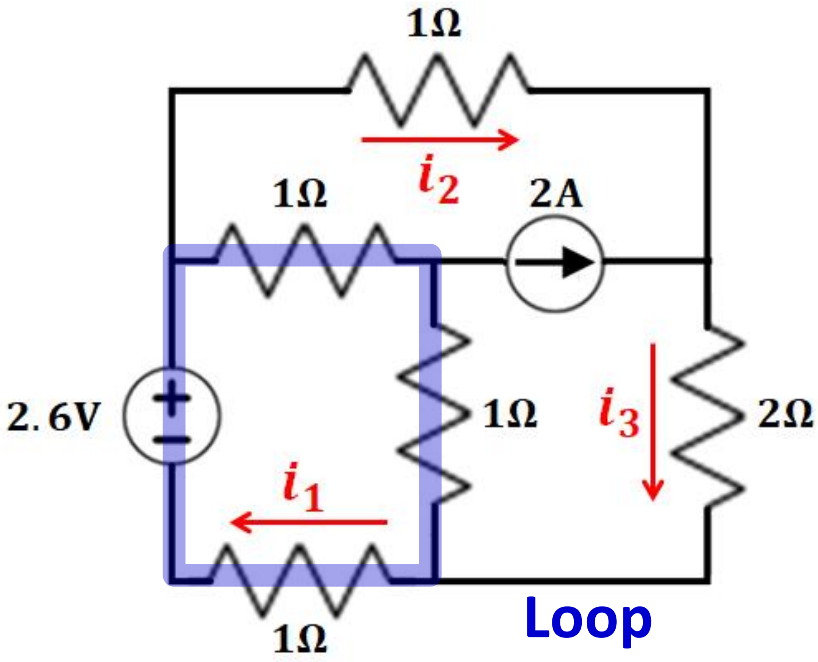
Ground reference
is positive terminal

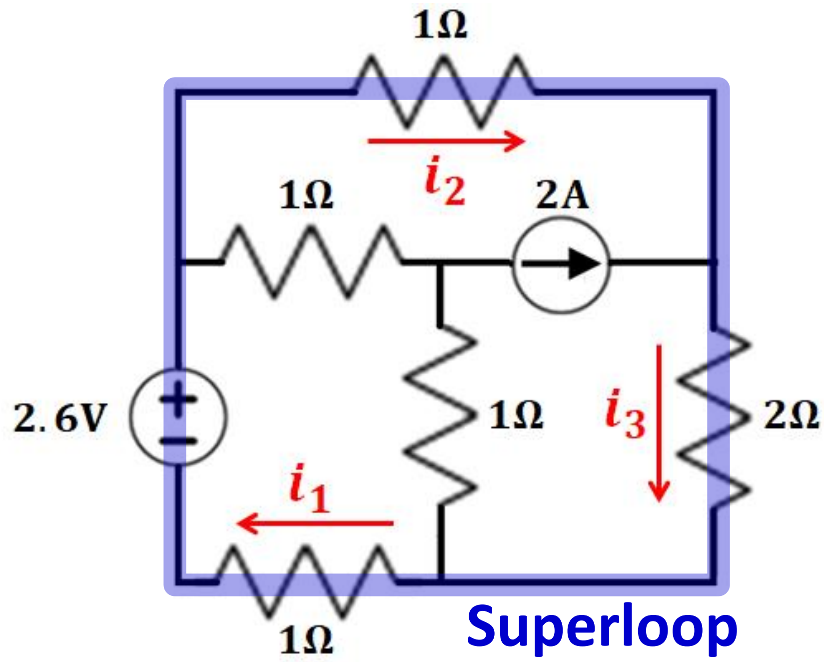


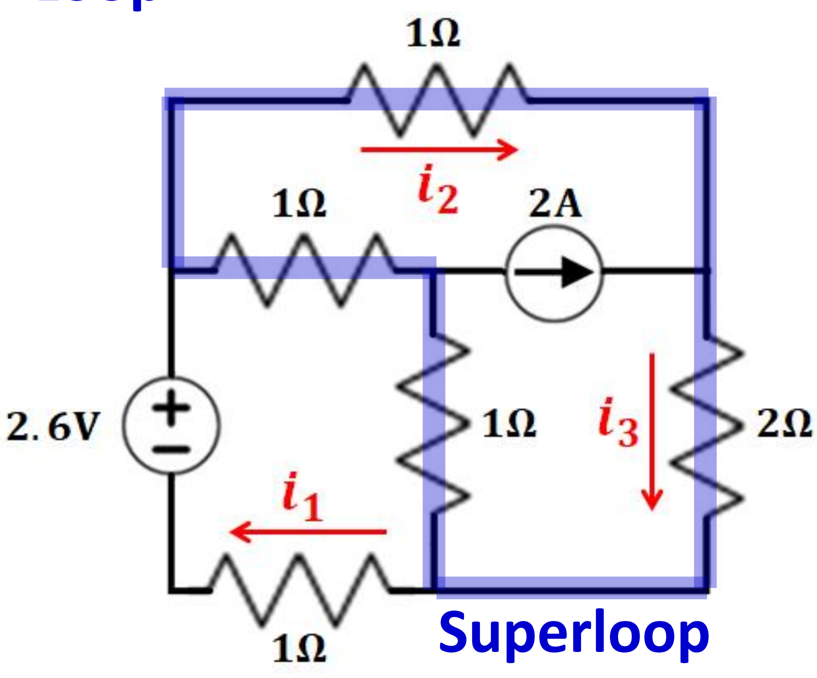
Example – Obtain the unknown currents i_1, i_2, i_3

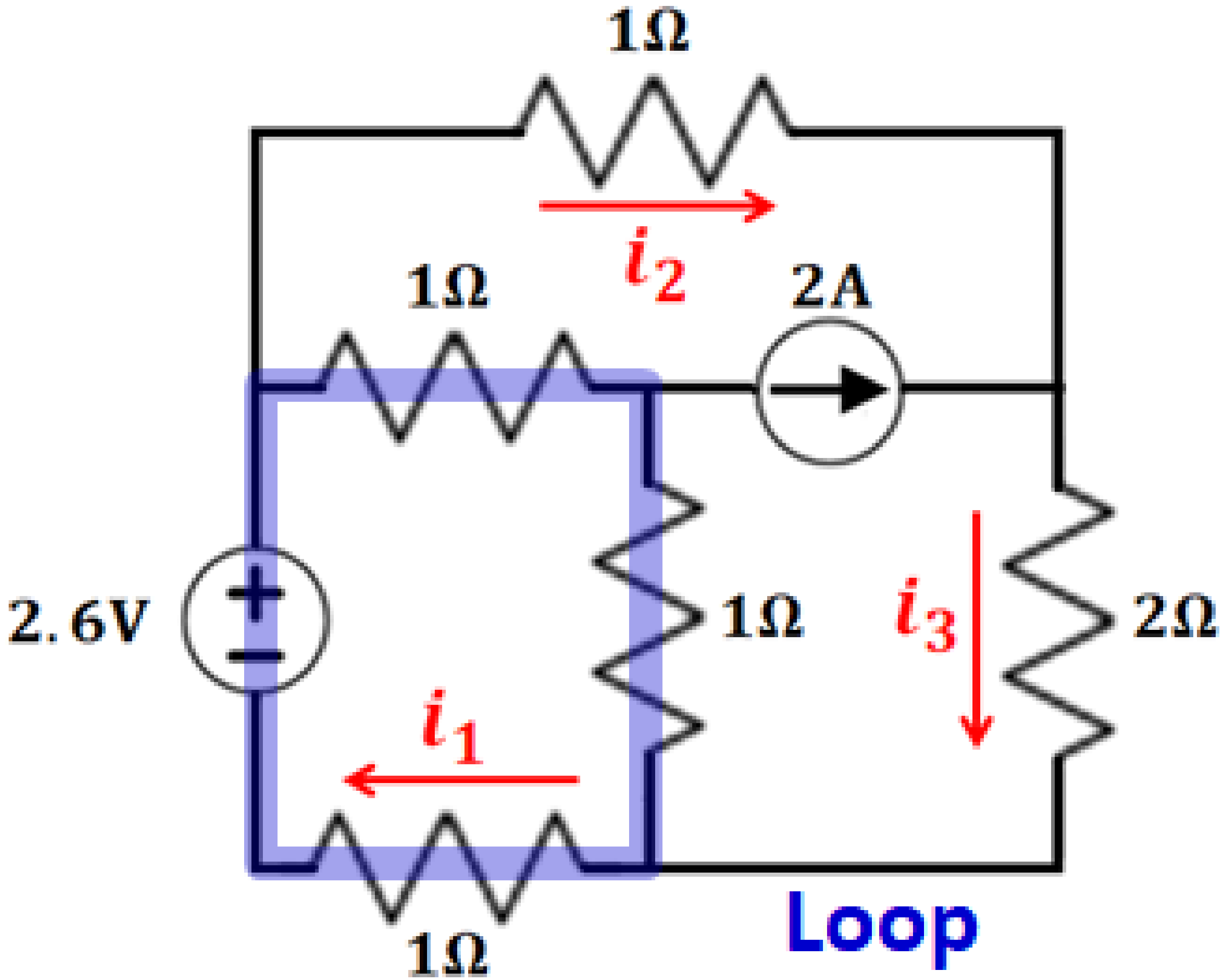


Possible Loops & Superloops

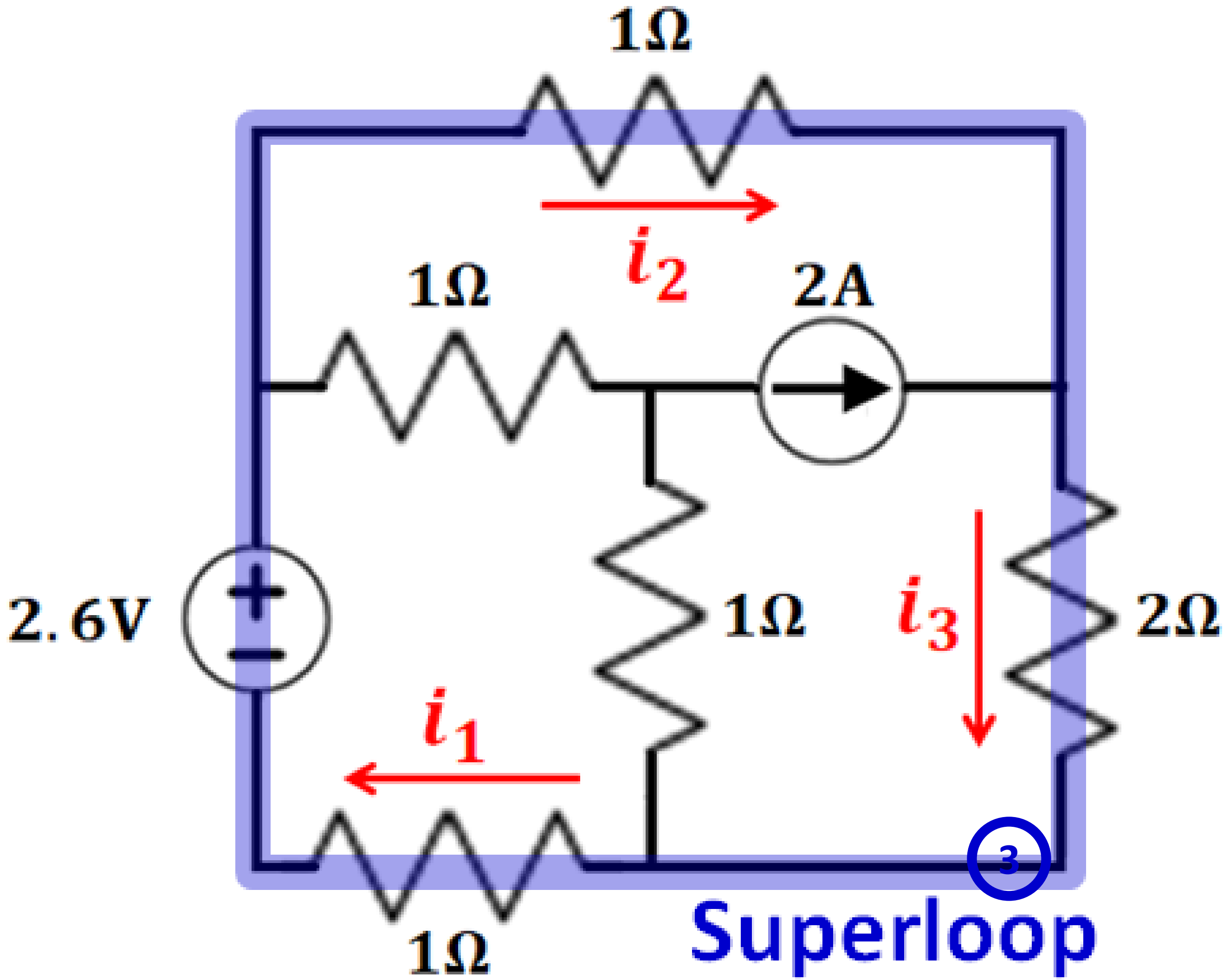




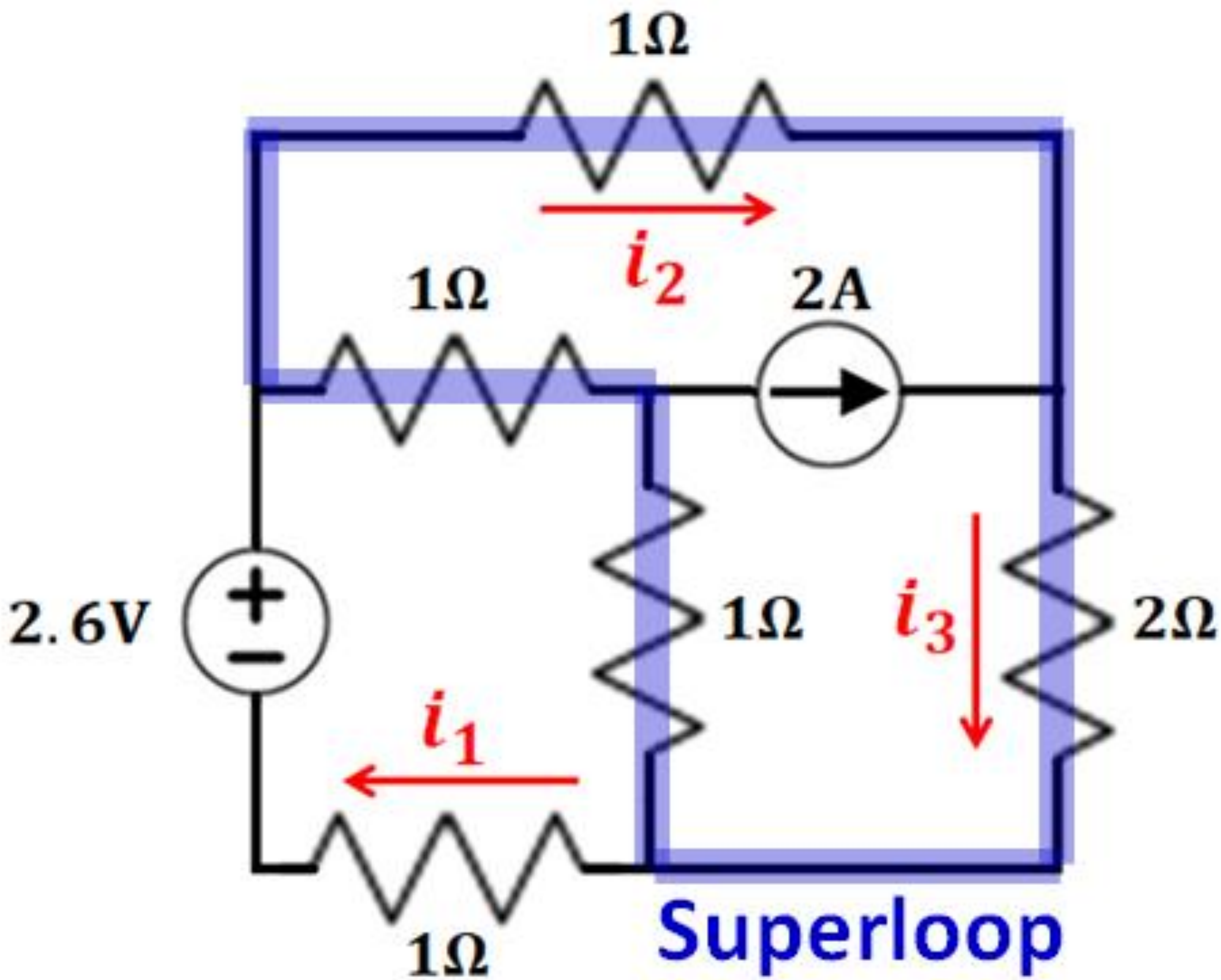




① $$-2.6\text{V} + 1\Omega(i_1 - i_2) + 1\Omega(i_1 - i_3) + 1\Omega i_1 = 0$$



2 $$-2.6\text{V} + 1\Omega\, i_2 + 2\Omega\, i_3 + 1\Omega\, i_1 = 0$$



③ $$1\Omega\, i_2 + 2\Omega\, i_3 + 1\Omega(i_3 - i_1) + 1\Omega(i_2 - i_1) = 0$$

① $-2.6\text{V} + 1\Omega(i_1 - i_2) + 1\Omega(i_1 - i_3) + 1\Omega i_1 = 0$

② $-2.6\text{V} + 1\Omega\, i_2 + 2\Omega\, i_3 + 1\Omega\, i_1 = 0$

③ $1\Omega\, i_2 + 2\Omega\, i_3 + 1\Omega(i_3 - i_1) + 1\Omega(i_2 - i_1) = 0$

④ $i_2 + 2A = i_3$ Add equation for current source

Divide equations ① ② ③ by Ω, all units become Amperes.

① $-2.6 + (i_1 - i_2) + (i_1 - i_3) + i_1 = 0$

② $-2.6 + i_2 + 2\, i_3 + i_1 = 0$

③ $i_2 + 2\, i_3 + (i_3 - i_1) + (i_2 - i_1) = 0$

④ $i_2 + 2 = i_3$

After simplifications

① $-2.6 + 3i_1 - i_2 - i_3 = 0$

② $-2.6 + i_2 + 2\,i_3 + i_1 = 0$

③ $2i_2 + 3\,i_3 - 2i_1 = 0$

④ $i_2 + 2 = i_3$

We have three unknowns, only two of the first three equations are needed

① $-2.6 + 3i_1 - i_2 - i_3 = 0$

③ $2i_2 + 3\,i_3 - 2i_1 = 0$

④ $i_2 + 2 = i_3$

Solve by substitution

④ $$i_3 = i_2 + 2$$

③ $$2i_2 + 3(i_2 + 2) - 2i_1 = 0$$

$$5i_2 + 6 - 2i_1 = 0 \rightarrow i_1 = 2.5i_2 + 3$$

① $$-2.6 + 3i_1 - i_2 - i_2 - 2 = 0$$

$$-4.6 + 3i_1 - 2\,i_2 = 0$$

$$-4.6 + 7.5i_2 + 9 - 2i_2 = 0$$

$$4.4 + 5.5i_2 = 0$$

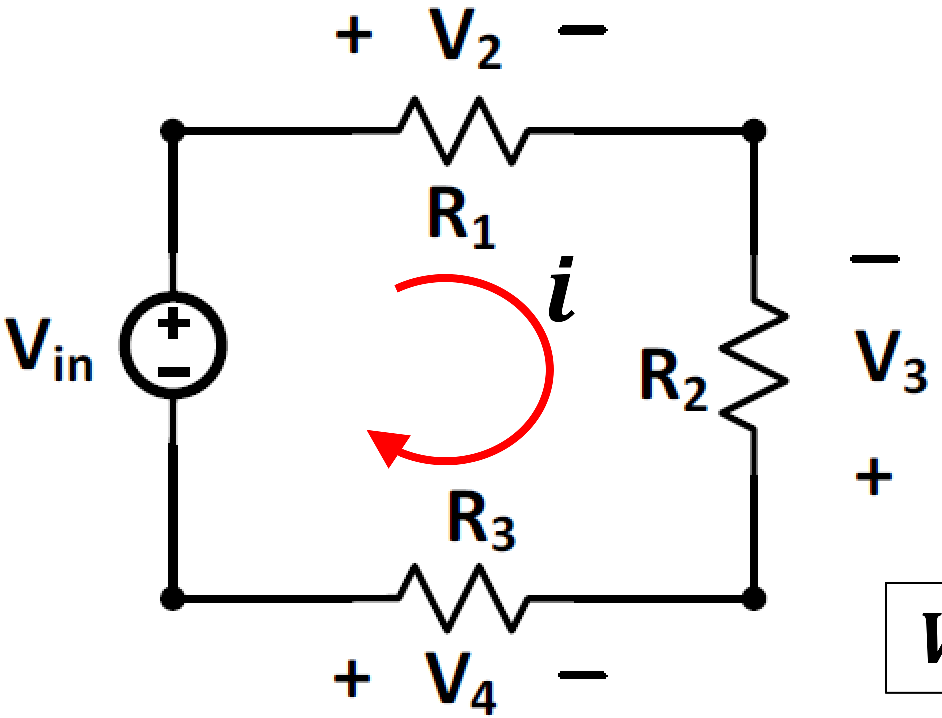
$i_2 = -0.8\ \mathrm{A}$	$i_1 = 1\mathrm{A}$	$i_3 = 1.2\mathrm{A}$

Verification: Substitute the results into loop and superloop KVL equations. The left hand sides should give zero.

WS 2.1

$$V_{in} = 9\text{V}$$

$$R_1 = R_2 = R_3 = 3\text{k}\Omega$$



$$R_{eq} = 9\text{k}\Omega$$

$$i = \frac{V_{in}}{R_{eq}} = 1\text{ mA}$$

Ohm's law

$$V_n = i\,R_n = 1\text{m} \times 3\text{k} = 3\text{V}$$

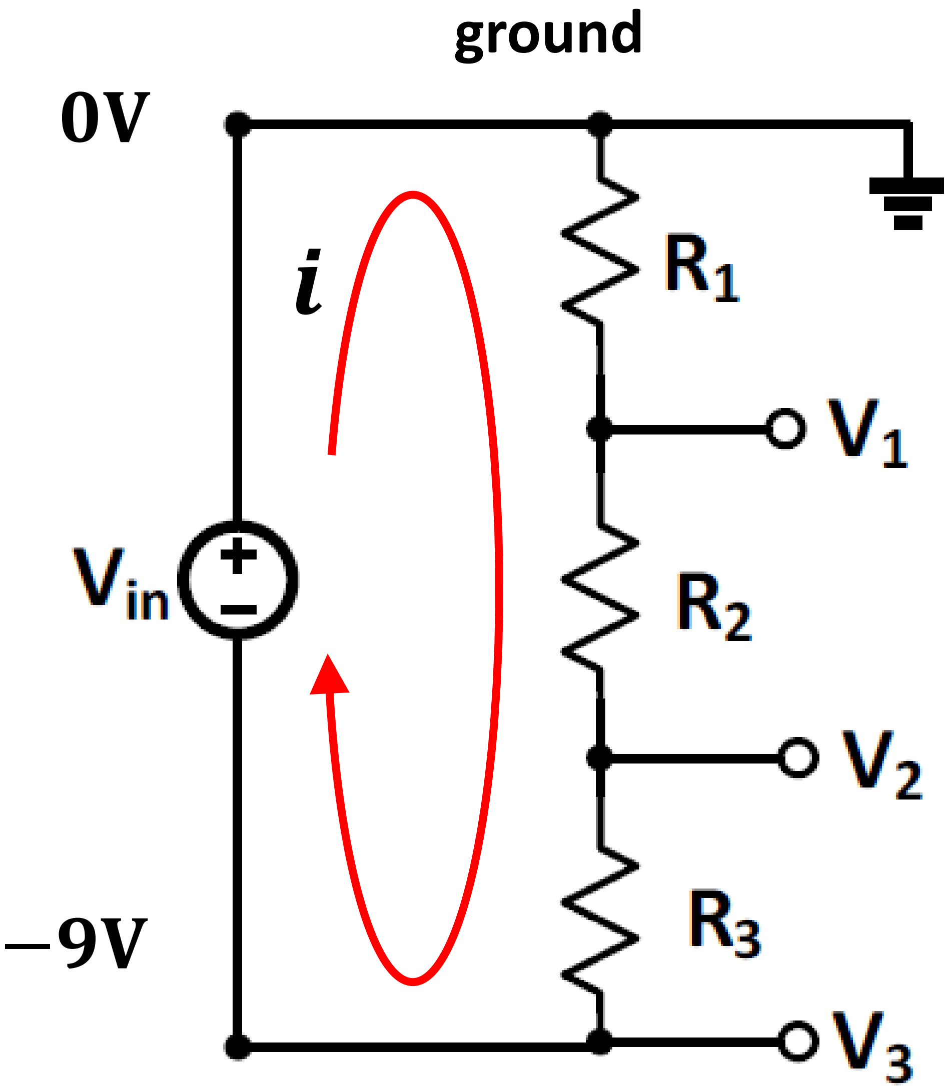
Equal resistors, by simple symmetry

$$\Rightarrow \quad V_2 = -V_3 = -V_4 = \frac{V_{in}}{3} = 3\text{V}$$

WS 2.2

$V_{in} = 9\text{V}$

$R_1 = R_2 = R_3 = 3\text{k}\Omega$



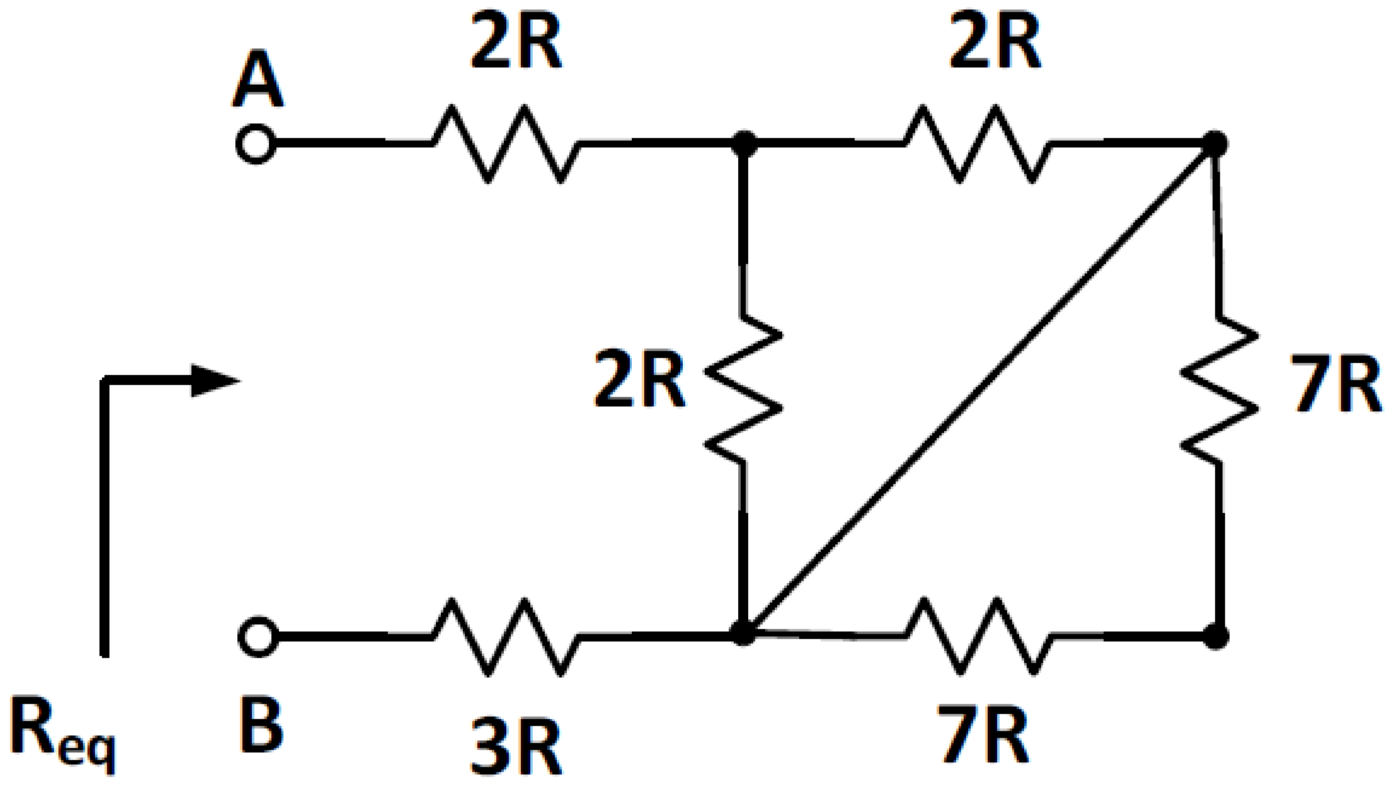
$V_1 = -3\text{V}$

$V_2 = -6\text{V}$

$V_3 = -9\text{V}$

Same circuit as in the previous problem, with a specified ground reference.

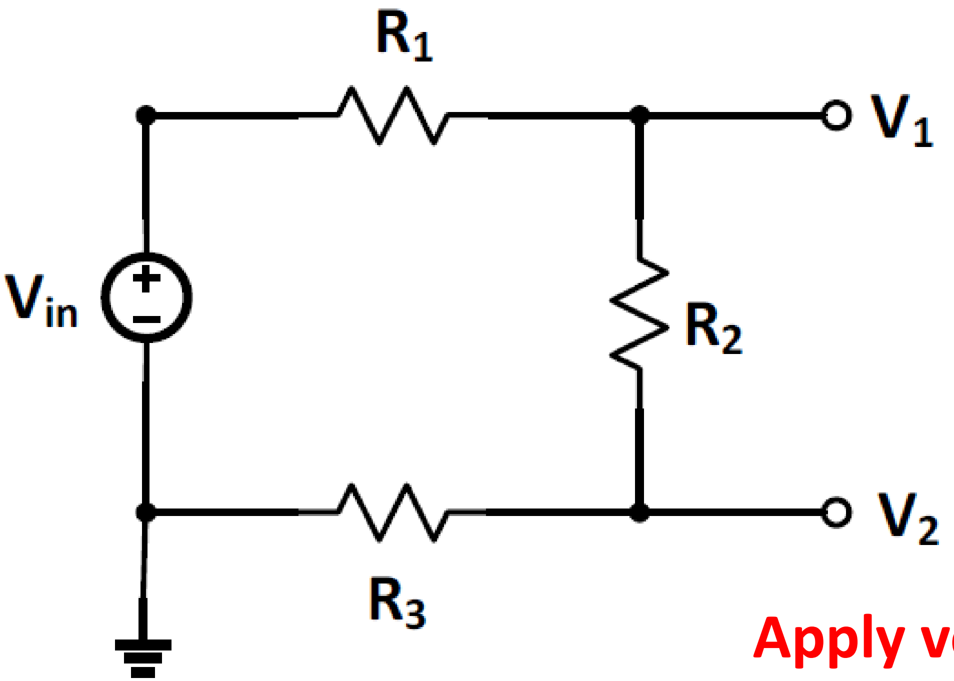
WS 2.3



$$R_{eq} = 2R + 2R//2R + 3R = (2 + 1 + 3)R = 6R$$

WS 2.4

Express V_1 and V_2 in terms of voltage V_{in} and resistors R_1, R_2, R_3



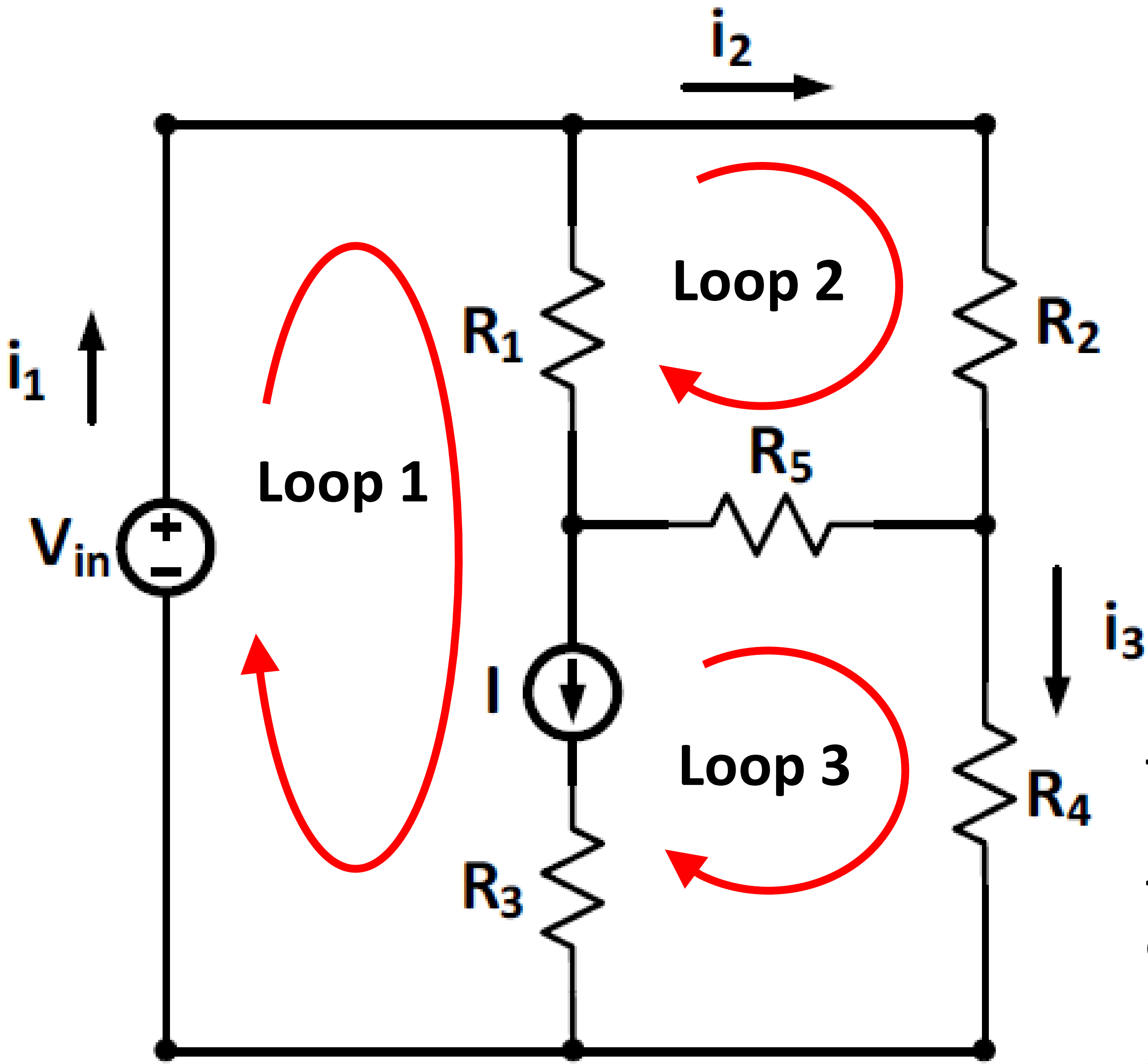
Another variation of the same circuit. Voltages are with respect to ground.

Apply voltage divider rules

$$V_1 = V_{in} \frac{R_2 + R_3}{R_1 + R_2 + R_3}$$

$$V_2 = V_{in} \frac{R_3}{R_1 + R_2 + R_3}$$

WS 2.5 – Solve with Loop Analysis



$V_{in} = 7\text{V}$

$I = 7\text{mA}$

$R_1 = 1\text{k}\Omega$

$R_2 = 2\text{k}\Omega$

$R_3 = 2\text{k}\Omega$

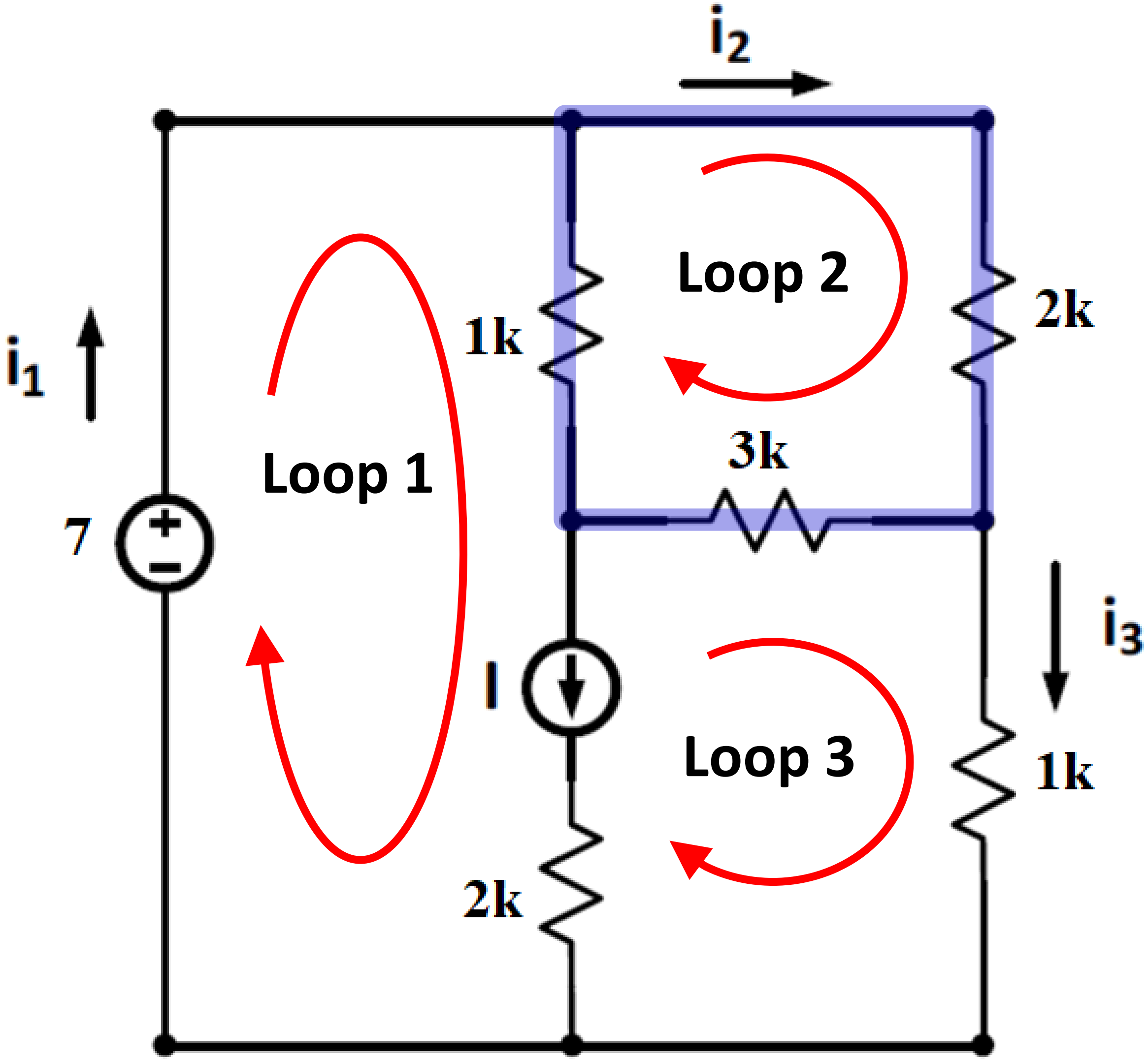
$R_4 = 1\text{k}\Omega$

$R_5 = 3\text{k}\Omega$

There is a current source between two loops. It is hard to write separate loop equations without introducing new variables (e.g., voltage across the current source)

Start with Loop 2

$$2\text{k}\, i_2 + 3\text{k}(i_2 - i_3) + 1\text{k}(i_2 - i_1) = 0$$



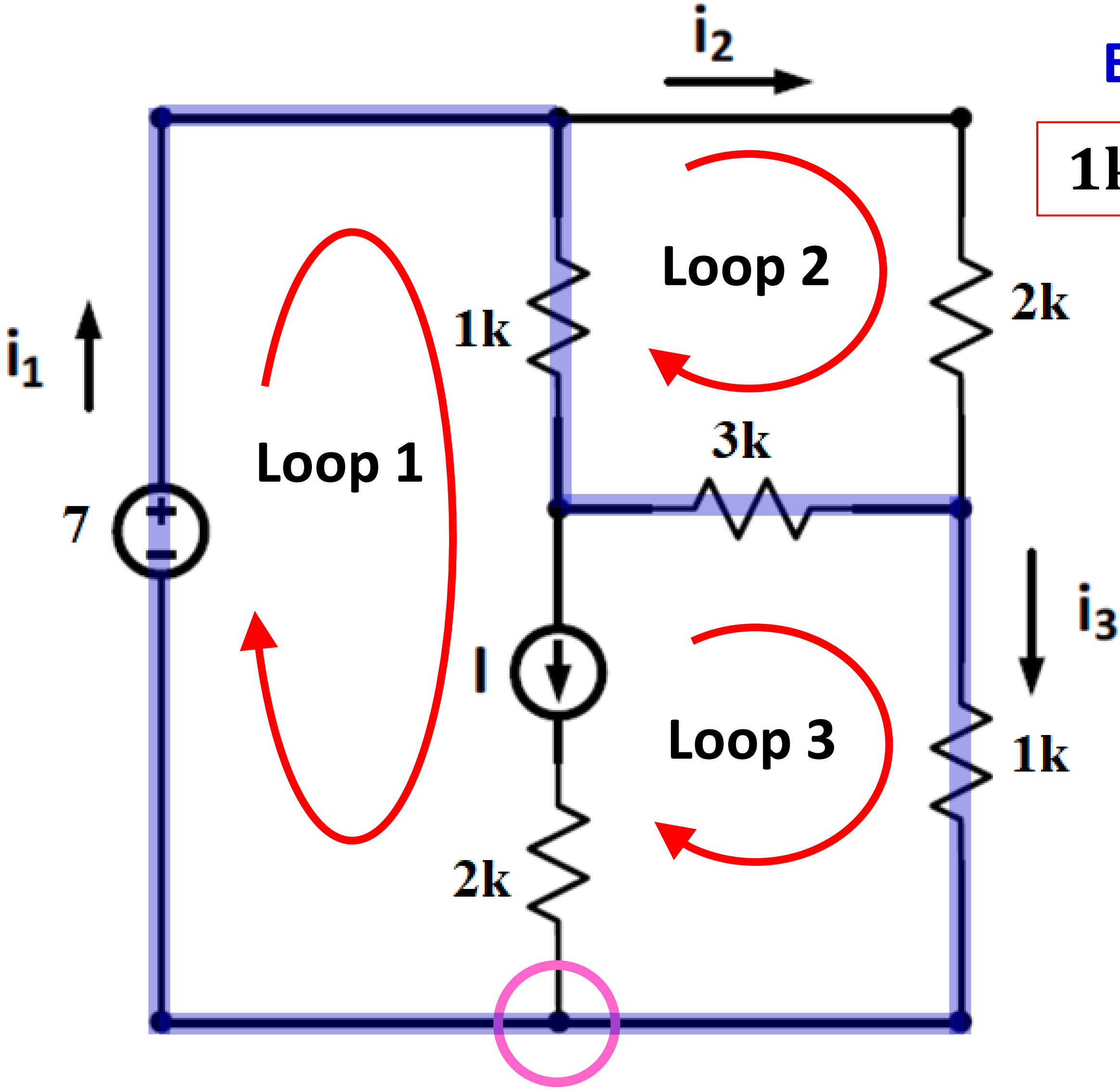
Eq (1)

$$-i_1 + 6\, i_2 - 3 i_3 = 0$$

Cannot write separate equations for Loop 1 and Loop 3.

Choose a "Superloop"!

Superloop equation $1k\,(i_1 - i_2) + 3k(i_3 - i_2) + 1k\,i_3 - 7 = 0$



Eq (2)

$$1k\,i_1 - 4k\,i_2 + 4k\,i_3 = 7$$

Finally, an equation including the current source at the bottom node ○

Eq (3)

$$i_1 = i_3 + 7mA$$

since we had the input $I = 7mA$

$$1\text{k}\ i_1 - 4\text{k}\ i_2 + 4\text{k}\ i_3 = 7\ \text{V}$$ Eq (2)

Divide by 1kΩ → simplifies to: $i_1 - 4\ i_2 + 4\ i_3 = 7\ \text{mA}$

Solve the system

$$i_1 - i_3 = 7\text{mA}$$ Eq (3)

$$-i_1 + 6\ i_2 - 3i_3 = 0$$ Eq (1)

$$i_1 - 4\ i_2 + 4\ i_3 = 7\ \text{mA}$$ Eq (2)

$$i_1 - 4\ i_2 + 4\ i_3 = i_1 - i_3 \Rightarrow i_2 = \frac{5}{4}\ i_3$$

$$-i_1 + 6\frac{5}{4}\ i_3 - 3i_3 = 0 \Rightarrow i_1 = \frac{9}{2}\ i_3$$

$$\frac{9}{2}\ i_3 - i_3 = 7\text{mA}$$

$i_3 = 2\text{mA}$ $\quad$ $i_1 = 9\text{mA}$

$i_2 = 2.5\text{mA}$