ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 6

MWF – 12:00pm

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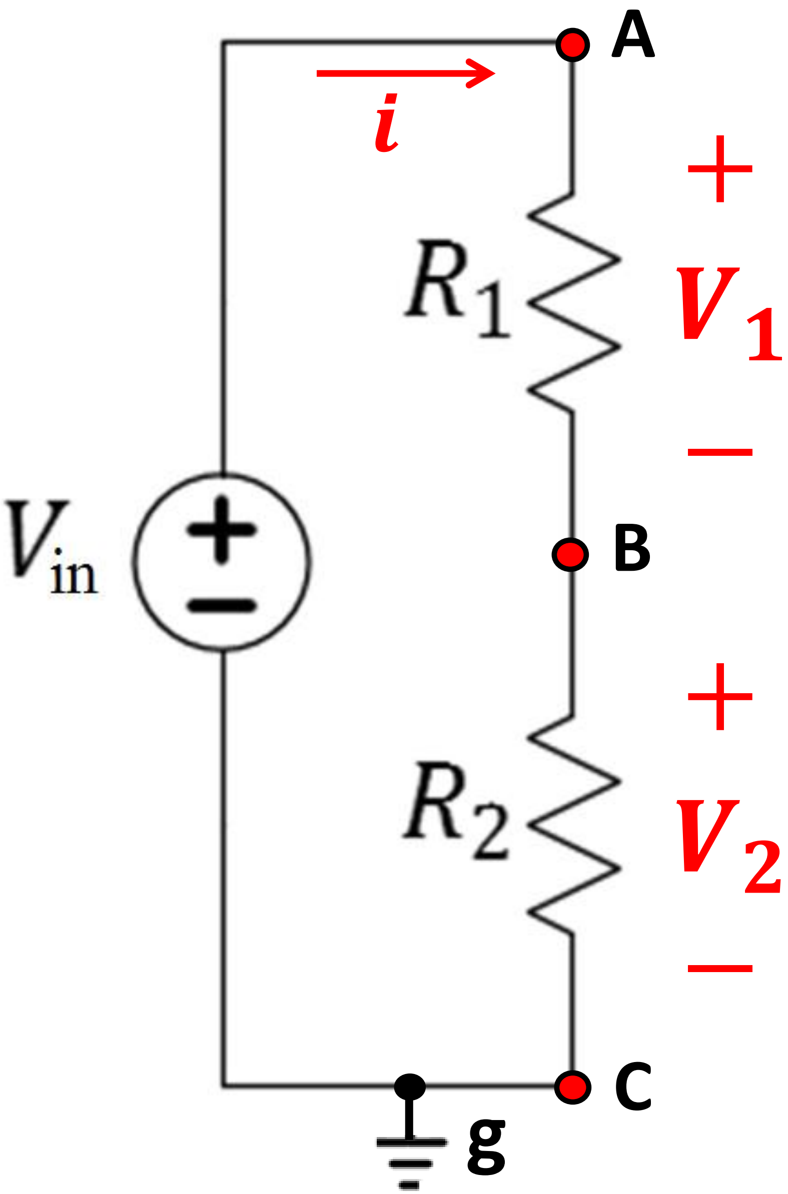
2062 ECE Building

Lecture 6 – Summary

Learning Objectives

1. More practice on Superloops
2. Define Kirchhoff's current law (KCL)
3. Understand current division formula
4. Introduce node voltage analysis method to compute node voltages

Voltage Divider



$$V_1 = V_{AB} \quad \& \quad V_2 = V_{BC}$$

$$i = \frac{V_{in}}{R_1 + R_2}$$

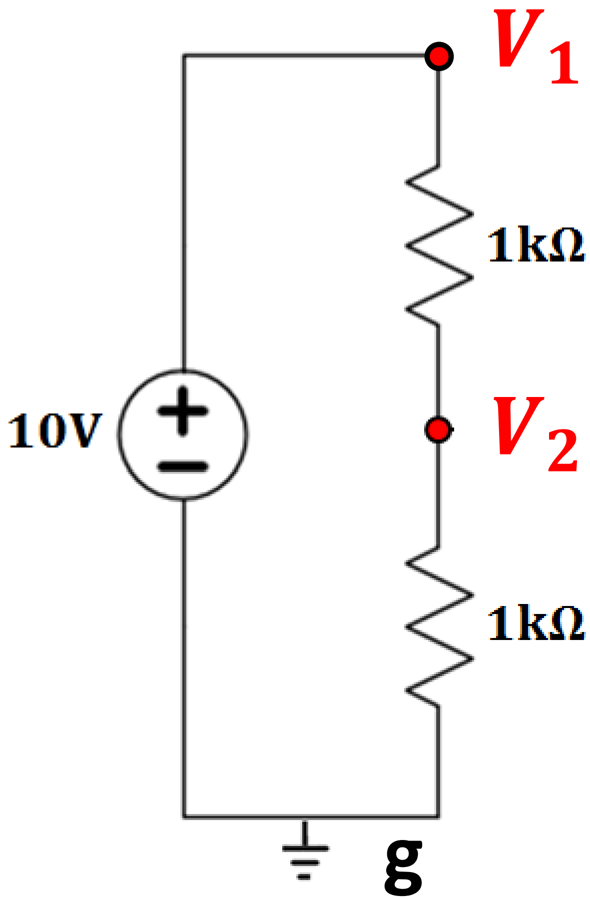
$$V_{AB} = i_{AB} R_1 = i R_1$$

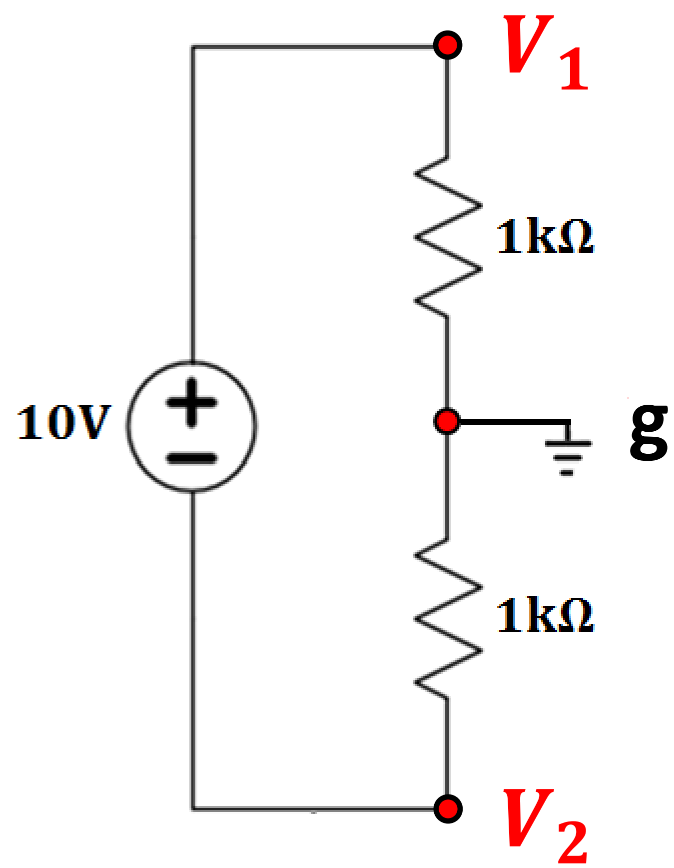
$$V_1 = \frac{V_{in} R_1}{R_1 + R_2}$$

$$V_{BC} = i_{BC} R_2 = i R_2$$

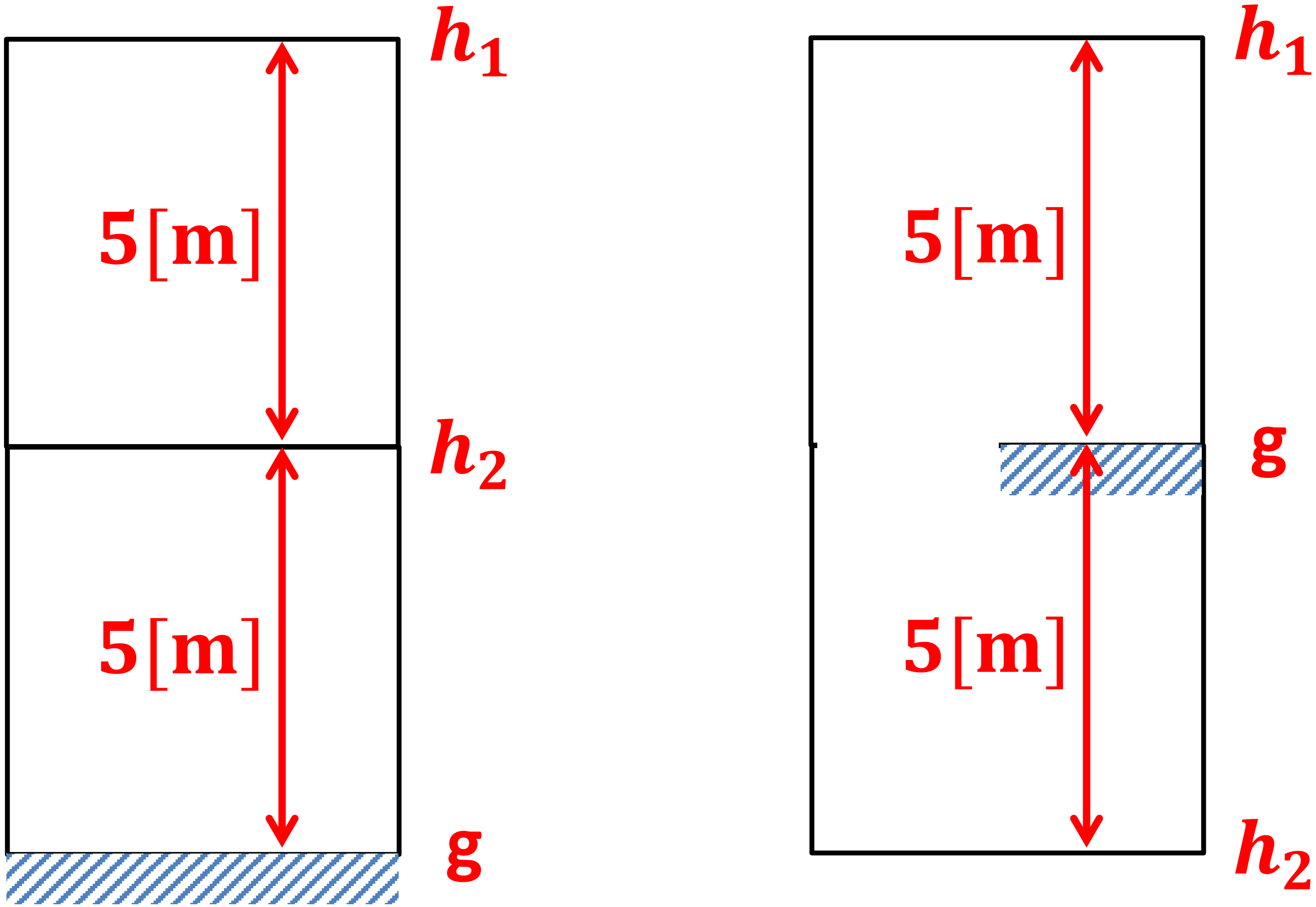
$$V_2 = \frac{V_{in} R_2}{R_1 + R_2}$$

Example – Find V_1 and V_2 for these two cases



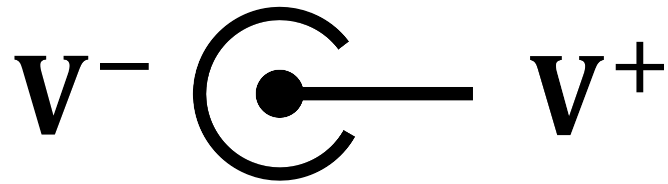


Mechanical analogy

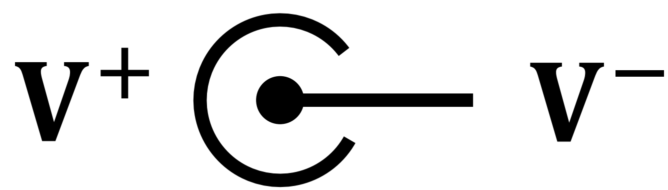


Have you ever noticed the polarity markings on d.c. power supplies?

Ground reference
is negative terminal

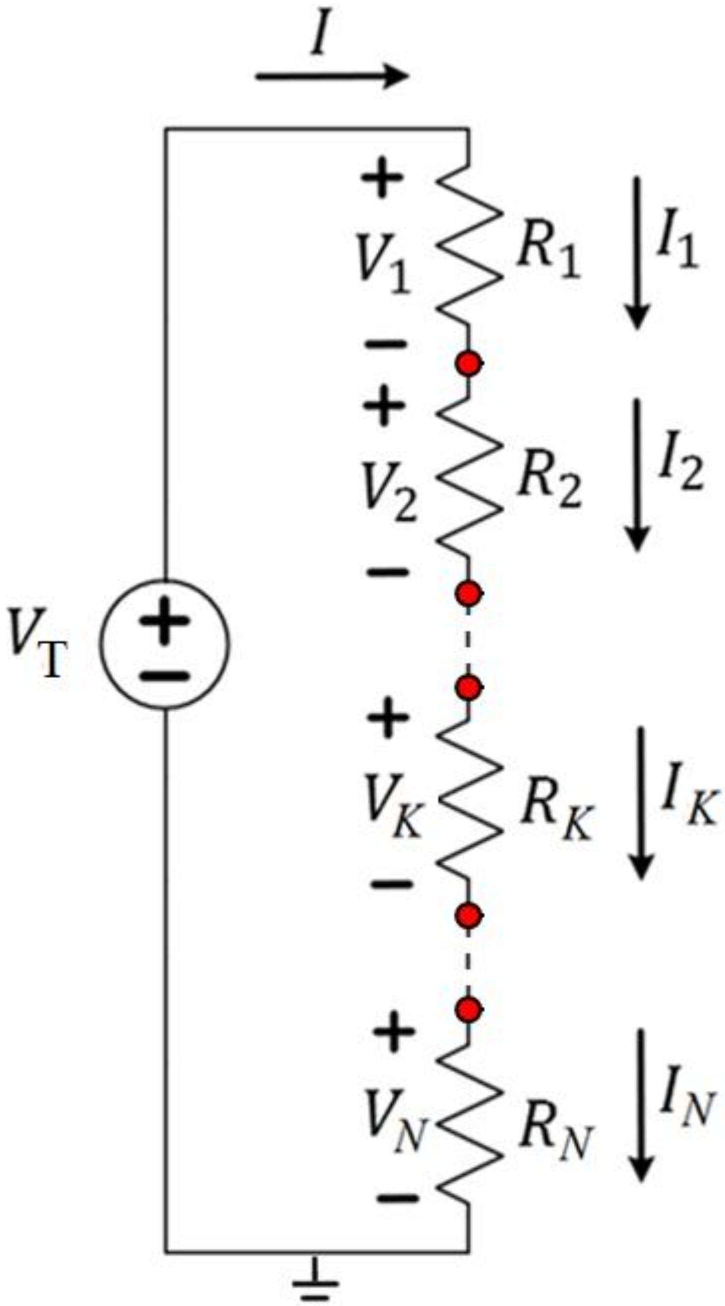


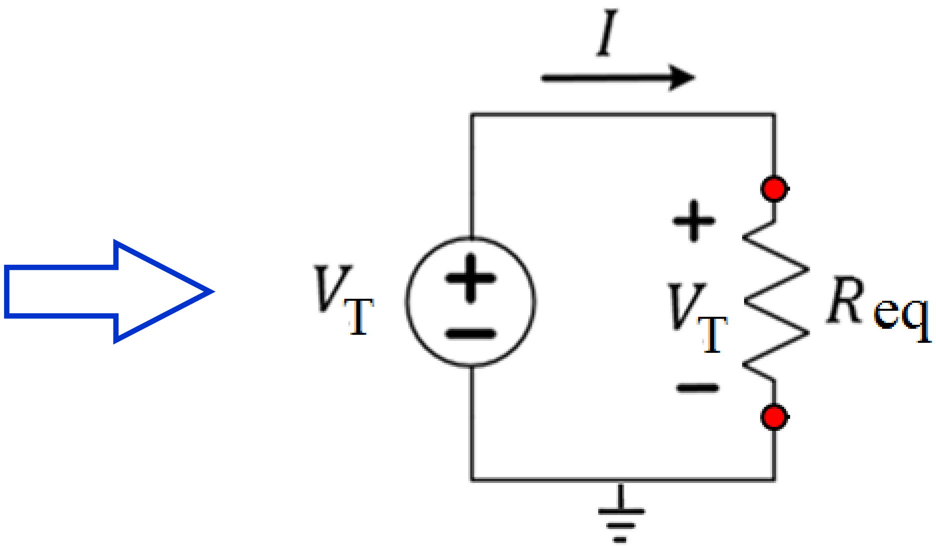
Ground reference
is positive terminal



Multiple resistors

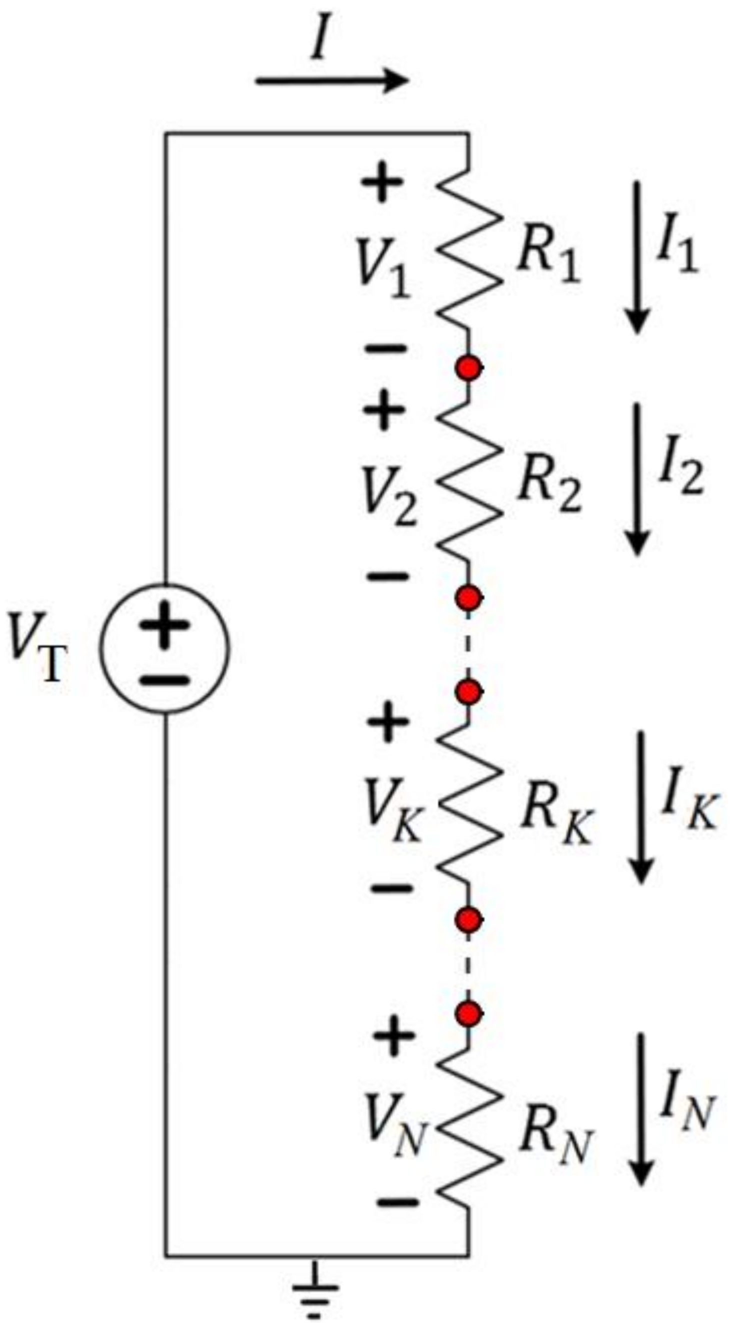
$$R_{eq} = R_1 + R_2 + \cdots + R_N$$





$$V_T = \sum_K V_K$$

$$I_K = I$$



Multiple resistors

$$R_{eq} = R_1 + R_2 + \cdots + R_N$$

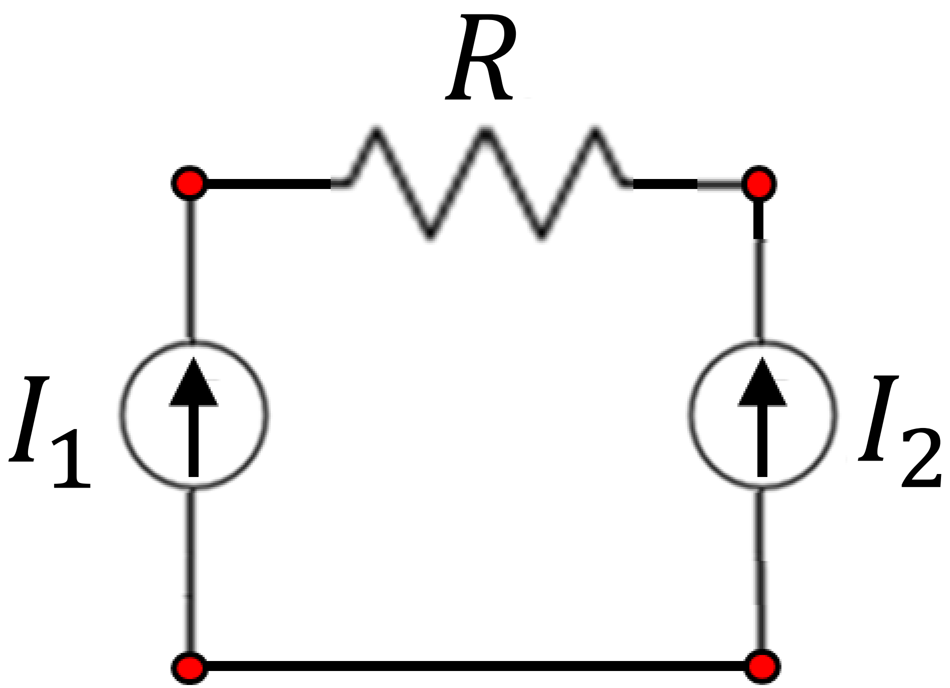
$$I_K = I \rightarrow \frac{V_K}{I_K} = \frac{V_T}{R_{eq}}$$

Across each resistor there is a voltage drop

$$V_k = \frac{R_k}{R_{eq}} \cdot V_T$$

Interesting student's question after class on Friday

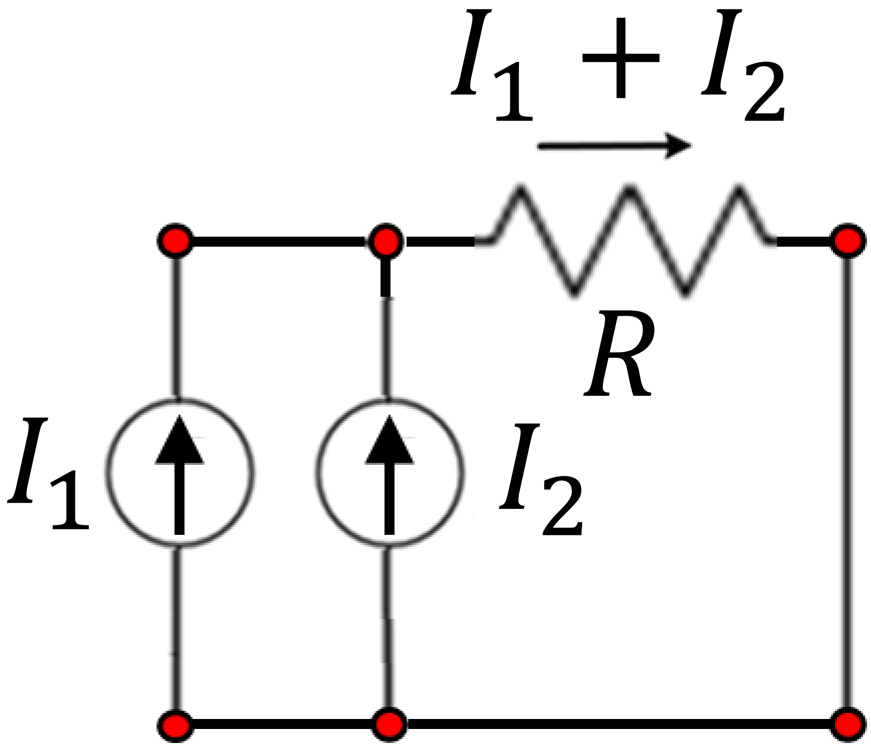
Ideal Current Sources in series?



The two current sources "fight" with each other to establish their own current in the loop.

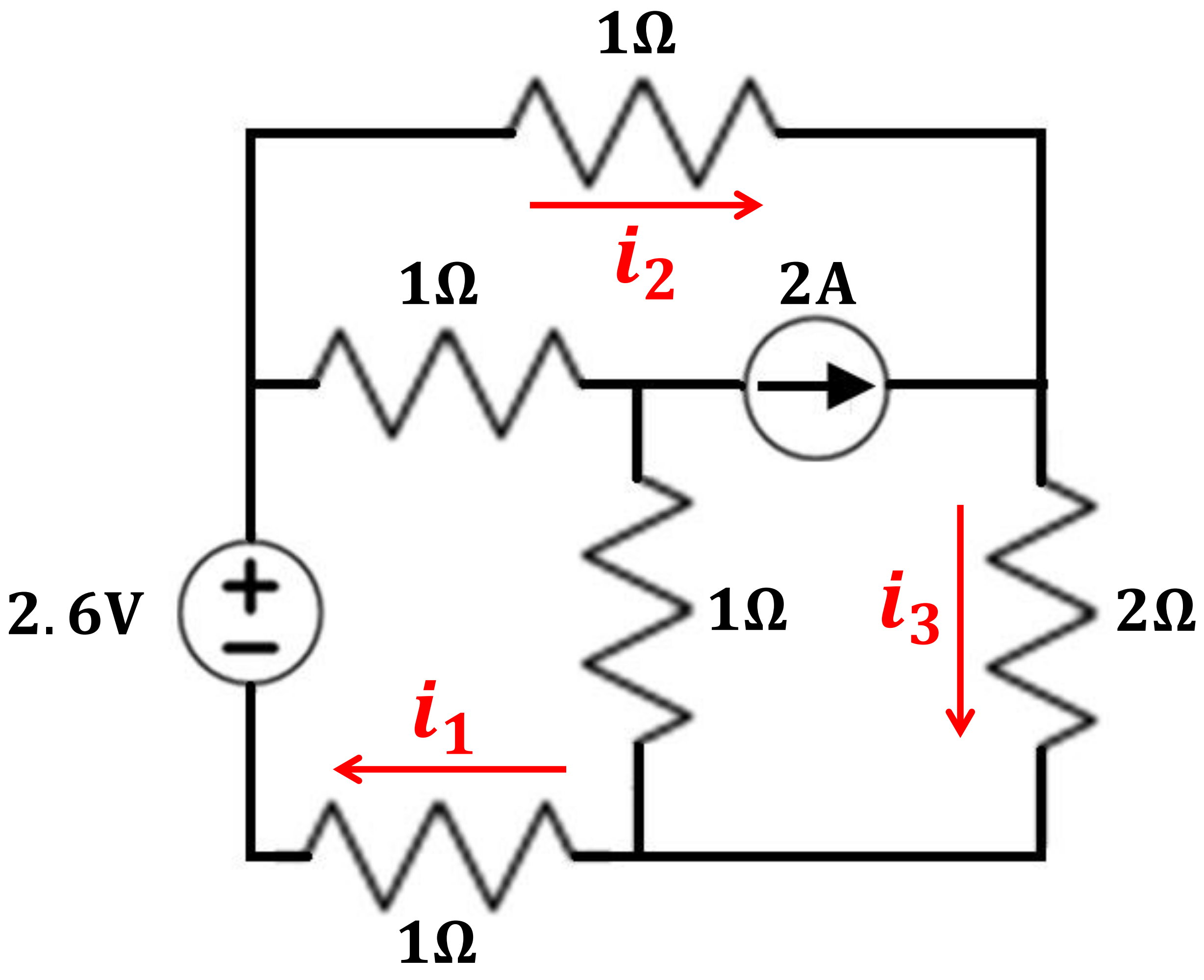
This is a configuration to avoid

Current Sources in parallel add up

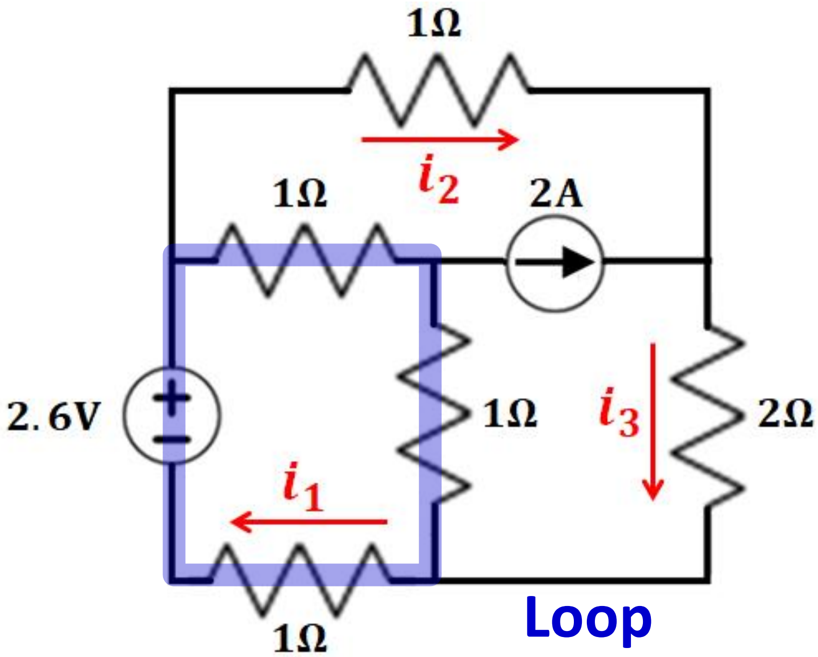


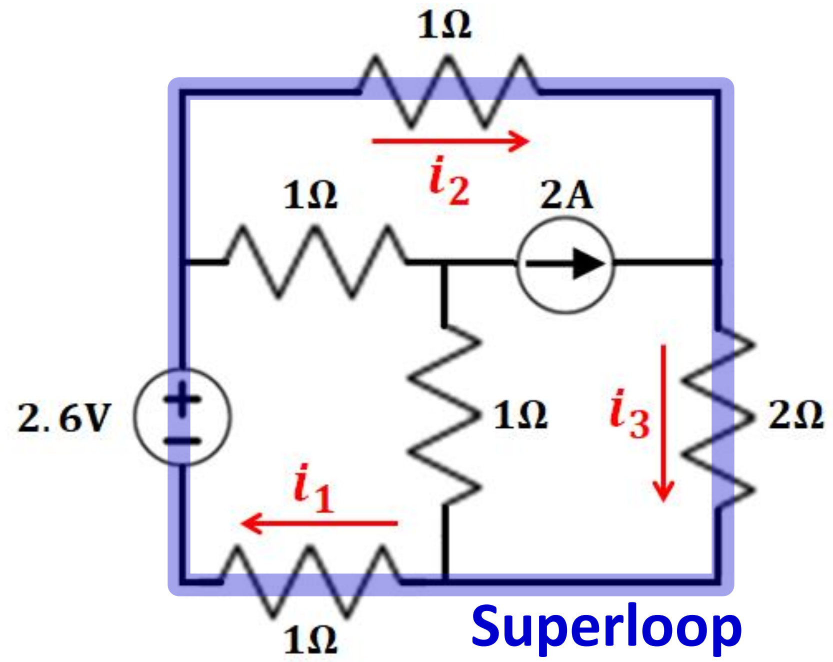
Superloop Example

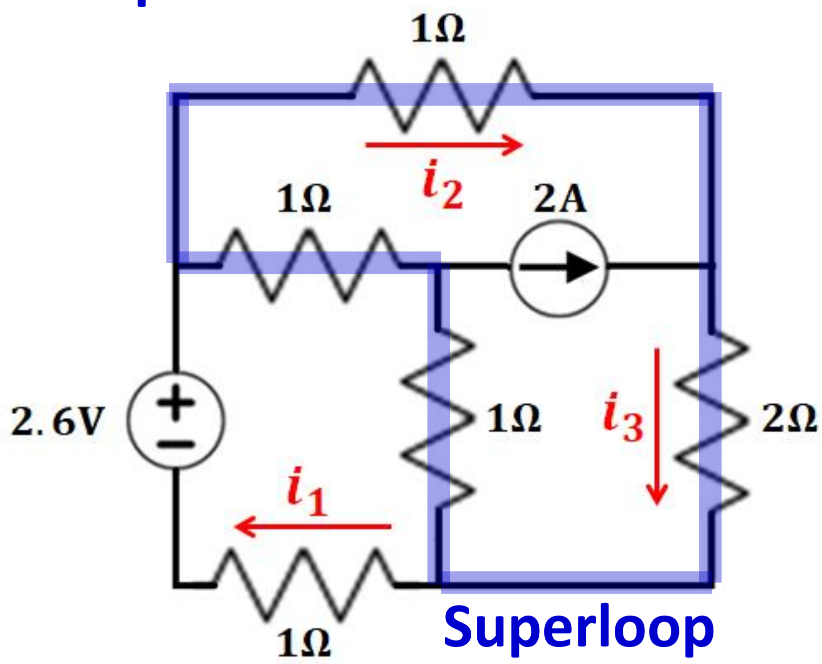
Example – Obtain the unknown currents i_1, i_2, i_3

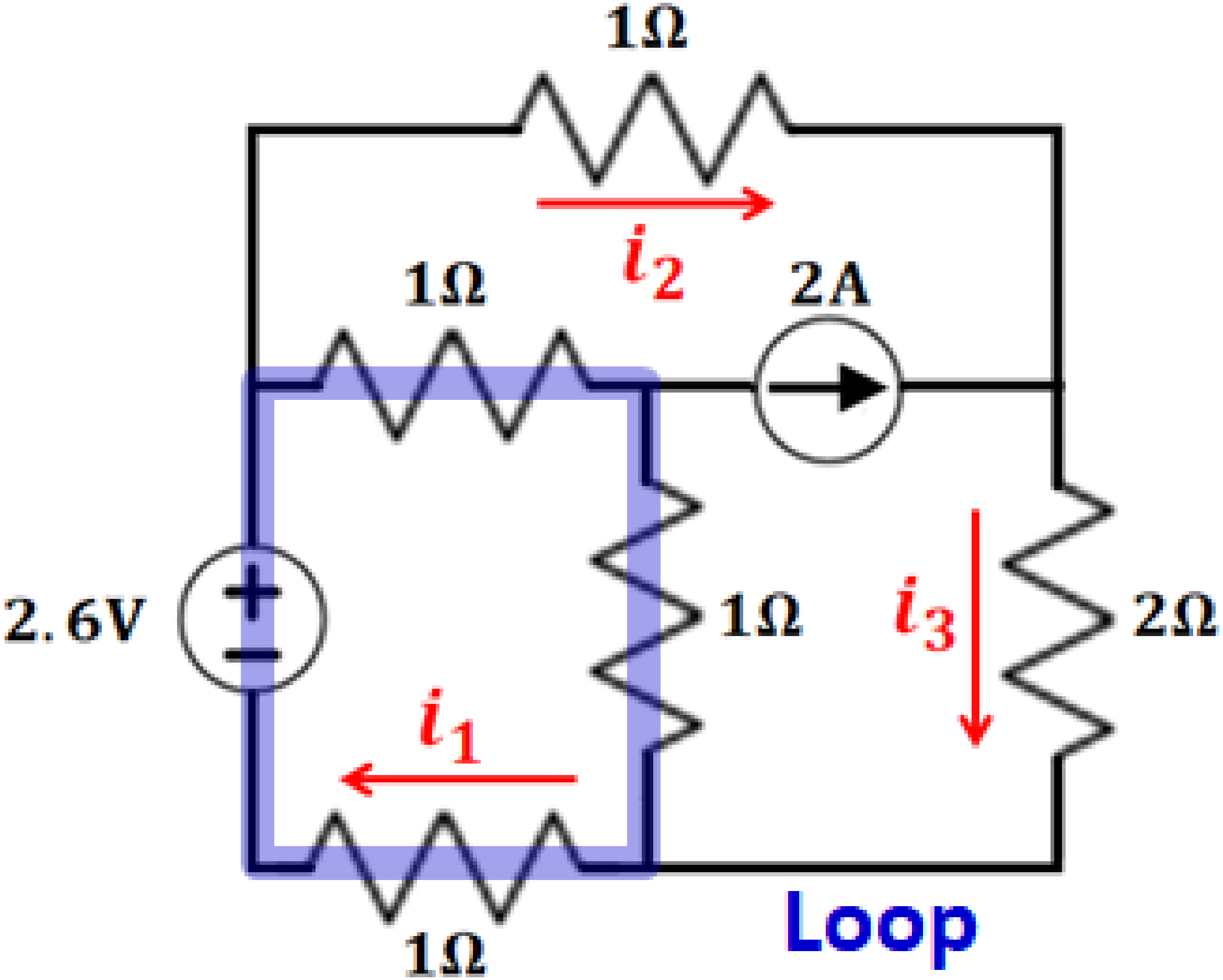


Possible Loops & Superloops

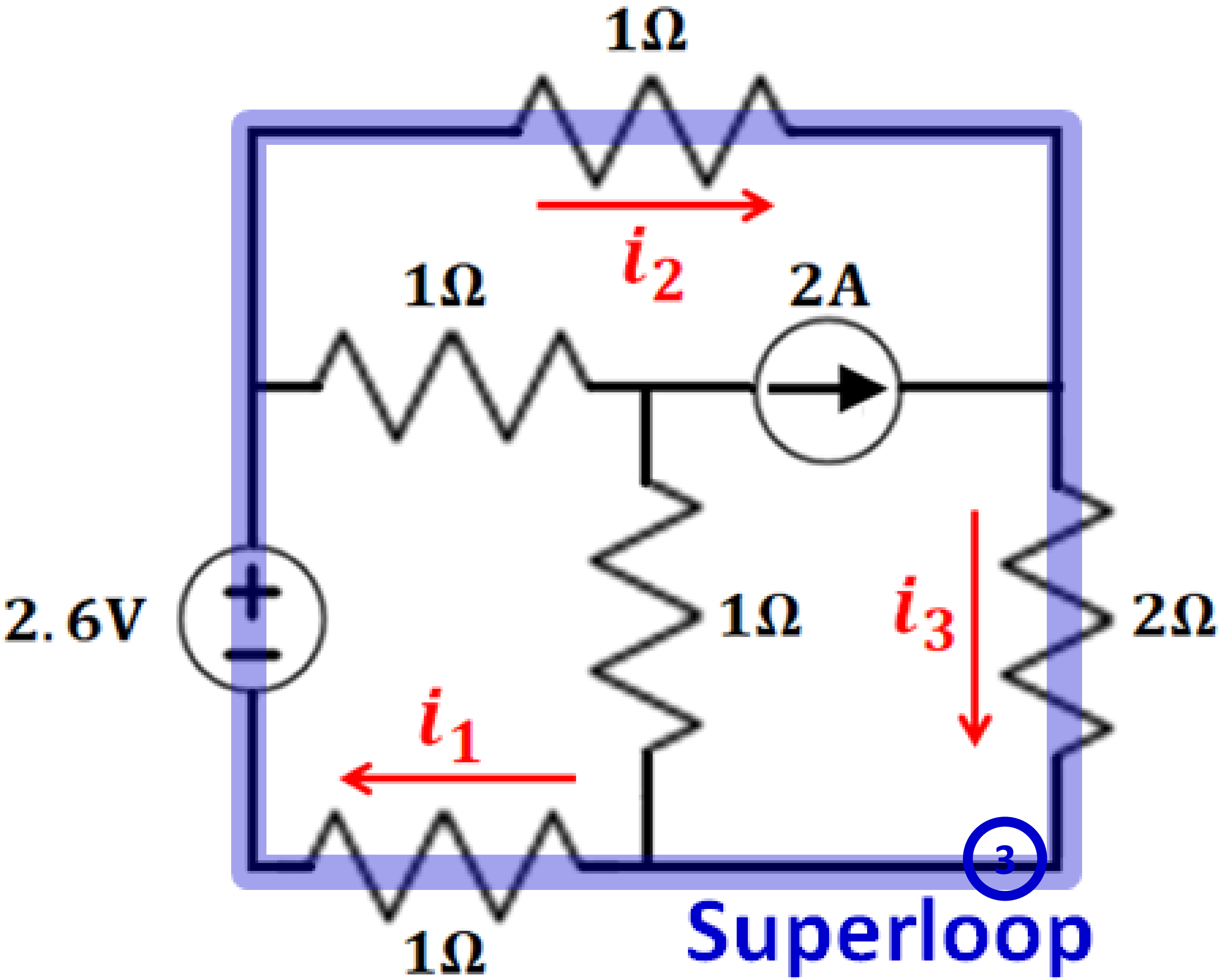




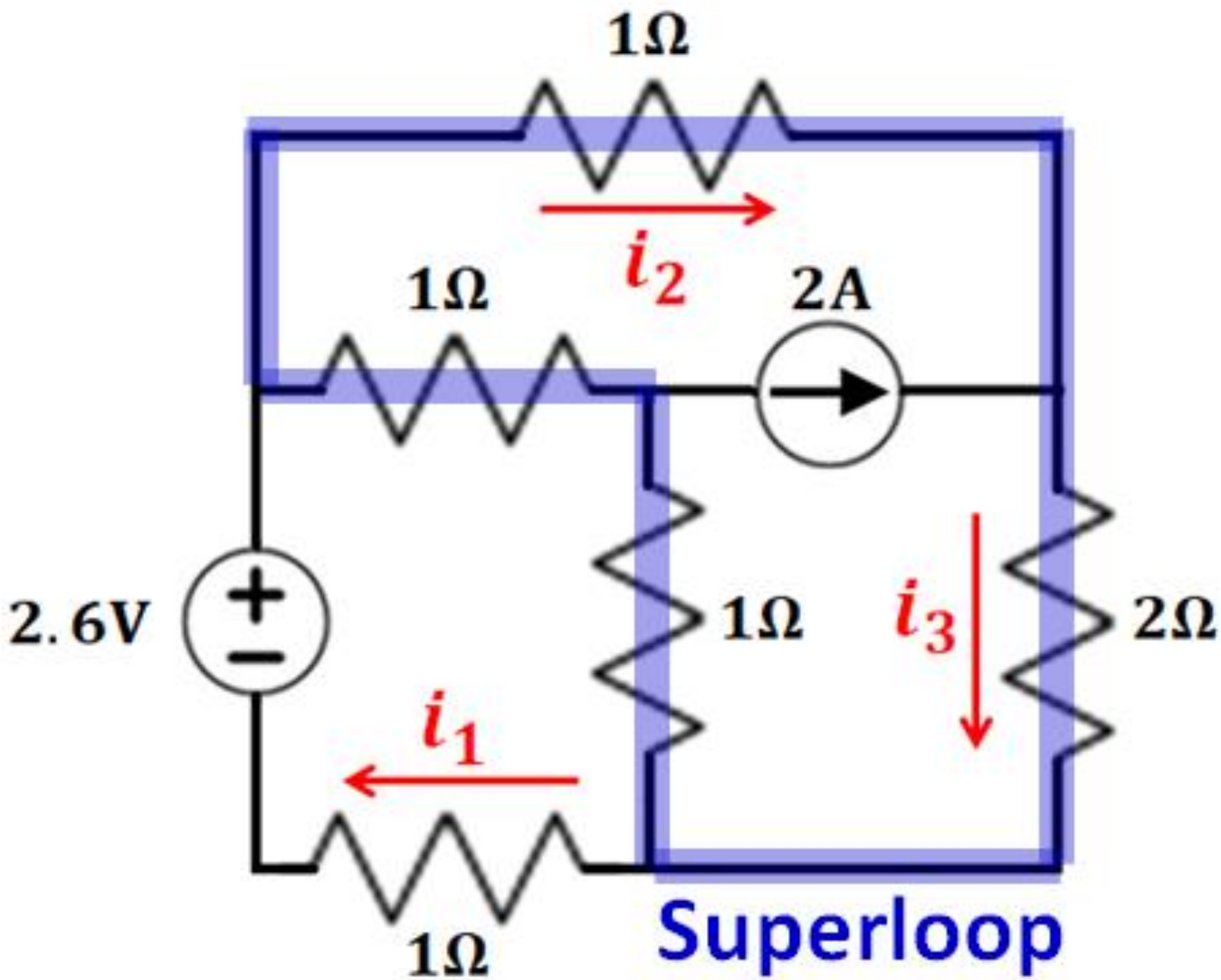




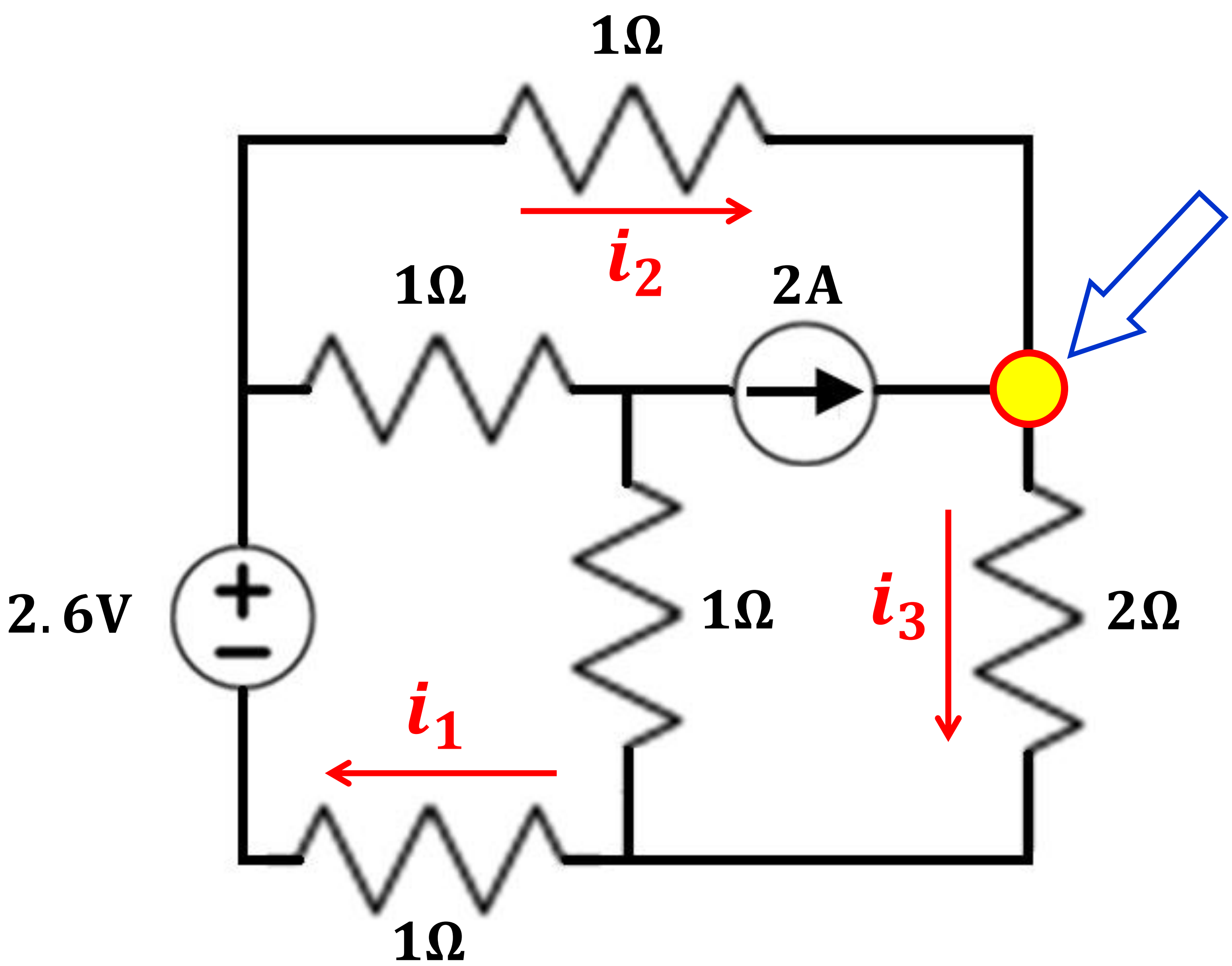
① $-2.6\text{V} + 1\Omega(i_1 - i_2) + 1\Omega(i_1 - i_3) + 1\Omega i_1 = 0$



② $$-2.6\text{V} + 1\Omega\, i_2 + 2\Omega\, i_3 + 1\Omega\, i_1 = 0$$



③ $$1\Omega\, i_2 + 2\Omega\, i_3 + 1\Omega(i_3 - i_1) + 1\Omega(i_2 - i_1) = 0$$



4 $i_2 + 2A = i_3$ Add equation for current source

① $-2.6\text{V} + 1\Omega(i_1 - i_2) + 1\Omega(i_1 - i_3) + 1\Omega i_1 = 0$

② $-2.6\text{V} + 1\Omega\, i_2 + 2\Omega\, i_3 + 1\Omega\, i_1 = 0$

③ $1\Omega\, i_2 + 2\Omega\, i_3 + 1\Omega(i_3 - i_1) + 1\Omega(i_2 - i_1) = 0$

④ $i_2 + 2A = i_3$ **Add equation for current source**

Divide equations ① ② ③ by Ω, all units become Amperes.

① $-2.6 + (i_1 - i_2) + (i_1 - i_3) + i_1 = 0$

② $-2.6 + i_2 + 2\, i_3 + i_1 = 0$

③ $i_2 + 2\, i_3 + (i_3 - i_1) + (i_2 - i_1) = 0$

④ $i_2 + 2 = i_3$

After simplifications

① $$-2.6 + 3i_1 - i_2 - i_3 = 0$$

② $$-2.6 + i_2 + 2\,i_3 + i_1 = 0$$

③ $$2i_2 + 3\,i_3 - 2i_1 = 0$$

④ $$i_2 + 2 = i_3$$

We have three unknowns, only two of the first three equations are needed

① $$-2.6 + 3i_1 - i_2 - i_3 = 0$$

③ $$2i_2 + 3\,i_3 - 2i_1 = 0$$

④ $$i_2 + 2 = i_3$$

Solve by substitution

④ $$i_3 = i_2 + 2$$

③ $$2i_2 + 3(i_2 + 2) - 2i_1 = 0$$

$$5i_2 + 6 - 2i_1 = 0 \rightarrow i_1 = 2.5i_2 + 3$$

① $$-2.6 + 3i_1 - i_2 - i_2 - 2 = 0$$

$$-4.6 + 3i_1 - 2\, i_2 = 0$$

$$-4.6 + 7.5i_2 + 9 - 2i_2 = 0$$

$$4.4 + 5.5i_2 = 0$$

$i_2 = -0.8\ A$	$i_1 = 1A$	$i_3 = 1.2A$

Verification: Substitute the results into loop and superloop KVL equations. The left hand sides should give zero.

Kirchhoff Current Law (KCL)

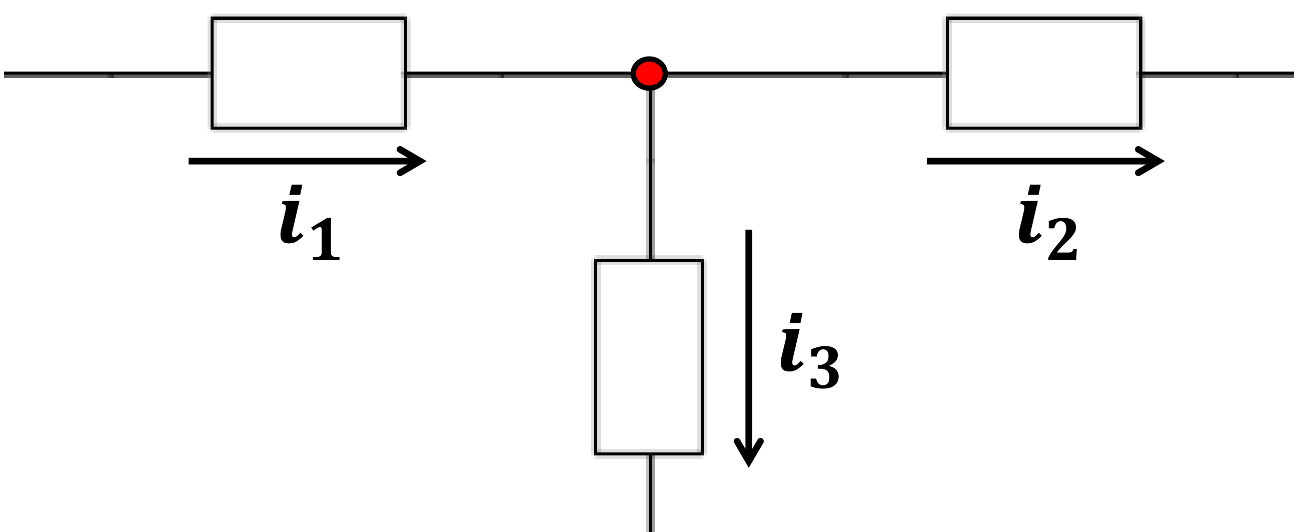
KVL states that the algebraic sum of current entering or leaving a node is zero (conservation of charge).

$$i_1 = i_2 + i_3$$

current in = current out

$$\text{KCL)} \quad -i_1 + i_2 + i_3 = 0$$

(Convention: current into node is negative, current out is positive)



From Lecture 5

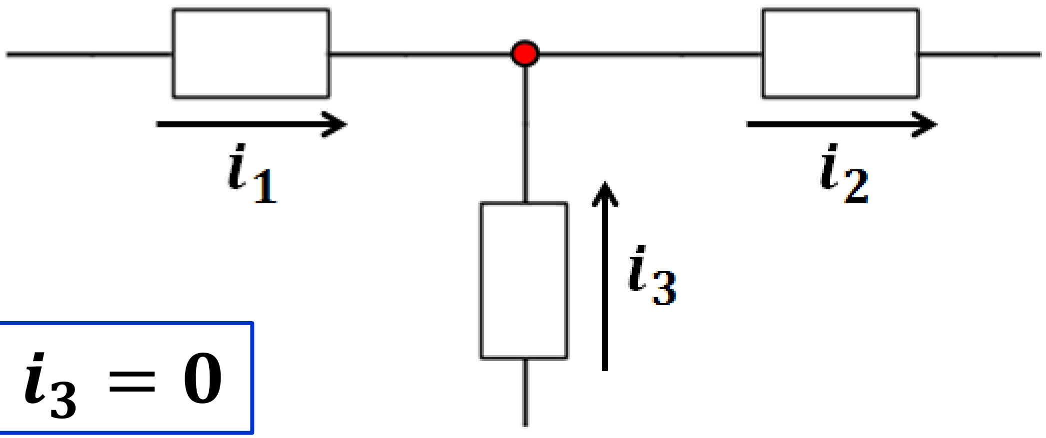
Kirchhoff Current Law (KCL) – General Advice

When solving for currents in a simple circuit, it is always good to assign the current direction arrows following the natural flow of the current.

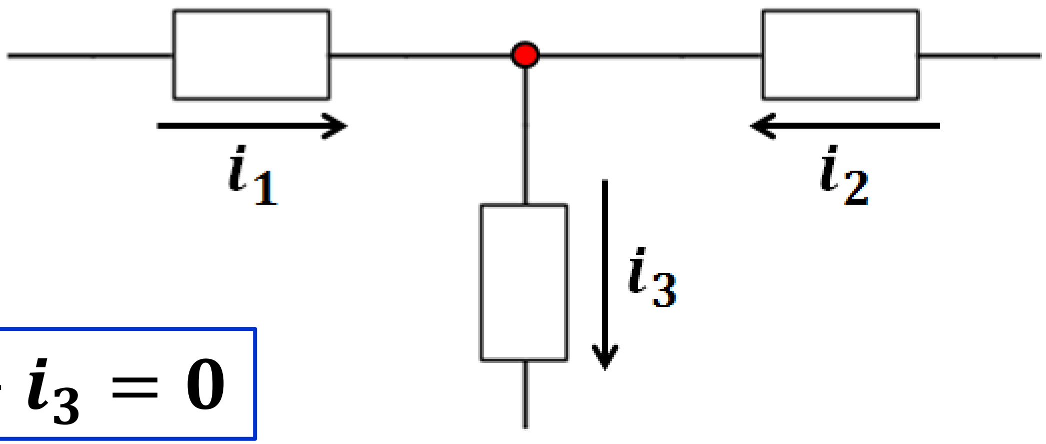
However, in a complicated circuit it may not be easy to predict the current flow intuitively. So, when setting up the problem, just assign reference directions for all currents.

In the end, if it turns out that an actual current should point in the opposite direction than guessed, the solution method will just give a "negative current" with respect to the reference arrow.

Some more practice with actual currents

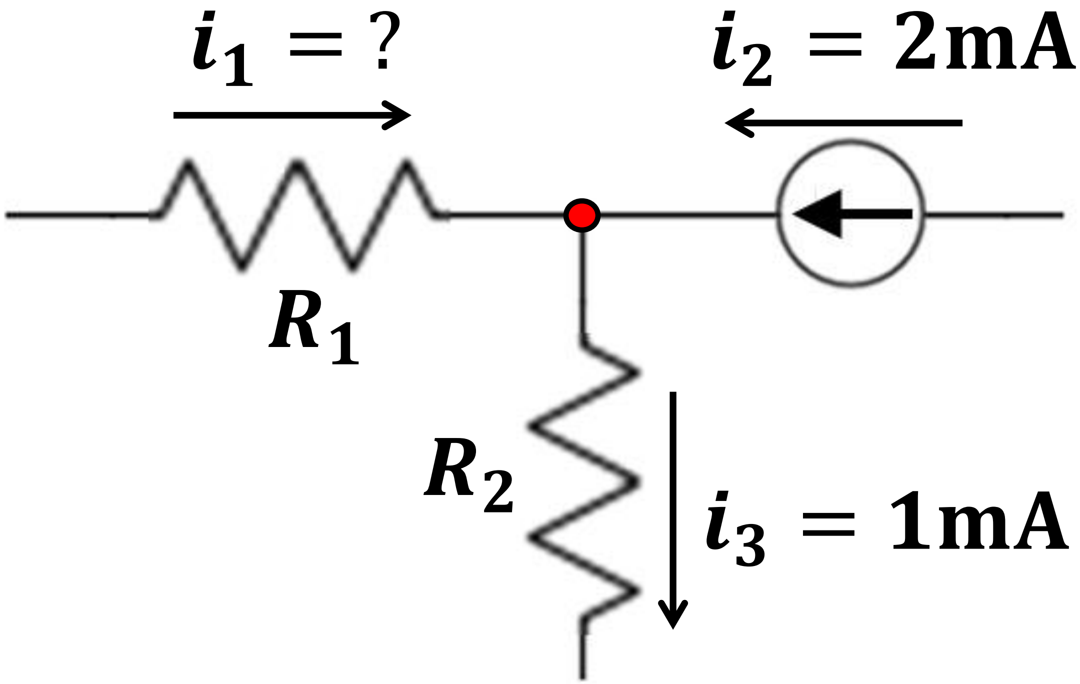


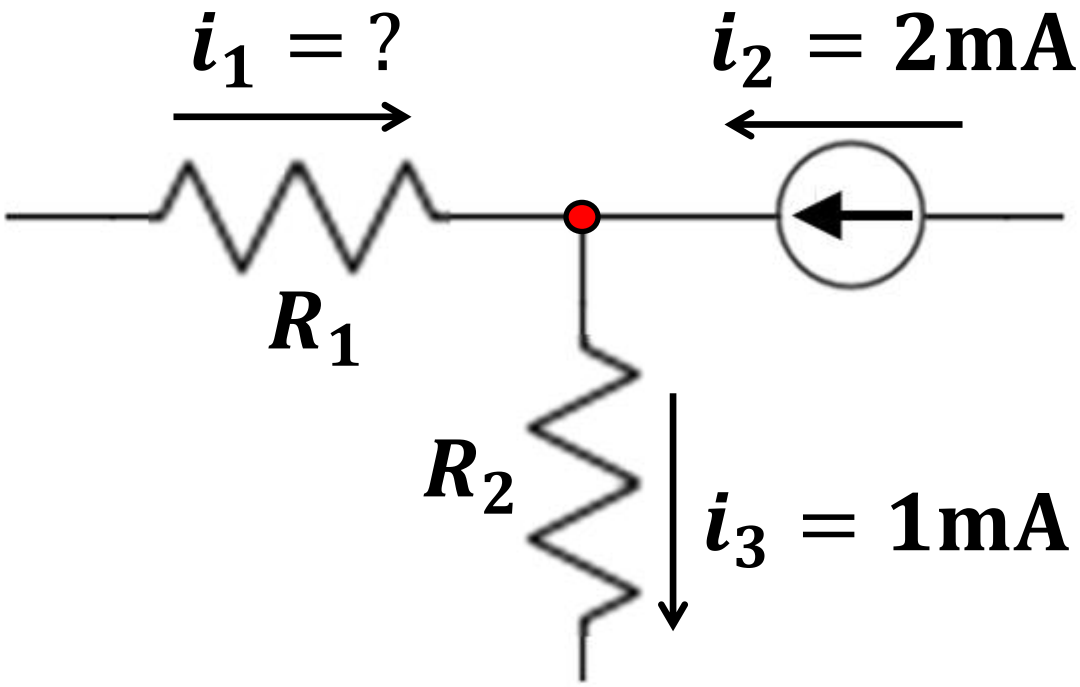
KCL) $-i_1 + i_2 - i_3 = 0$



KCL) $-i_1 - i_2 + i_3 = 0$

Example – Compute current i_1 with KCL





KCL) $-i_1 - i_2 + i_3 = 0$

$-i_1 + 1\text{m} - 2\text{m} = 0$ $\quad i_1 = -1\text{mA}$

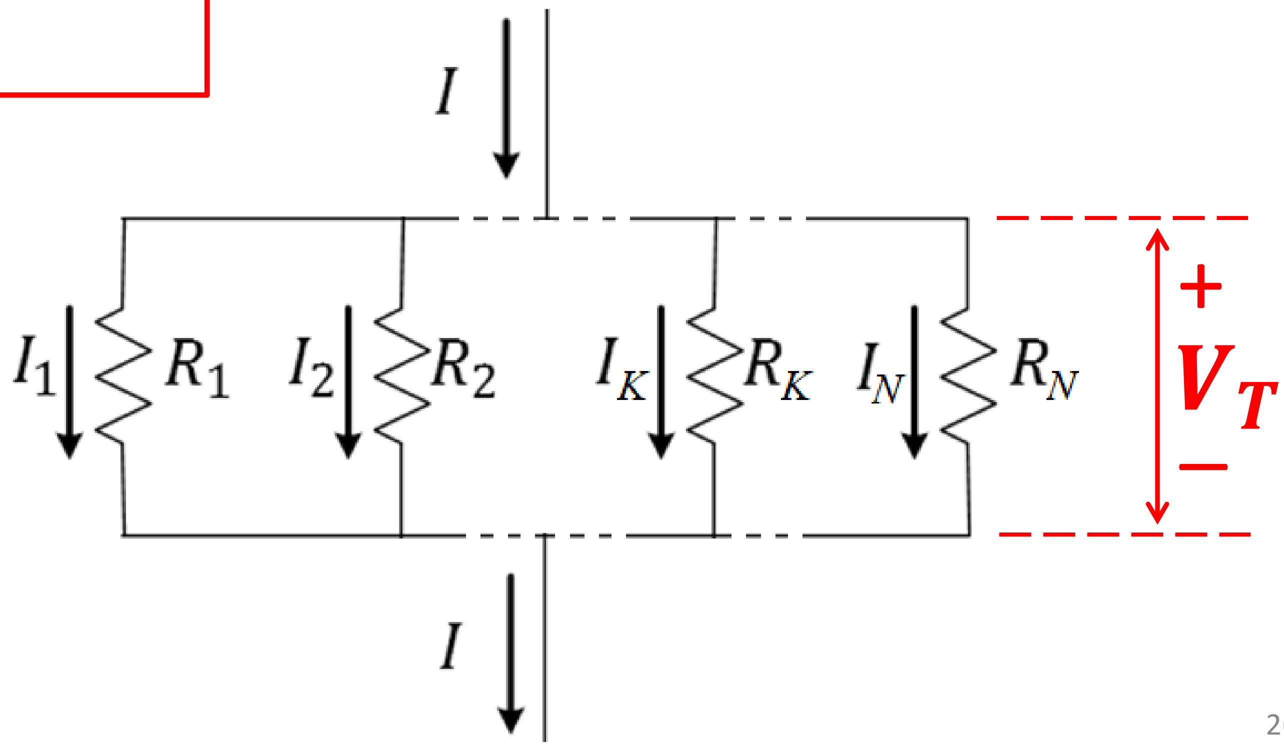
As you can see i_1 is negative. But if $i_3 = 3\text{mA}$

$-i_1 + 3\text{m} - 2\text{m} = 0$ $\quad i_1 = 1\text{mA}$

Current divider rule

When a current divides into two or more paths, more current will favor the paths of lowest resistance.

$$I_k = \frac{R_{eq}}{R_k} \cdot I$$



Simple Proof

$$R_{eq} = \left[\sum_K \frac{1}{R_K}\right]^{-1} \rightarrow \sum_K \frac{1}{R_K} = \frac{1}{R_{eq}}$$

$$I = \sum_K I_K = \sum_K \frac{1}{R_K} V_T = \frac{V_T}{R_{eq}}$$

$$I_k = \frac{V_T}{R_K}$$

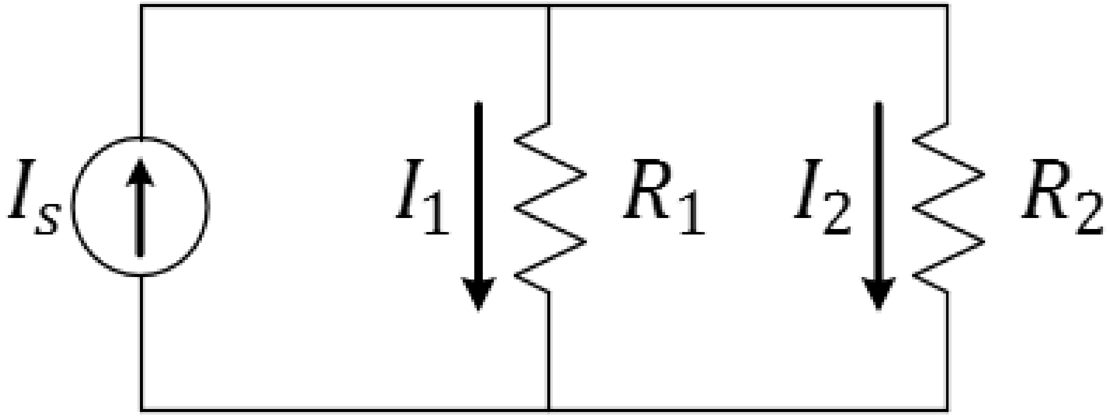
$$V_T = R_{eq}\, I$$

$$\Rightarrow$$

$$I_k = \frac{R_{eq}}{R_K}\, I$$

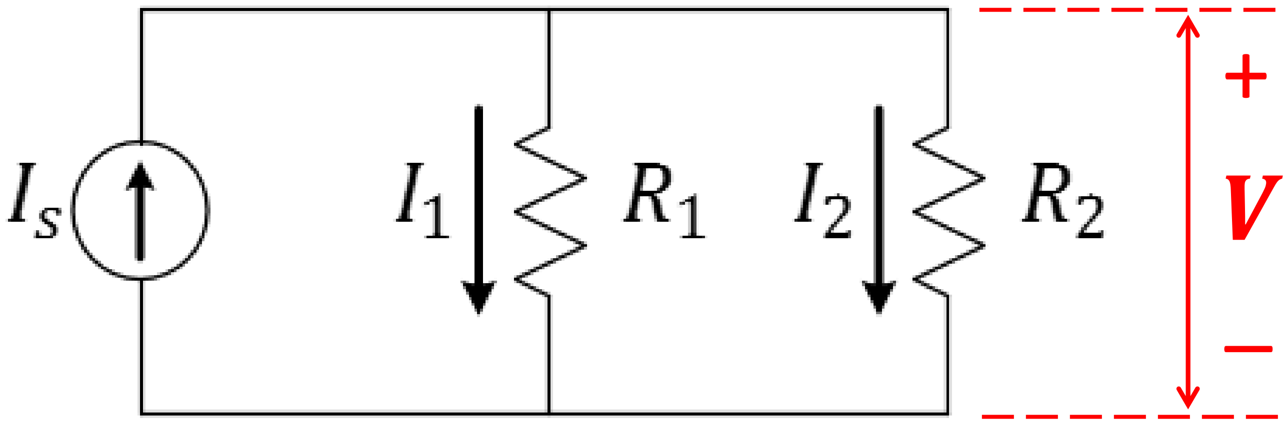
Question on Power

In a parallel connection, does a smaller or a larger resistor absorb more power?



Question on Power

In a parallel connection, does a smaller or larger resistor absorb more power?

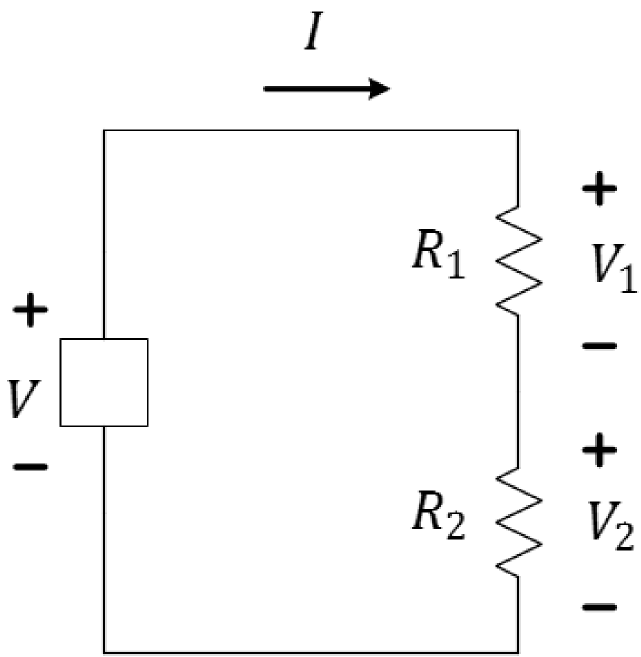


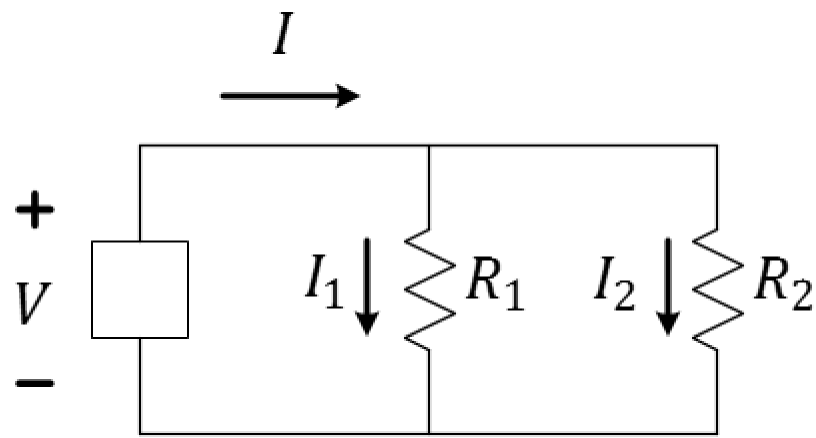
Since Power = Voltage × Current and V is the same, the smaller resistor with more current absorbs more power.

$$P = V \times I = R\,I^2 = \frac{V^2}{R}$$

Inversely proportional to *R*

Voltage Division and Current Division for Two Resistors





$$V_1 = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V$$

$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{R_1}{R_1 + R_2} I$$

Derivation for two parallel resistors

$$I_k = \frac{R_{eq}}{R_K} I$$

$$I_1 = \frac{R_{eq}}{R_1} I = \frac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}{R_1} I = \frac{\left(\frac{R_2 + R_1}{R_1 R_2}\right)^{-1}}{R_1} I$$

$$= \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1} I = \frac{R_2}{R_1 + R_2} I$$

“Node Voltage Analysis”

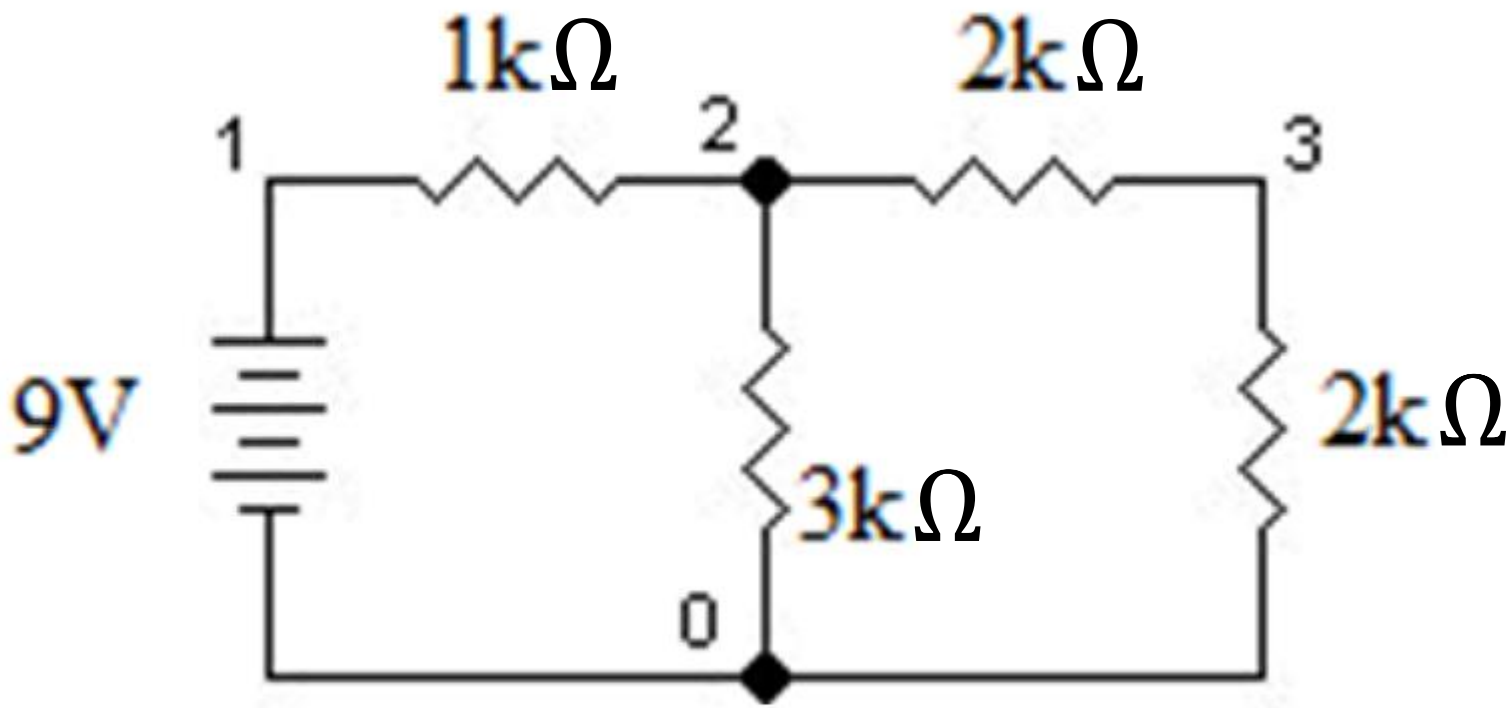
(based on KCL)

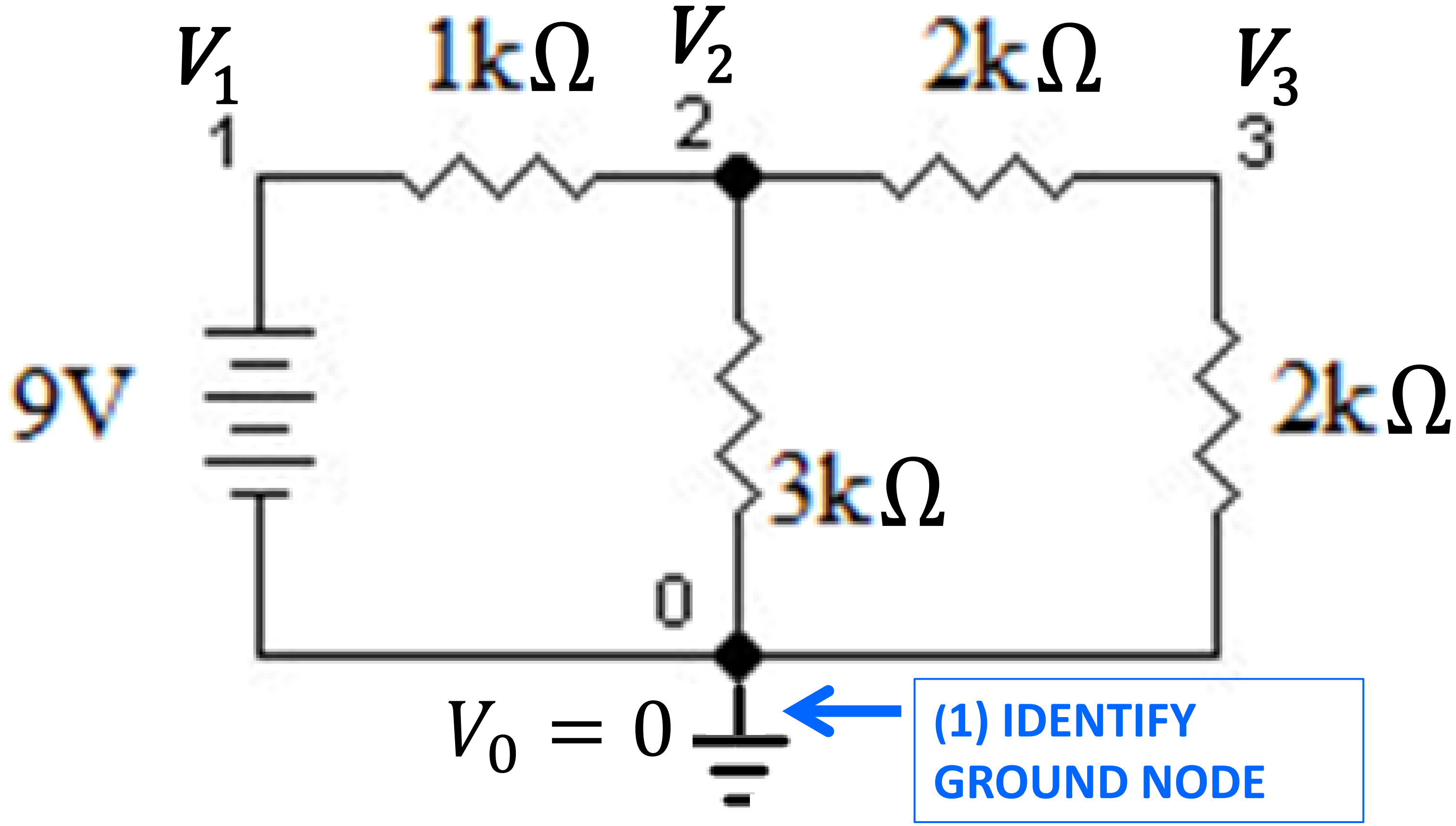
Here, we solve for voltage at nodes

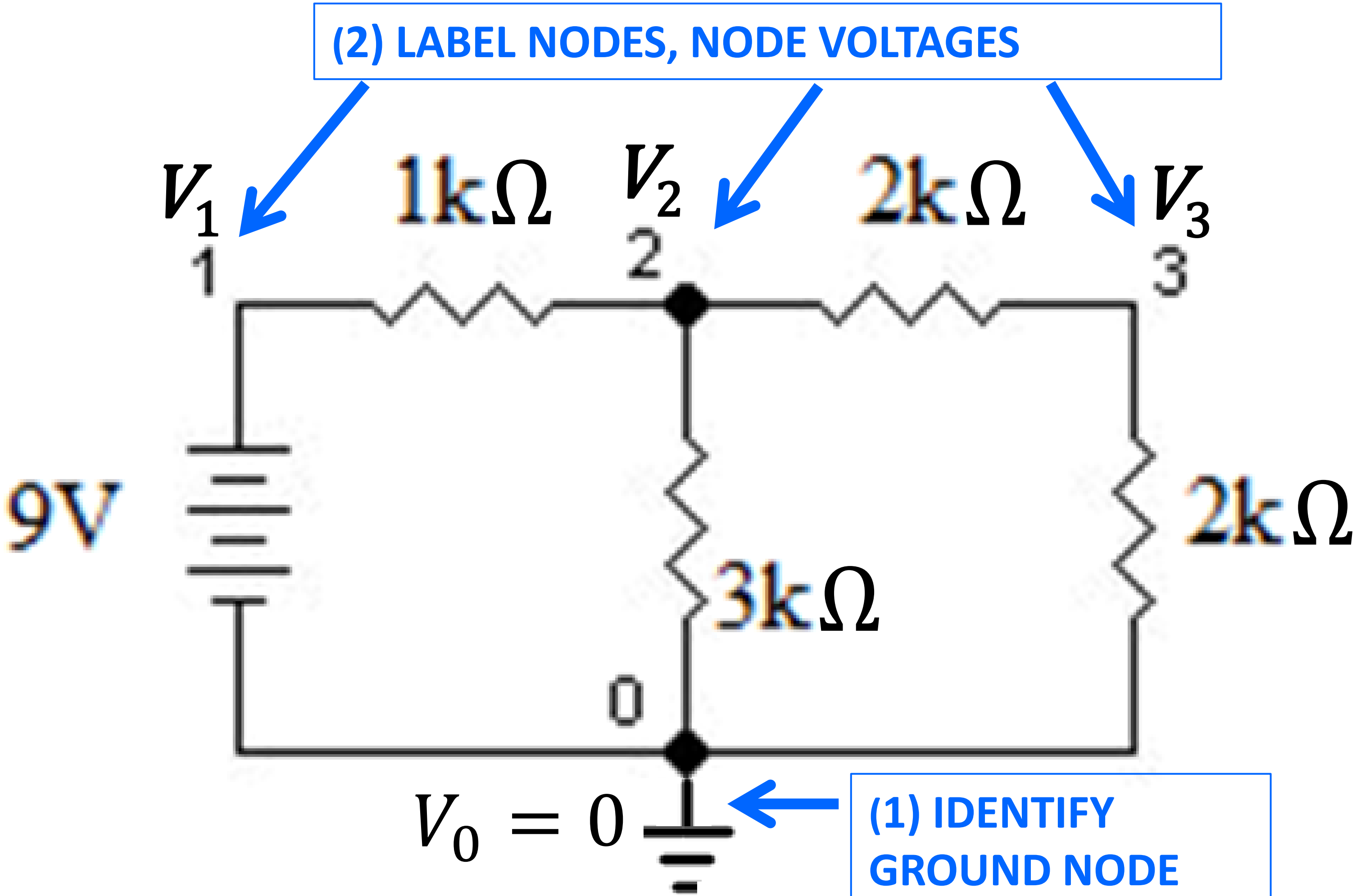
STEPS

- **Identify a node as reference ground ($V = 0$)**
- **Identify all other nodes and label them.**
- **Set up KCL at nodes**
- **Solve node equations to obtain voltages**

Let’s look at examples in detail.


1kΩ
2kΩ
1
2
3
9V
3kΩ
2kΩ
0

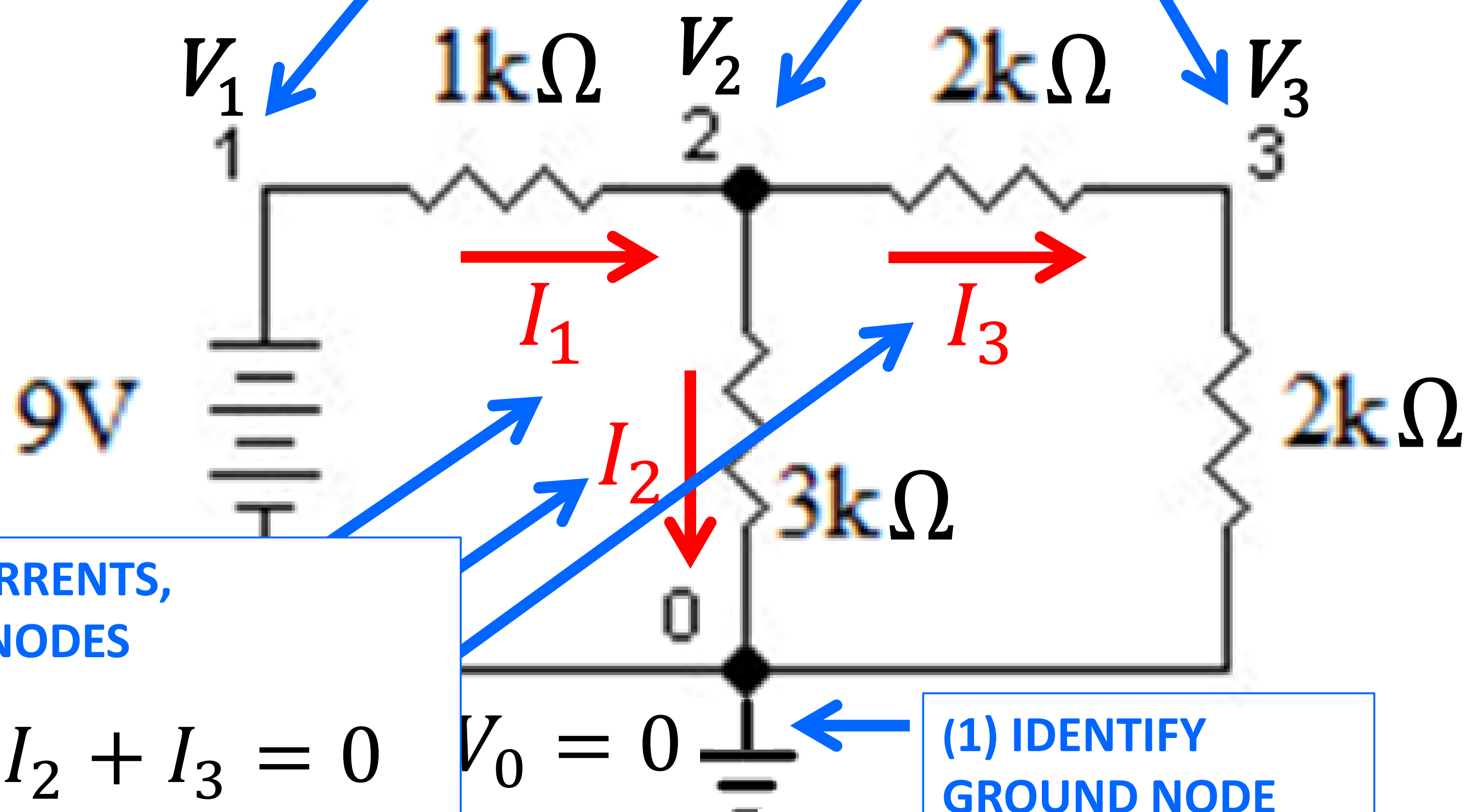

V_1
1
1kΩ
V_2
2
2kΩ
V_3
3
9V
3kΩ
2kΩ
0
$V_0 = 0$
(1) IDENTIFY
GROUND NODE



(2) LABEL NODES, NODE VOLTAGES
V_1
1
$1k\Omega$
V_2
2
$2k\Omega$
V_3
3
9V
$3k\Omega$
$2k\Omega$
0
$V_0 = 0$
(1) IDENTIFY GROUND NODE

You could now assign a fixed reference for currents. This is also good to implement computer solvers.

(2) LABEL NODES, NODE VOLTAGES



(3) LABEL CURRENTS, USE KCL AT NODES

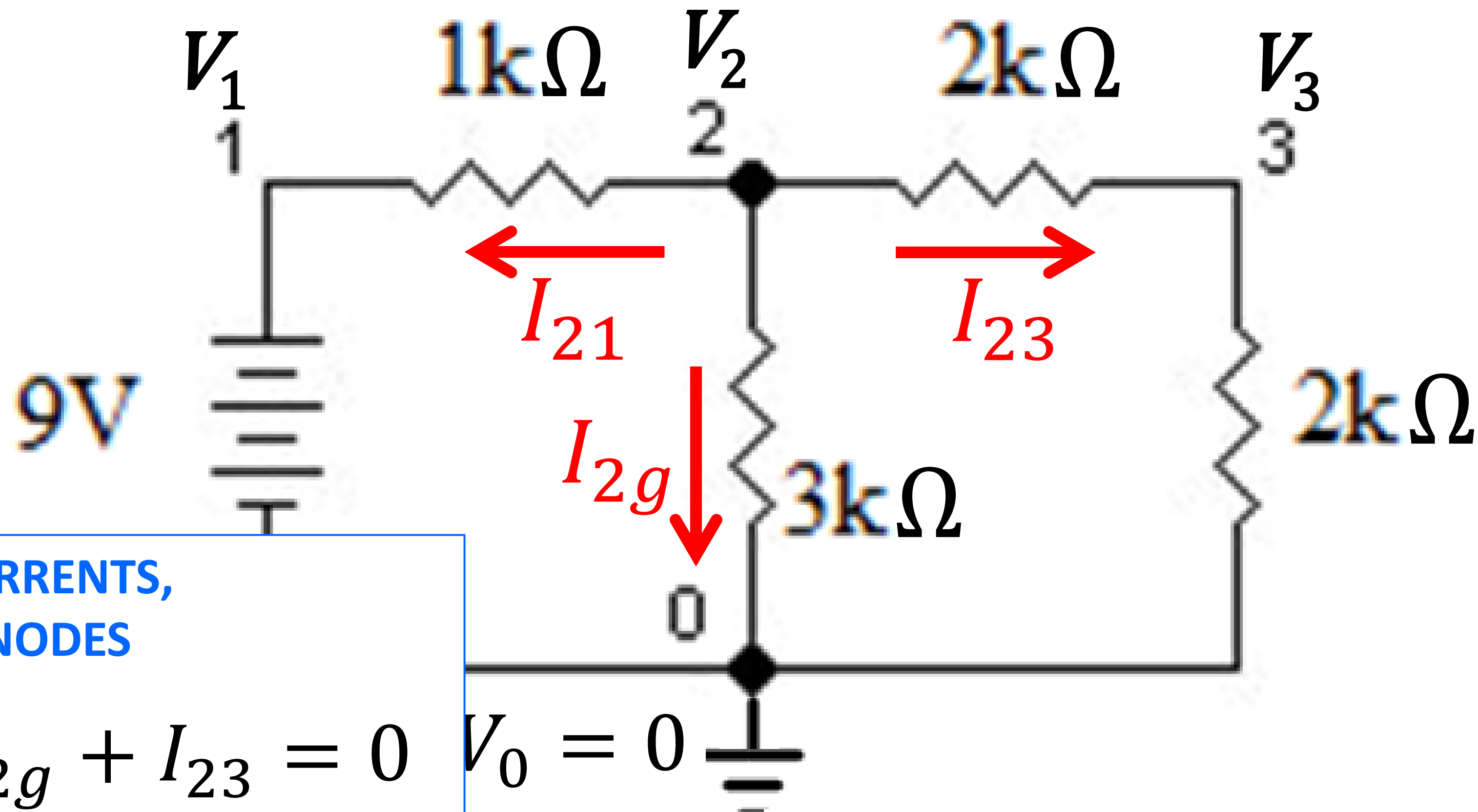
$$-I_1 + I_2 + I_3 = 0$$

(1) IDENTIFY GROUND NODE

(4) USE VOLTAGES TO FIND CURRENTS

$$I_1 = \frac{V_1 - V_2}{1k}; \; I_2 = \frac{V_2}{3k}; \; I_3 = \frac{V_2 - V_3}{2k}$$

You could also define currents using indices between a specific node and neighboring ones without specifying a fixed reference. In this case it is good to write KCL with all outgoing currents.

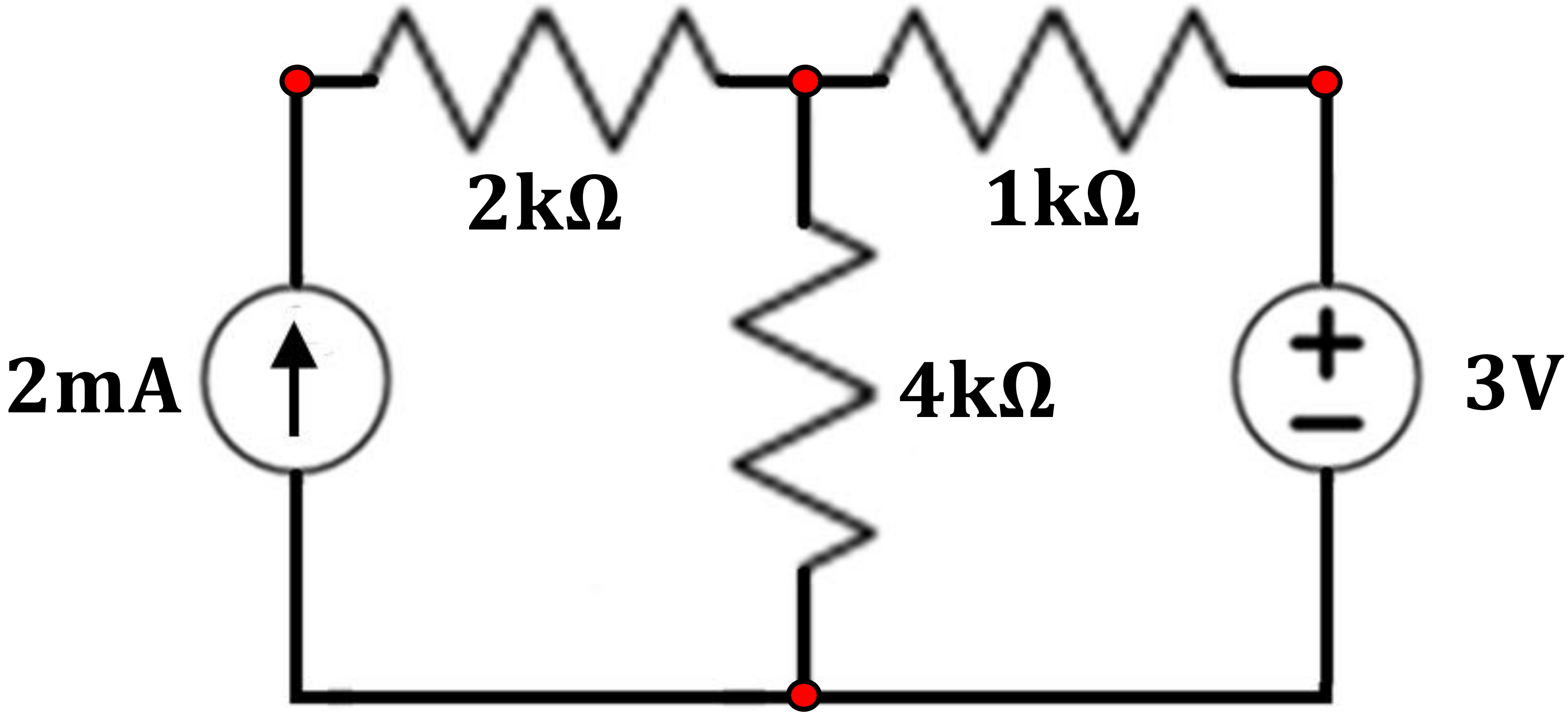


(3) LABEL CURRENTS, USE KCL AT NODES

$$I_{21} + I_{2g} + I_{23} = 0$$

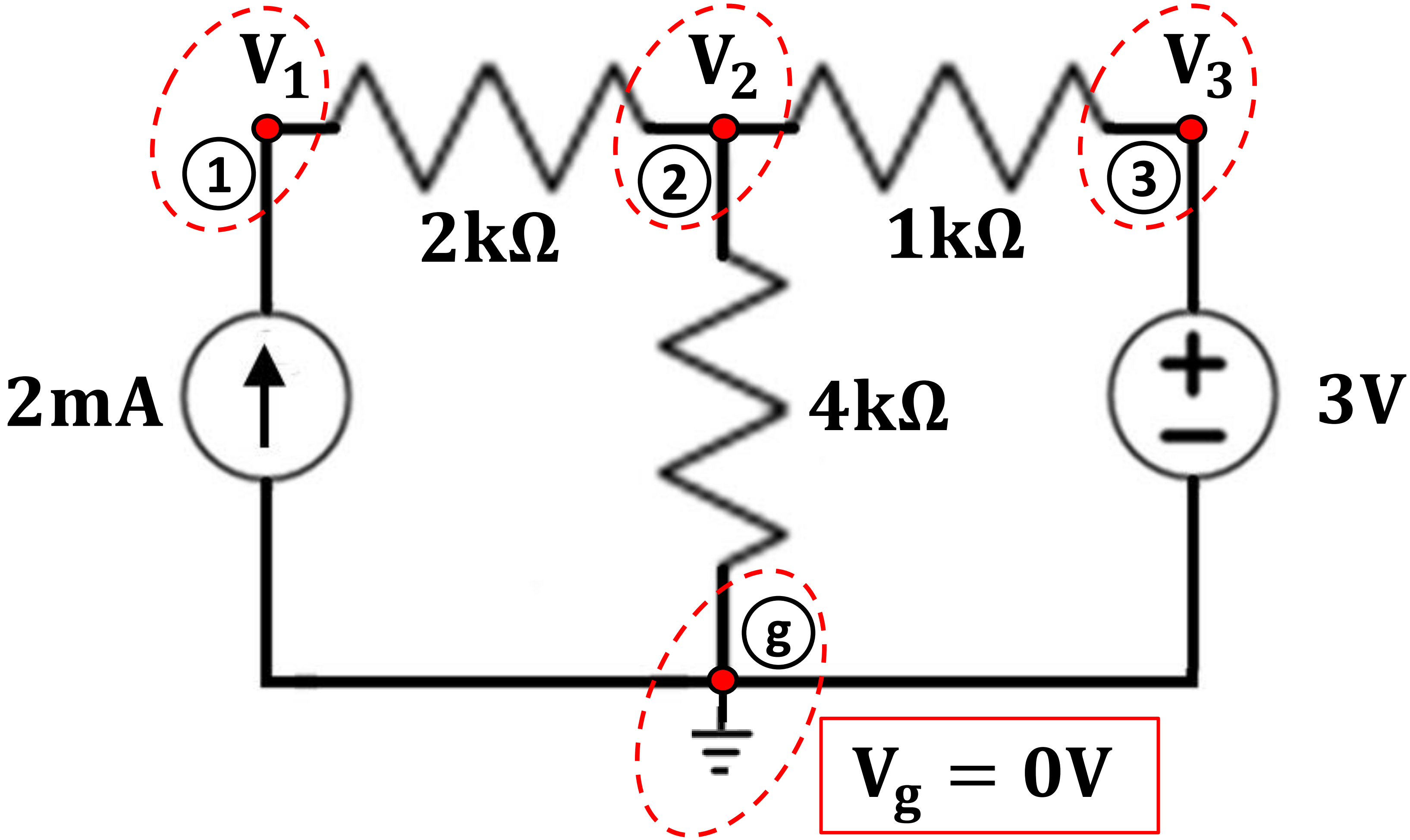
(4) USE VOLTAGES TO FIND CURRENTS

$$I_{21} = \frac{V_2 - V_1}{1k};\ I_{2g} = \frac{V_2 - 0}{3k};\ I_{23} = \frac{V_2 - V_3}{2k}$$

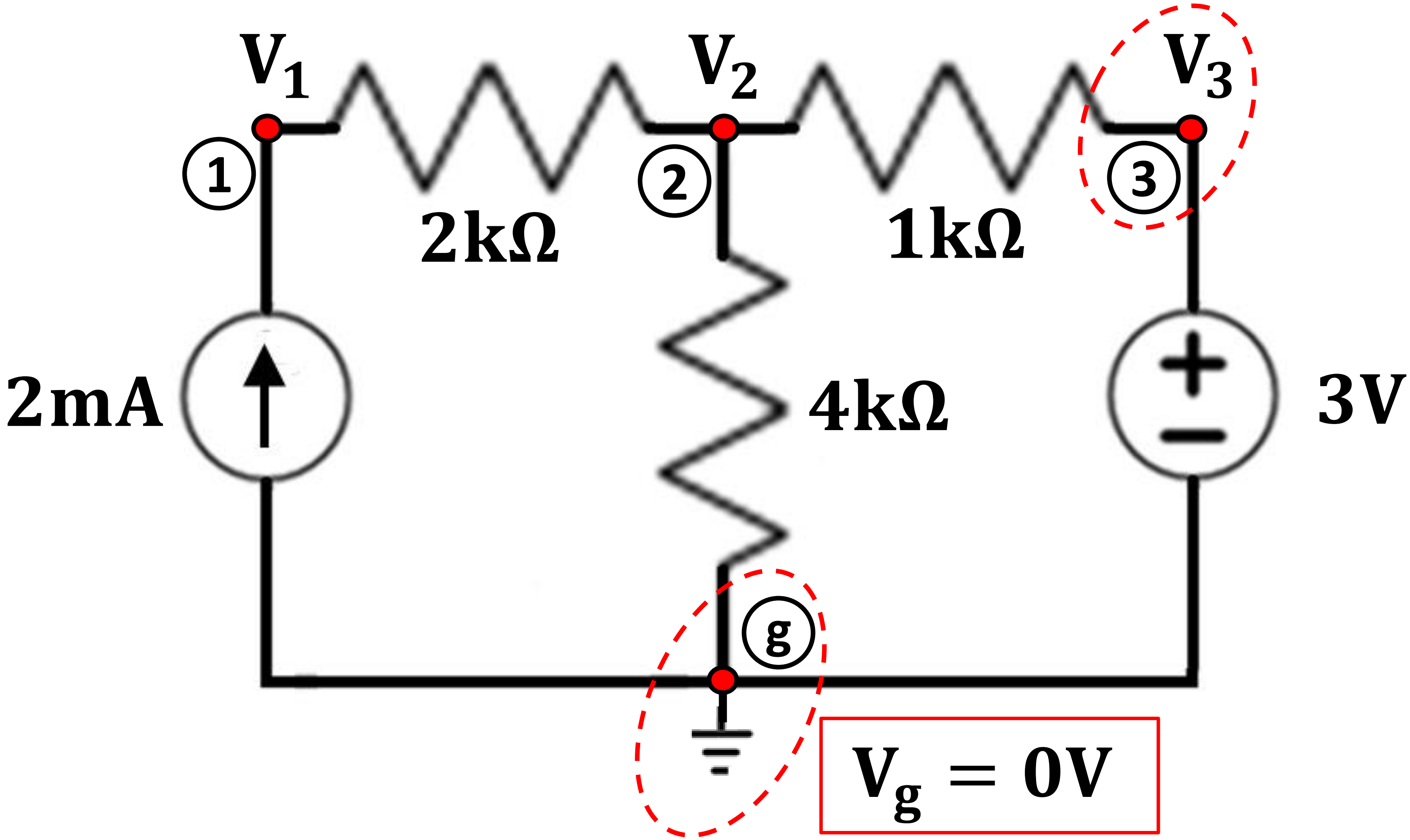


Example – Determine Voltages at circuit nodes

We will identify currents between neighboring nodes

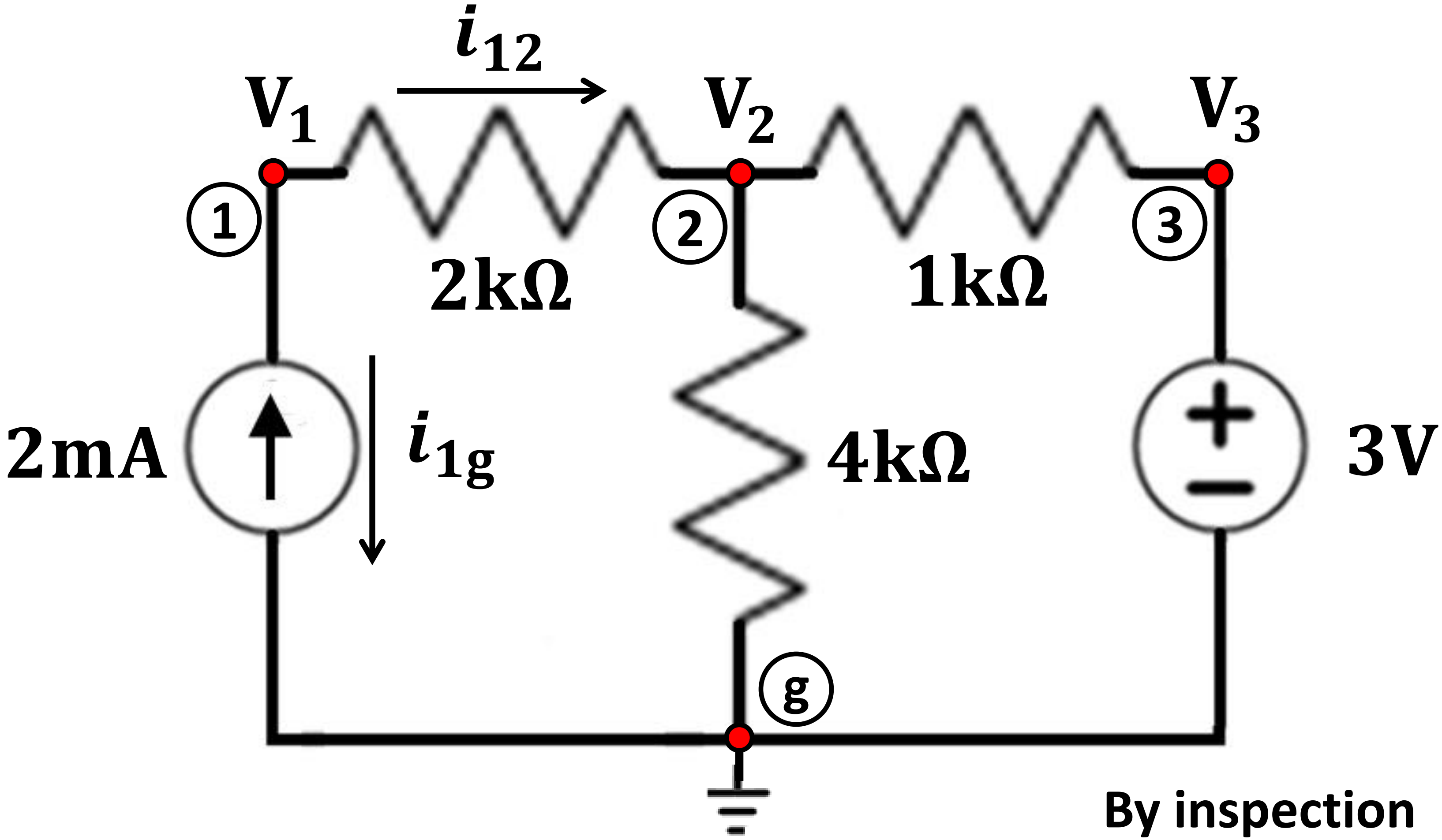


Choice of ground node at the terminal of a voltage source is a good strategy.



By inspection, $V_3 = 3V$. Need to find V_1 and V_2.

$$V_{3g} = V_3 - V_g = 3 - 0 \rightarrow V_3 = 3V$$

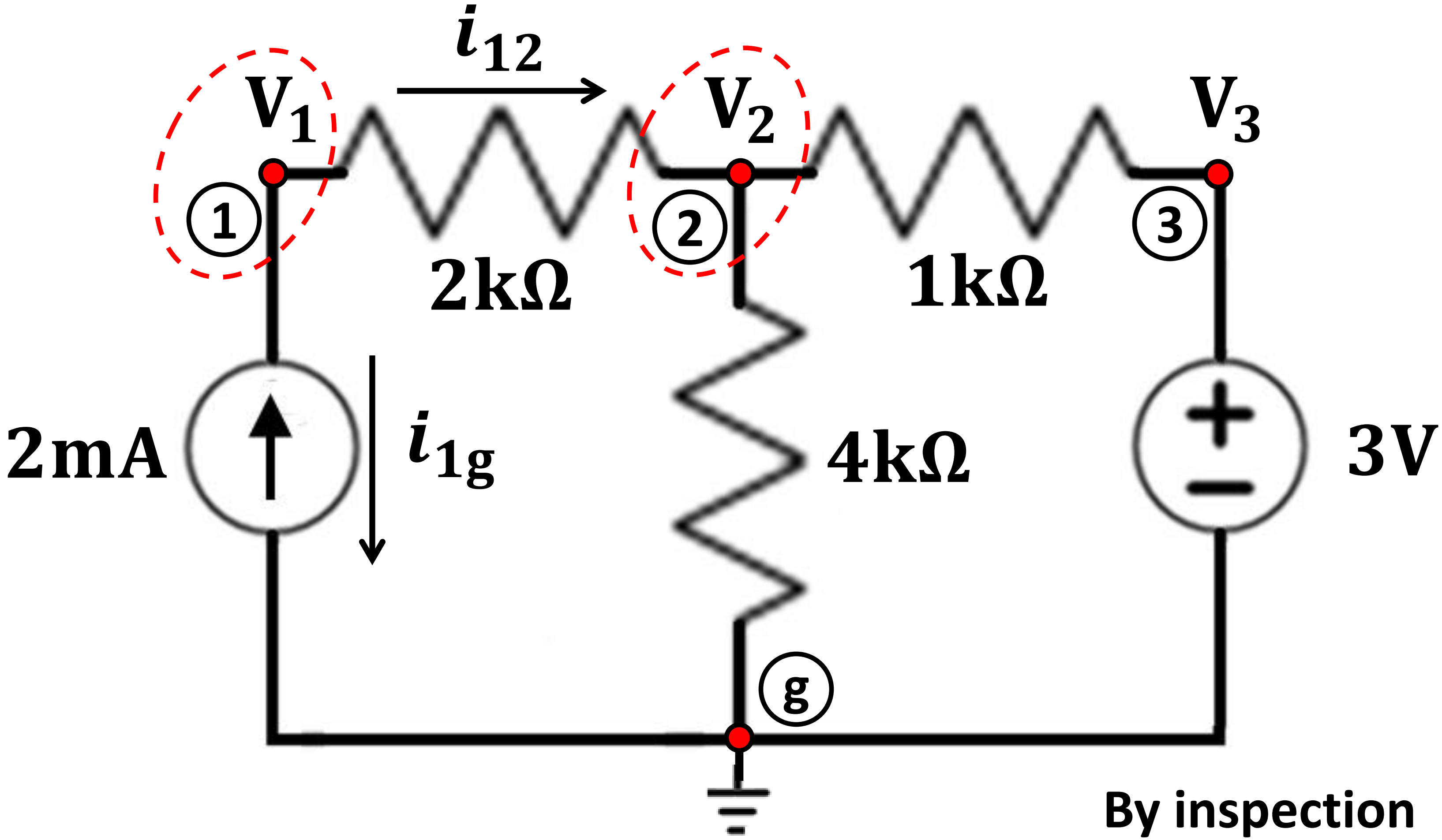


By inspection

KCL at Node 1: $i_{1g} + i_{12} = 0.$

$i_{1g} = -2\text{mA}$

You may formulate the KCL equation in different equivalent ways, but it is good to have a consistent method.

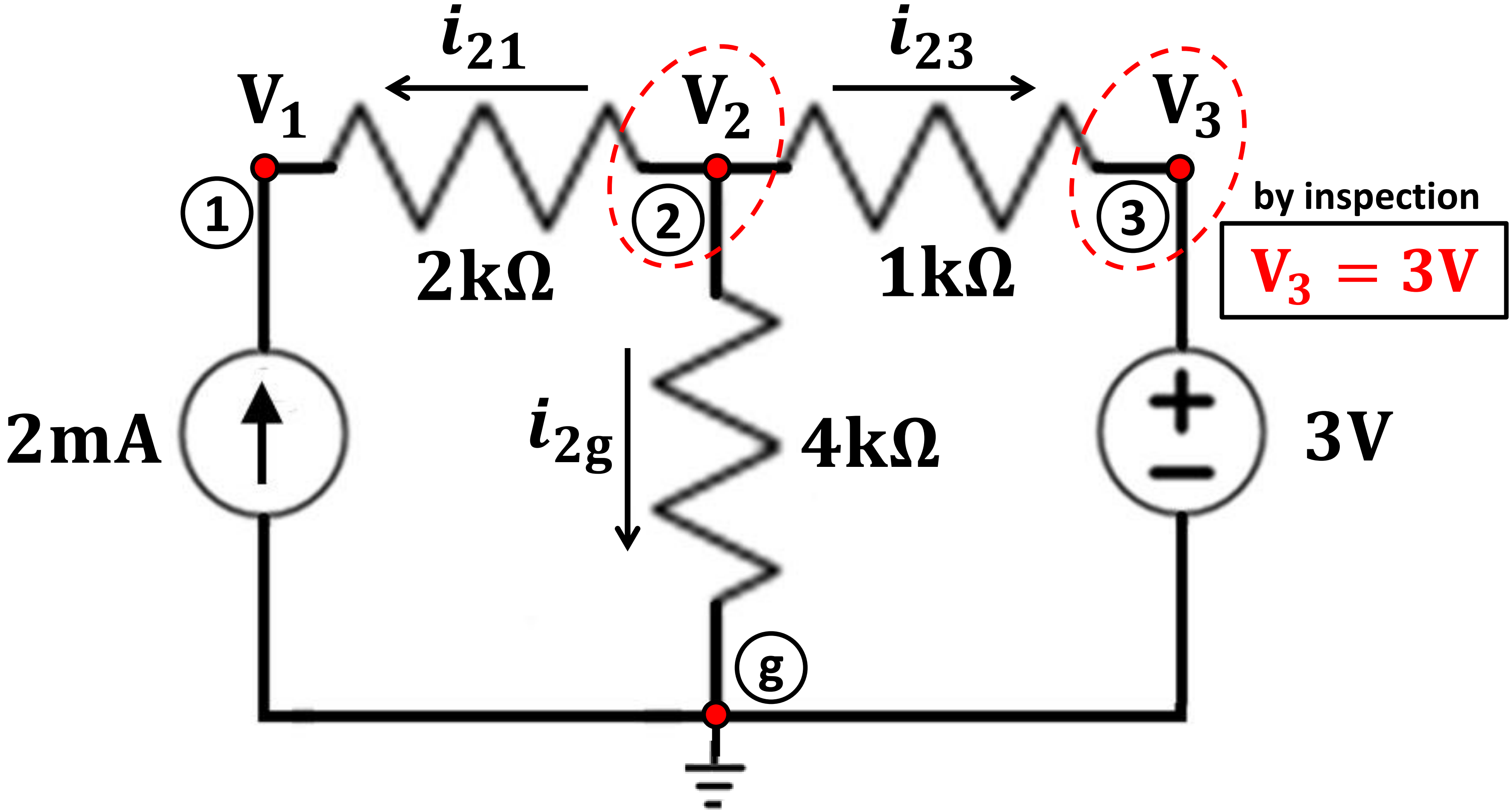


By inspection

$$i_{1g} = -2\text{mA}$$

KCL at Node 1: $i_{1g} + i_{12} = 0.$

Ohm's Law: $i_{12} = 2\text{mA} = \frac{V_{12}}{2\text{k}\Omega} = \frac{V_1 - V_2}{2\text{k}}$



KCL at Node 2: $i_{21} + i_{23} + i_{2g} = 0.$

Ohm's Law: $i_{21} = \frac{V_2 - V_1}{2k}$; $i_{2g} = \frac{V_2 - 0}{4k}$; $i_{23} = \frac{V_2 - V_3}{1k}$

$$\frac{V_2 - V_1}{2k} = i_{21} = -i_{12} = -2mA \rightarrow \frac{V_2 - V_1}{2} = -2V$$

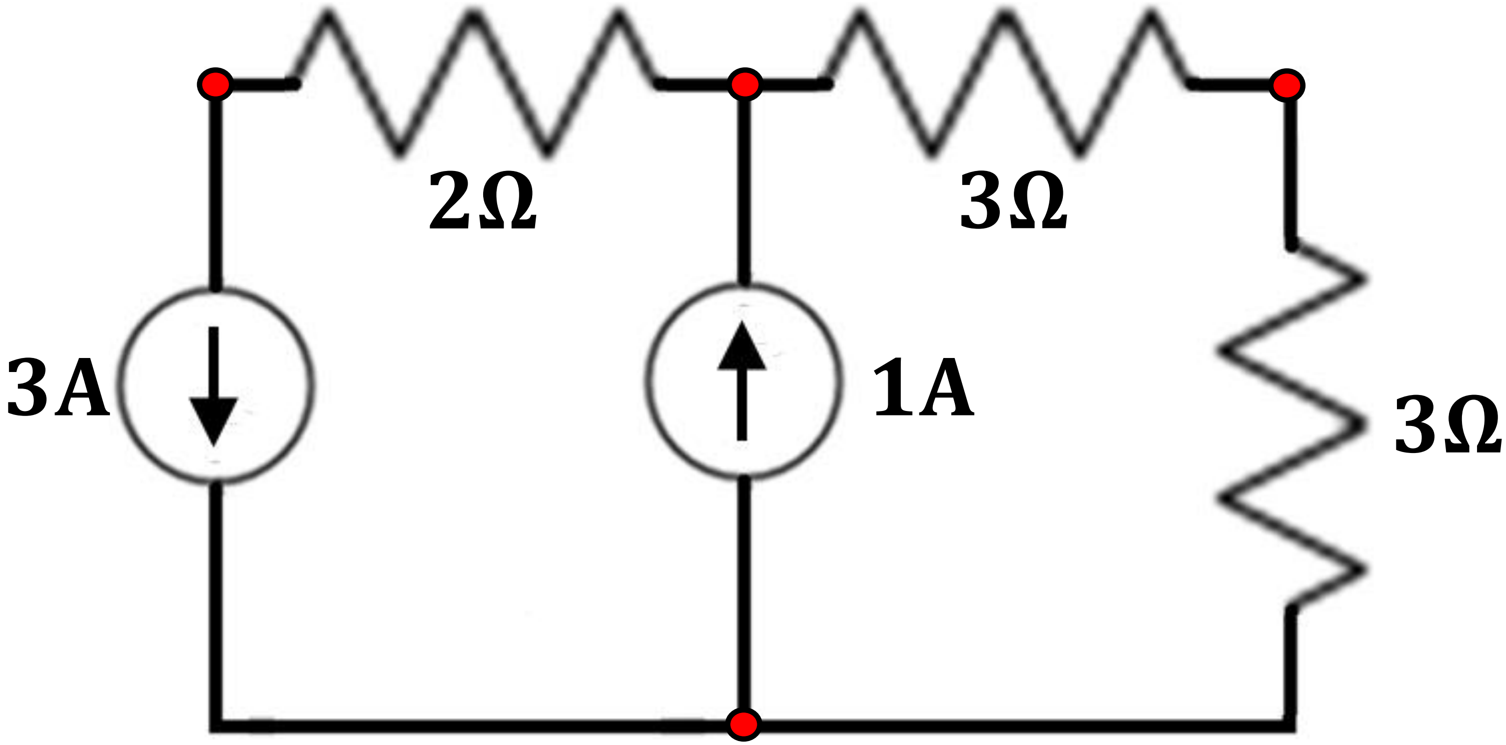
Node 1 $$2mA = \frac{V_{12}}{2k\Omega} = \frac{V_1 - V_2}{2k} \rightarrow V_1 - V_2 = 4$$

Node 2 $$\frac{V_2 - V_1}{2k} + \frac{V_2 - 0}{4k} + \frac{V_2 - V_3}{1k} = 0$$

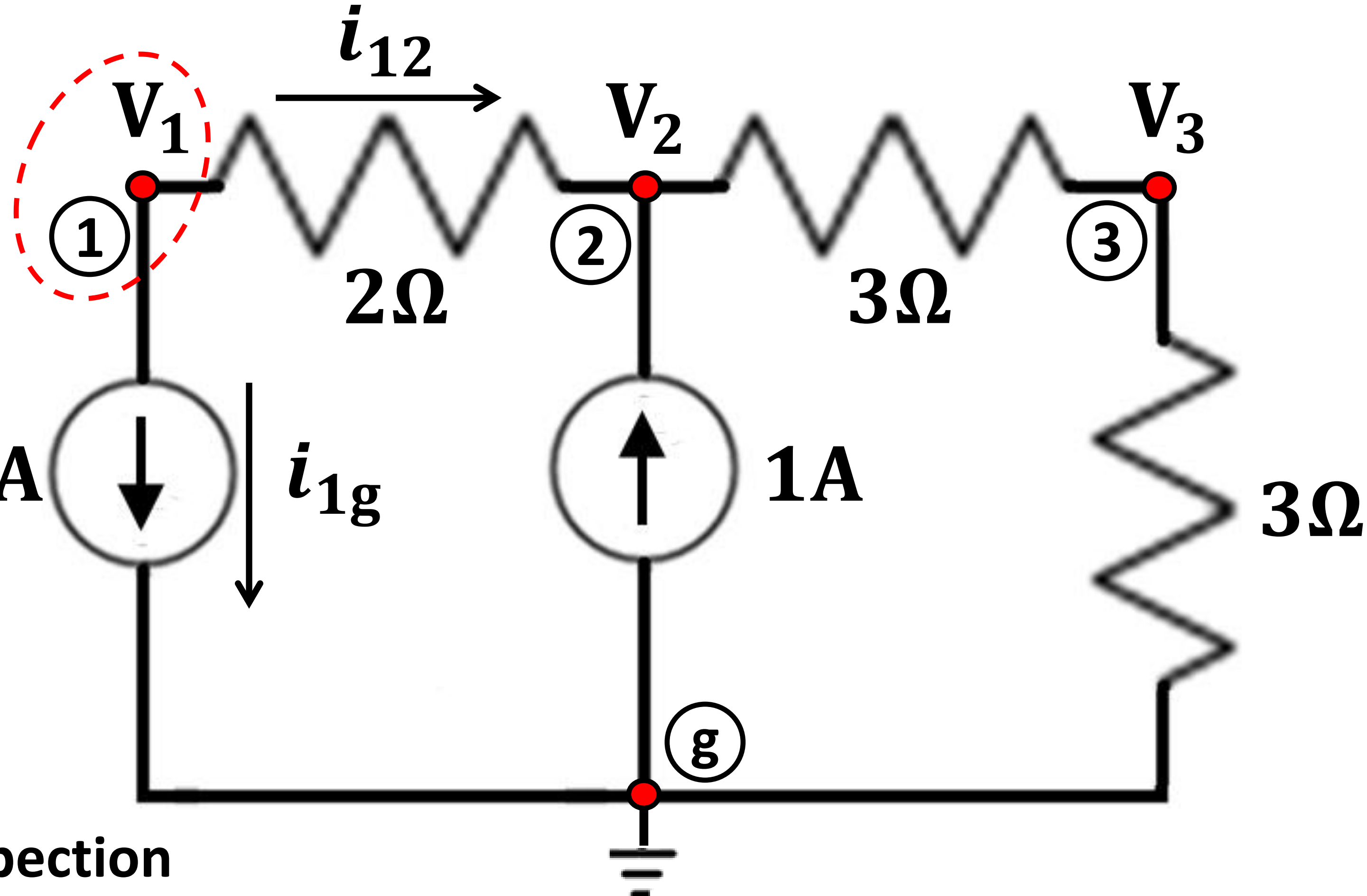
$$-2 + \frac{V_2}{4} + V_2 - 3 = 0 \rightarrow \frac{5}{4}V_2 = 5 \rightarrow \boxed{V_2 = 4V}$$

Node 3 $\boxed{V_3 = 3V}$

$\boxed{V_1 = 8V}$



Example – Determine Voltages at circuit nodes



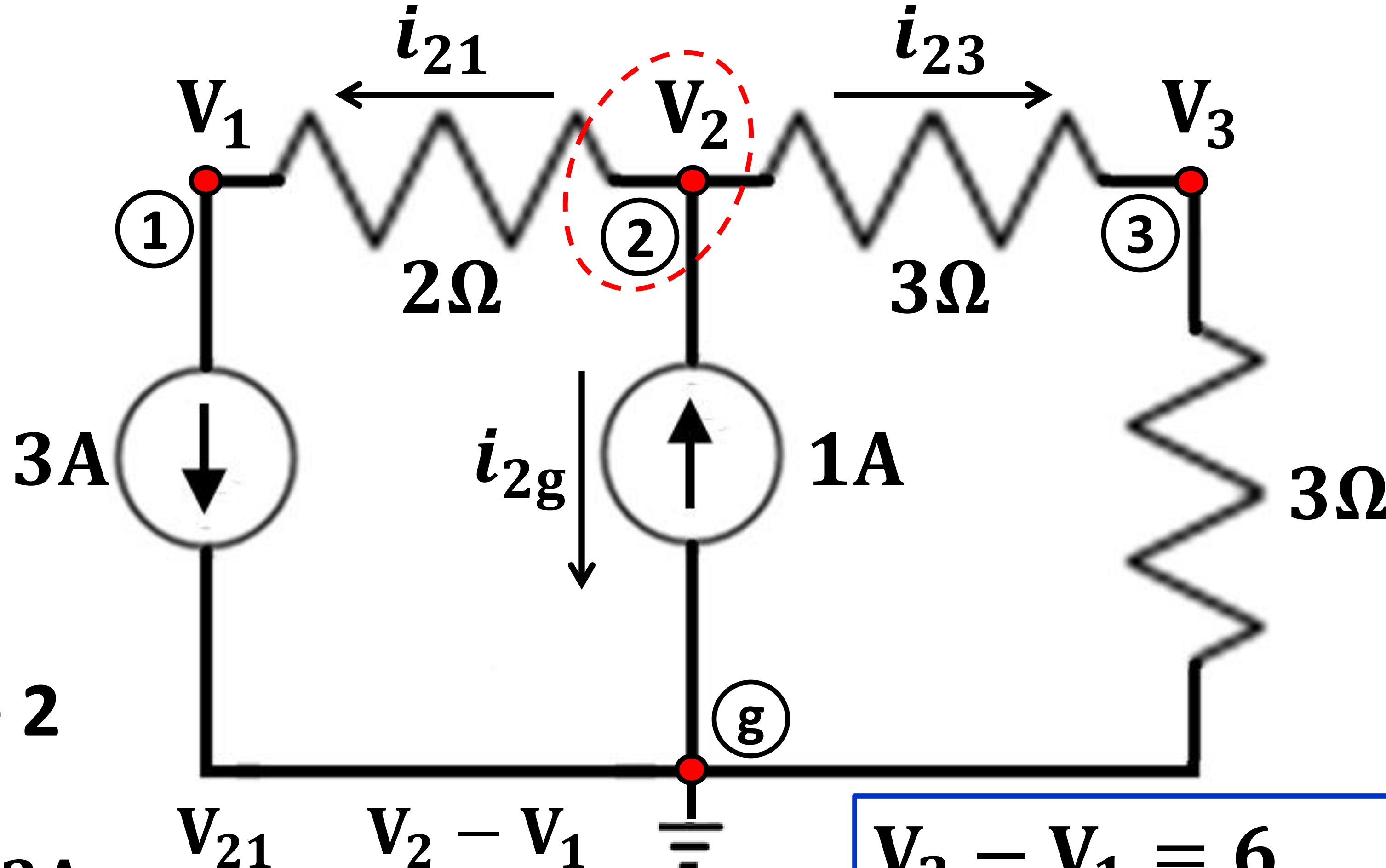
Node 1

By inspection

$$i_{1g} = -i_{12} = 3A$$

$$i_{12} = -3A = \frac{V_{12}}{2\Omega} = \frac{V_1 - V_2}{2}$$

$$V_2 - V_1 = 6\text{ V}$$



Node 2

$$i_{21} = 3A = \frac{V_{21}}{2\Omega} = \frac{V_2 - V_1}{2}$$

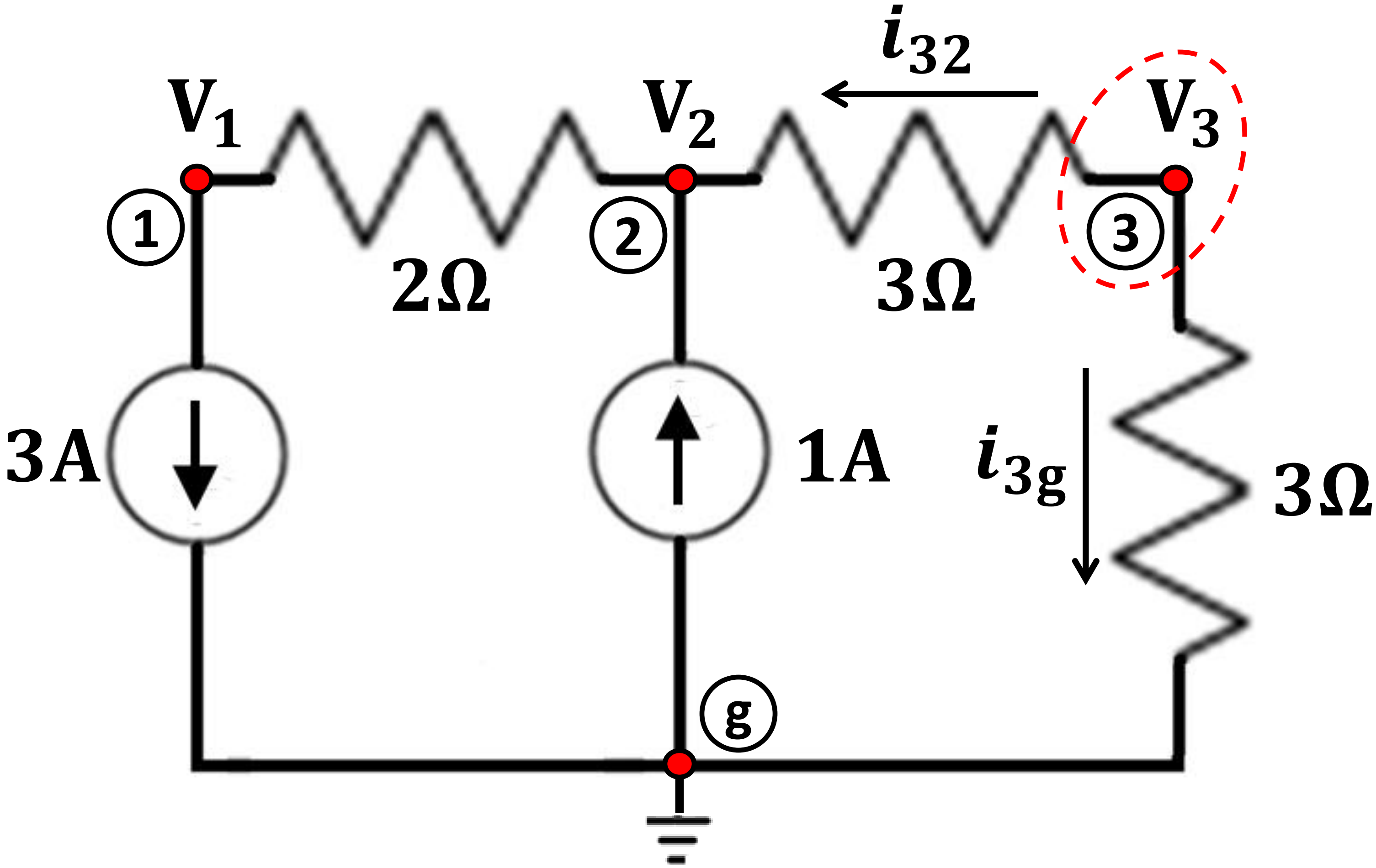
$$i_{23} = \frac{V_{23}}{3\Omega} = \frac{V_2 - V_3}{3}$$

By inspection $i_{2g} = -1A$

$$V_2 - V_1 = 6$$

$$i_{21} + i_{2g} + i_{23} = 0$$

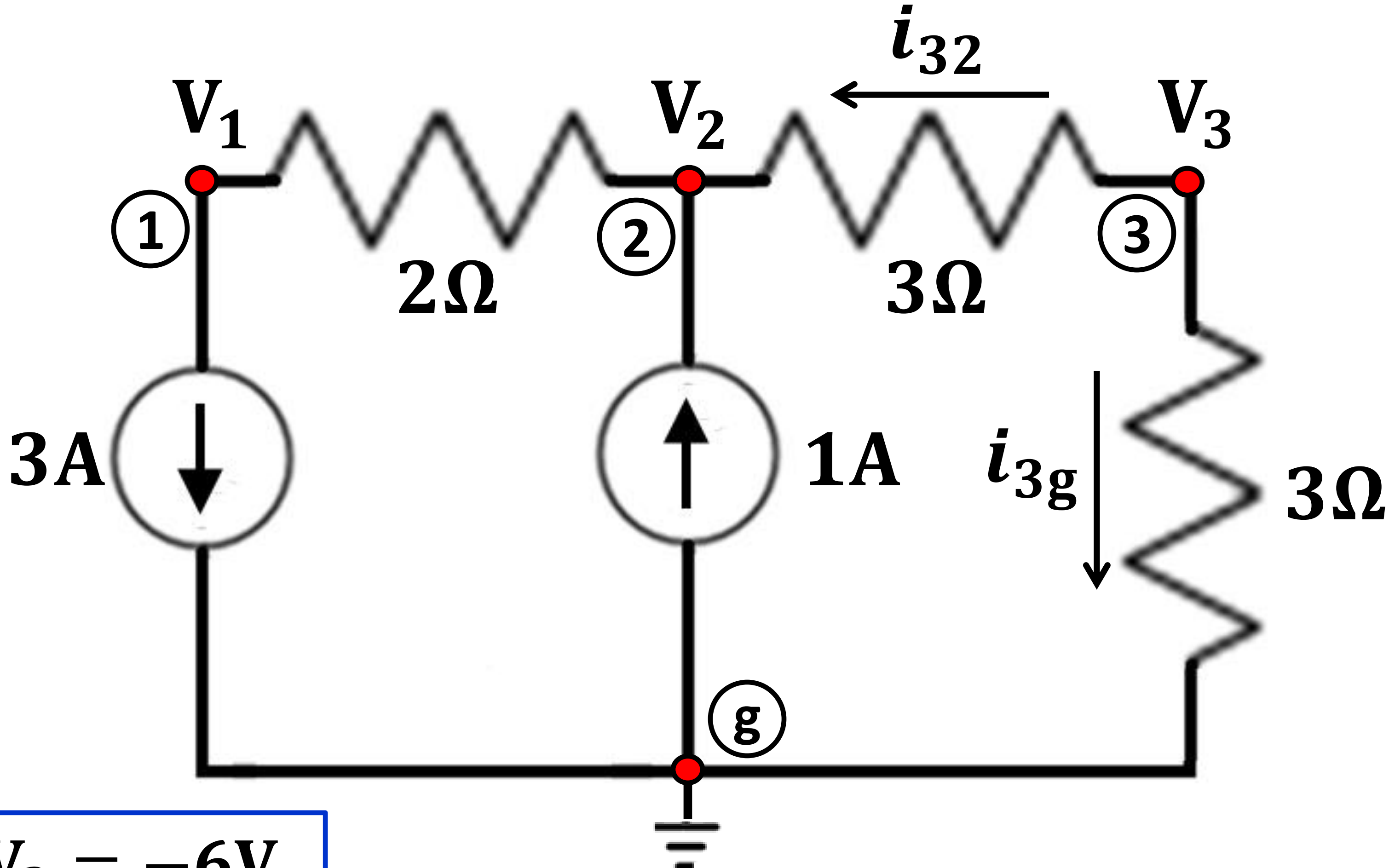
$$\frac{V_2 - V_1}{2} + \frac{V_2 - V_3}{3} - 1 = 0$$



Node 3 $i_{3g} = -i_{32} = \frac{V_{3g}}{3\Omega} = \frac{V_3 - 0}{3}$

ⓖ $i_{3g} = -3A + 1A = -2A$

$$V_3 = i_{3g} \times 3\Omega = -2 \times 3 = -6V$$

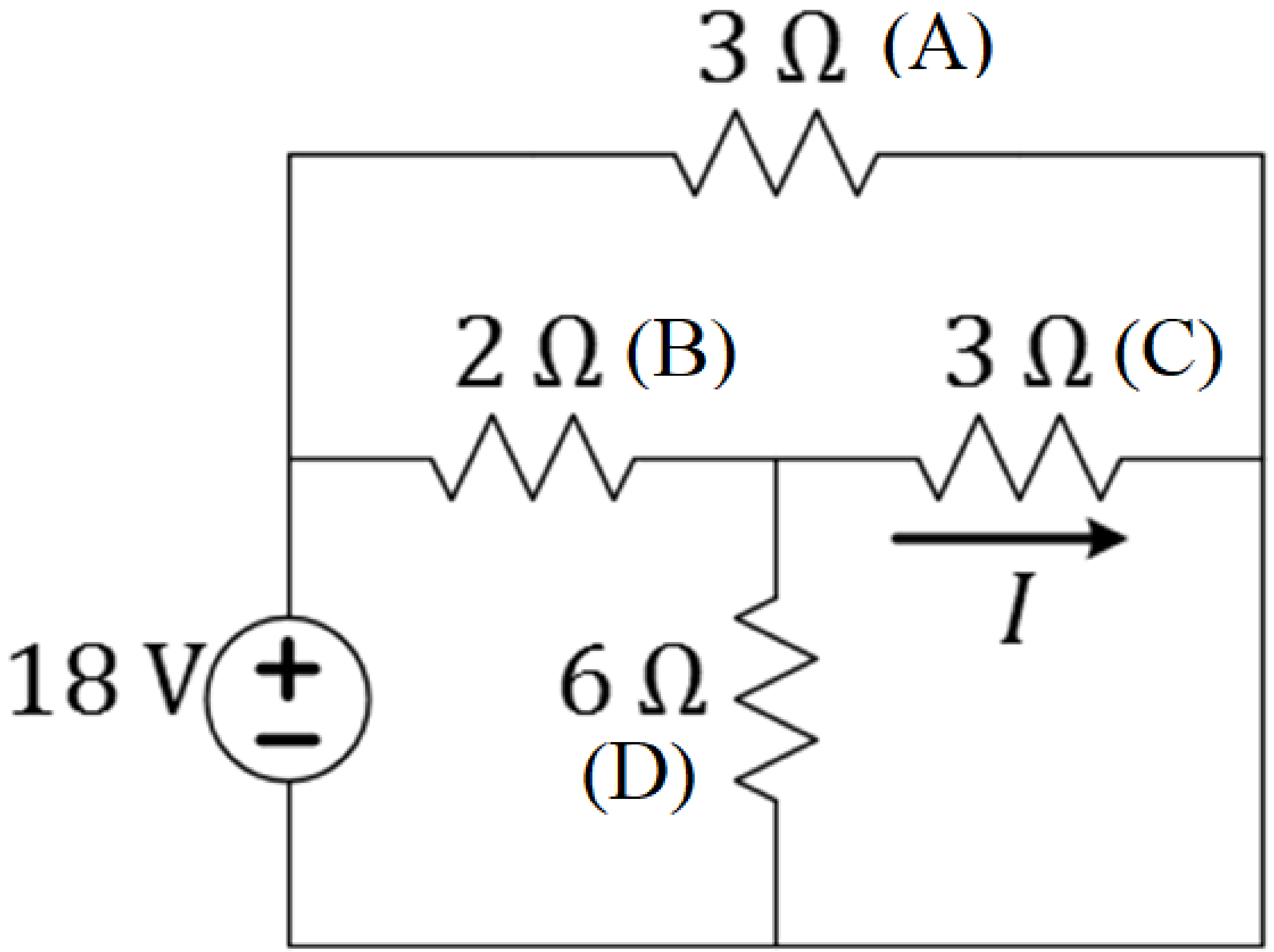


$$V_3 = -6V$$

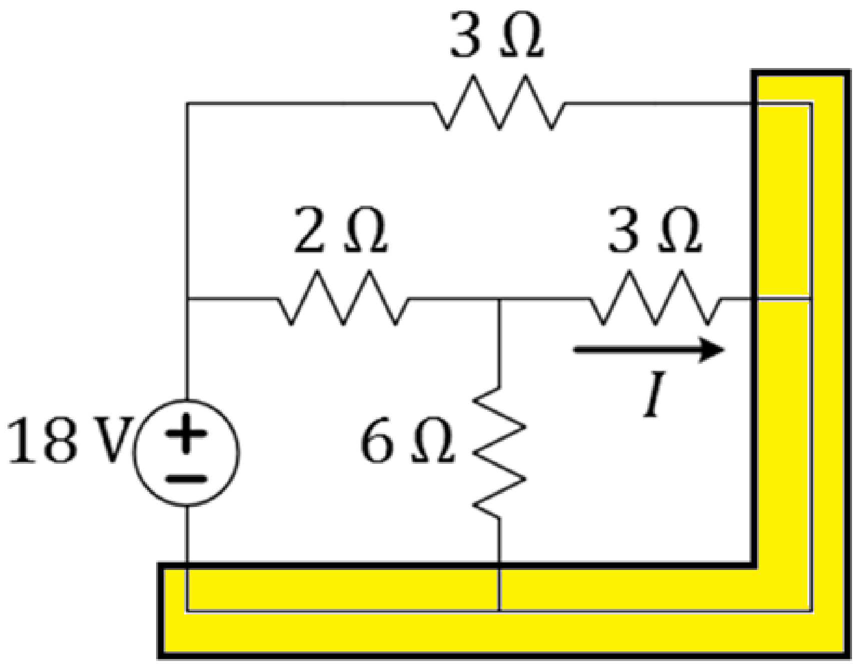
$$V_2 = V_3 + i_{3g} \times 3\Omega = -6V - 2A \times 3\Omega = -12V$$

$$V_1 = V_2 + i_{12} \times 2\Omega = -12V - 3A \times 2\Omega = -18V$$

Find the labelled current *I*



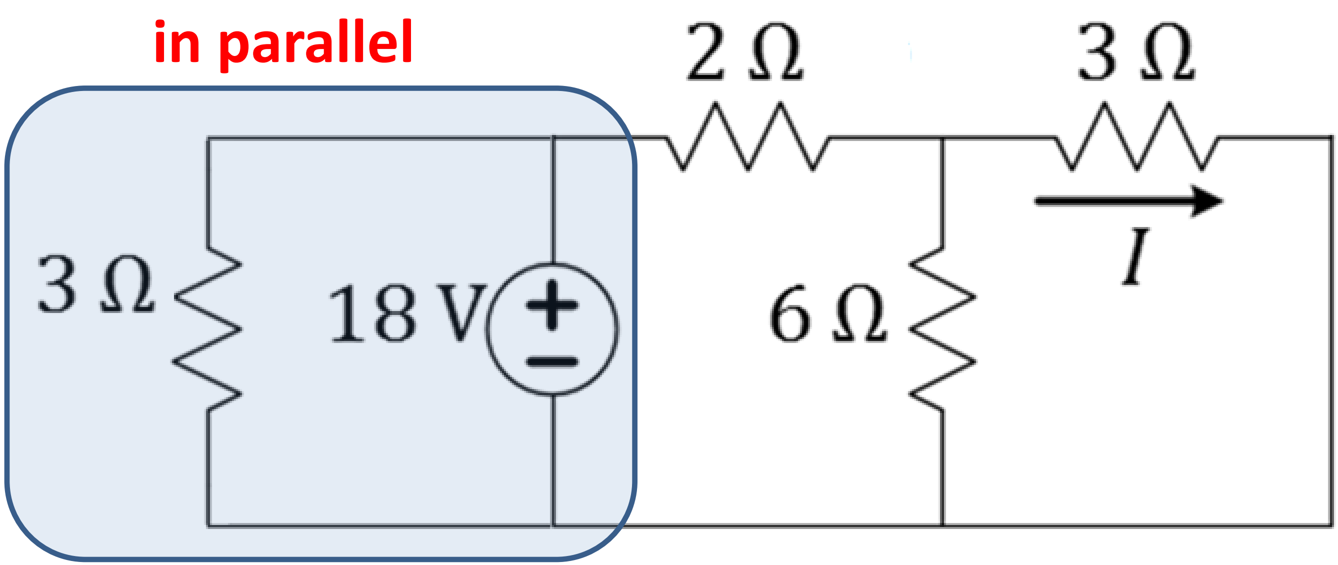
Q: Which resistor is in parallel with the voltage source?



All these wires are at the same potential

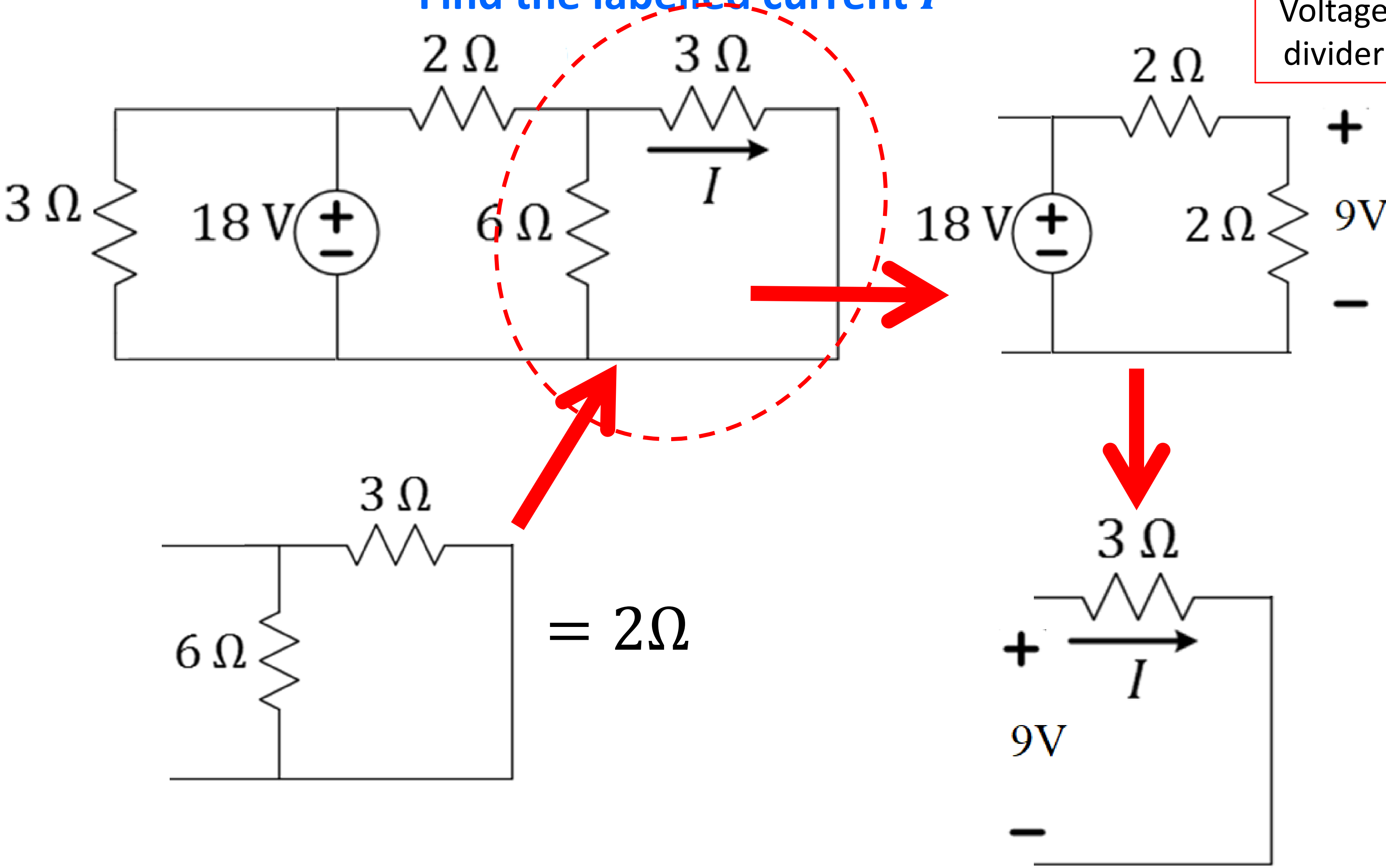
This problem can be solved very quickly without node voltage analysis

We can rearrange the diagram as



Find the labelled current I

Voltage divider



$$I = 3A$$