ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 7

MWF – 12:00pm

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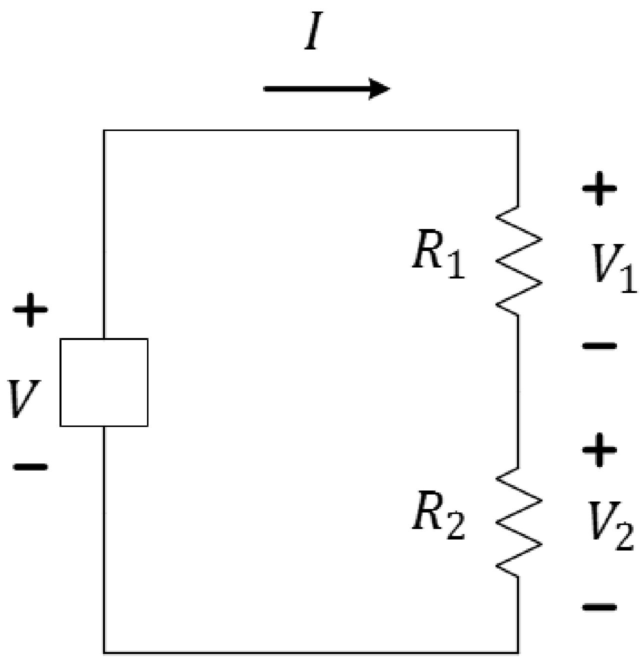
2062 ECE Building

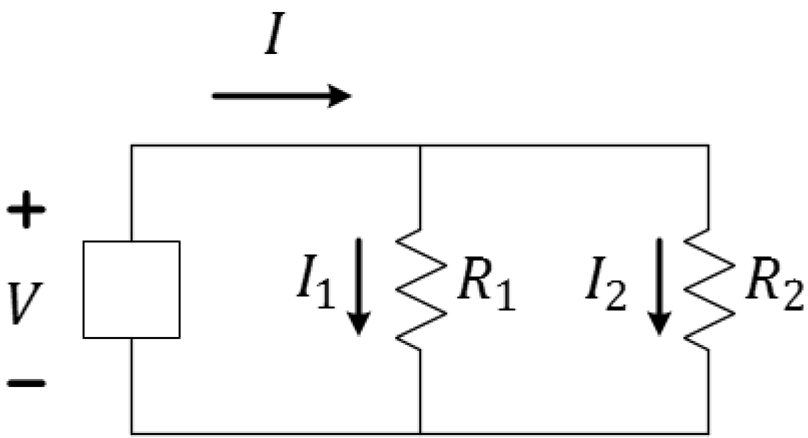
Lecture 7 – Summary

Learning Objectives

1. Node analysis method to compute node voltages
2. Introduce the concept or "supernodes" to treat circuit branches with floating voltage sources (if time allows)

Voltage Division and Current Division for Two Resistors





$$V_1 = \frac{R_1}{R_1 + R_2} V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V$$

$$I_1 = \frac{R_2}{R_1 + R_2} I$$

$$I_2 = \frac{R_1}{R_1 + R_2} I$$

Derivation for two parallel resistors

$$I_k = \frac{R_{eq}}{R_K} I$$

$$I_1 = \frac{R_{eq}}{R_1} I = \frac{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}{R_1} I = \frac{\left(\frac{R_2 + R_1}{R_1 R_2}\right)^{-1}}{R_1} I$$

$$= \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1} I = \frac{R_2}{R_1 + R_2} I$$

“Node Voltage Analysis”

(based on KCL)

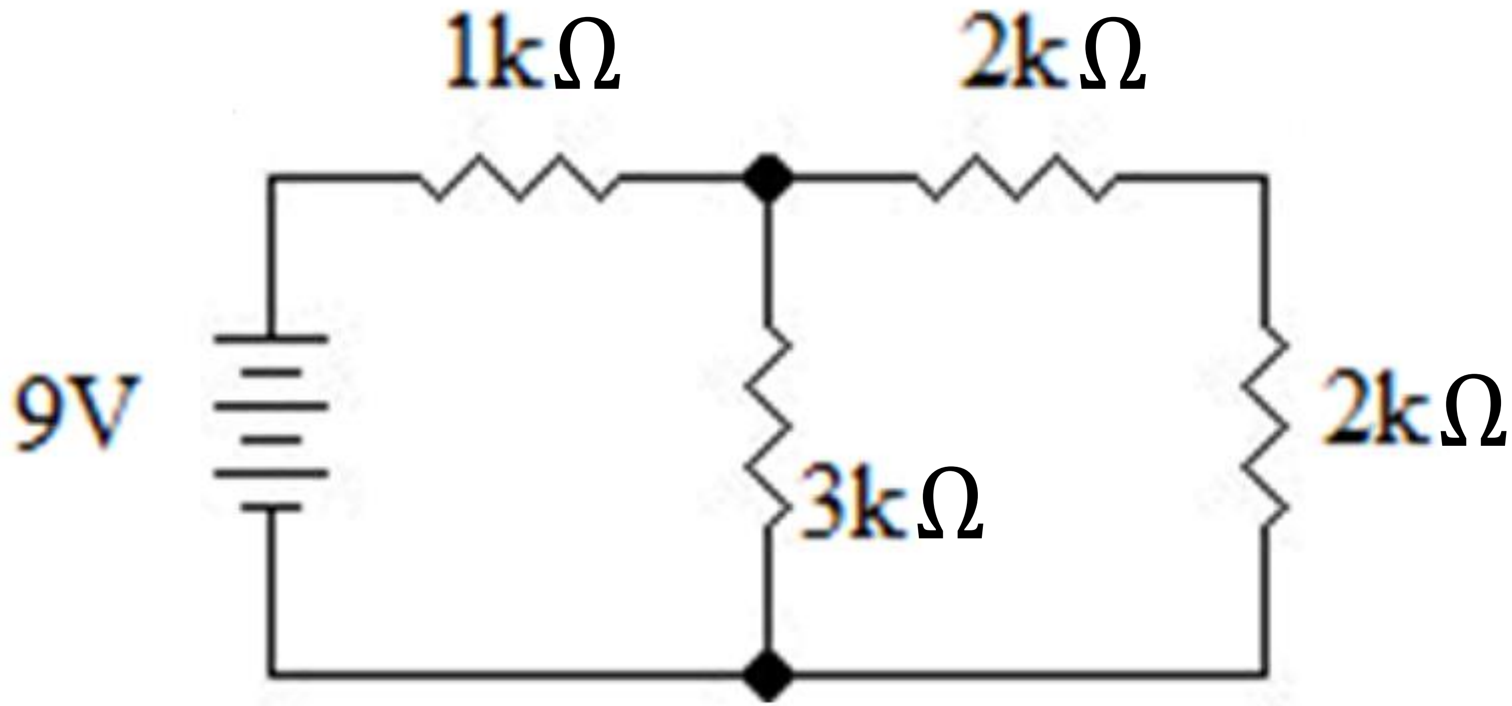
Here, we solve for voltage at nodes

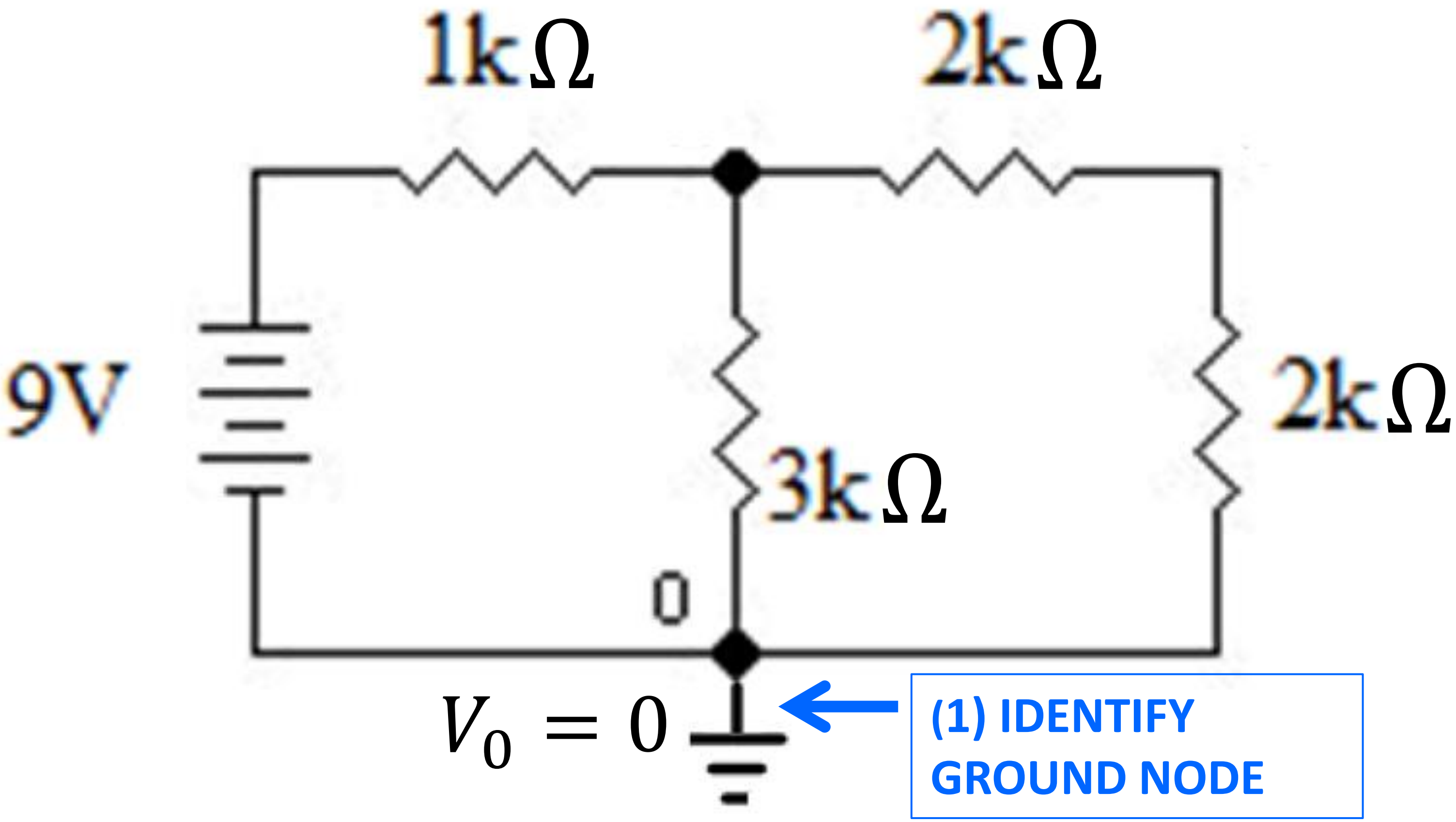
STEPS

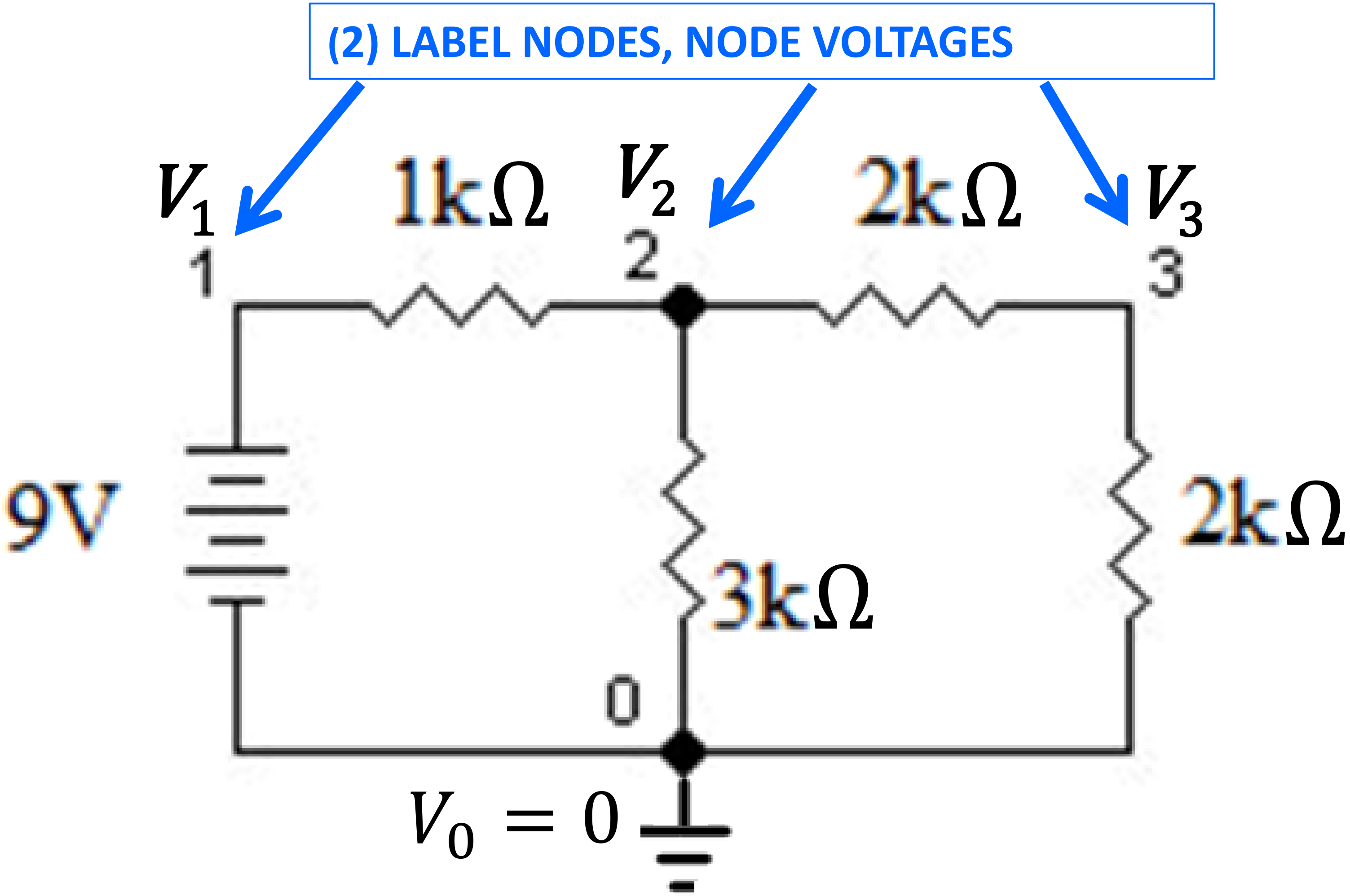
- **Identify a node as reference ground ($V = 0$)**
- **Identify all other nodes and label them.**
- **Set up KCL equations at nodes (using Ohm’s law to write currents in terms of voltages)**
- **Solve node equations to obtain voltages**

Let’s look at examples in detail.

As a start, a very simple prototype

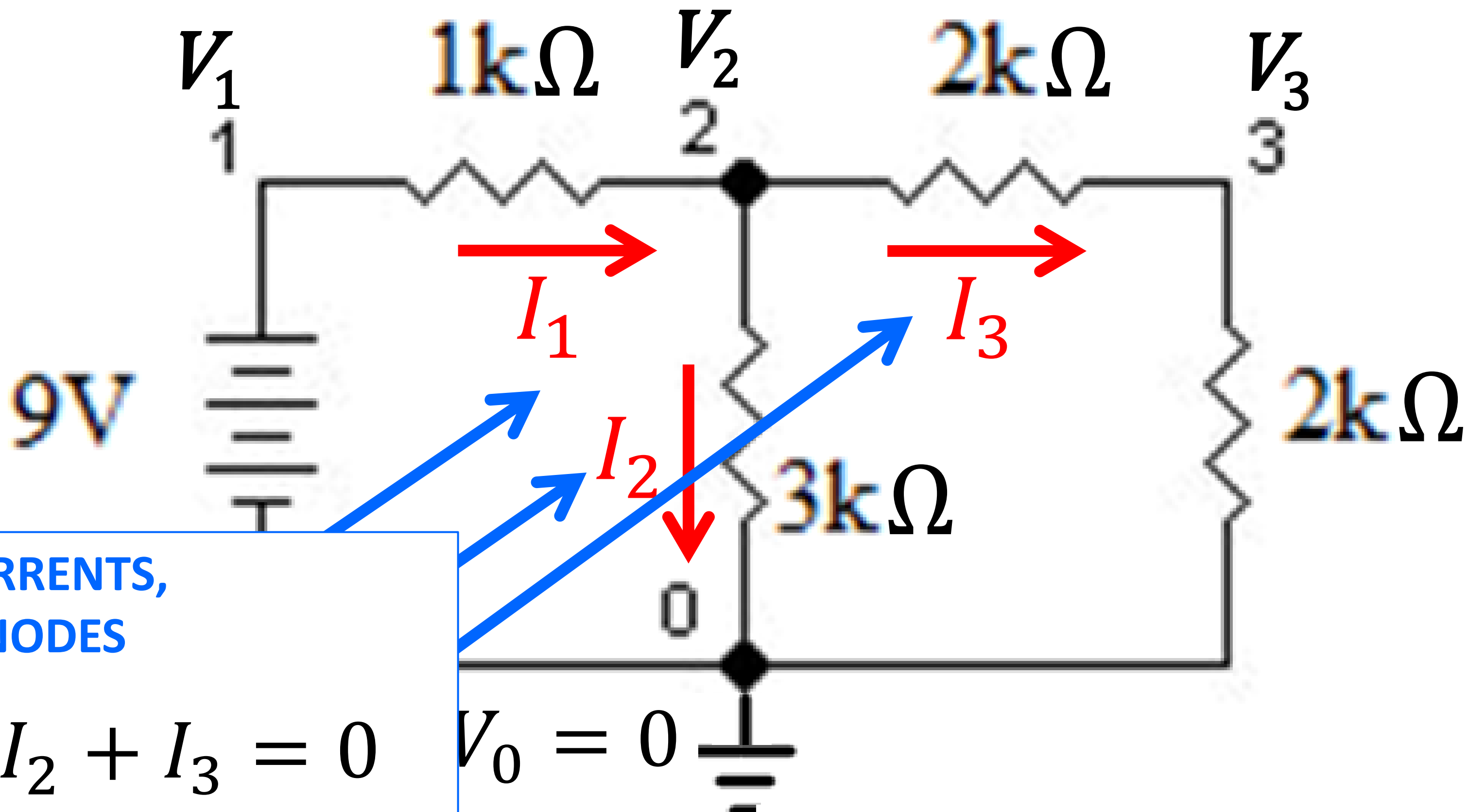



1kΩ
2kΩ
9V
3kΩ
2kΩ
0
$V_0 = 0$
(1) IDENTIFY GROUND NODE



(2) LABEL NODES, NODE VOLTAGES
V_1
$1k\Omega$
V_2
$2k\Omega$
V_3
1
2
3
9V
$3k\Omega$
$2k\Omega$
0
$V_0 = 0$

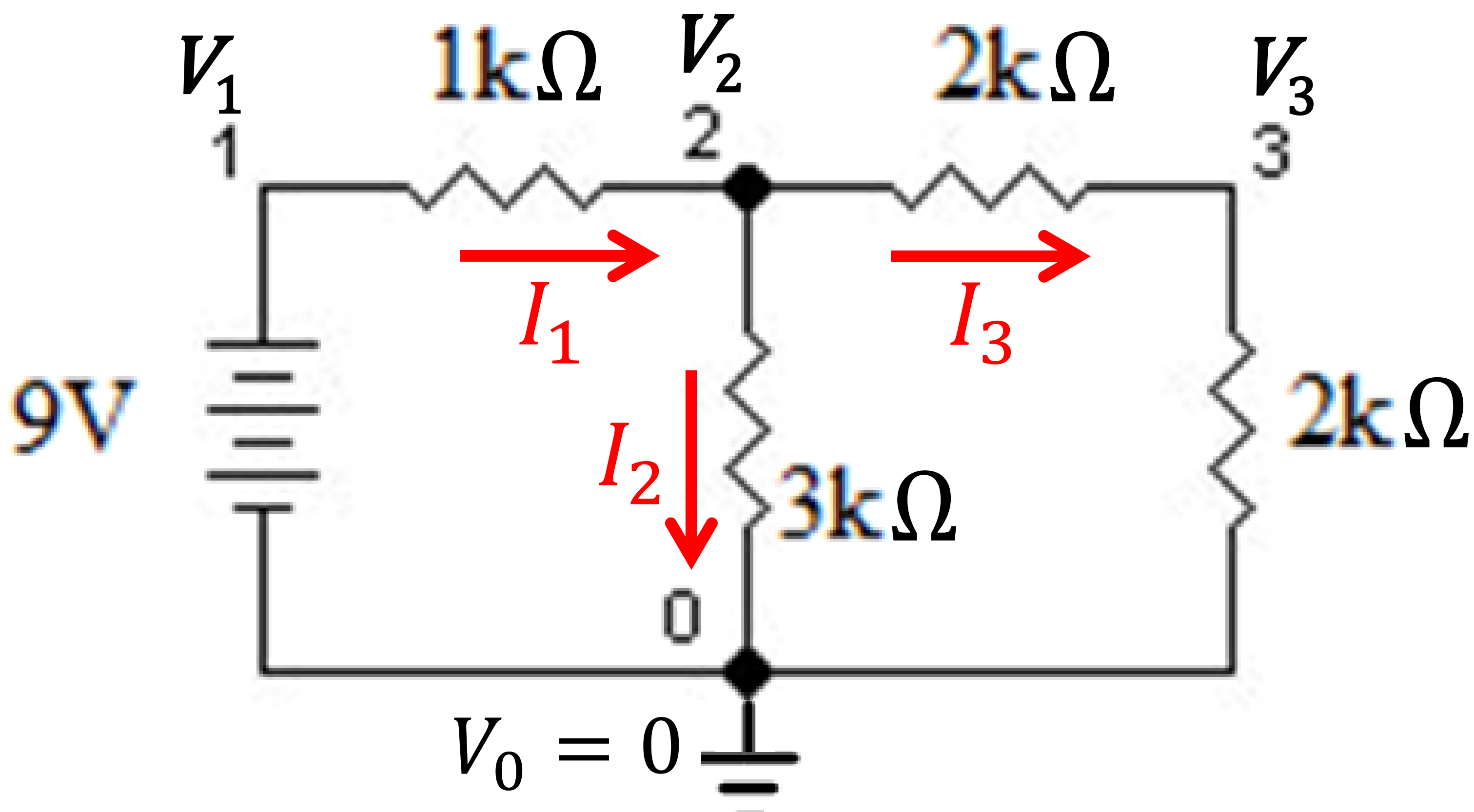
You could now assign a <u>fixed reference</u> for currents. This is also good to implement computer solvers.



(3) LABEL CURRENTS,
USE KCL AT NODES

$$-I_1 + I_2 + I_3 = 0$$

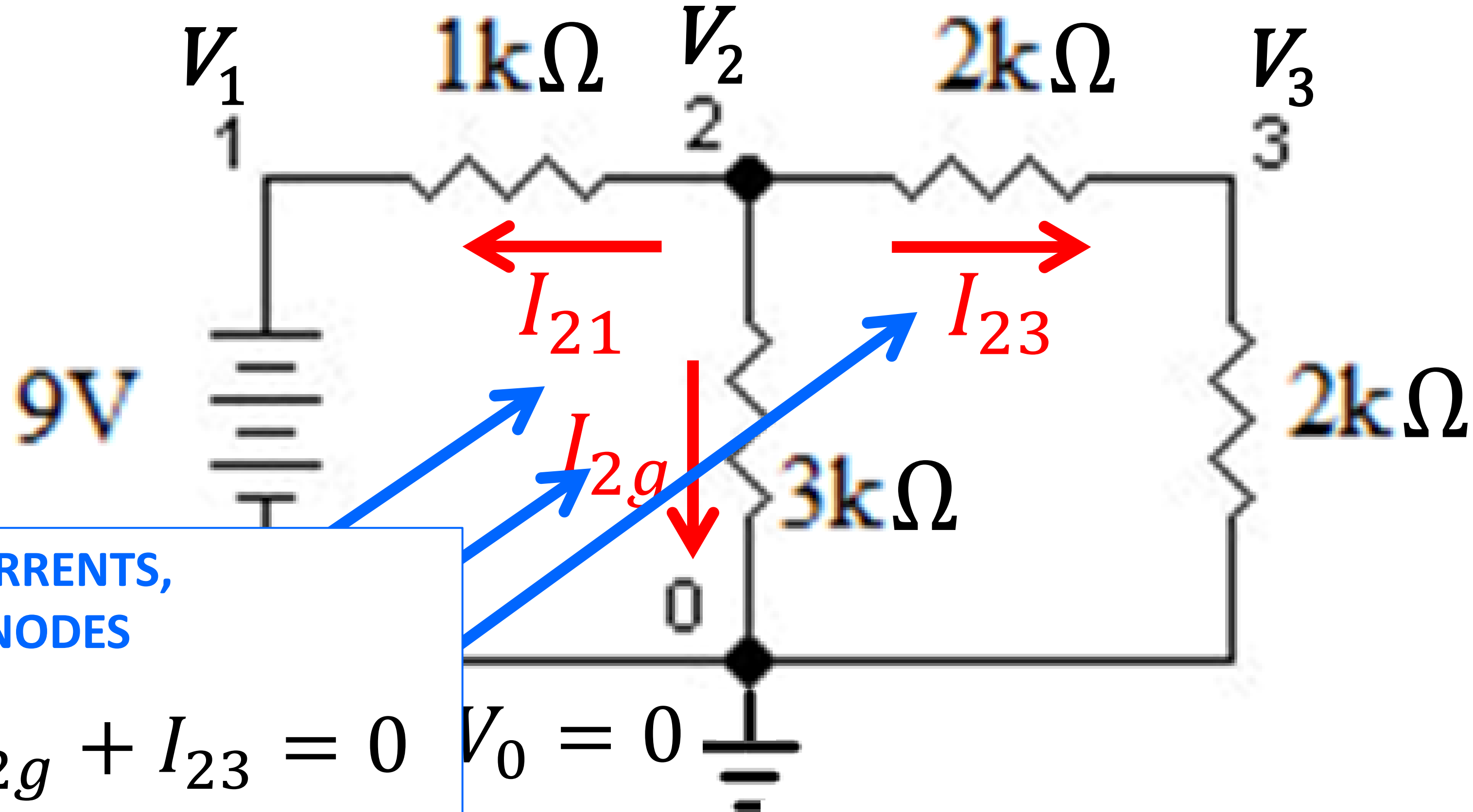
You could now assign a fixed reference for currents. This is also good to implement computer solvers.



(4) USE VOLTAGES TO SPECIFY CURRENTS

$$I_1 = \frac{V_1 - V_2}{1k\Omega}; I_2 = \frac{V_2}{3k\Omega}; I_3 = \frac{V_2 - V_3}{2k\Omega}$$

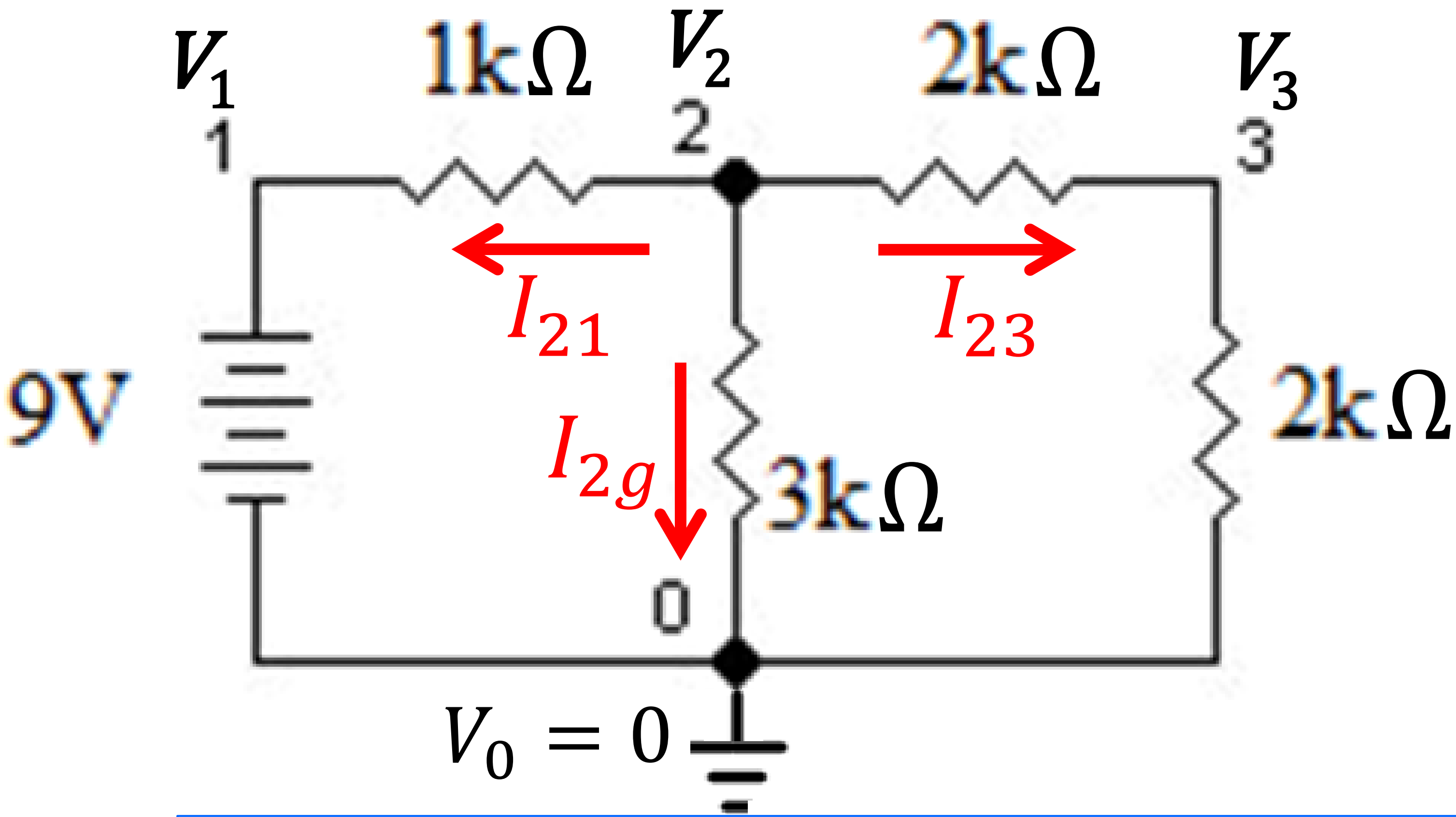
You could also define currents using indices between a specific node and neighboring ones <u>without specifying a fixed reference</u>. In this case it is good to write KCL with all outgoing currents.



(3) LABEL CURRENTS,
USE KCL AT NODES

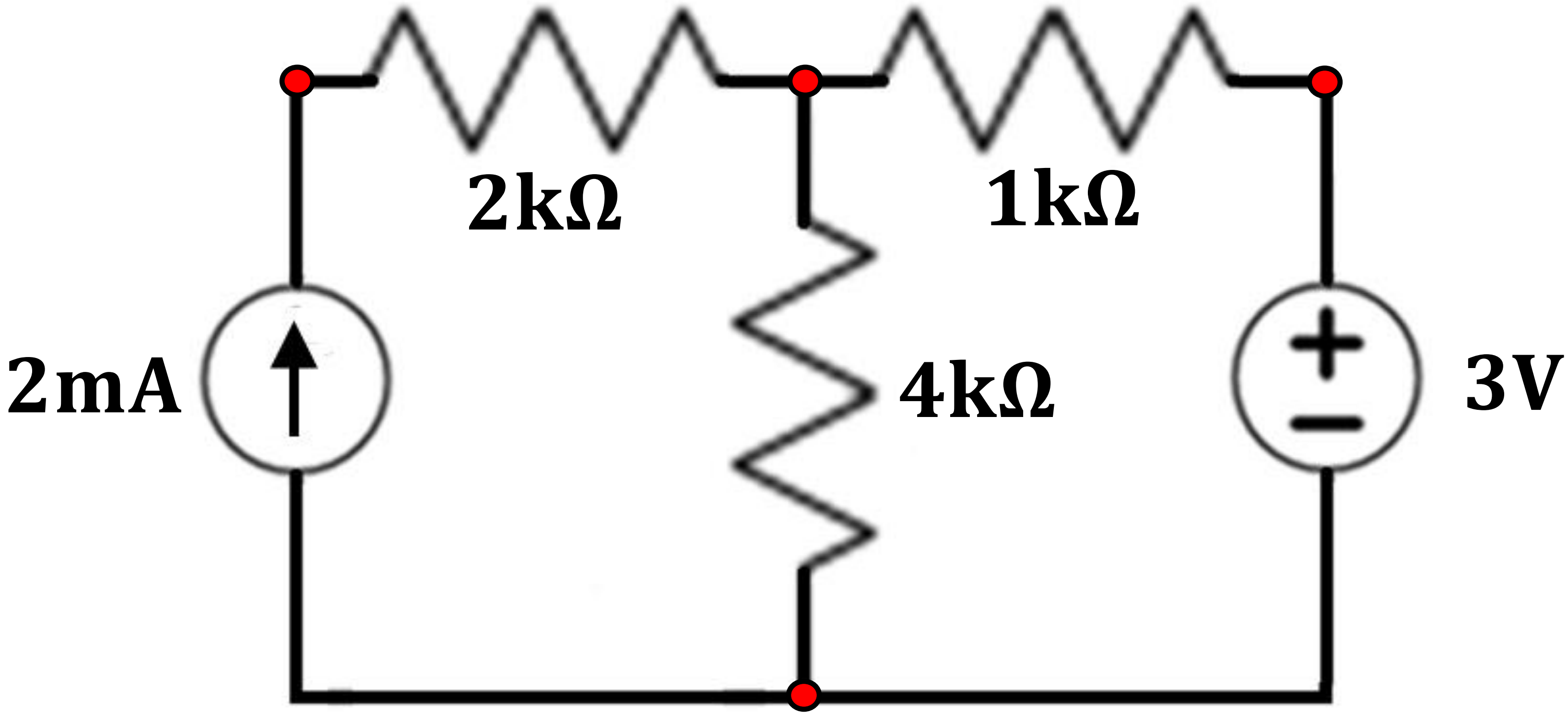
$$I_{21} + I_{2g} + I_{23} = 0$$

You could also define currents using indices between a specific node and neighboring ones <u>without specifying a fixed reference</u>. In this case it is good to write KCL with all outgoing currents.



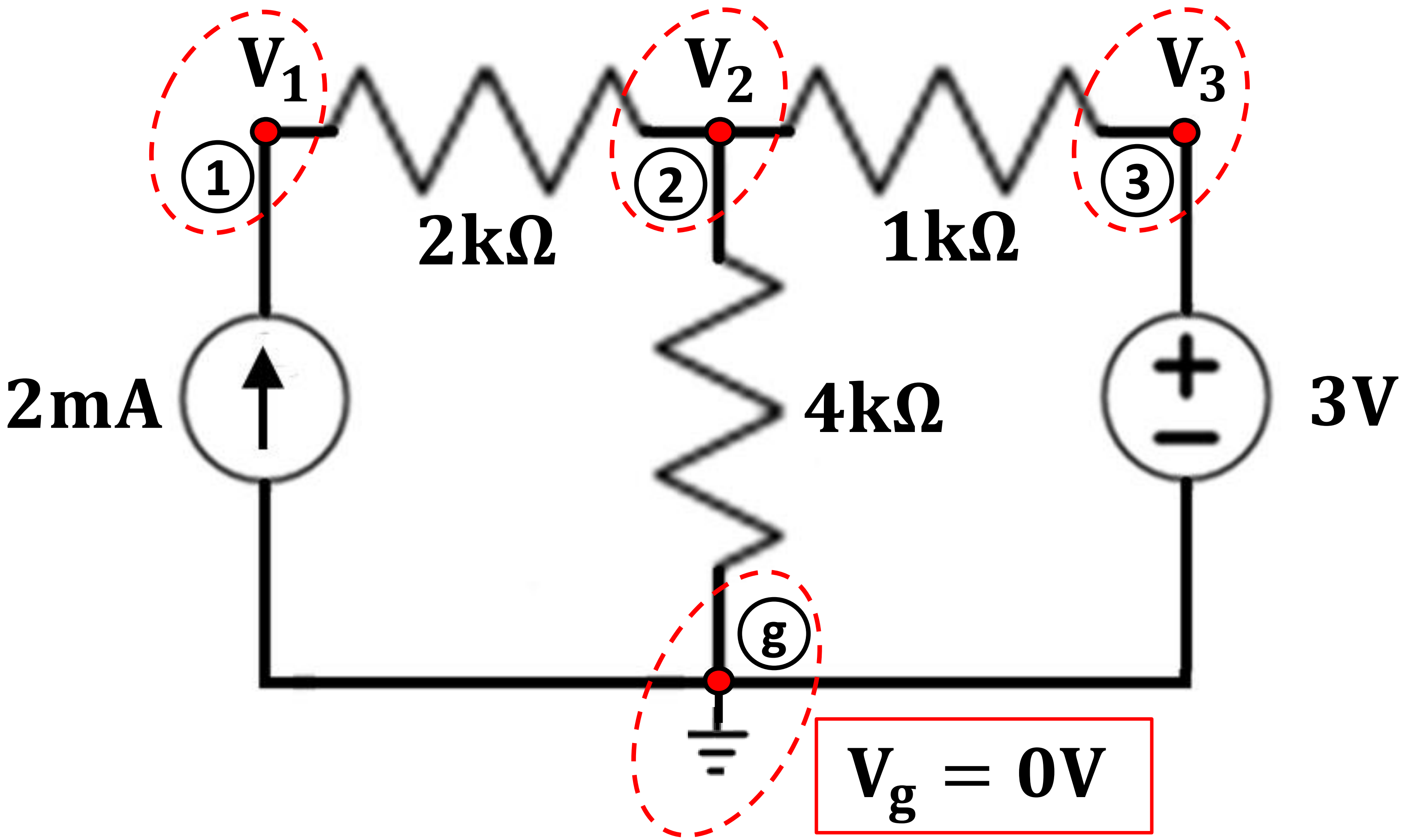
(4) USE VOLTAGES TO SPECIFY CURRENTS

$$I_{21} = \frac{V_2 - V_1}{1k\Omega}\,;\; I_{2g} = \frac{V_2 - 0}{3k\Omega}\,;\; I_{23} = \frac{V_2 - V_3}{2k\Omega}$$

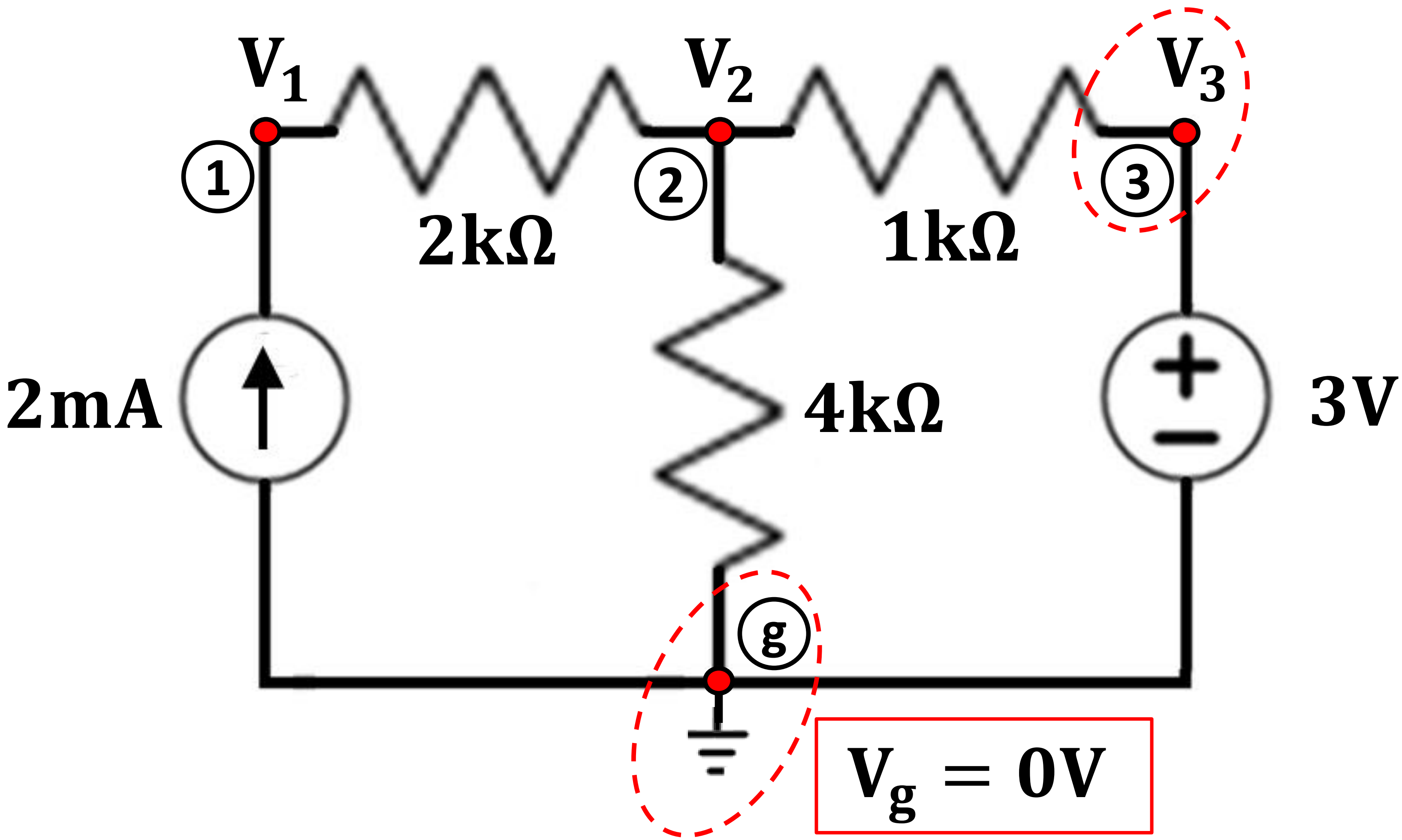


Example – Determine Voltages at circuit nodes

We will identify currents between neighboring nodes

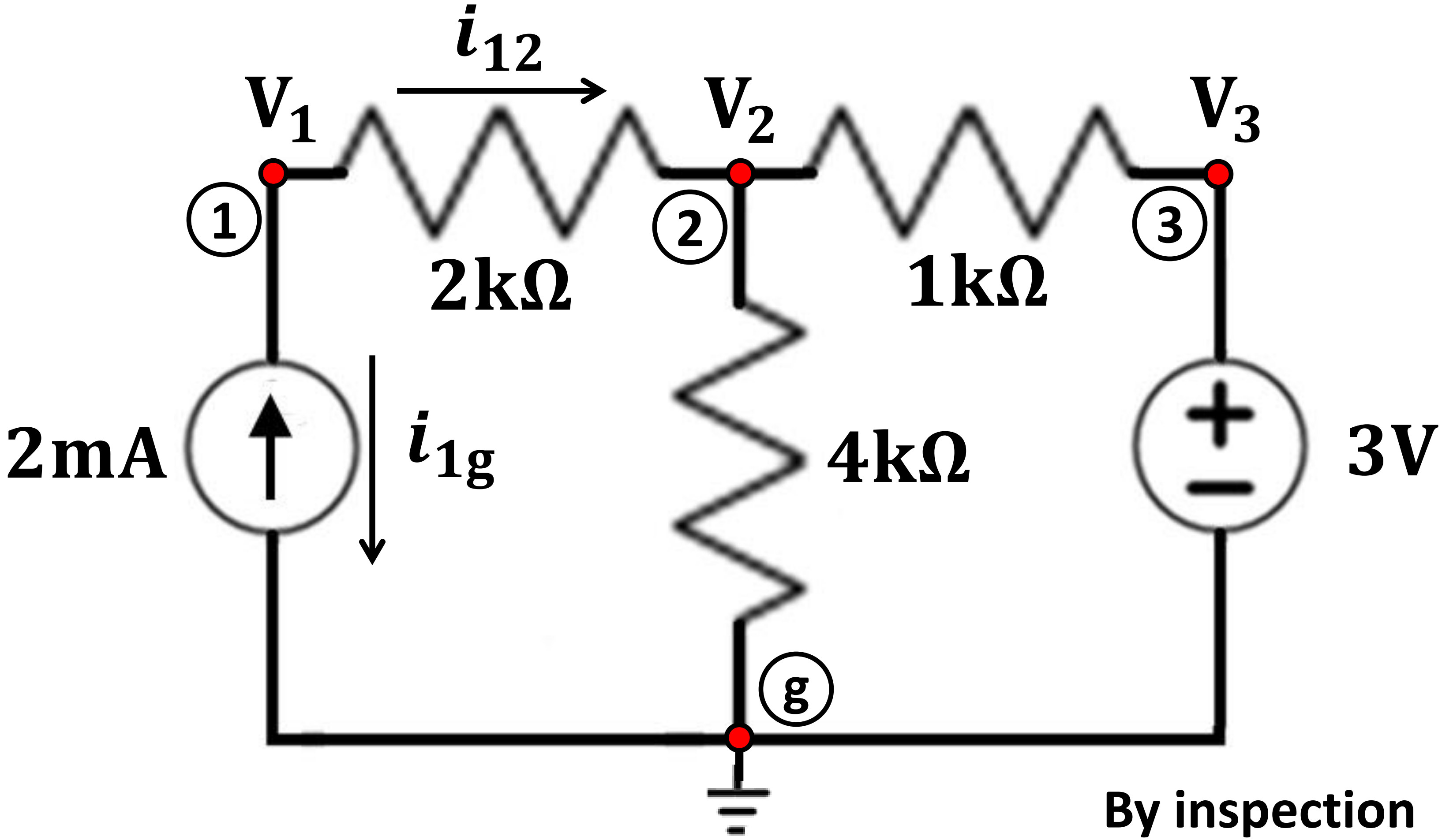


Choice of ground node at the terminal of a voltage source is a good strategy.



By inspection, $V_3 = 3\text{V}$. Need to find V_1 and V_2.

$$V_{3g} = V_3 - V_g = 3 - 0 \rightarrow \mathbf{V_3 = 3V}$$

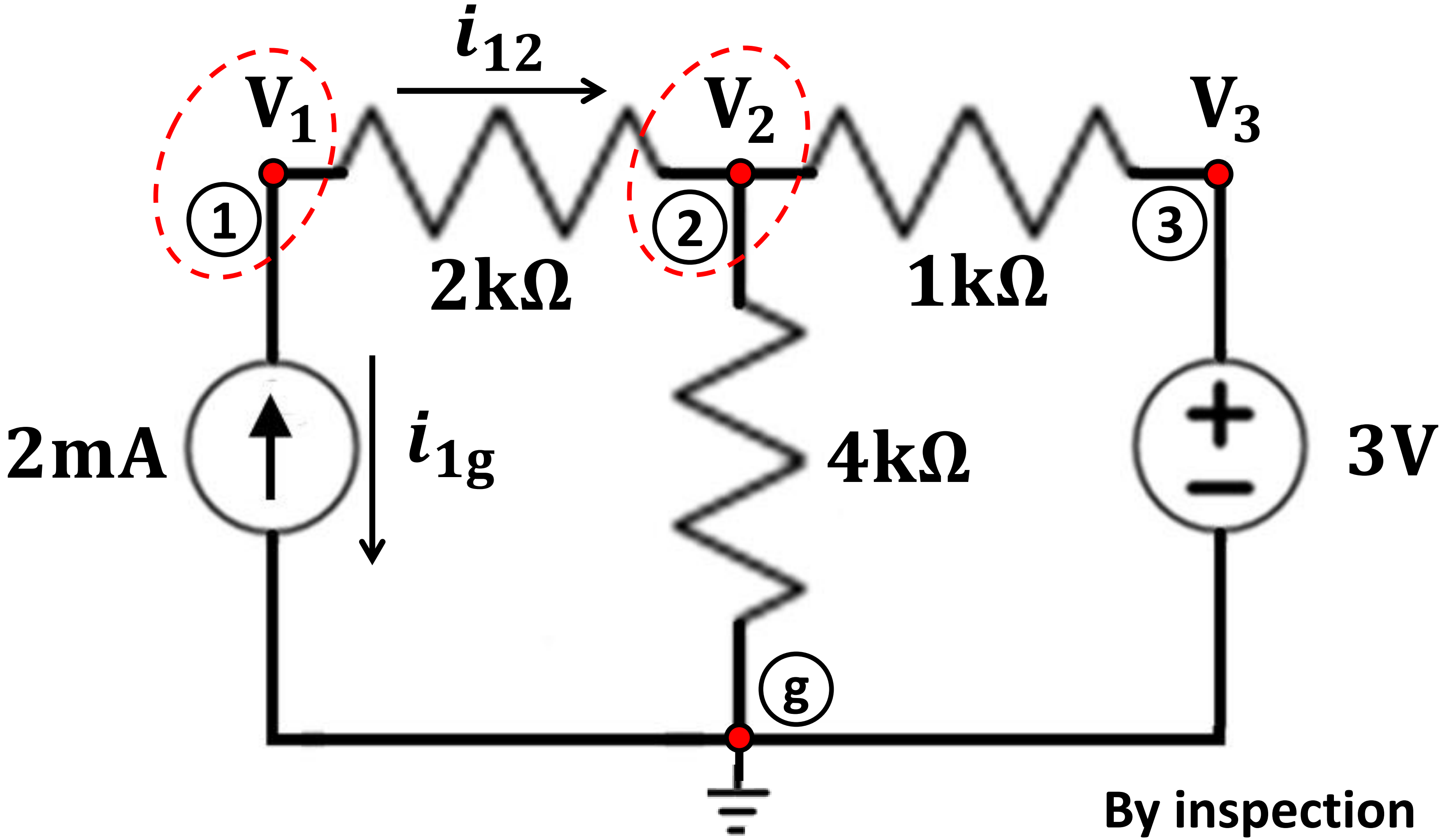


By inspection

$$i_{1g} = -2\text{mA}$$

KCL at Node 1: $i_{1g} + i_{12} = 0.$

You may formulate the KCL equation in different equivalent ways, but it is good to have a consistent method.

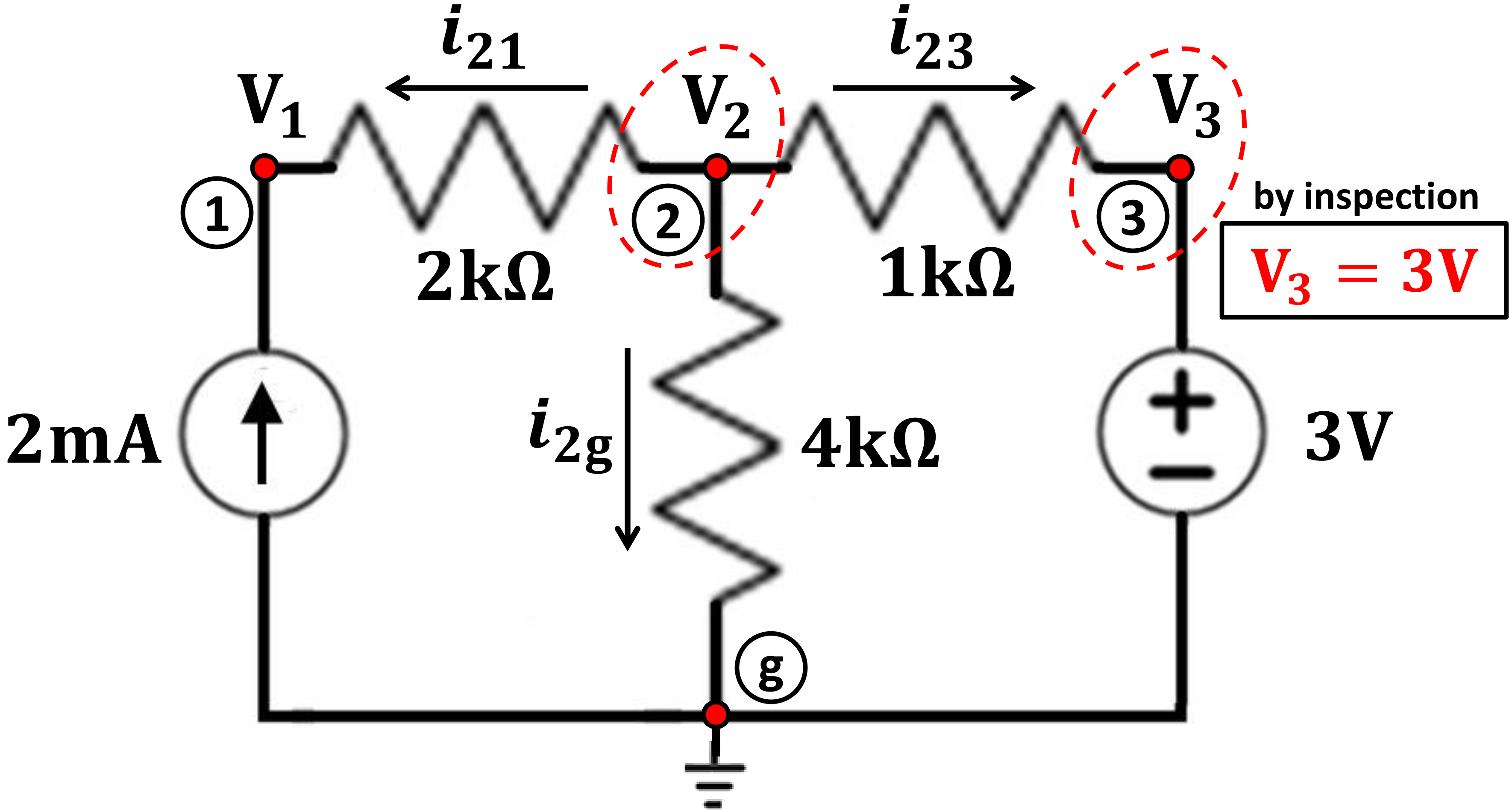


By inspection

$$i_{1g} = -2\text{mA}$$

KCL at Node 1: $i_{1g} + i_{12} = 0.$

Ohm's Law: $i_{12} = 2\text{mA} = \frac{V_{12}}{2k\Omega} = \frac{V_1 - V_2}{2k\Omega}$



KCL at Node 2: $i_{21} + i_{23} + i_{2g} = 0.$

Ohm's Law: $i_{21} = \frac{V_2 - V_1}{2k\Omega}$; $i_{2g} = \frac{V_2 - 0}{4k\Omega}$; $i_{23} = \frac{V_2 - V_3}{1k\Omega}$

$$\frac{V_2 - V_1}{2k\Omega} = i_{21} = -i_{12} = -2mA \rightarrow \frac{V_2 - V_1}{2} = -2V$$

Node 1 $$2mA = \frac{V_{12}}{2k\Omega} = \frac{V_1 - V_2}{2k\Omega} \rightarrow V_1 - V_2 = 4V$$

Node 2 $$\frac{V_2 - V_1}{2k\Omega} + \frac{V_2 - 0}{4k\Omega} + \frac{V_2 - V_3}{1k\Omega} = 0$$

$$-2 + \frac{V_2}{4} + V_2 - 3 = 0 \rightarrow \frac{5}{4}V_2 = 5 \rightarrow \boxed{V_2 = 4V}$$

Node 3 $$\boxed{V_3 = 3V}$$

$$\frac{V_2 - V_1}{2k\Omega} = i_{21} = -i_{12} = -2mA \rightarrow \frac{V_2 - V_1}{2} = -2V$$

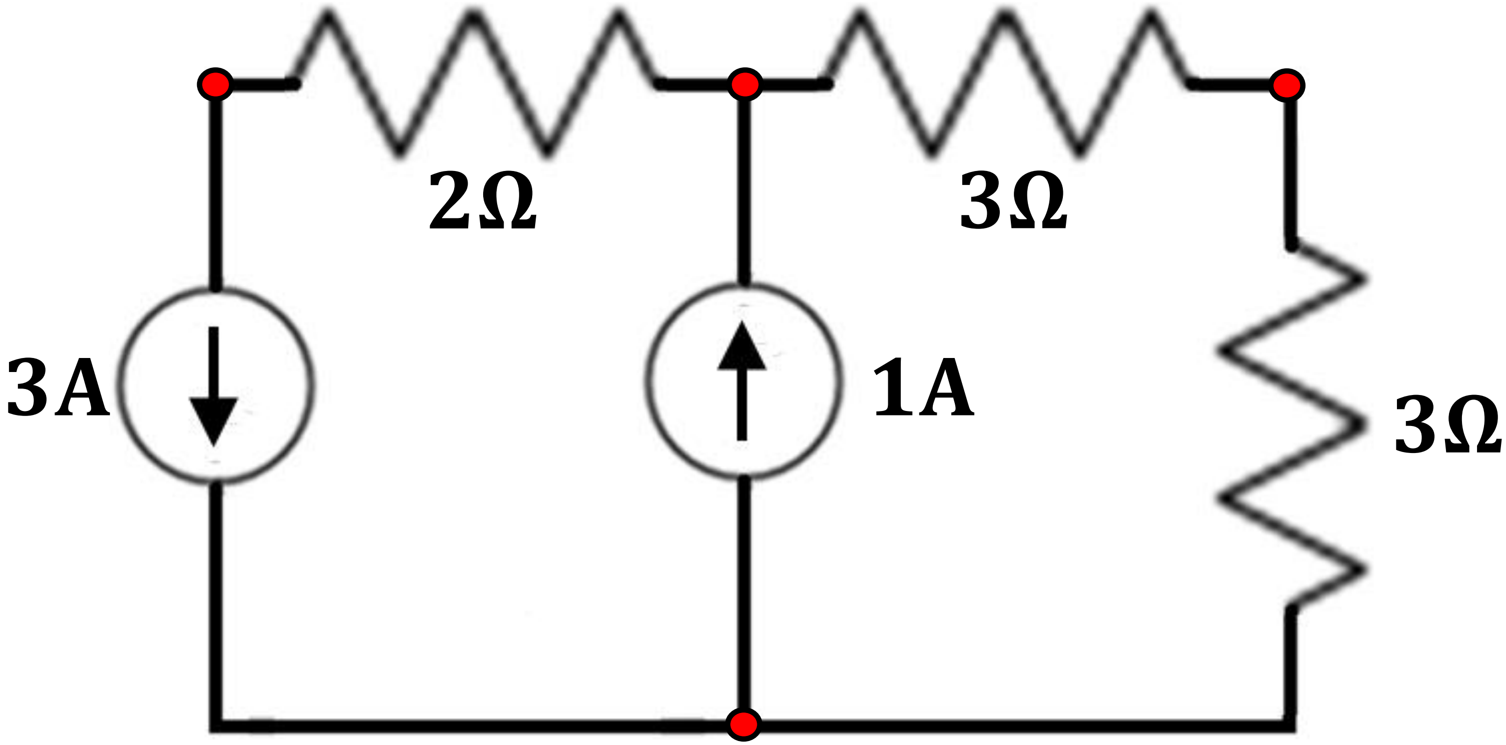
Node 1 $$2mA = \frac{V_{12}}{2k\Omega} = \frac{V_1 - V_2}{2k\Omega} \rightarrow V_1 - V_2 = 4V$$

Node 2 $$\frac{V_2 - V_1}{2k\Omega} + \frac{V_2 - 0}{4k\Omega} + \frac{V_2 - V_3}{1k\Omega} = 0$$

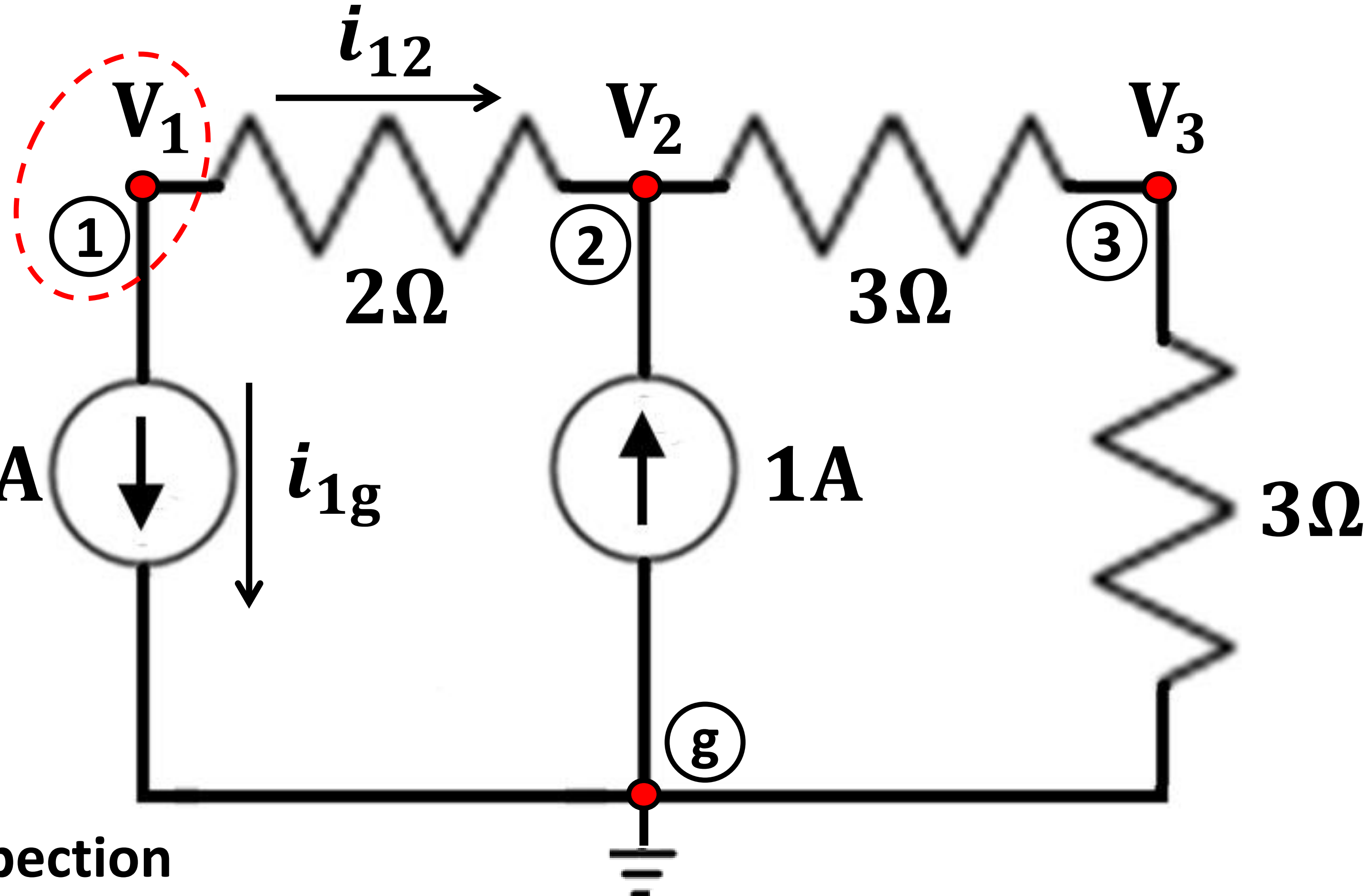
$$-2 + \frac{V_2}{4} + V_2 - 3 = 0 \rightarrow \frac{5}{4}V_2 = 5 \rightarrow \boxed{V_2 = 4V}$$

$$\boxed{V_1 = 8V}$$

Node 3 $$\boxed{V_3 = 3V}$$



Example – Determine Voltages at circuit nodes



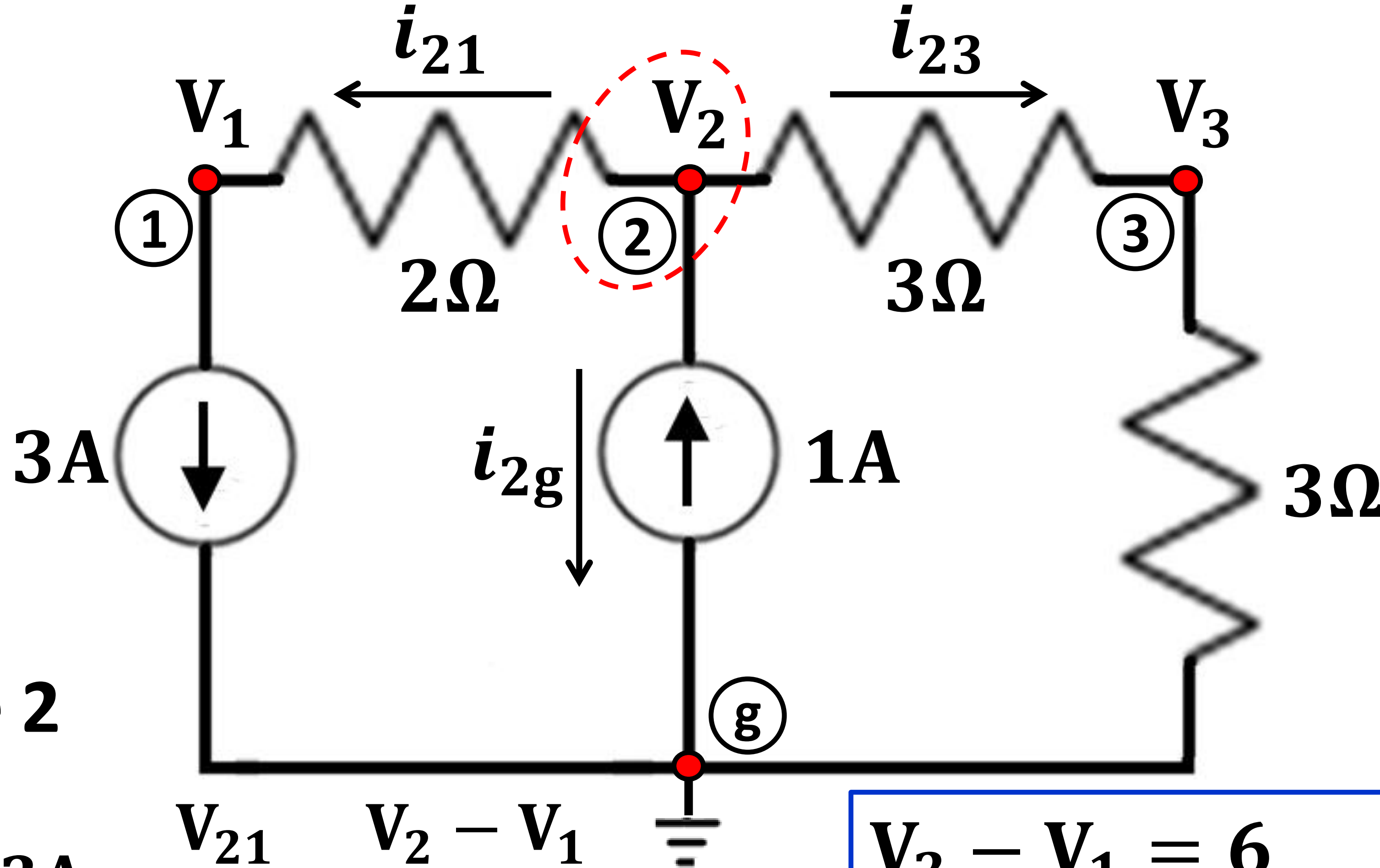
Node 1

By inspection

$$i_{1g} = -i_{12} = 3A$$

$$i_{12} = -3A = \frac{V_{12}}{2\Omega} = \frac{V_1 - V_2}{2\Omega}$$

$$V_2 - V_1 = 6$$



Node 2

$$i_{21} = 3A = \frac{V_{21}}{2\Omega} = \frac{V_2 - V_1}{2\Omega}$$

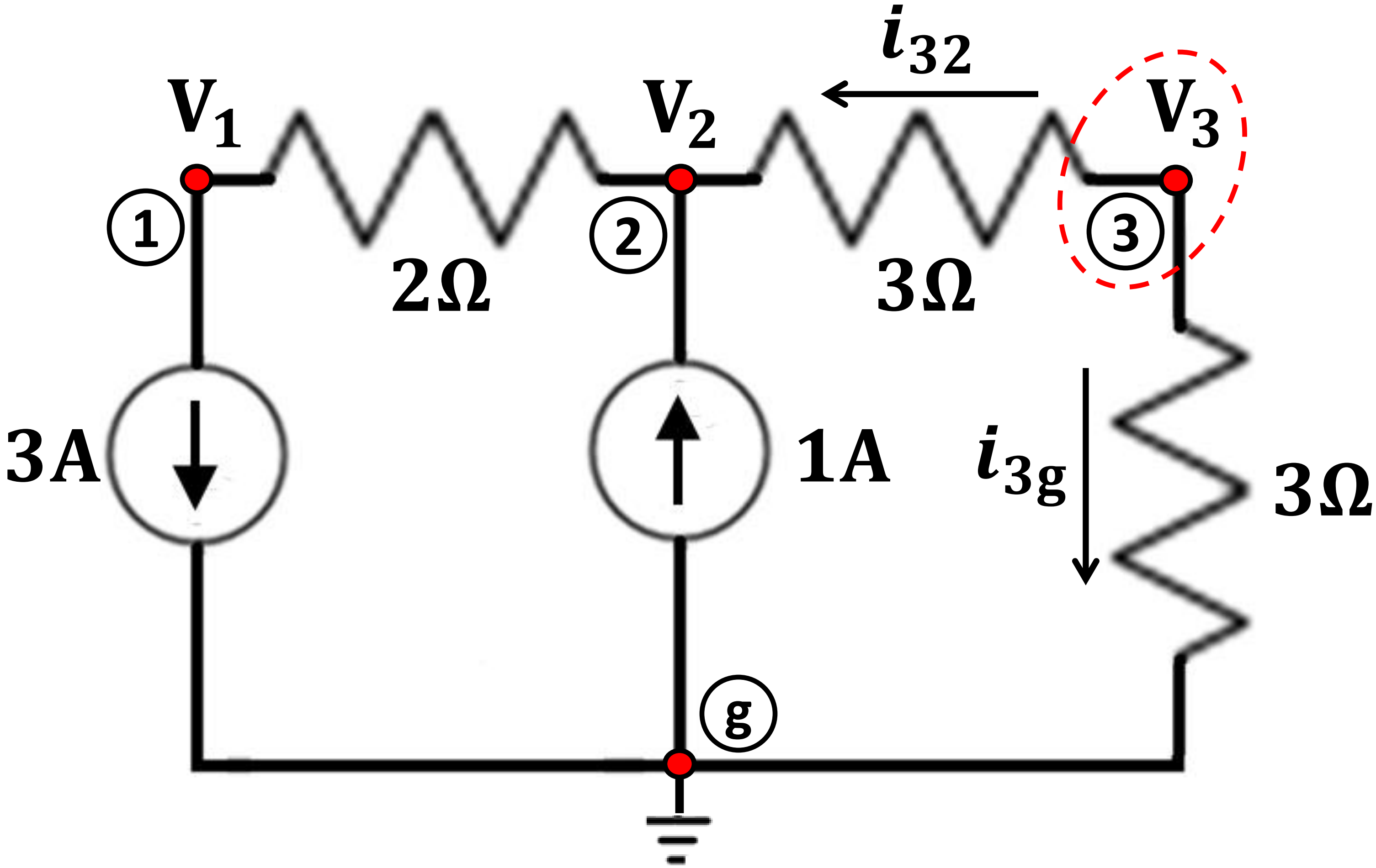
$$V_2 - V_1 = 6$$

$$i_{21} + i_{2g} + i_{23} = 0$$

$$i_{23} = \frac{V_{23}}{3\Omega} = \frac{V_2 - V_3}{3\Omega}$$

$$\frac{V_2 - V_1}{2\Omega} + \frac{V_2 - V_3}{3\Omega} - 1 = 0$$

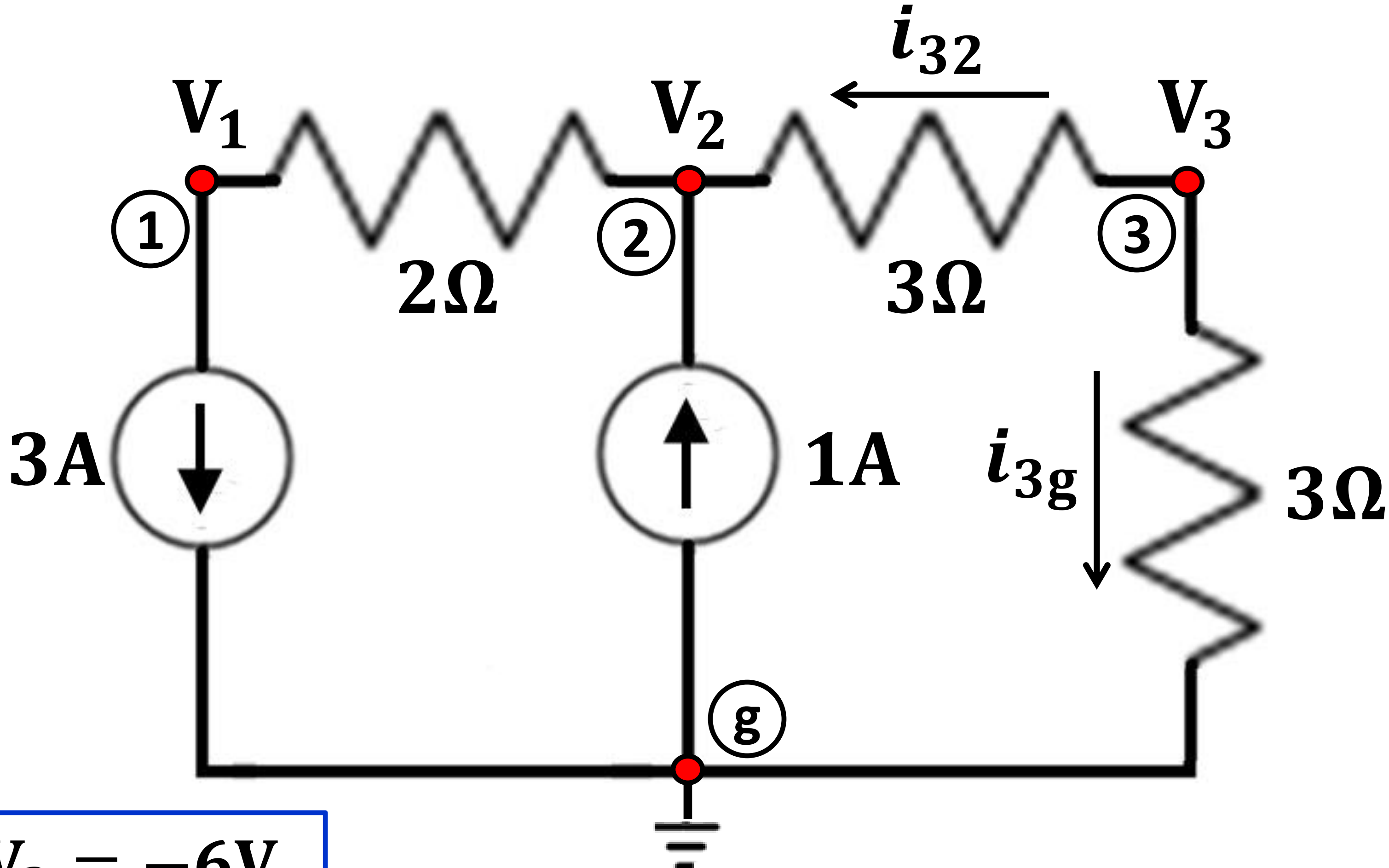
By inspection $i_{2g} = -1A$



Node 3 $i_{3g} = -i_{32} = \frac{V_{3g}}{3\Omega} = \frac{V_3 - 0}{3\Omega}$

Ⓖ $i_{3g} = -3A + 1A = -2A$

$$V_3 = i_{3g} \times 3\Omega = -2 \times 3 = -6V$$

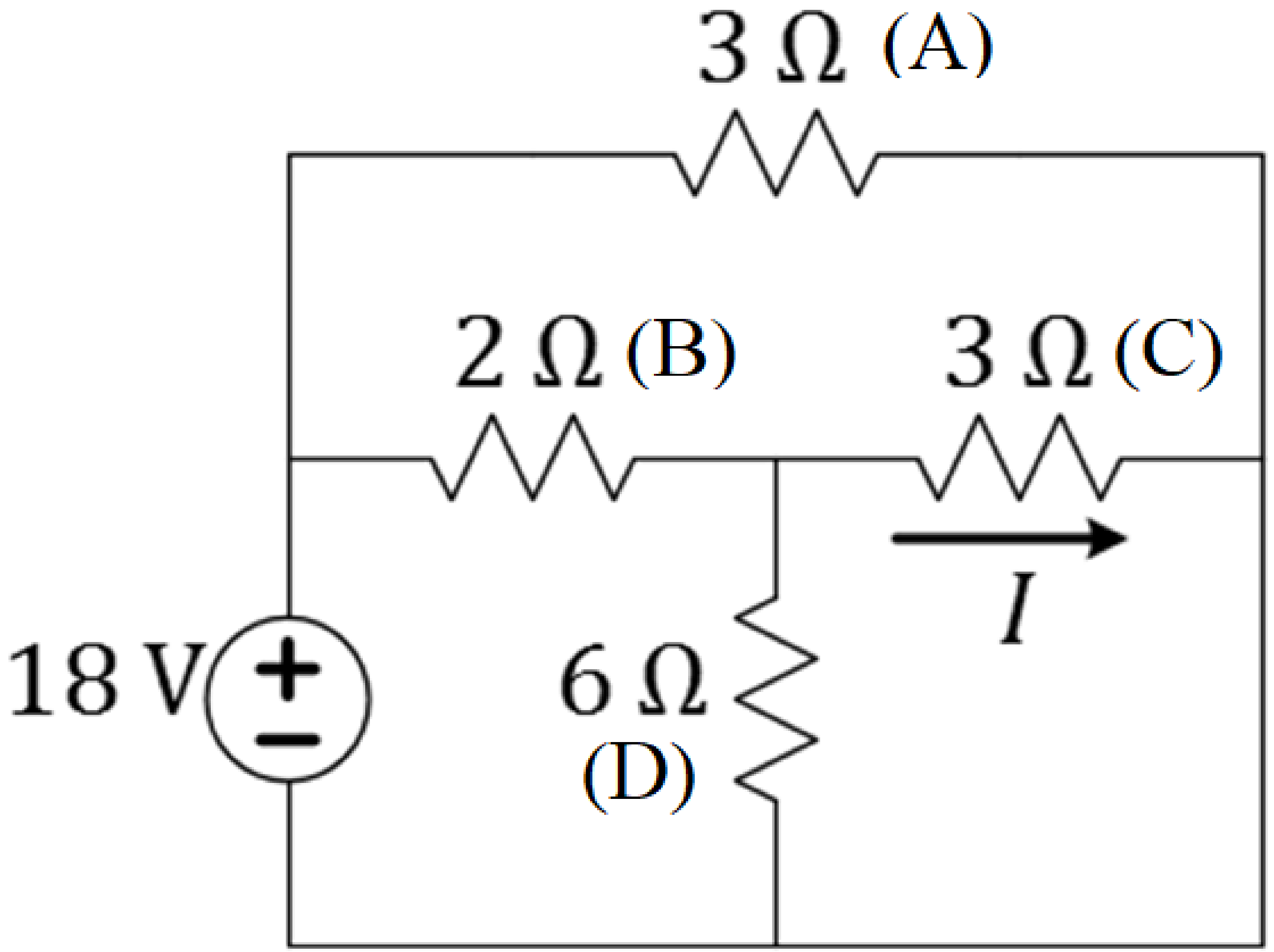


$$V_3 = -6V$$

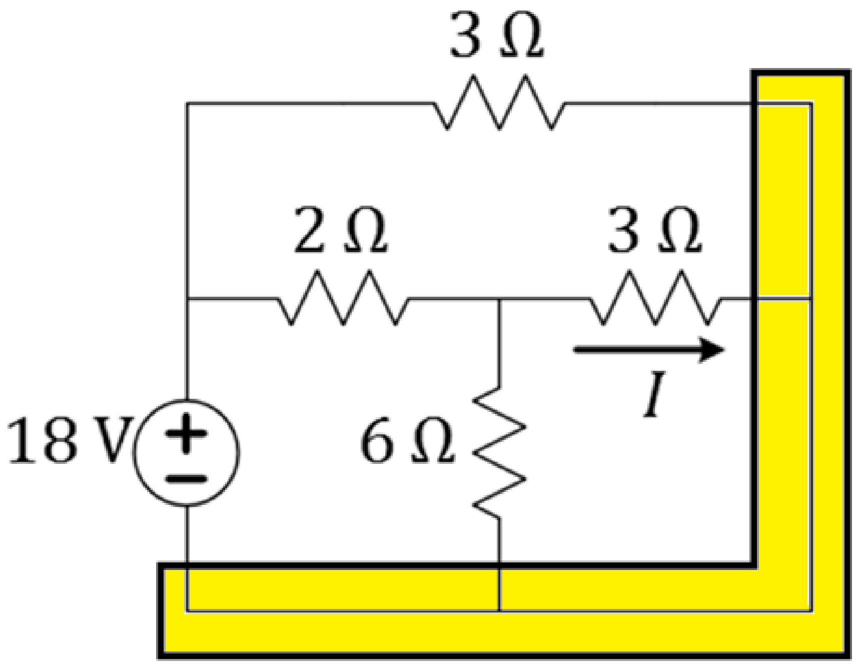
$$V_2 = V_3 + i_{3g} \times 3\Omega = -6V - 2A \times 3\Omega = -12V$$

$$V_1 = V_2 + i_{12} \times 2\Omega = -12V - 3A \times 2\Omega = -18V$$

Find the labelled current *I*



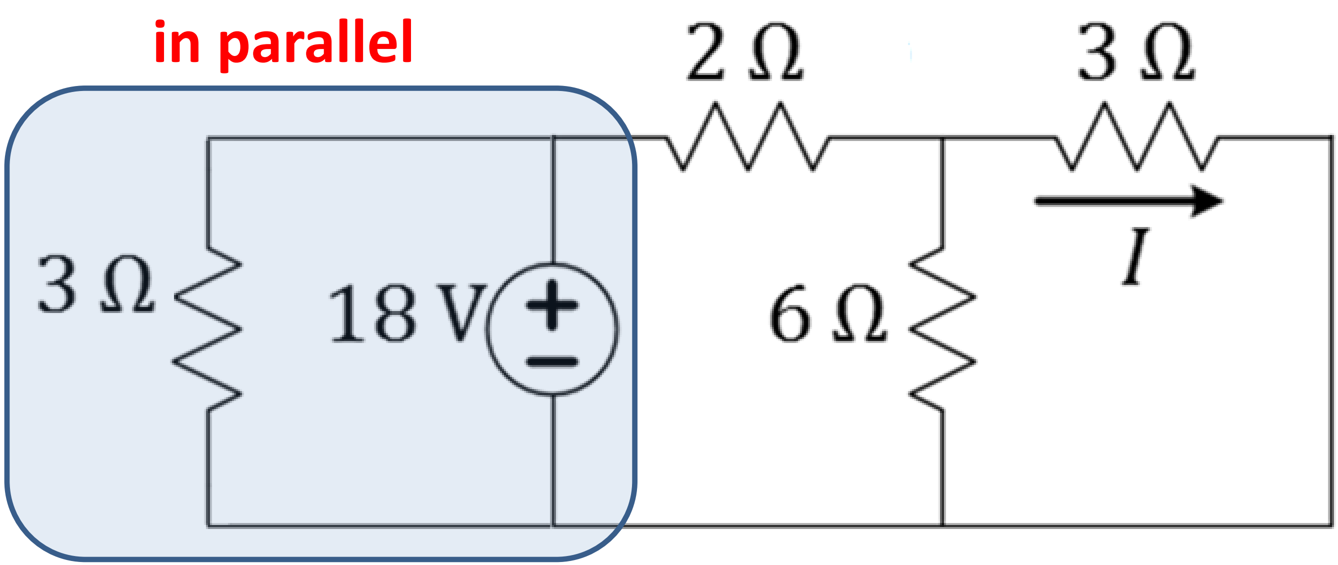
Q: Which resistor is in parallel with the voltage source?



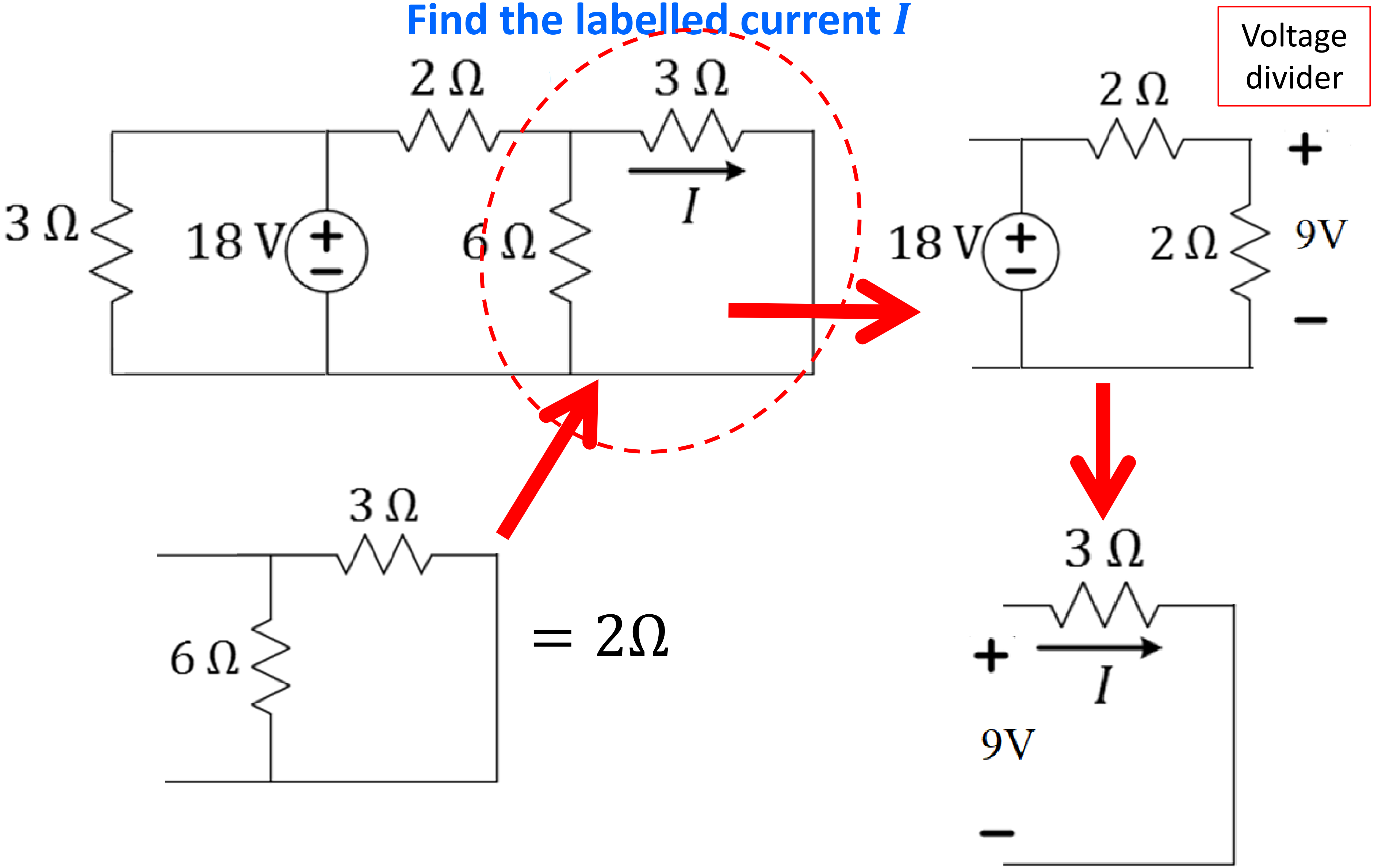
All these wires are at the same potential

This problem can be solved very quickly without node voltage analysis

We can rearrange the diagram as

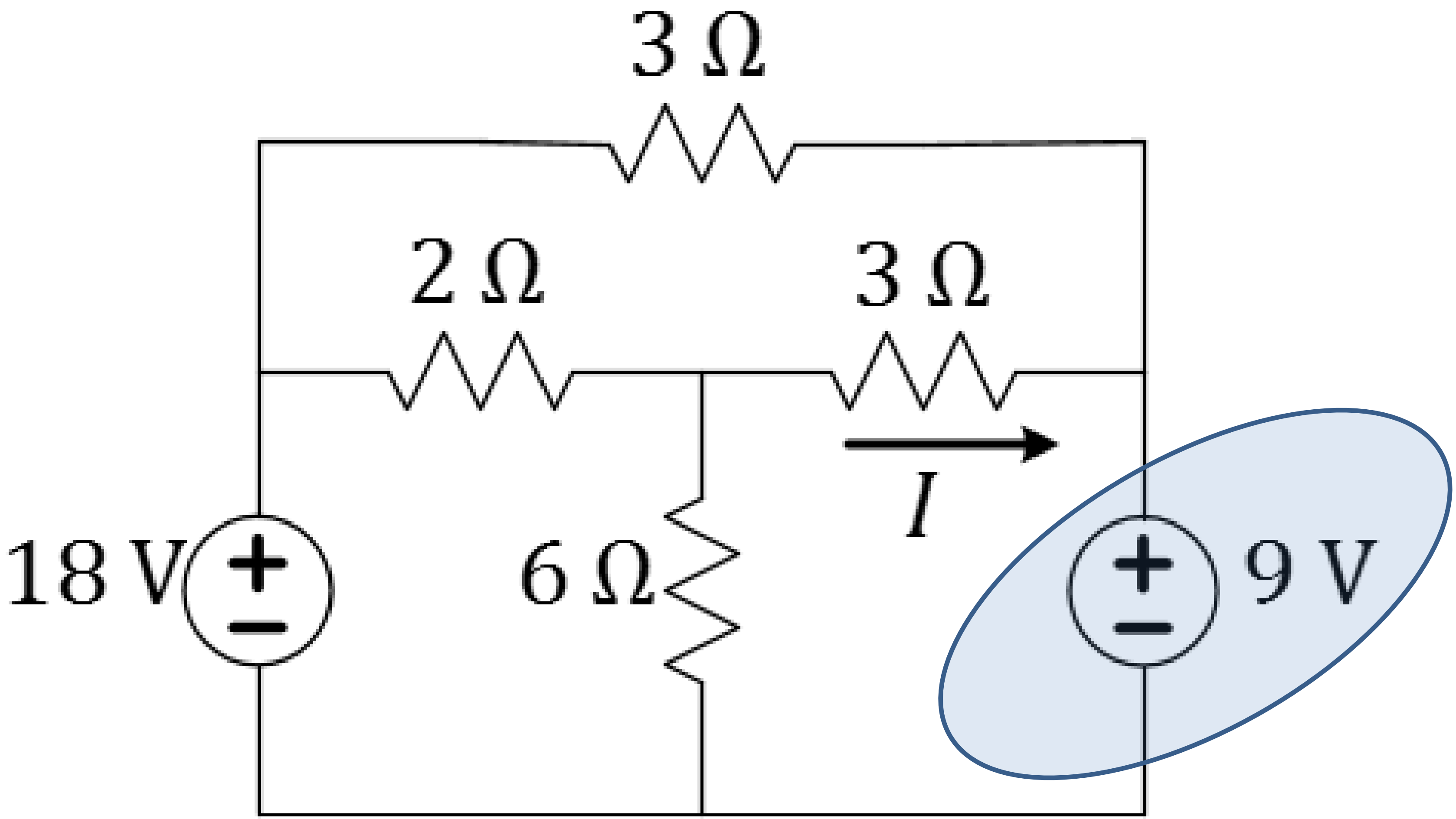


Find the labelled current I



$$I = 3\text{A}$$

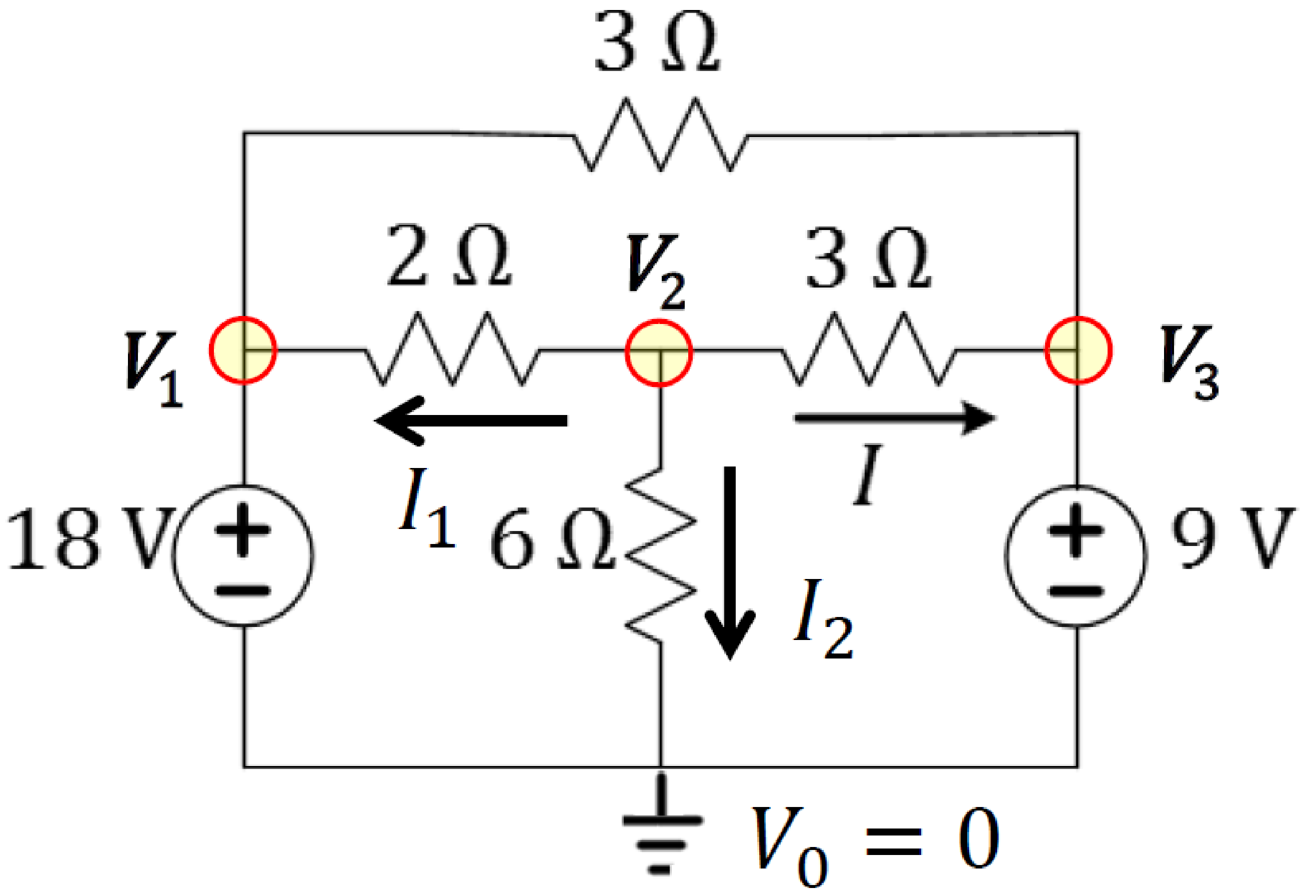
Find the labelled current I



Now there is a second voltage source in this branch. Node voltage analysis is a good approach.

In this example and in the following ones, we are going to specify <u>fixed references for the currents</u> in each of the circuit branches.

As mentioned earlier, this is a good approach for implementation of computer circuit solution using algorithms based on linear algebra.



$$I_1 = \frac{V_2 - V_1}{2}$$

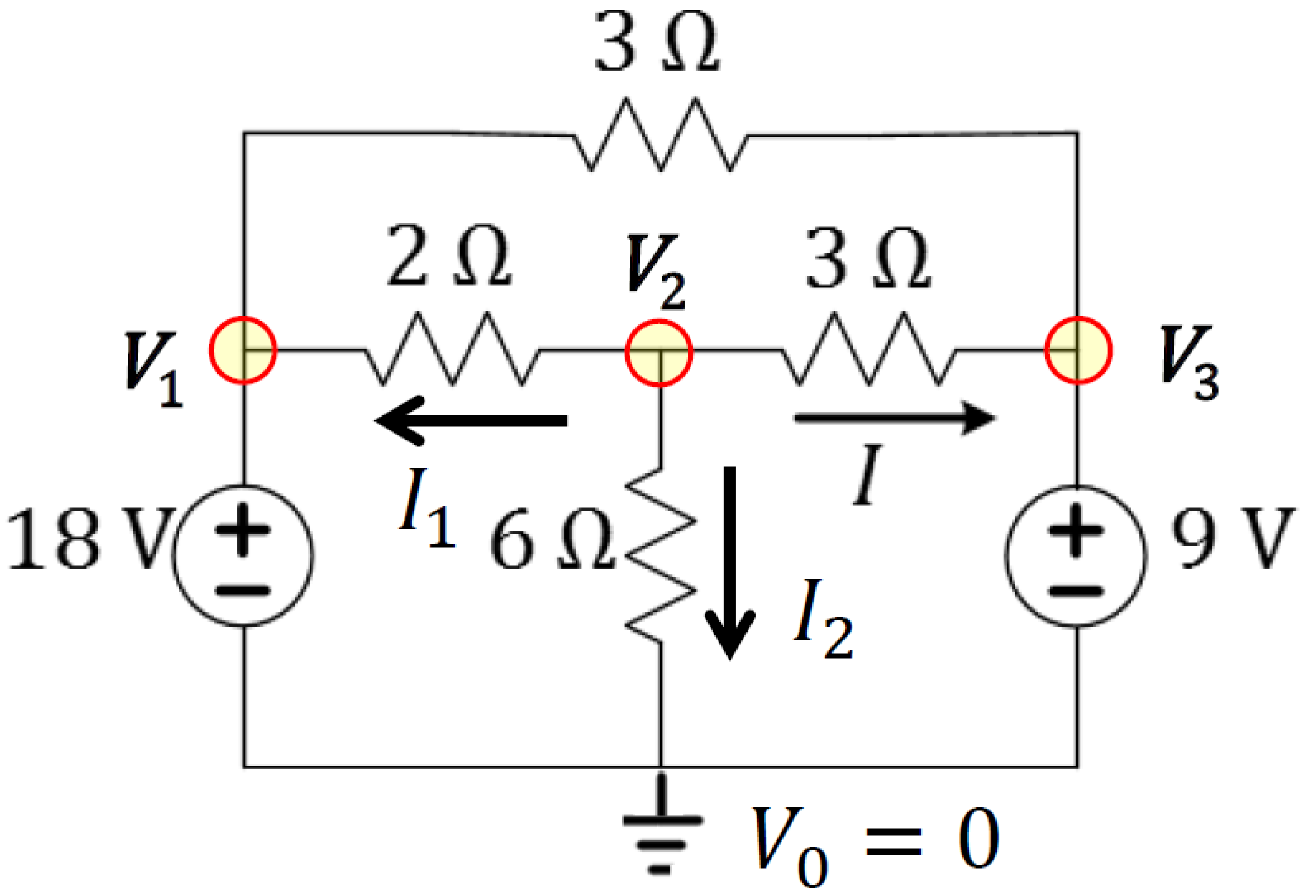
$$I_2 = \frac{V_2 - V_0}{6}$$

$$I = \frac{V_2 - V_3}{3}$$

Ground reference – zero potential

KCL – node 2

$$I_1 + I_2 + I = 0$$

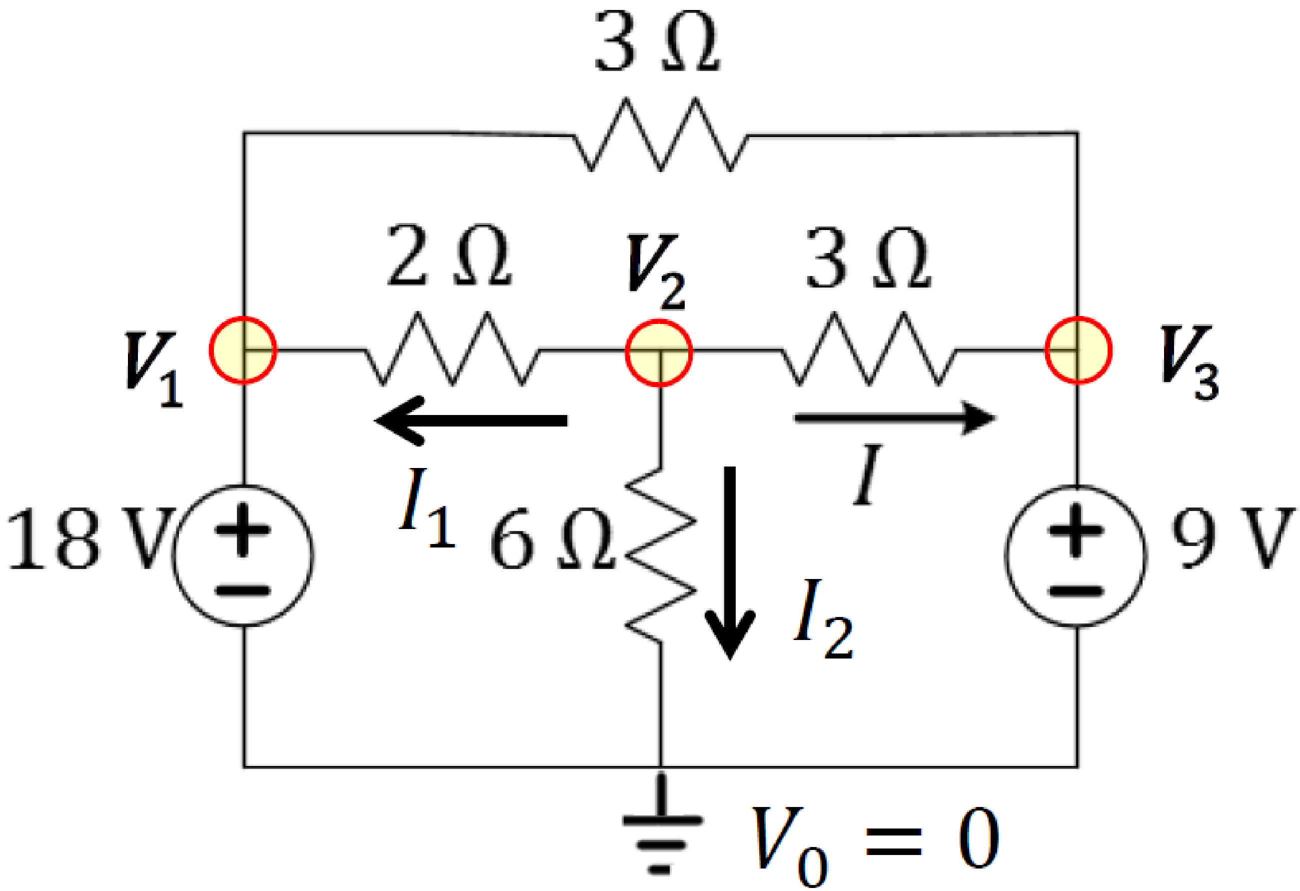


$$I_1 = \frac{V_2 - V_1}{2}$$

$$I_2 = \frac{V_2 - V_0}{6}$$

$$I = \frac{V_2 - V_3}{3}$$

Now we should be comfortable with the method, so we can write the currents directly in terms of Amperes, without having to write all the time Ω.



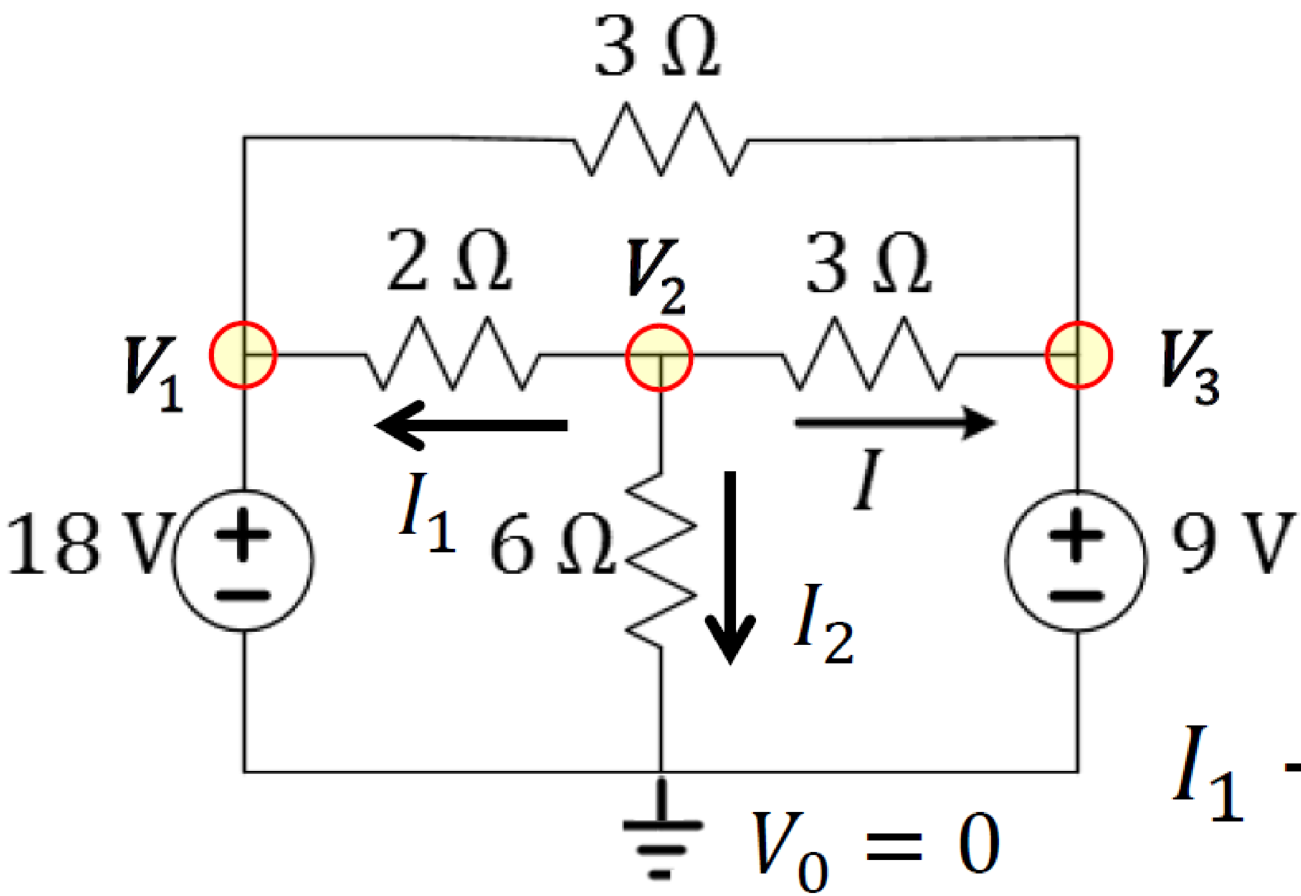
By inspection

$$V_1 = 18V$$

$$V_3 = 9V$$

We only need to solve for V_2.

With the loop method, we would need to write 3 loop equation!



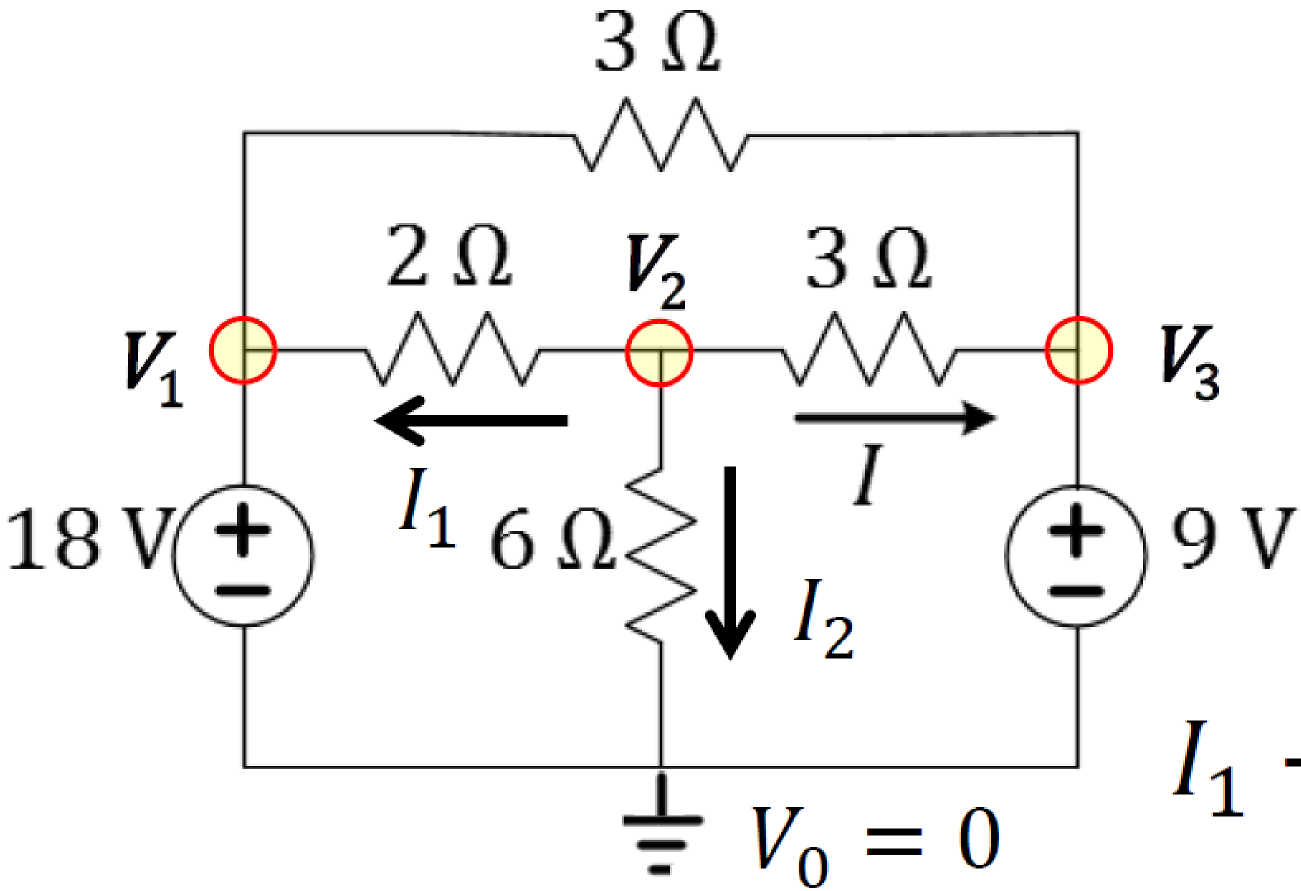
By inspection

$$V_1 = 18V$$

$$V_3 = 9V$$

$$I_1 + I_2 + I = 0$$

KCL



By inspection

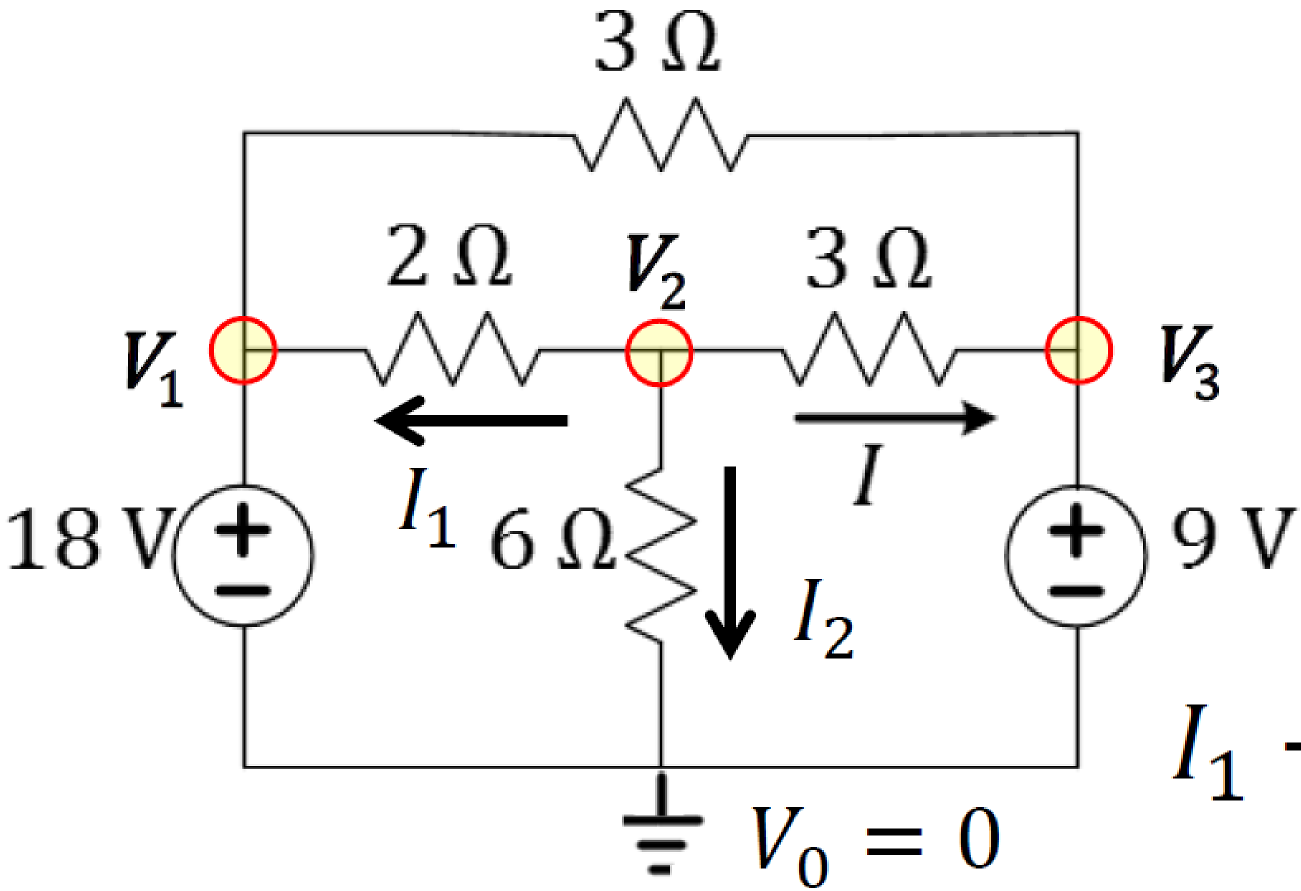
$$V_1 = 18\text{V}$$

$$V_3 = 9V$$

$$I_1 + I_2 + I = 0$$

$$\frac{V_2 - 18}{2} + \frac{V_2}{6} + \frac{V_2 - 9}{3} = 0 \Rightarrow \frac{V_2}{2} + \frac{V_2}{6} + \frac{V_2}{3} = 12$$

$$\Rightarrow \frac{3V_2}{6} + \frac{V_2}{6} + \frac{2V_2}{6} = 12 \Rightarrow V_2 = 12\text{V}$$



By inspection

$$V_1 = 18\text{V}$$

$$V_3 = 9V$$

$$V_2 = 12\text{V}$$

$$I_1 + I_2 + I = 0$$

$$I_1 = \frac{12 - 18}{2} = -3\text{A}$$

$$I_2 = \frac{12}{6} = 2\text{A}$$

Ohm's Law

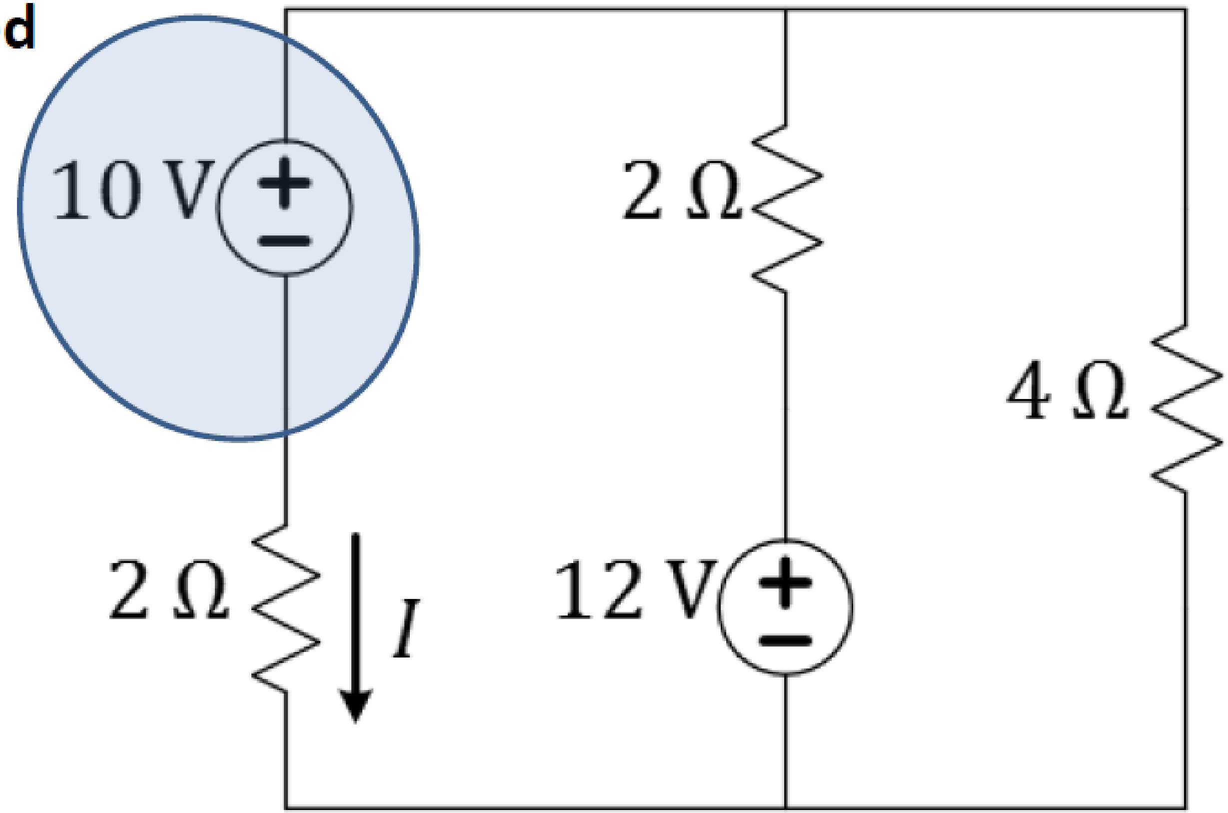
$$I = \frac{12 - 9}{3} = 1\text{A}$$

KCL

$$I = 3\text{A} - 2\text{A} = 1\text{A}$$

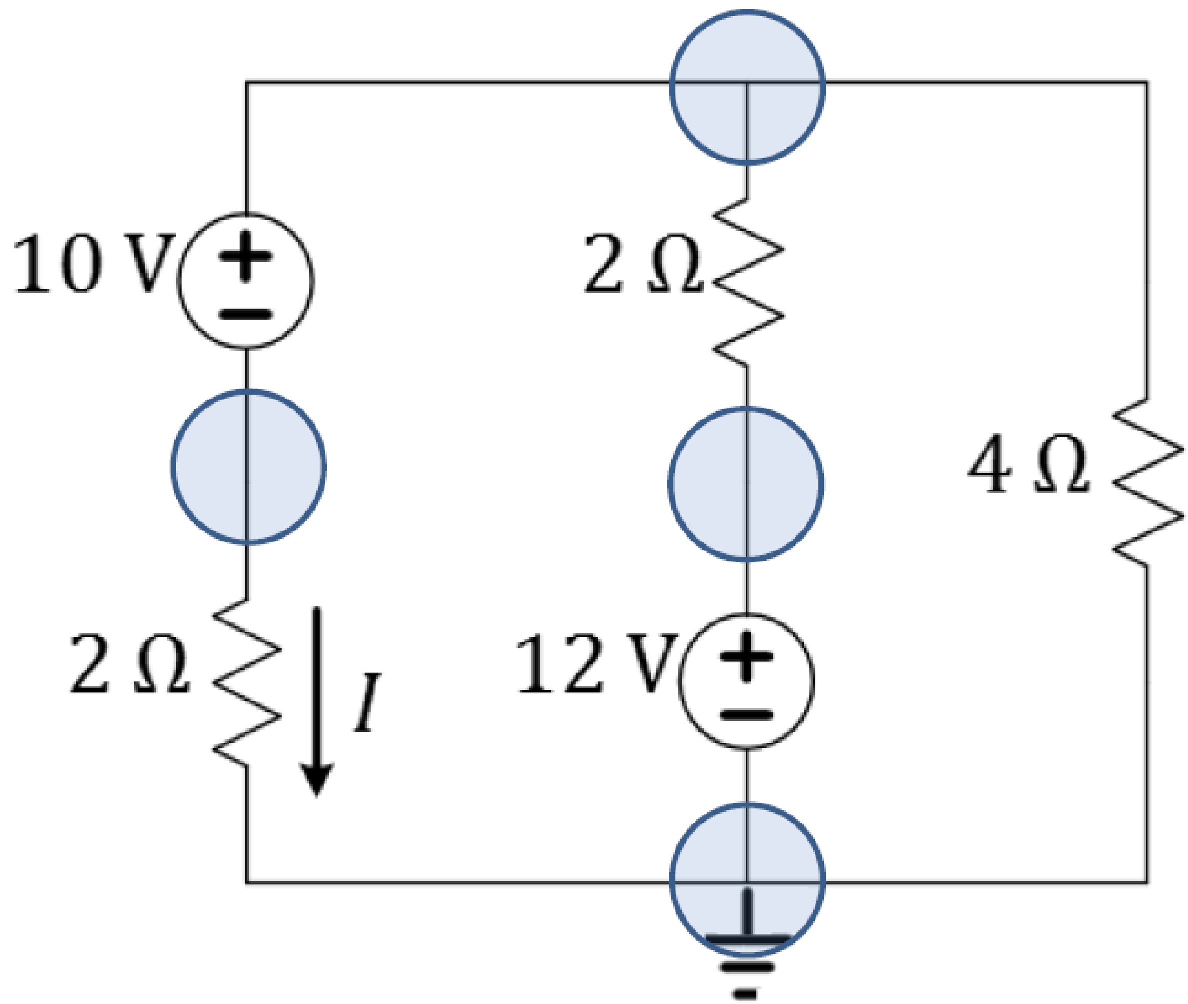
Floating voltage source

Not connected to ground



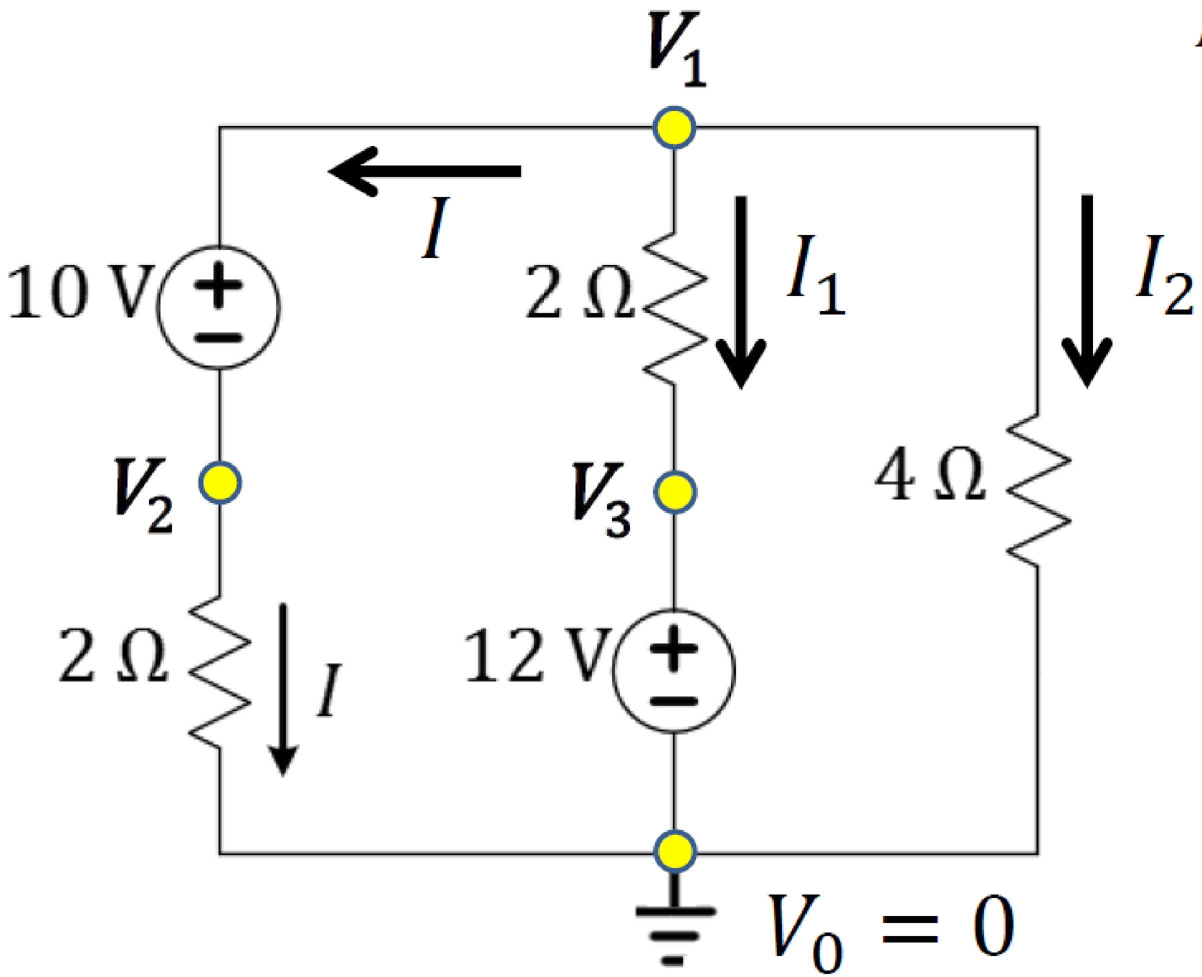
Find Current I

4 nodes



KCL – node 1

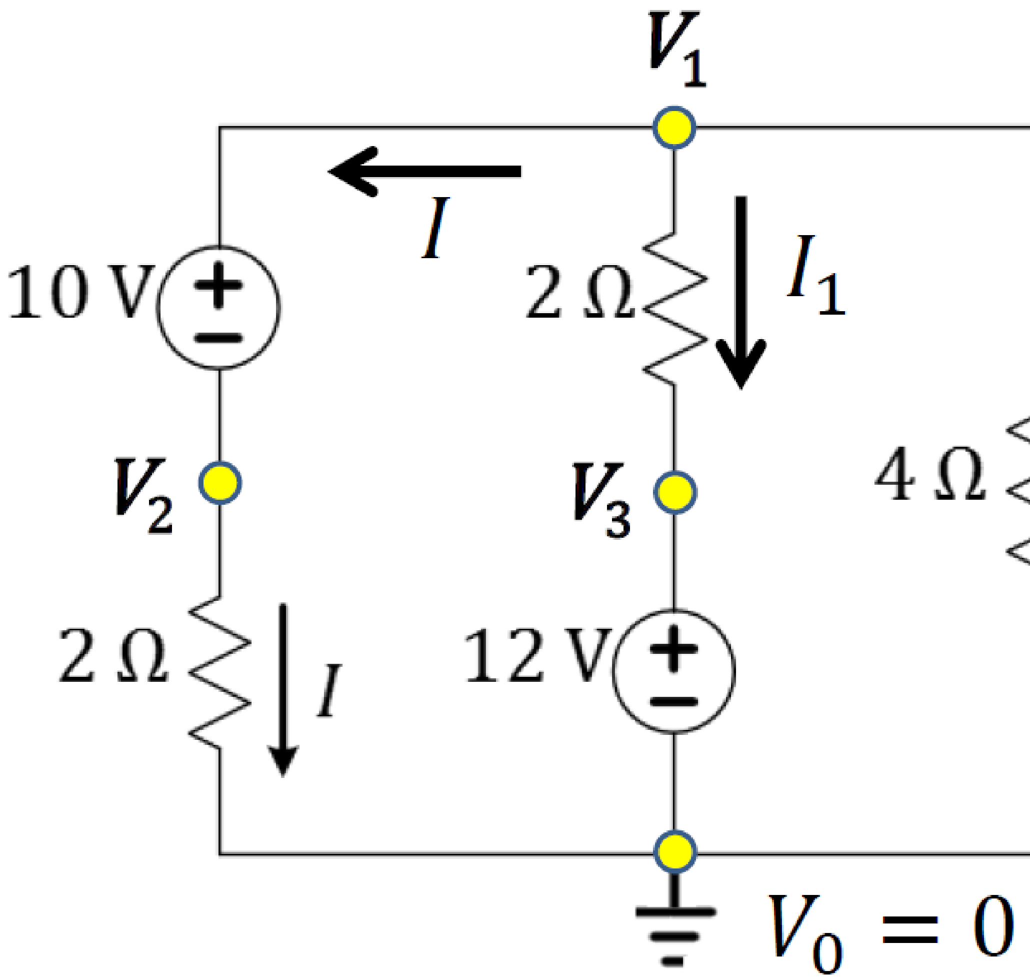
$$I_1 + I_2 + I = 0$$



$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_2 = \frac{V_1 - V_0}{4}$$

$$I = \frac{V_2 - V_0}{2}$$



KCL – node 1

$$I_1 + I_2 + I = 0$$

$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_2 = \frac{V_1 - V_0}{4}$$

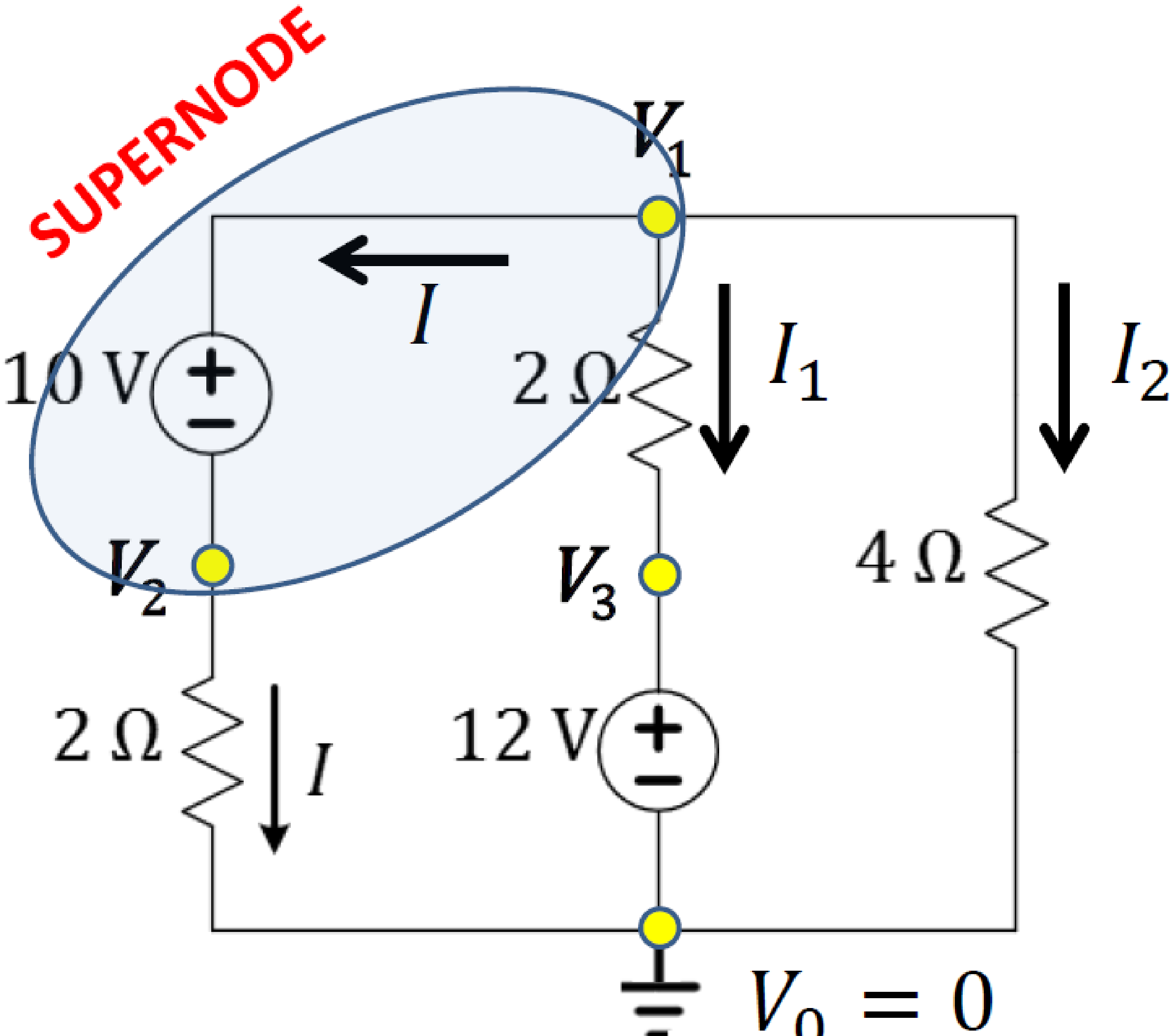
$$I = \frac{V_2 - V_0}{2}$$

What is V_2?

$$V_2 = V_1 - 10$$

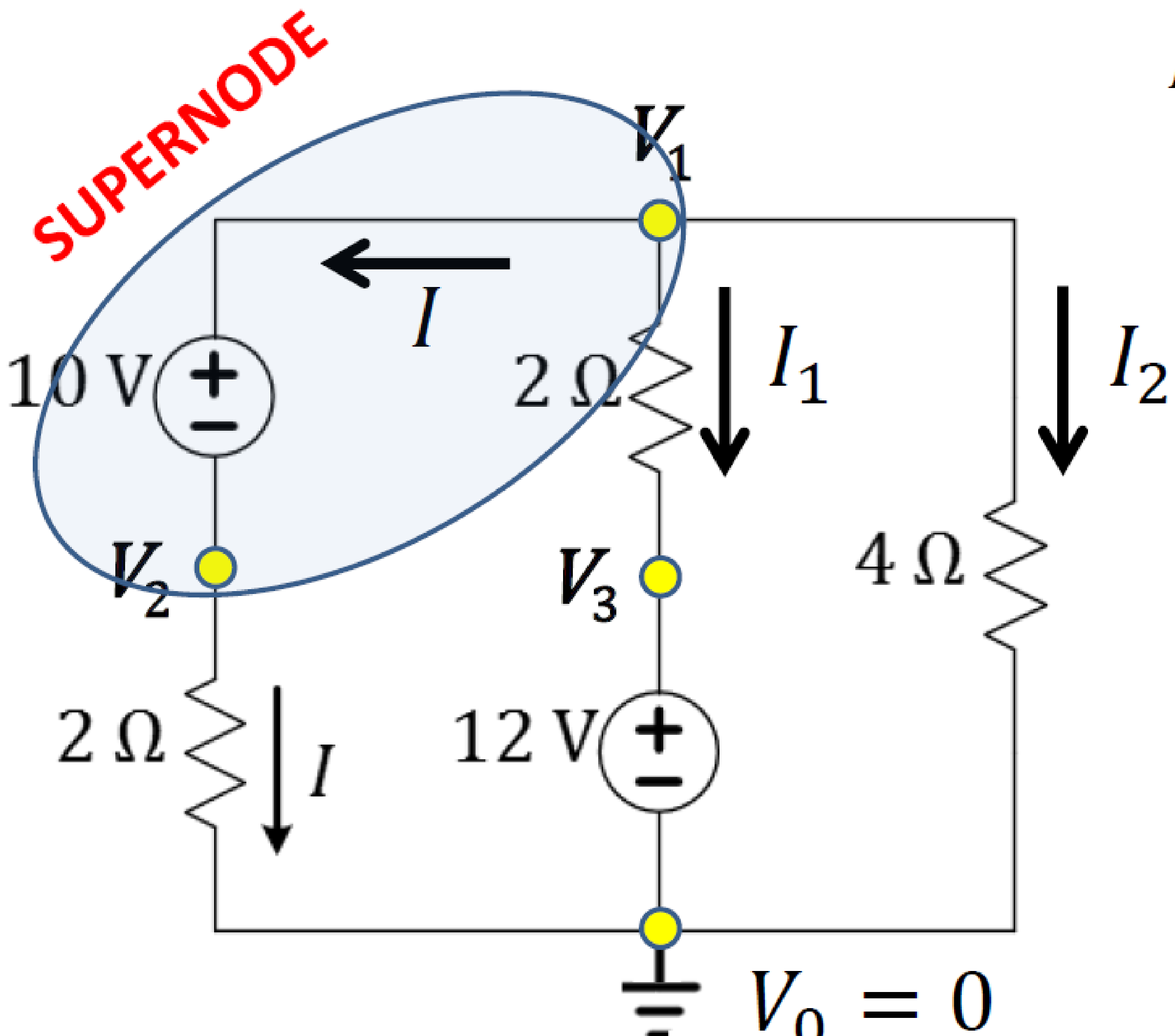
$$V_3 = 12V$$

By inspection



$$V_2 = V_1 - 10$$

$$V_3 = 12V$$



KCL – node 1

$$I_1 + I_2 + I = 0$$

$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_2 = \frac{V_1 - V_0}{4}$$

$$I = \frac{V_2 - V_0}{2}$$

$$V_2 = V_1 - 10$$

$$V_3 = 12V$$

$$I_1 = \frac{V_1 - V_3}{2} \qquad I_2 = \frac{V_1 - V_0}{4} \qquad I = \frac{V_2 - V_0}{2}$$

KCL – node 1 $\quad I_1 + I_2 + I = 0$

$$\frac{V_1 - 12}{2} + \frac{V_1}{4} + \frac{V_2}{2} = 0$$

$$-12 + \frac{3V_1}{2} + V_2 = 0$$

$$-12 + \frac{3V_1}{2} - V_1 - 10 = 0$$

$$V_1 = 8.8\text{V}$$

$$V_2 = V_1 - 10$$

$$V_3 = 12\text{V}$$

$$V_1 = 8.8\text{V}$$

$$V_2 = -1.2\text{V}$$

$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_2 = \frac{V_1 - V_0}{4}$$

$$I = \frac{V_2 - V_0}{2}$$

$$I_1 = \frac{V_1 - V_3}{2} = -\frac{3.2}{2} = -1.6\text{A}$$

$$I_2 = \frac{V_1}{4} = \frac{8.8}{4} = 2.2A$$

$$I = \frac{V_2}{2} = -0.6\text{A}$$

Verify KCL

$$I = -I_1 - I_2 \Rightarrow I = 1.6 - 2.2 = -0.6\text{A}$$

$V_2 = V_1 - 10$

$V_3 = 12\text{V}$

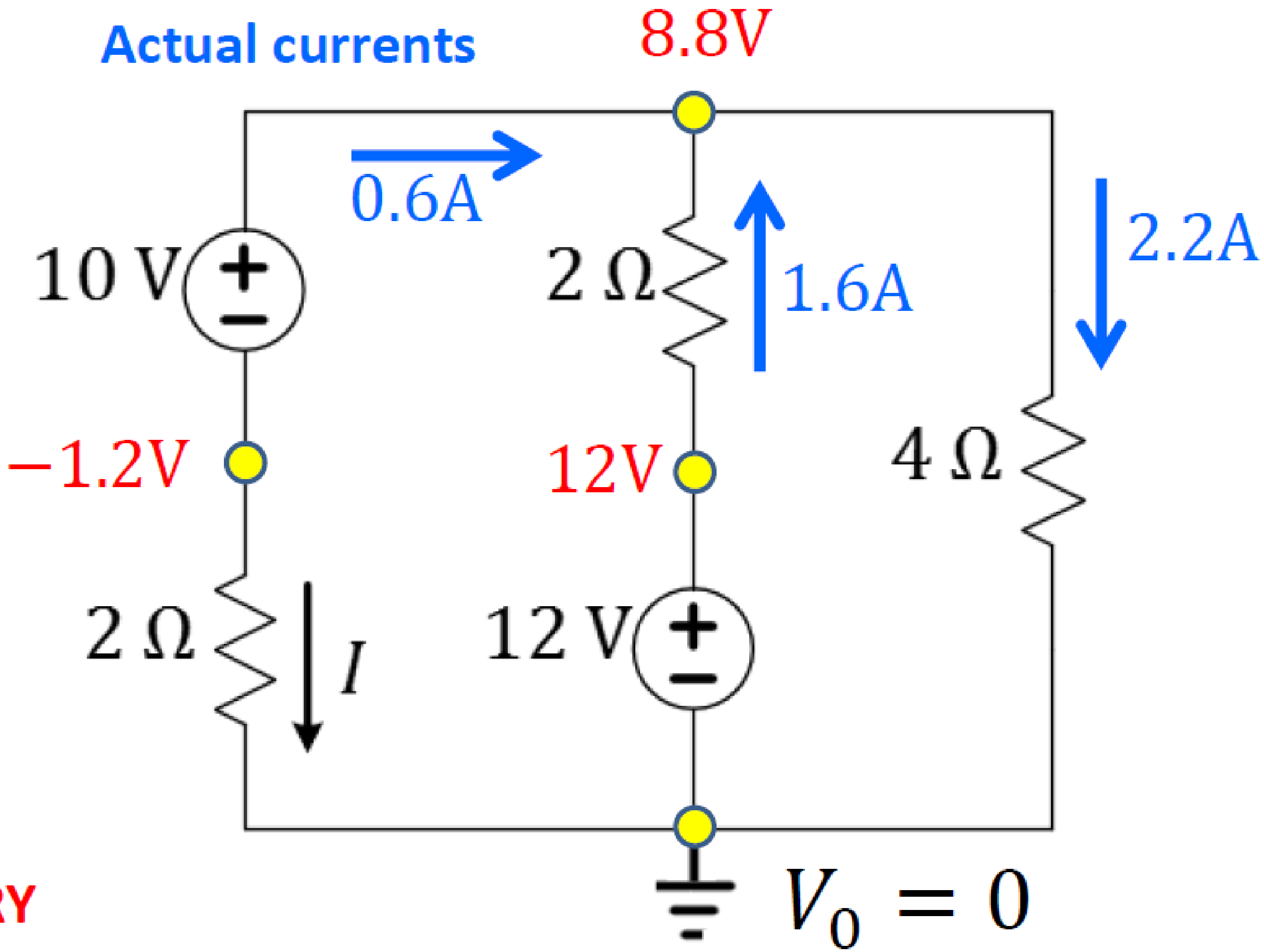
$V_1 = 8.8\text{V}$

$V_2 = -1.2\text{V}$

$I_1 = -1.6\text{A}$

$I_2 = 2.2\text{A}$

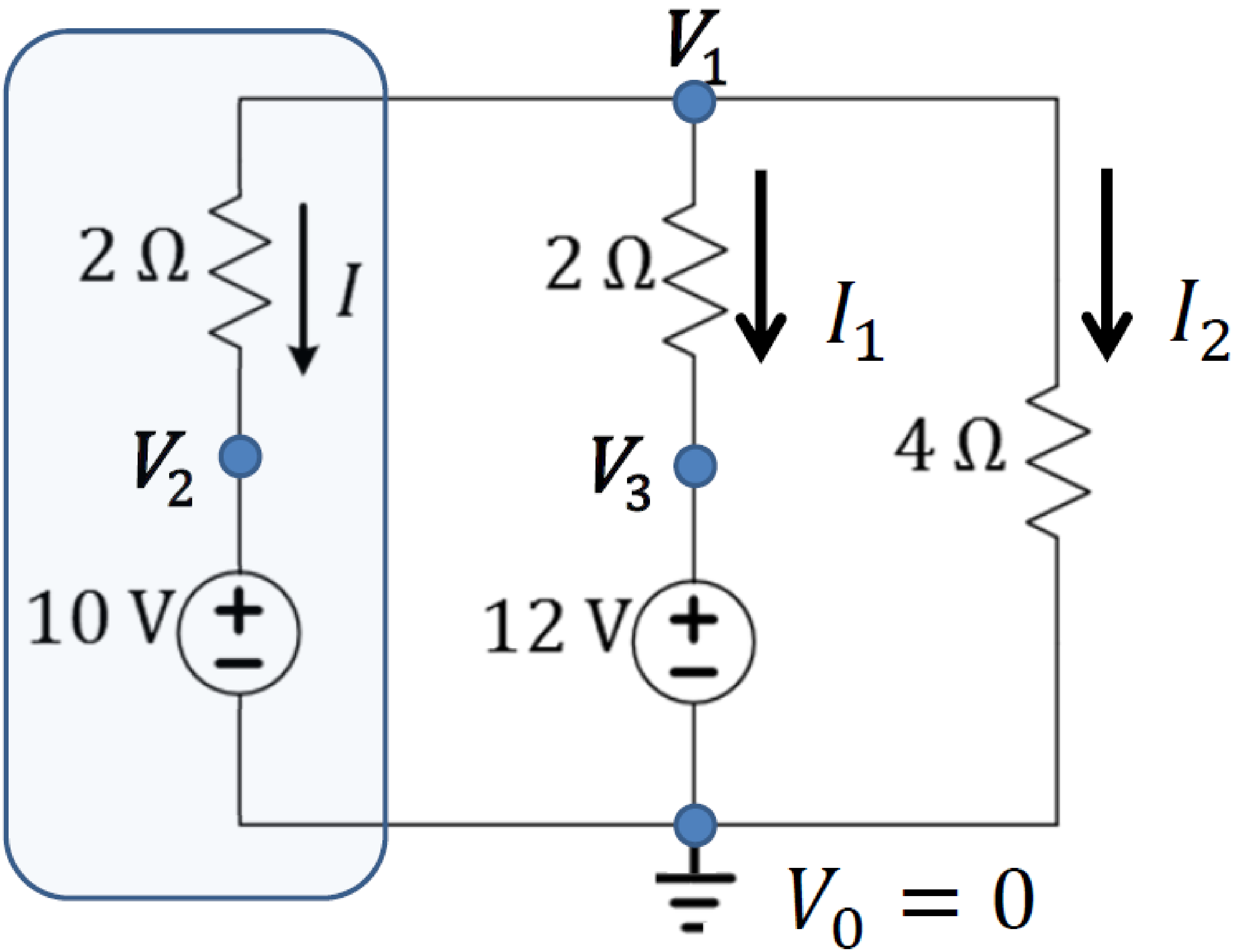
$I = -0.6\text{A}$



SUMMARY

What if we swap elements about node 2?

$V_2 = 10$ | $V_3 = 12V$ | $V_1 = 8.8V$ | $V_1 - V_2 = -1.2V$



Same result for currents, no need for supernode

$$I_1 = -1.6\text{A}$$

$$I_2 = 2.2\text{A}$$

$$I = -0.6\text{A}$$