# ECE 205 “Electrical and Electronics Circuits”

**Spring 2024 – LECTURE 8**

MWF – 12:00pm

**Prof. Umberto Ravaioli**

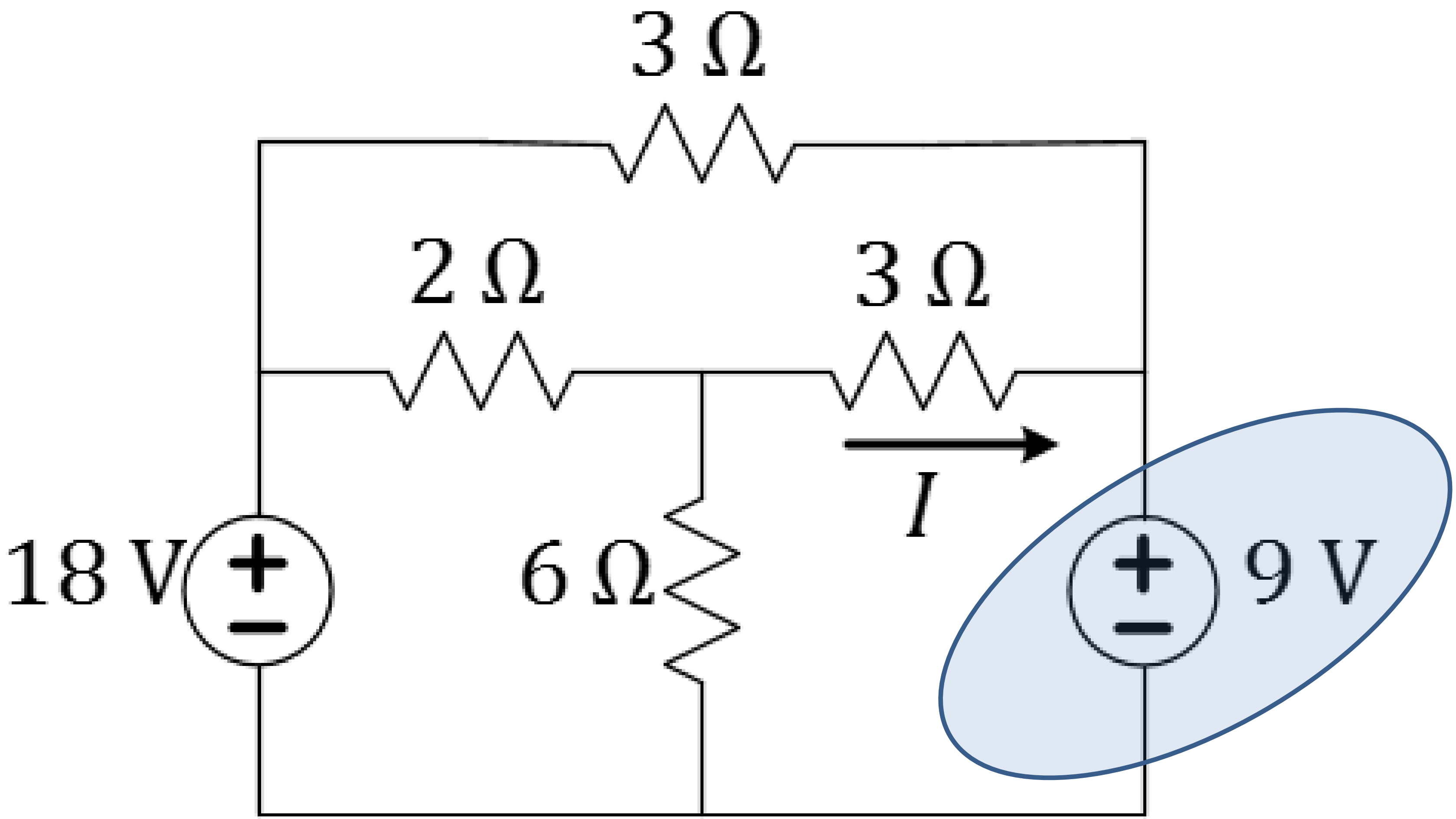
2062 ECE Building

# Lecture 8 – Summary

## Learning Objectives

1. More practice with the node analysis method to compute node voltages
2. Supernodes
3. Introduce concept of equivalent circuit

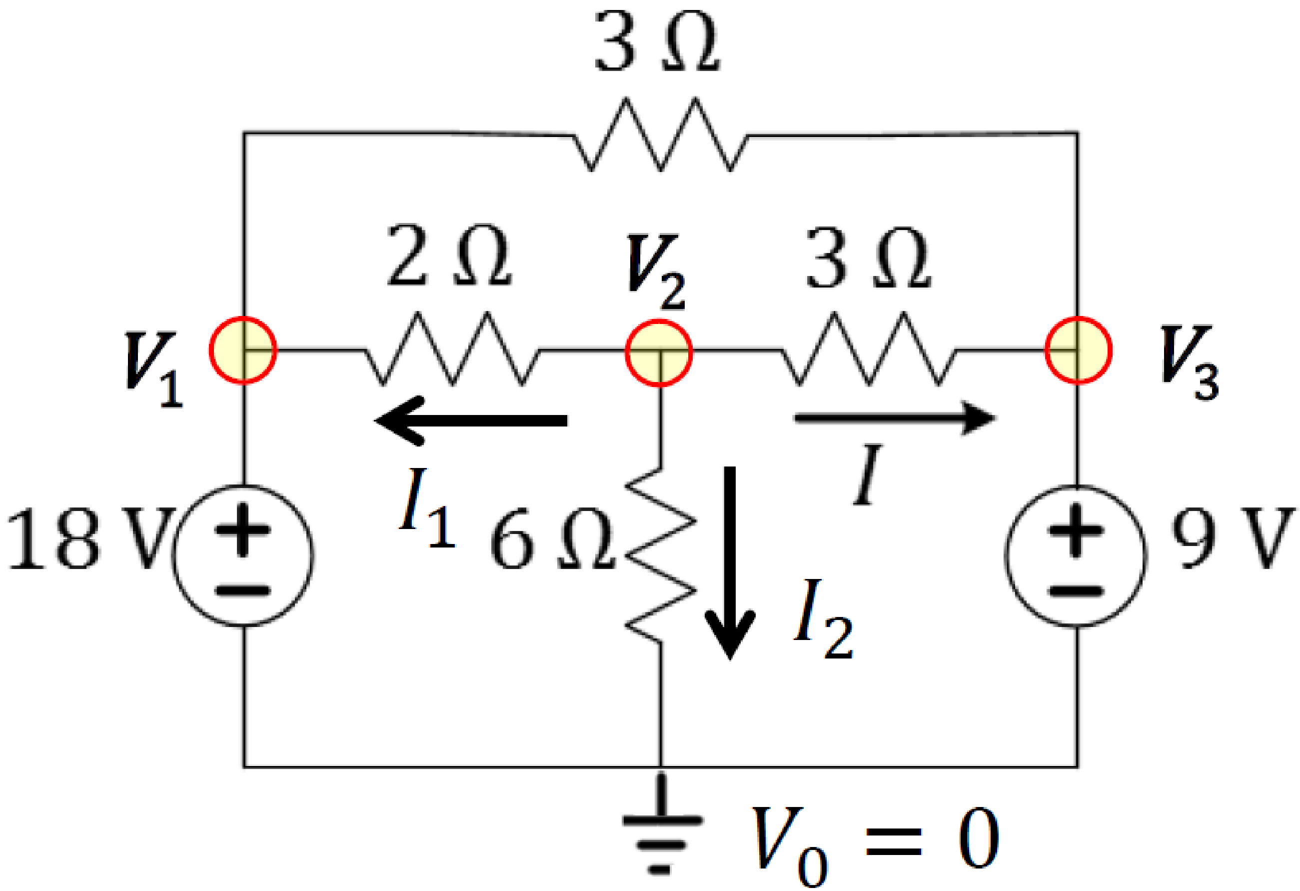
# Find the labelled current *I*




**Now there is a second voltage source in this branch. Node voltage analysis is a good approach.**

**In this example and in the following ones, we are going to specify fixed references for the currents in each of the circuit branches.**

**As mentioned earlier, this is a good approach for implementation of computer circuit solution using algorithms based on linear algebra.**




$$I_1 = \frac{V_2 - V_1}{2}$$

$$I_2 = \frac{V_2 - V_0}{6}$$

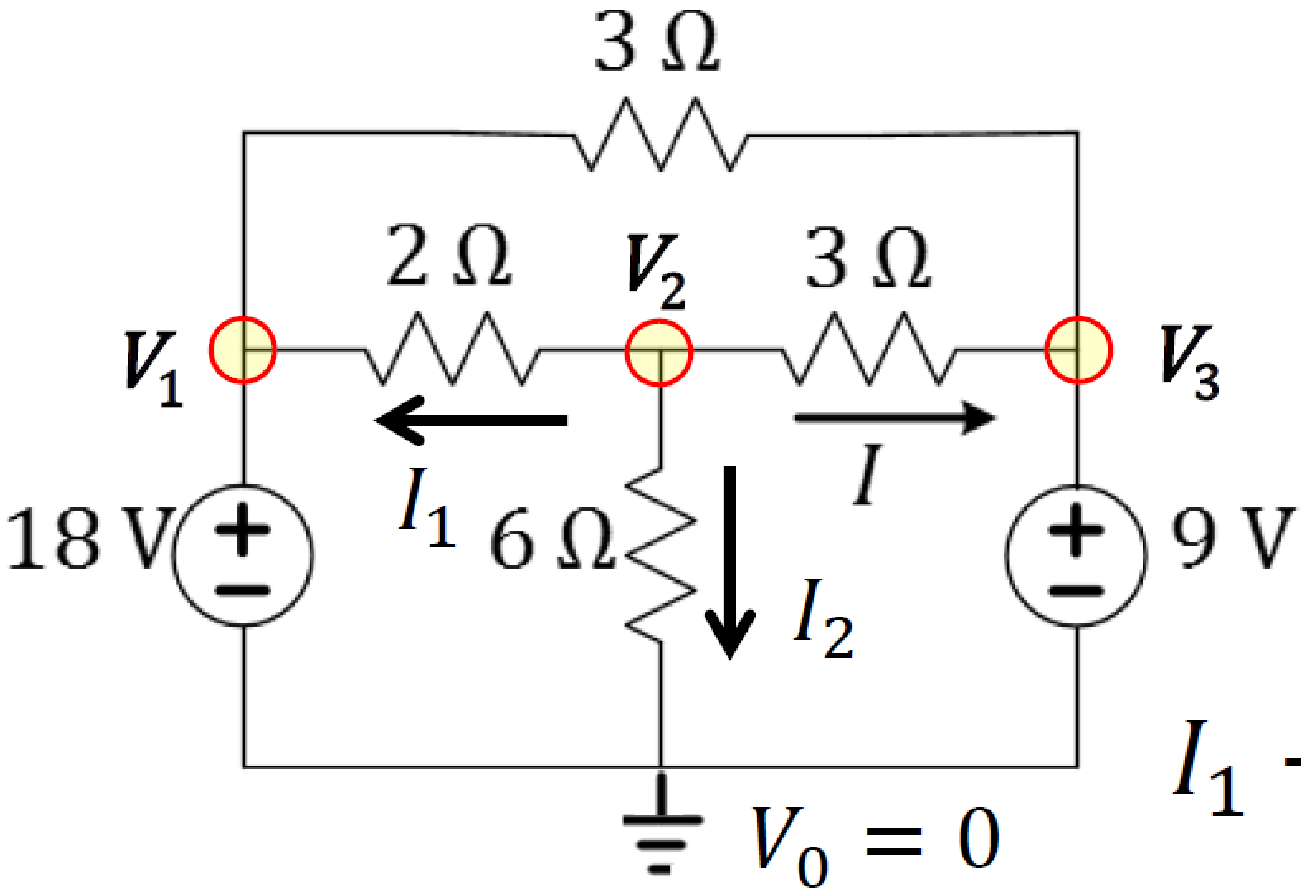
$$I = \frac{V_2 - V_3}{3}$$

Ground reference – zero potential

KCL – node 2

$$I_1 + I_2 + I = 0$$

**By inspection**



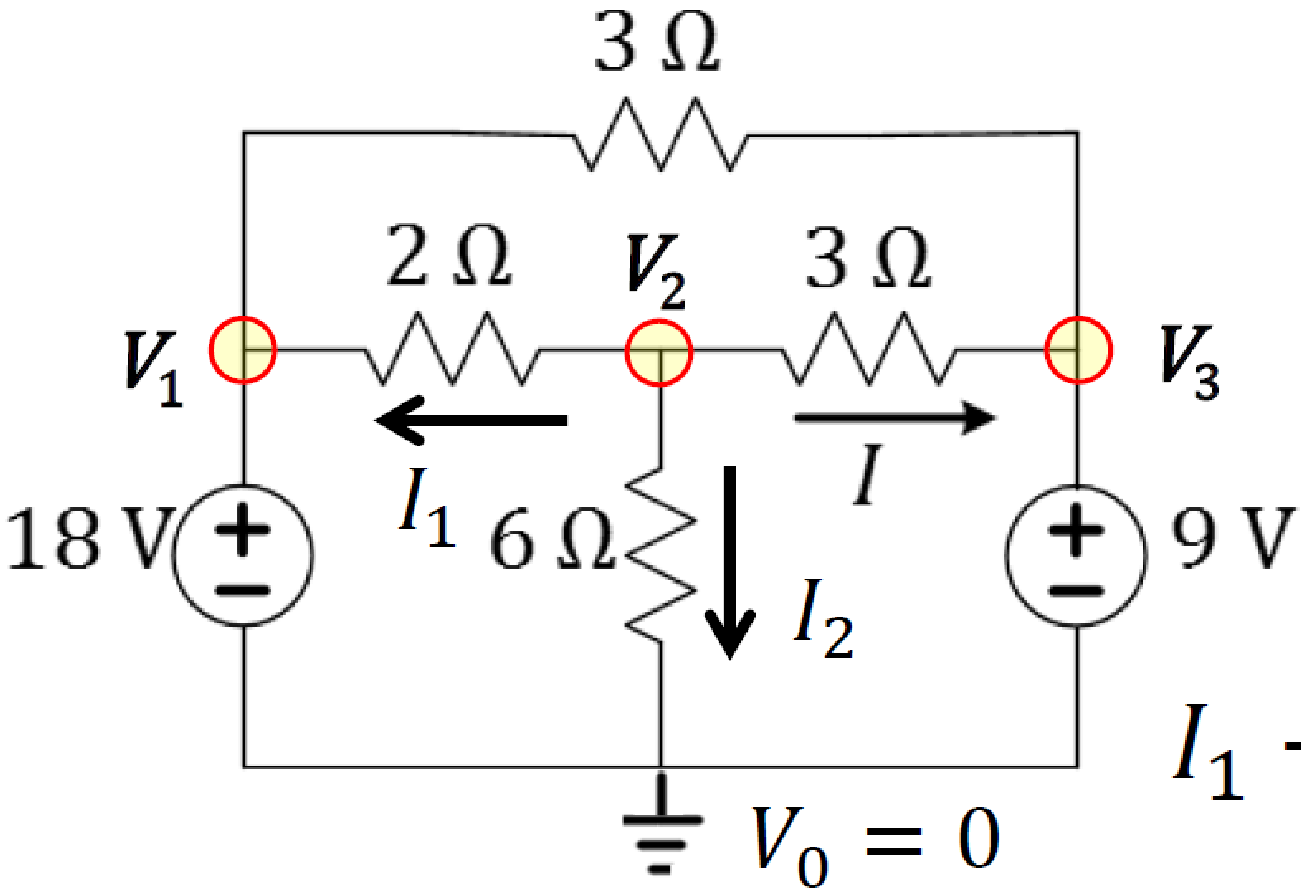

$V_1 = 18\text{V}$

$V_3 = 9V$

$$I_1 + I_2 + I = 0$$

$$\frac{V_2 - 18}{2} + \frac{V_2}{6} + \frac{V_2 - 9}{3} = 0 \Rightarrow \frac{V_2}{2} + \frac{V_2}{6} + \frac{V_2}{3} = 12$$

$$\Rightarrow \frac{3V_2}{6} + \frac{V_2}{6} + \frac{2V_2}{6} = 12 \Rightarrow V_2 = 12\text{V}$$




**By inspection**

$$V_1 = 18\text{V}$$

$$V_3 = 9V$$

$$V_2 = 12\text{V}$$

$$I_1 + I_2 + I = 0$$

$$I_1 = \frac{12 - 18}{2} = -3\text{A}$$

$$I_2 = \frac{12}{6} = 2\text{A}$$

**Ohm's Law**

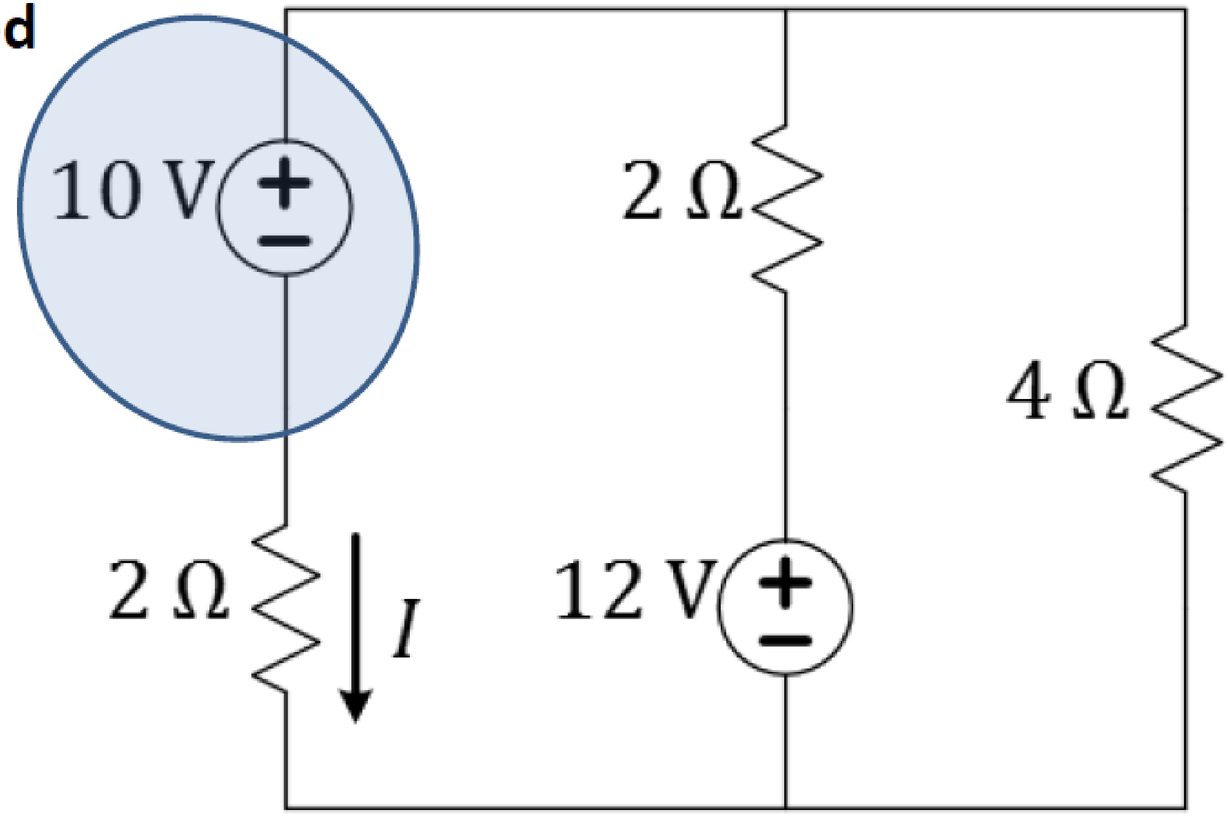
$$I = \frac{12 - 9}{3} = 1\text{A}$$

**KCL**

$$I = 3\text{A} - 2\text{A} = 1\text{A}$$

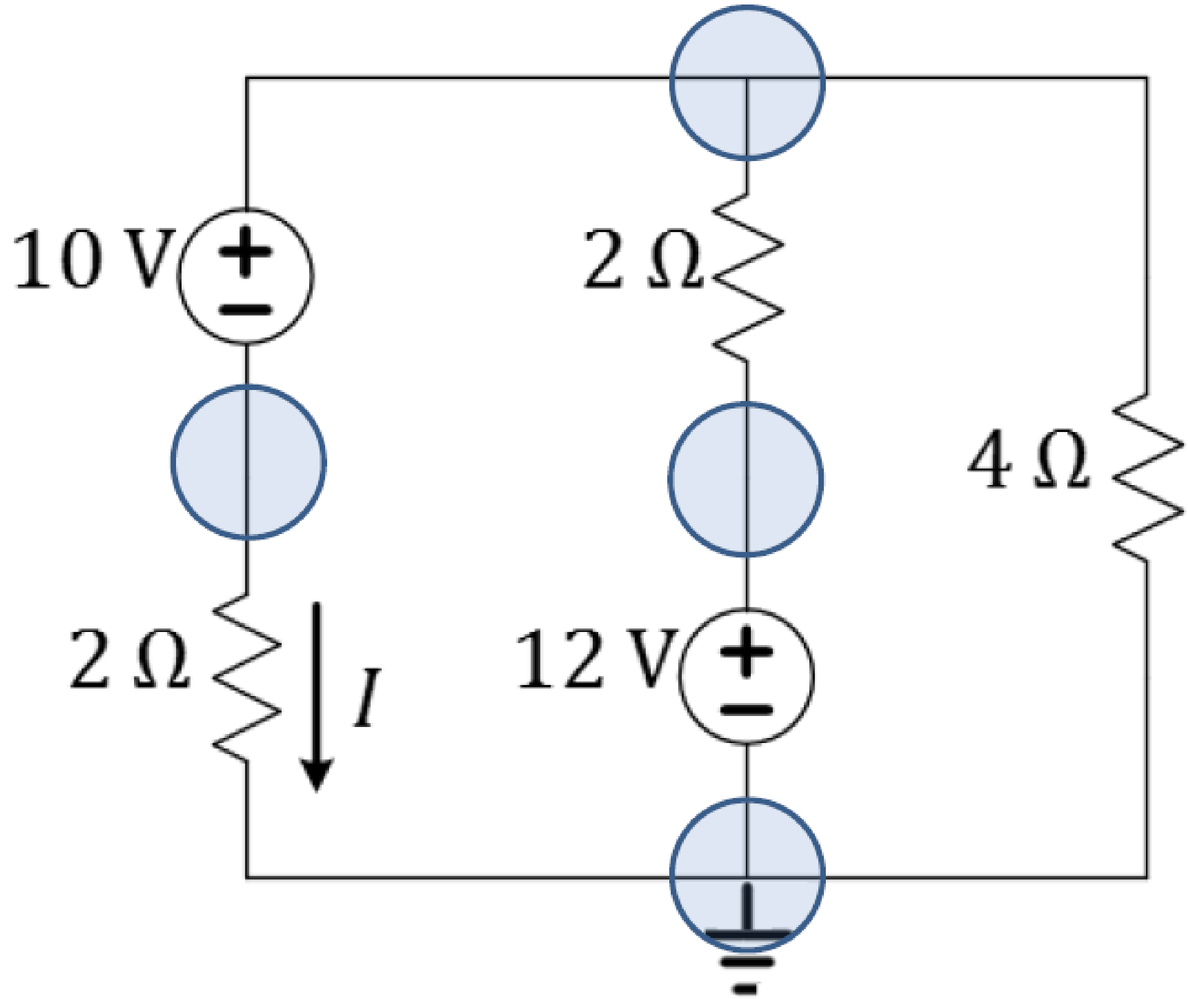
# Floating voltage source

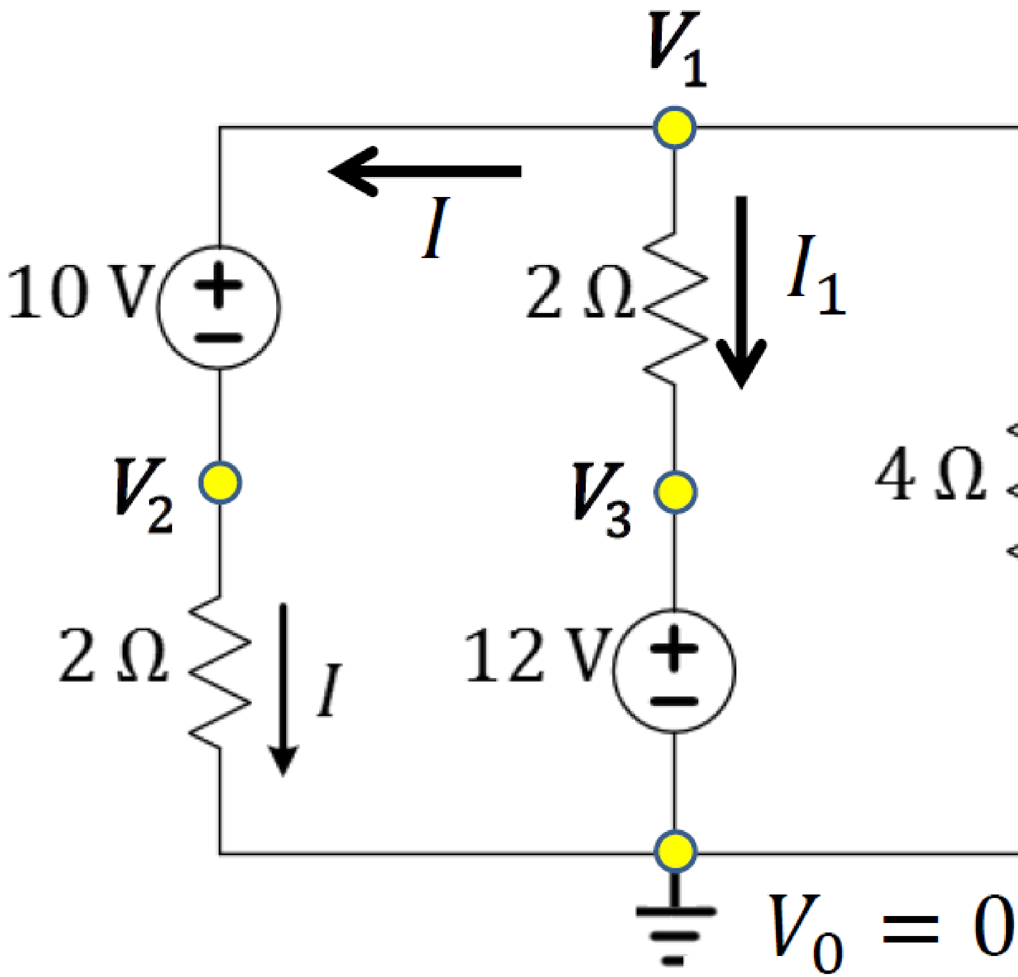
Not connected to ground



# Find Current $I$

**4 nodes**






KCL – node 1

$$I_1 + I_2 + I = 0$$

$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_2 = \frac{V_1 - V_0}{4}$$

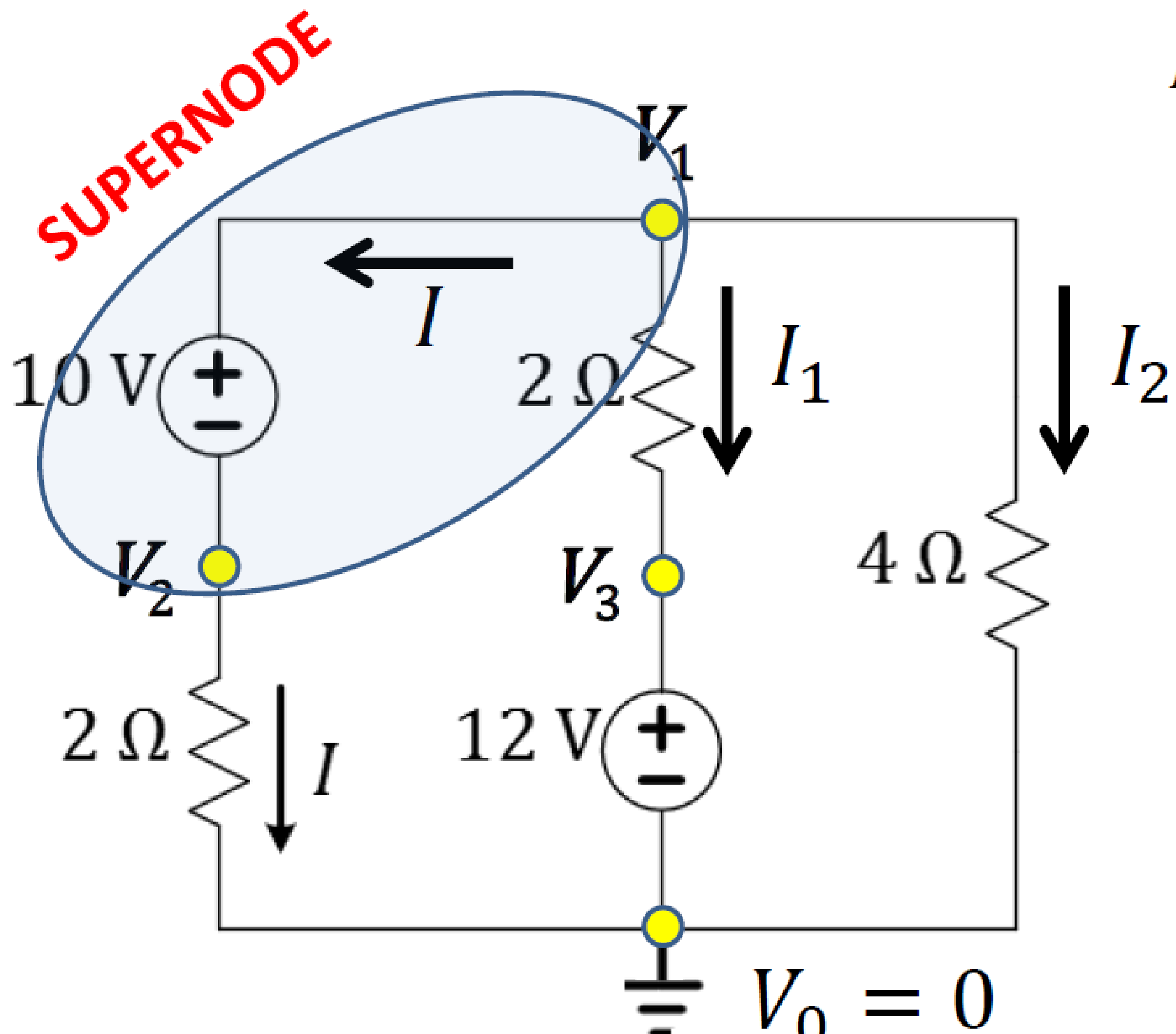
$$I = \frac{V_2 - V_0}{2}$$

$$V_2 = V_1 - 10$$

$$V_3 = 12V$$

KCL – node 1

$$I_1 + I_2 + I = 0$$




$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_2 = \frac{V_1 - V_0}{4}$$

$$I = \frac{V_2 - V_0}{2}$$

$$\boxed{V_2 = V_1 - 10} \qquad \boxed{V_3 = 12V}$$

$$I_1 = \frac{V_1 - V_3}{2} \qquad I_2 = \frac{V_1 - V_0}{4} \qquad I = \frac{V_2 - V_0}{2}$$

KCL – node 1 $\quad I_1 + I_2 + I = 0$

$$\frac{V_1 - 12}{2} + \frac{V_1}{4} + \frac{V_2}{2} = 0$$

$$-12 + \frac{3V_1}{2} + V_2 = 0$$

$$-12 + \frac{3V_1}{2} + V_1 - 10 = 0 \qquad \boxed{V_1 = 8.8\text{V}}$$

$V_2 = V_1 - 10$

$V_3 = 12\text{V}$

$V_1 = 8.8\text{V}$

$V_2 = -1.2\text{V}$

$$I_1 = \frac{V_1 - V_3}{2}$$

$$I_2 = \frac{V_1 - V_0}{4}$$

$$I = \frac{V_2 - V_0}{2}$$

$$I_1 = \frac{V_1 - V_3}{2} = -\frac{3.2}{2} = -1.6\text{A}$$

$$I_2 = \frac{V_1}{4} = \frac{8.8}{4} = 2.2A$$

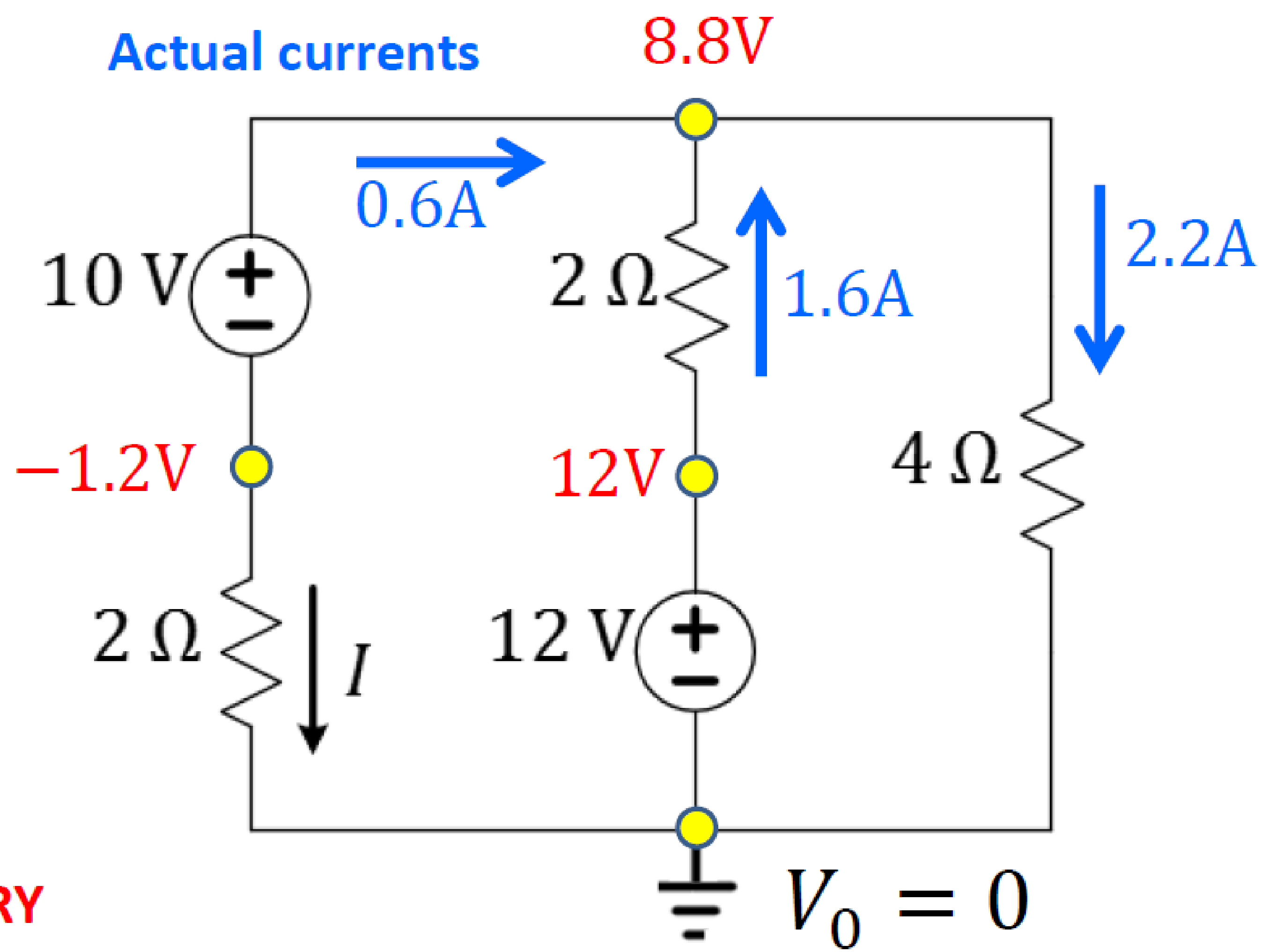
$$I = \frac{V_2}{2} = -0.6\text{A}$$

Verify KCL

$I = -I_1 - I_2$ ⇨ $I = 1.6 - 2.2 = -0.6\text{A}$

$V_2 = V_1 - 10$ | $V_3 = 12\text{V}$ | $V_1 = 8.8\text{V}$ | $V_2 = -1.2\text{V}$

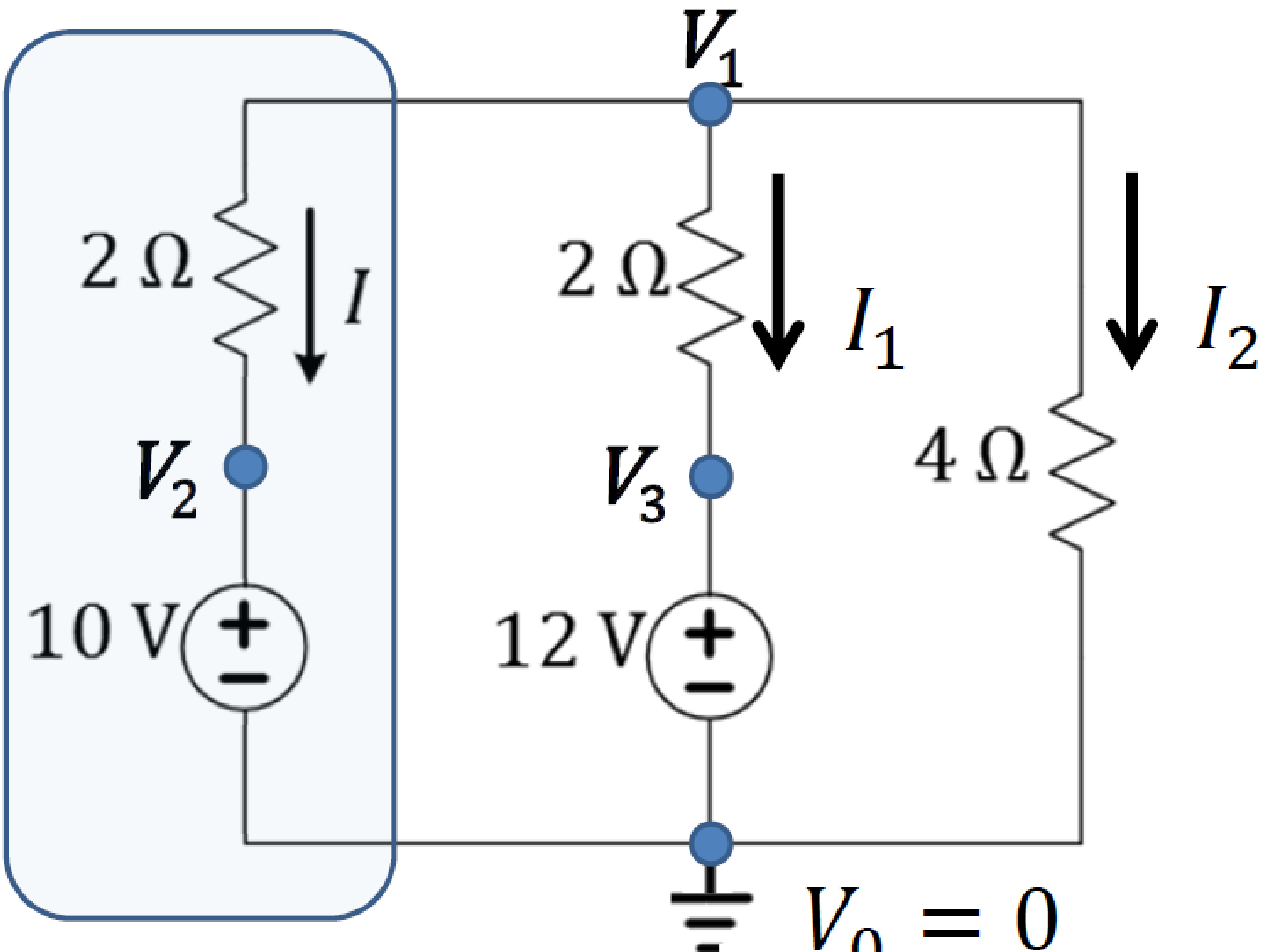
$I_1 = -1.6\text{A}$ | $I_2 = 2.2\text{A}$ | $I = -0.6\text{A}$




**SUMMARY**

# What if we swap elements about node 2?

$V_2 = 10$ | $V_3 = 12V$ | $V_1 = 8.8V$ | $V_1 - V_2 = -1.2V$




**Same result for currents, no need for supernode**

$$I_1 = -1.6\text{A}$$

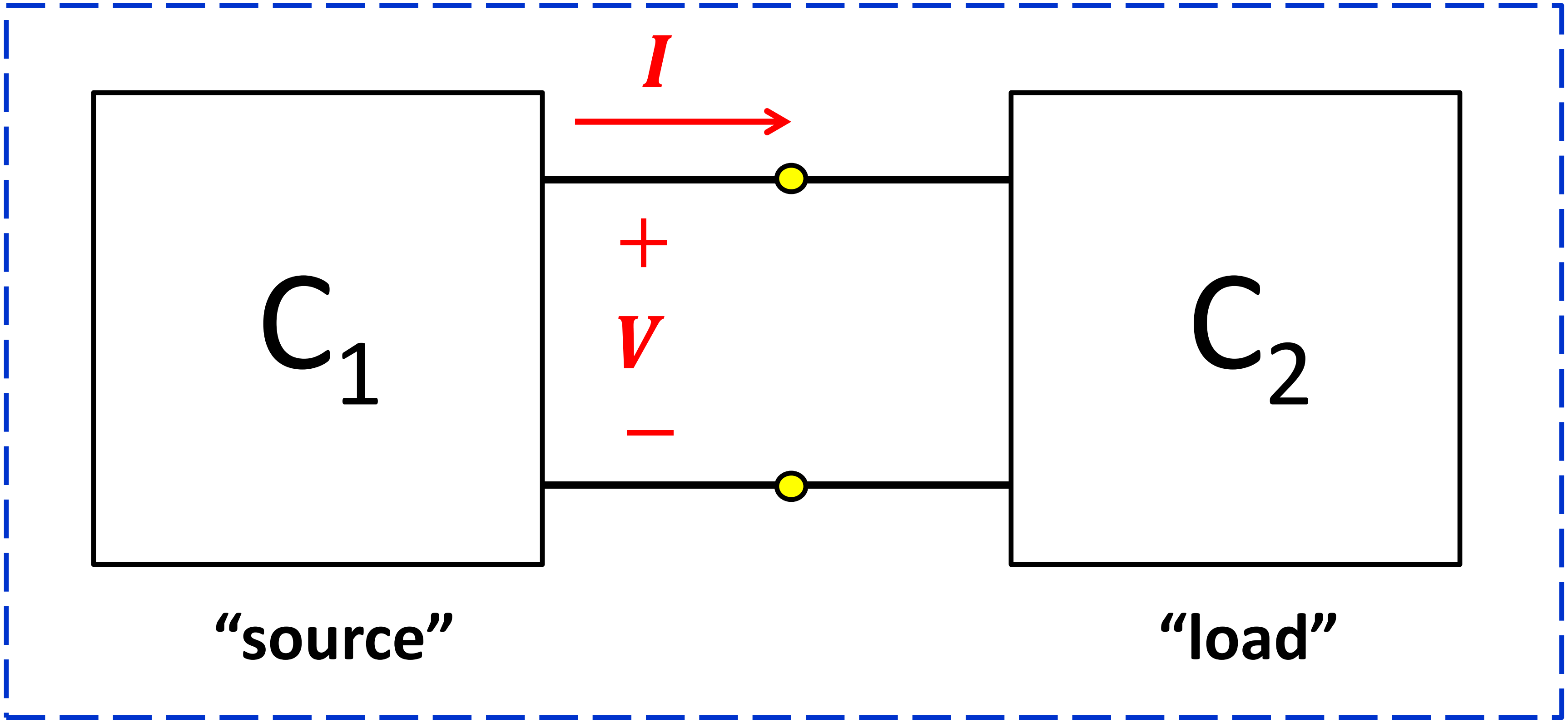
$$I_2 = 2.2\text{A}$$

$$I = -0.6\text{A}$$

# Introduction to Equivalent Circuits

# Circuit decomposition

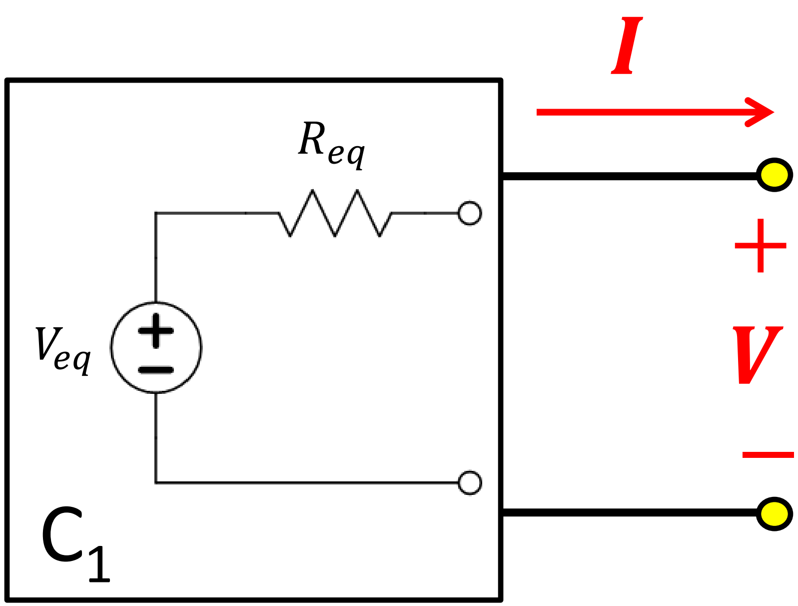
**Usually, an independent circuit can be decomposed into two sub-circuits, connected at two nodes and identifiable as a "source" and a "load".**



# Equivalent circuits

**The source sub-circuit is a "black box" identified by the voltage and output current at its terminals.**

**Assuming a "linear circuit" we can represent it as an equivalent ideal voltage generator connected in series to an equivalent internal resistance.**




"source"

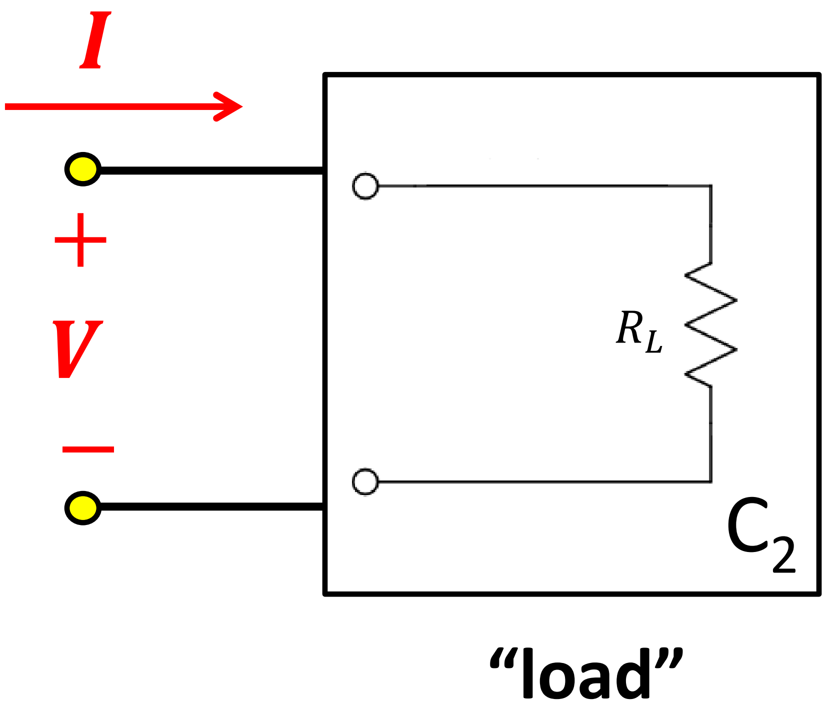
The equivalent source has the same terminal behavior of the original source sub-circuit

# Equivalent circuits

**The load sub-circuit is also a "black box" identified by the voltage and input current at its terminals.**

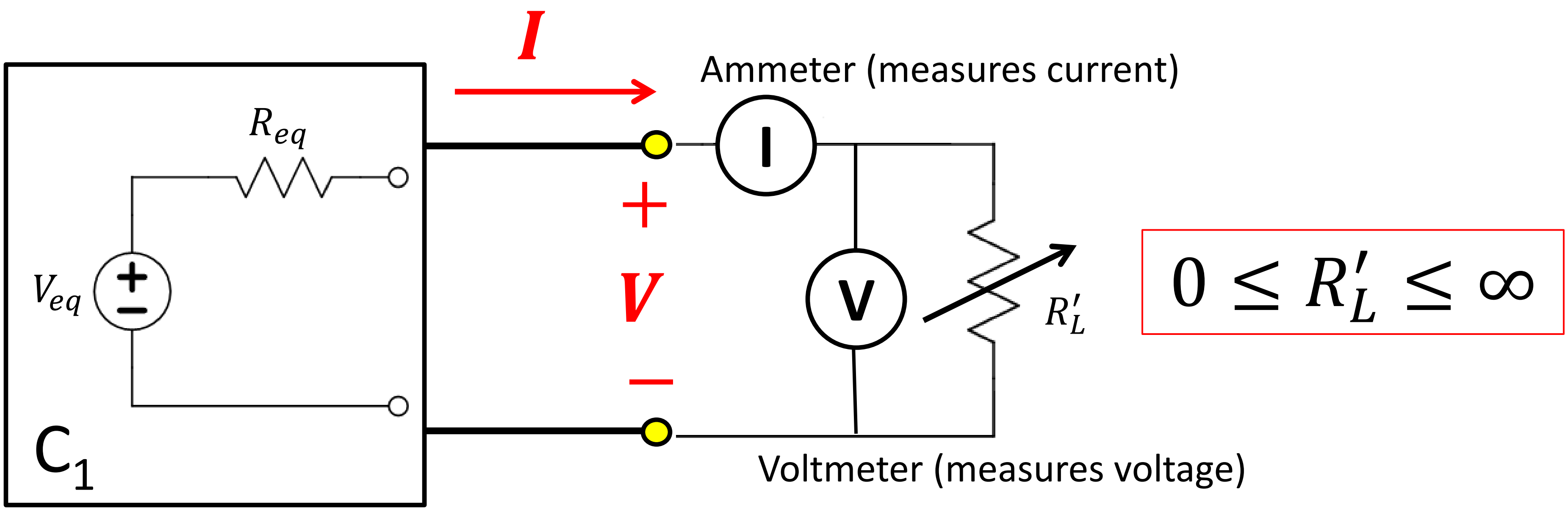
**We could represent it as an equivalent resistor.**

The equivalent load has the same terminal behavior of the original load sub-circuit



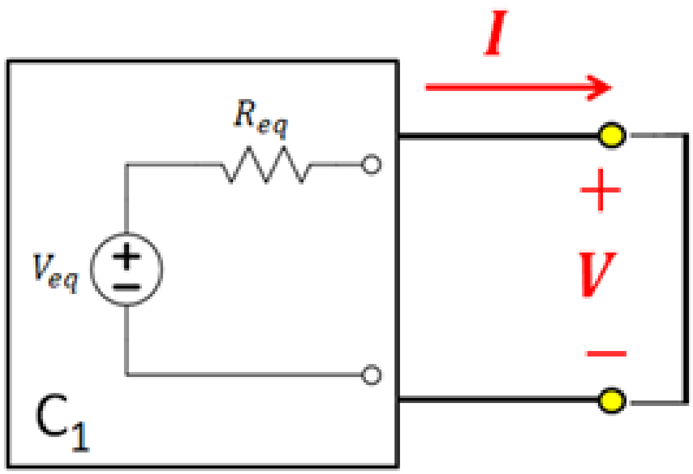
# Equivalent circuits

**To characterize the source sub-circuit, let's connect it to a variable resistor load to record the behavior at the terminals with a measurement.**




**"source"**

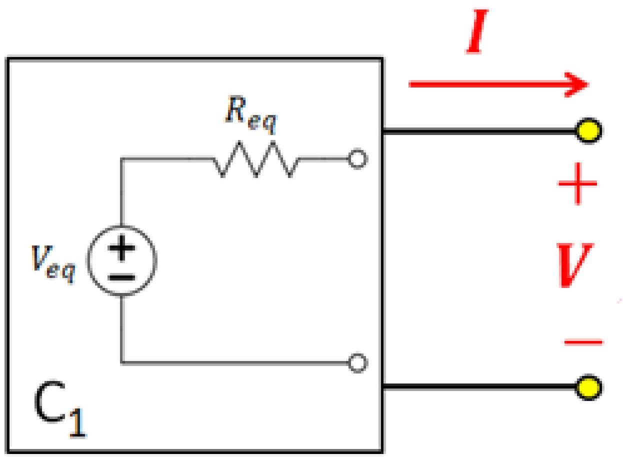
# Limit cases




$R_L' = 0$ (short circuit)

$$I_{sc} = \frac{V_{eq}}{R_{eq}}$$

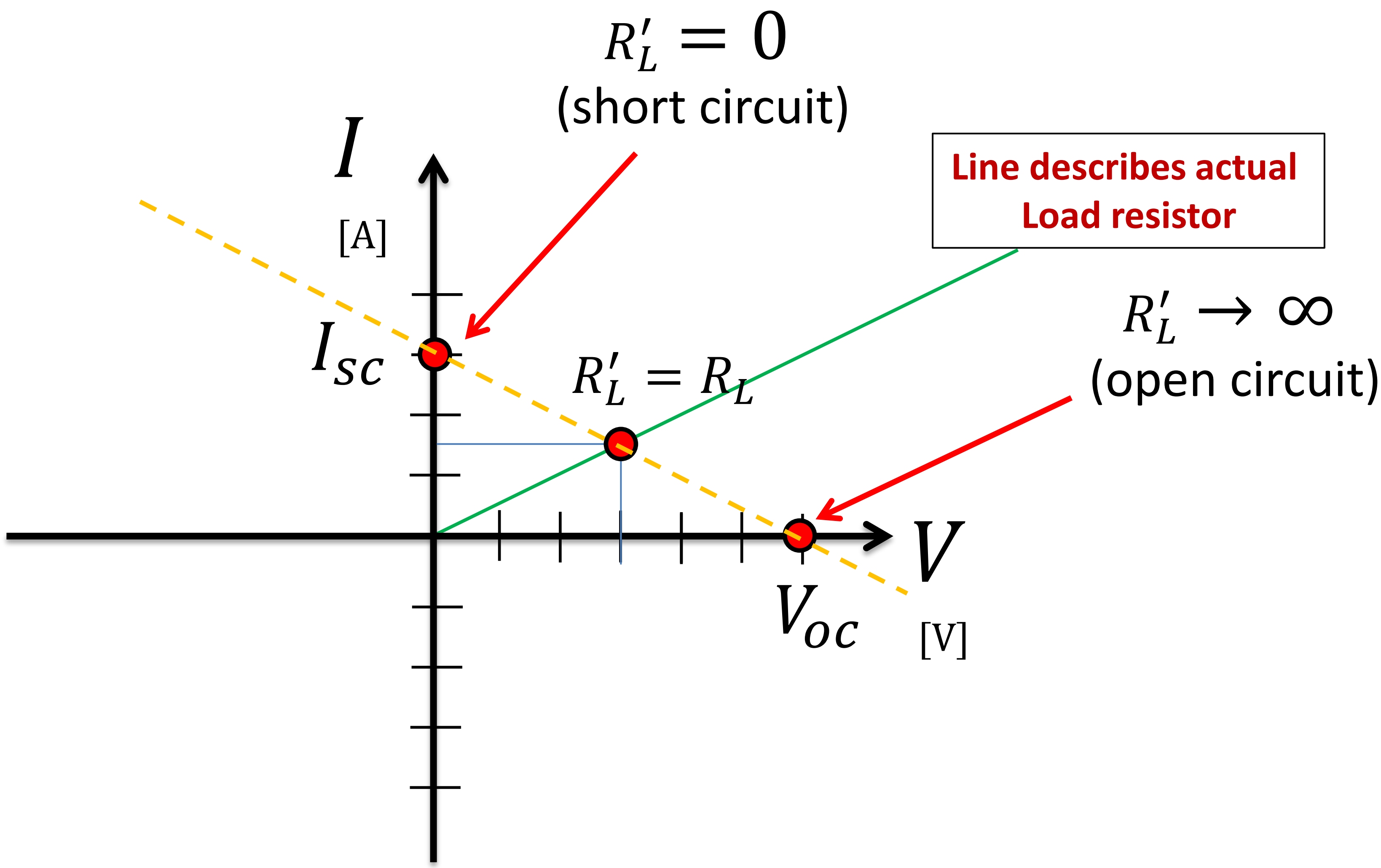
$$R_{eq} = \frac{V_{eq}}{I_{sc}}$$



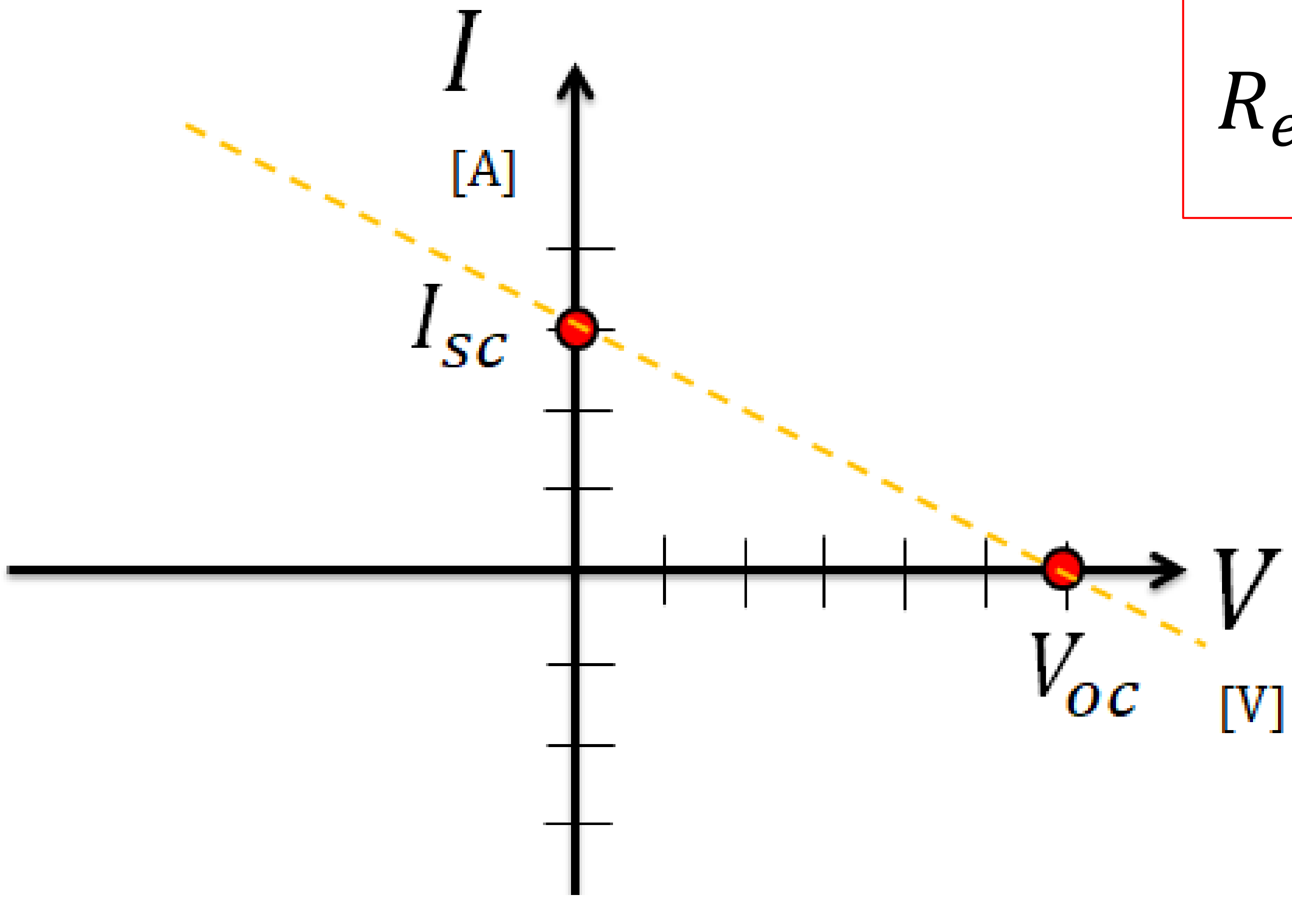

$R_L' \to \infty$ (open circuit)

$$V_{eq} = V_{oc}$$

# I-V Curve



$R_L' = 0$
(short circuit)
Line describes actual Load resistor
$I$
[A]
$I_{sc}$
$R_L' = R_L$
$R_L' \to \infty$
(open circuit)
$V$
$V_{oc}$
[V]



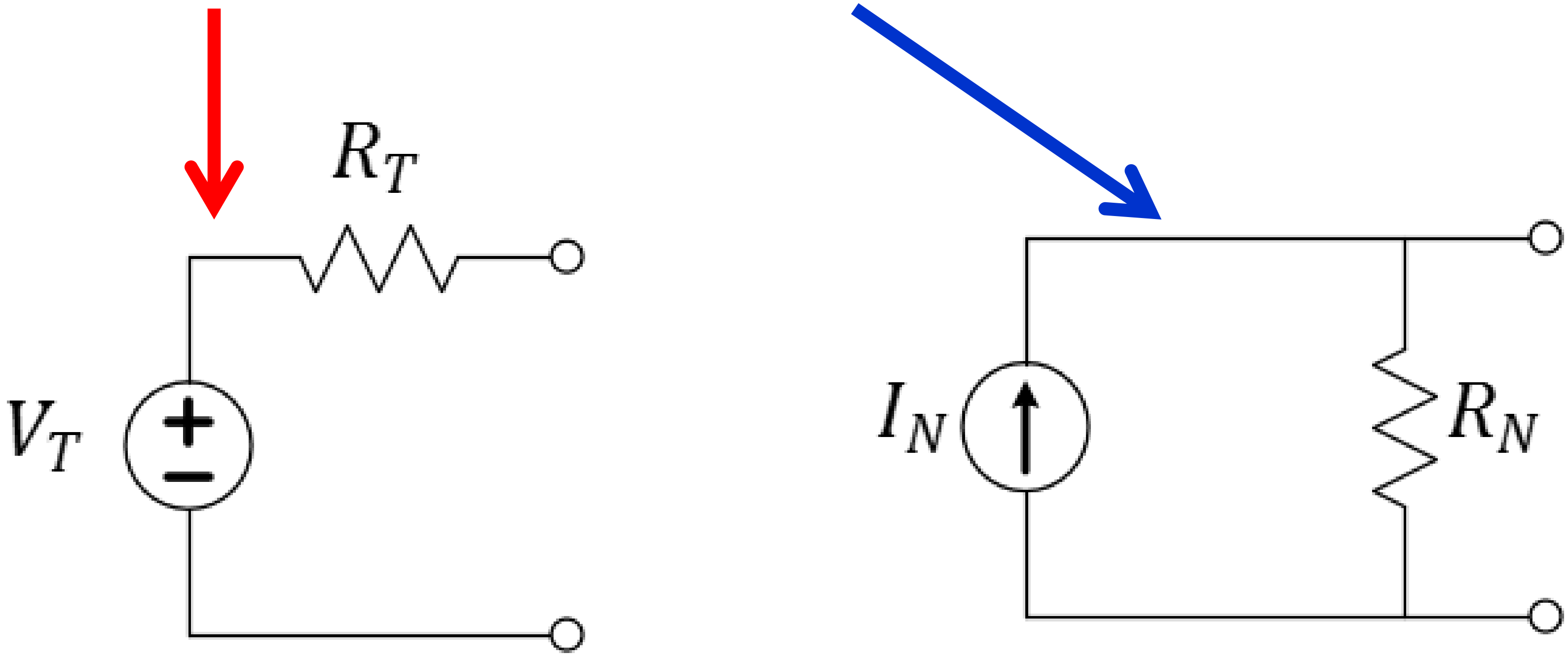

$$R_{eq} = \frac{V_{oc}}{I_{sc}}$$

$$I = I_{sc} - \frac{I_{sc}}{V_{oc}} V = I_{sc} - \frac{1}{R_{eq}} V$$

**This equation contains all the information on how the source circuit interacts with other circuits**

**We can formulate equivalent circuits with a voltage source or with a current source, producing the same terminals equation**

# Thevenin and Norton Equivalents




**Both represent the terminal equation**

$$I = I_{sc} - \frac{I_{sc}}{V_{oc}} V = I_{sc} - \frac{1}{R_{eq}} V$$