ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 9

MWF – 12:00pm

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2062 ECE Building

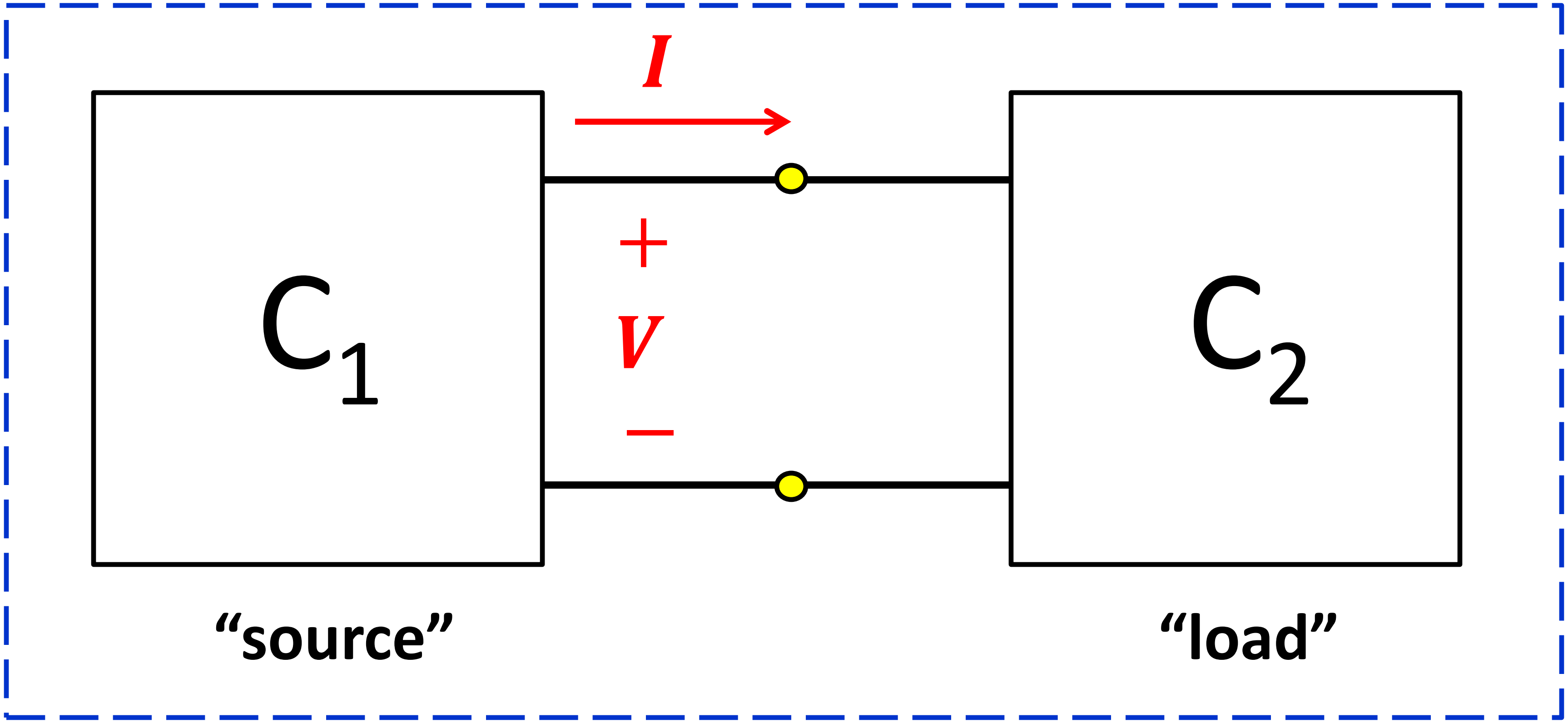
Lecture 9 – Summary

Learning Objectives

1. Understand equivalent circuits
2. Define the Thevenin equivalent for a source
3. Define the Norton equivalent for a source
4. Obtain the equivalent circuit

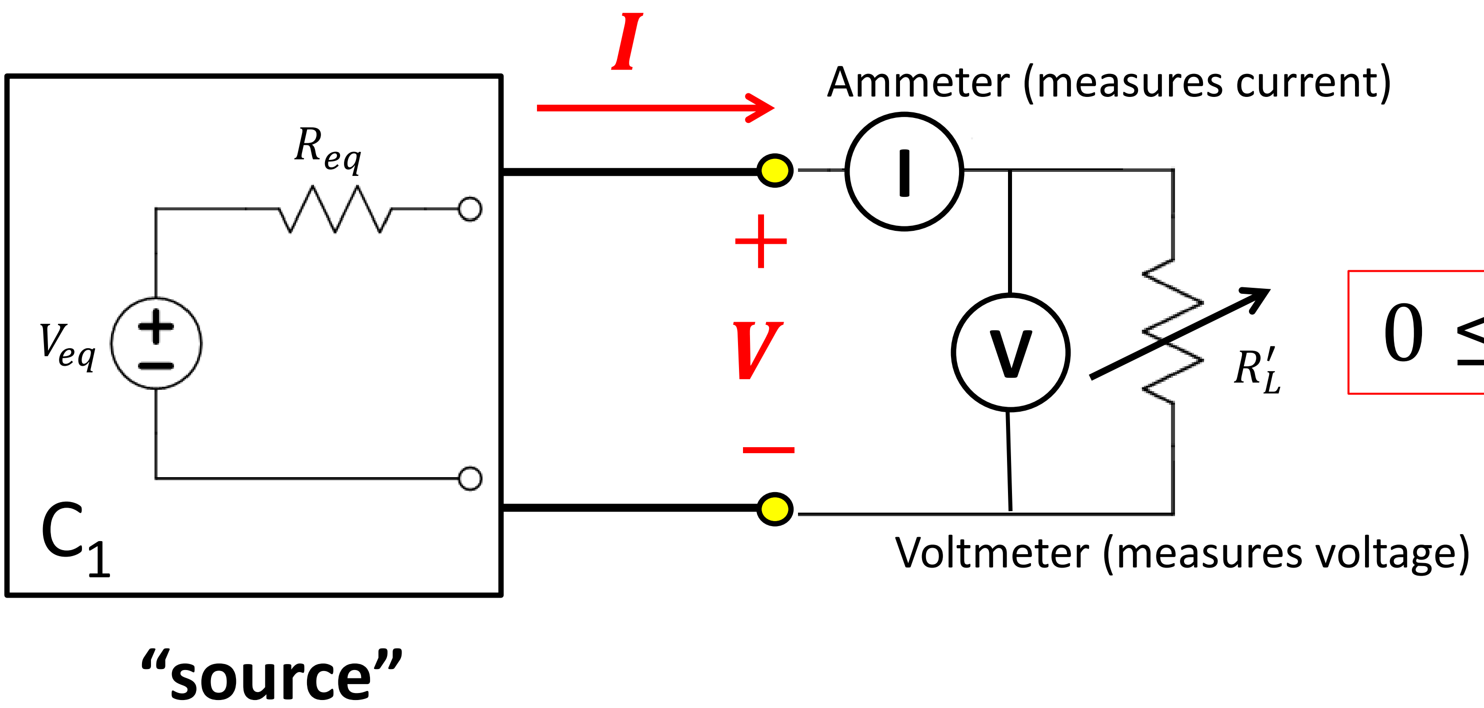
Circuit decomposition

Usually, an independent circuit can be decomposed into two sub-circuits, connected at two nodes and identifiable as a "source" and a "load".



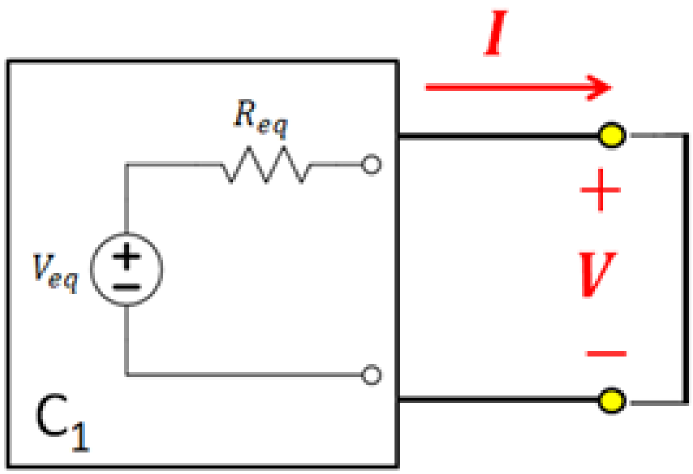
Equivalent circuits

To characterize the source sub-circuit, let's connect it to a variable resistor load to record the behavior at the terminals with a measurement.



$$0 \le R_L' \le \infty$$

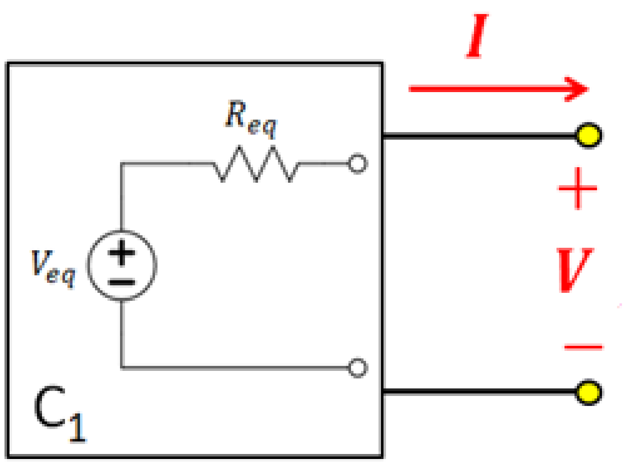
Limit cases



$R_L' = 0$ (short circuit)

$$I_{sc} = \frac{V_{eq}}{R_{eq}}$$

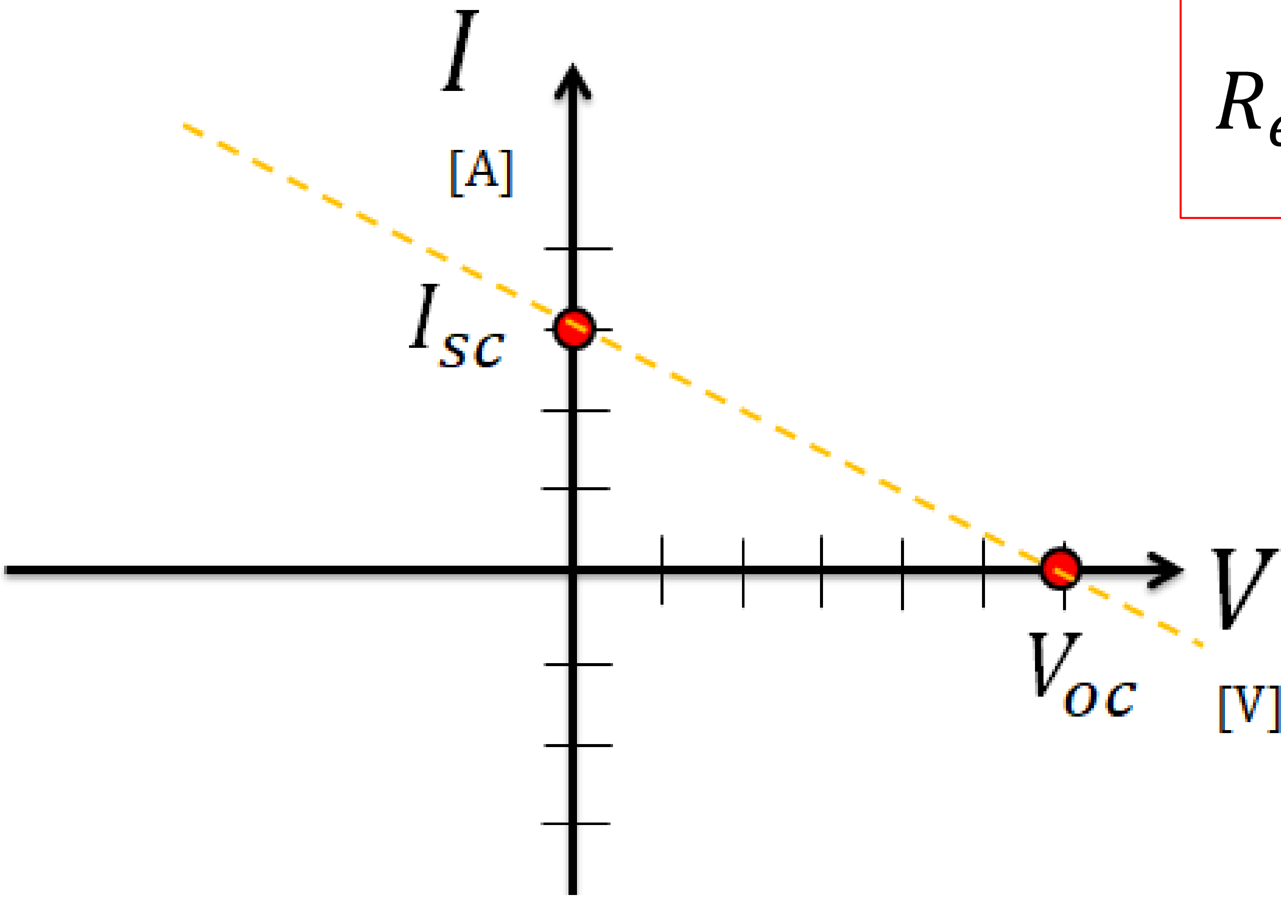
$$R_{eq} = \frac{V_{eq}}{I_{sc}}$$



$R_L' \rightarrow \infty$ (open circuit)

$$V_{eq} = V_{oc}$$

$$R_{eq} = \frac{V_{oc}}{I_{sc}}$$



$$I = I_{sc} - \frac{I_{sc}}{V_{oc}} V = I_{sc} - \frac{1}{R_{eq}} V$$

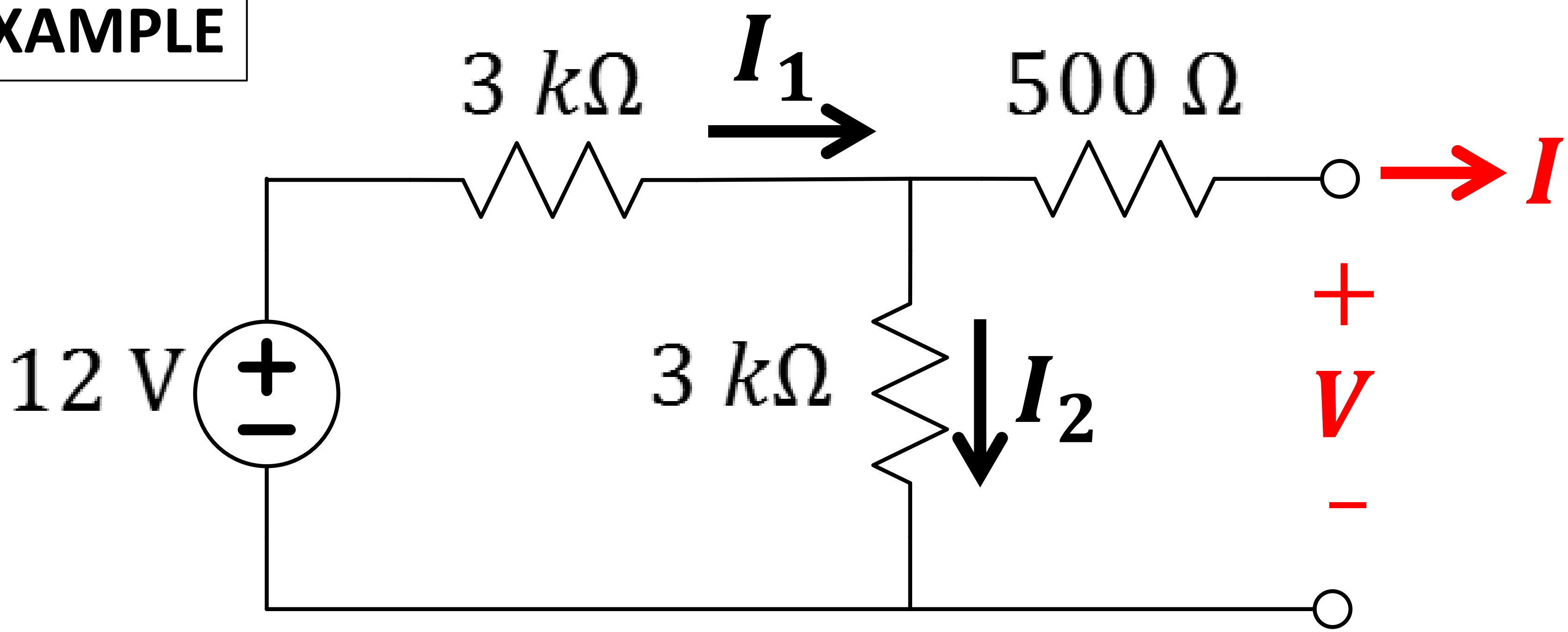
This equation contains all the information on how the source circuit interacts with other circuits

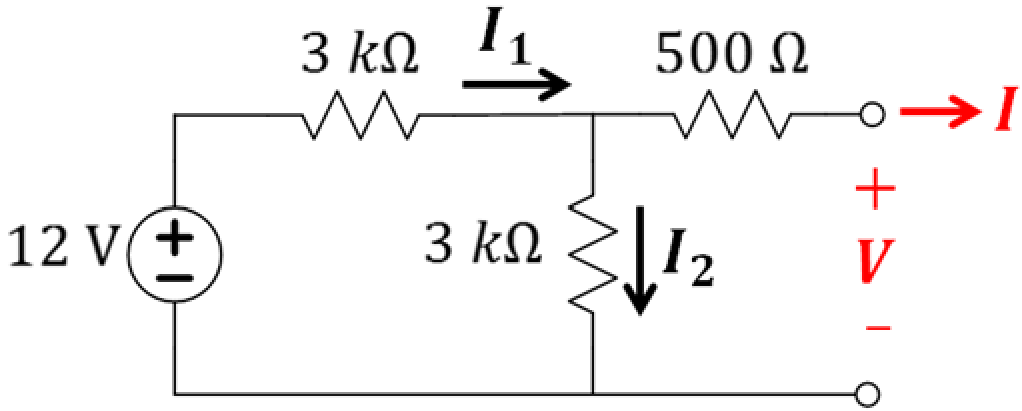
If the source circuit is known:

we can characterize the behavior of the circuit using KVL and KCL analysis…

… but, as we shall see, this may entail a lot of work!

EXAMPLE





Indeed, it is a lot work to find the terminals equation!

1 KCL $I_1 = I_2 + I$

KVL $12 = 3k \cdot I_1 + 3k \cdot I_2$

KVL $V = -500 \cdot I + 3k \cdot I_2$

2 Eliminate I_1 from 1st KVL

$I_1 = I_2 + I$

$12 = 3k(I_2 + I) + 3k \cdot I_2$

$12 = 6k \cdot I_2 + 3k \cdot I$

$6 - 1.5k \cdot I = 3k \cdot I_2$

3 Rewrite 2nd KVL

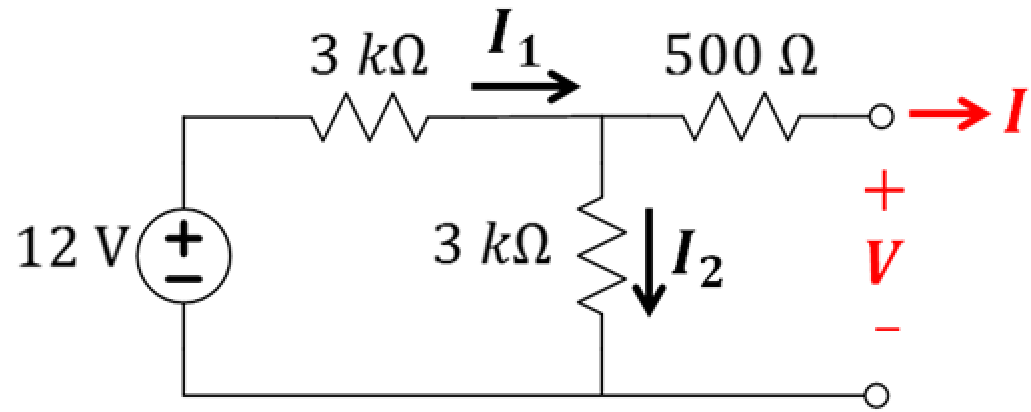
$V + 500 \cdot I = 3k \cdot I_2$

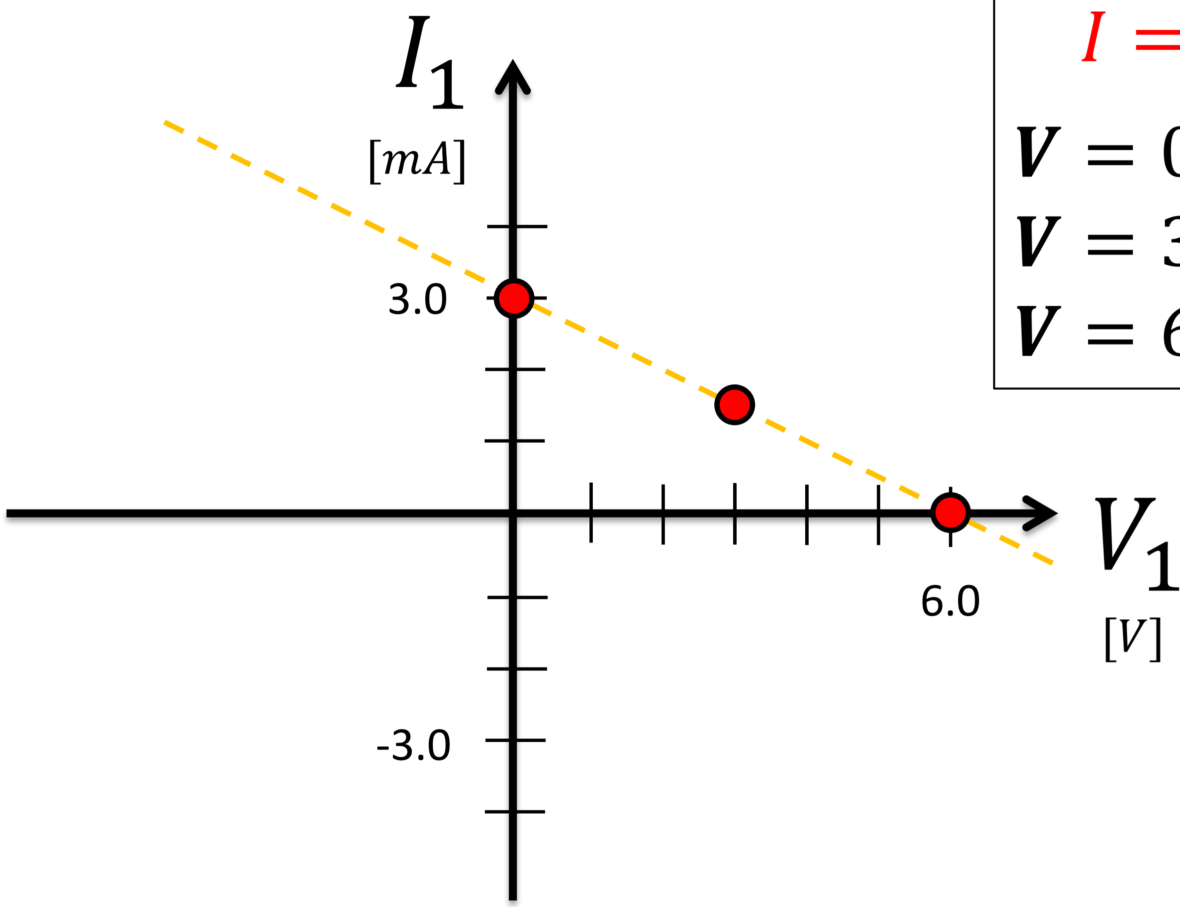
4 Eliminate I_2 from the KVL's

$$V + 500 \cdot I = 6 - 1.5k \cdot I$$

$$\frac{(6 - V)}{2k} = I$$

$$I = (-V/2 + 3)\ [mA]$$

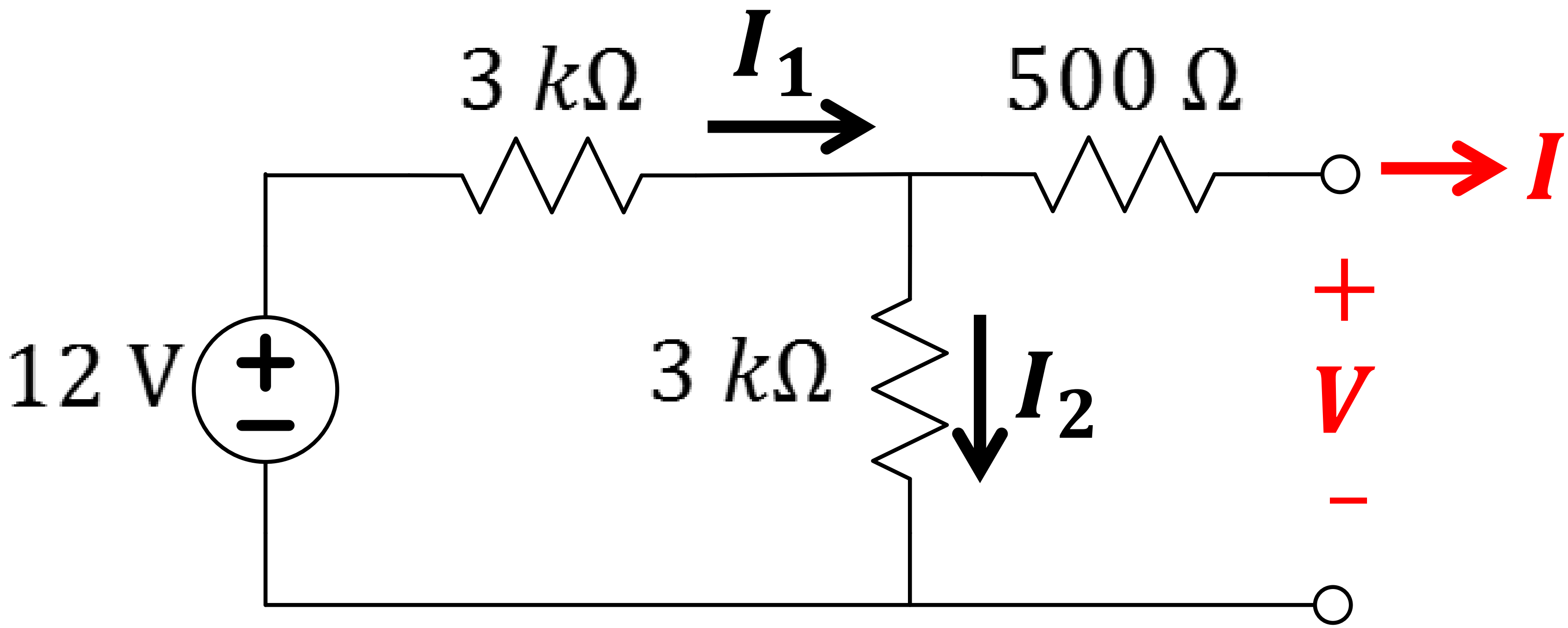




$$I = (-V/2 + 3)\ [mA]$$

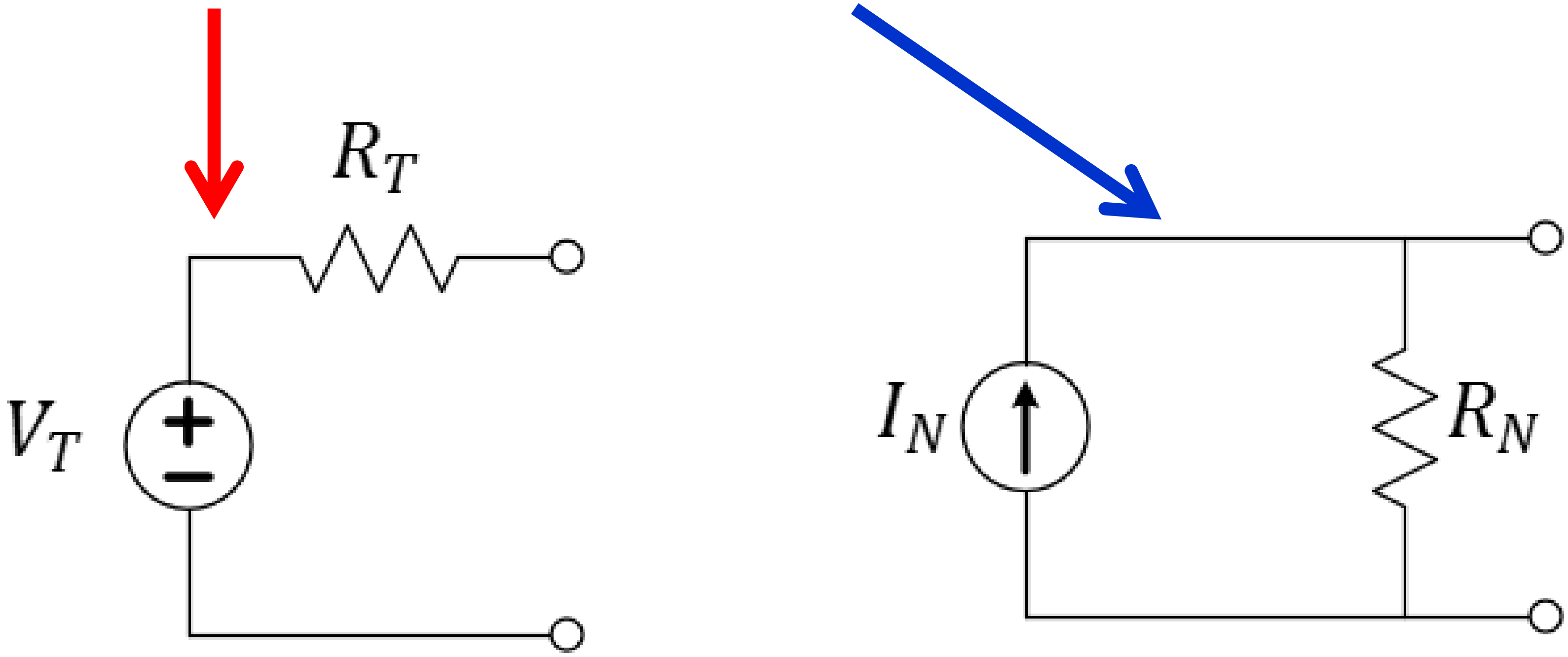
$$\boldsymbol{V} = 0V \quad \rightarrow \quad \boldsymbol{I} = 3\ mA$$
$$\boldsymbol{V} = 3V \quad \rightarrow \quad \boldsymbol{I} = 1.5\ mA$$
$$\boldsymbol{V} = 6V \quad \rightarrow \quad \boldsymbol{I} = 0$$

Even for a simple circuit like the one below, the KVL/KCL analysis is cumbersome. Can we simplify?



We can formulate equivalent circuits with a voltage source or with a current source, producing the same terminals equation

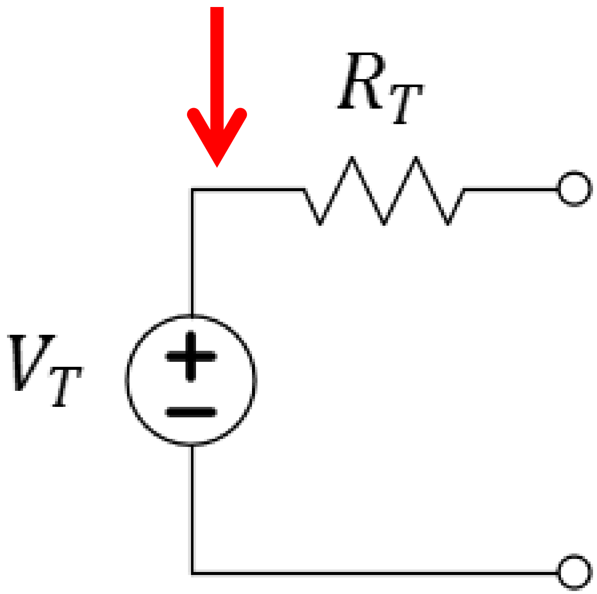
Thevenin and Norton Equivalents

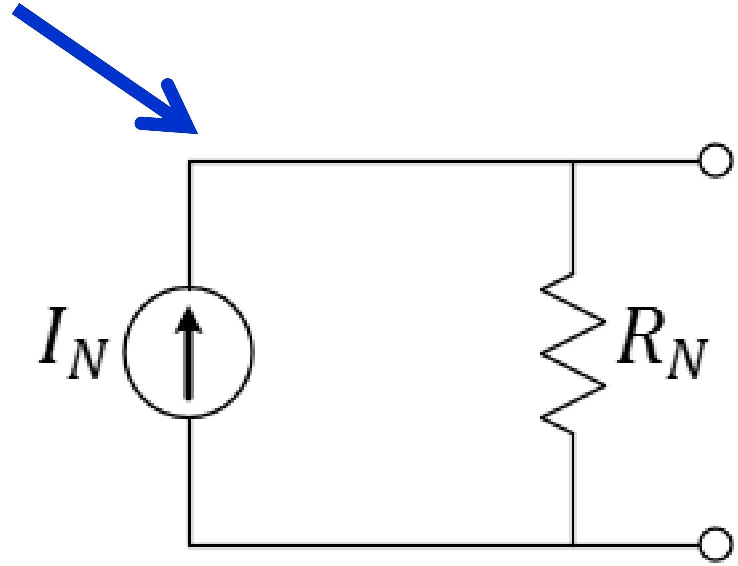


Both represent the terminals equation

$$I = I_{sc} - \frac{I_{sc}}{V_{oc}} V = I_{sc} - \frac{1}{R_{eq}} V$$

Thevenin and Norton Equivalents





$$V_{oc} = V_T$$

Thevenin voltage

$$R_{eq} = R_T$$

Thevenin resistance

$$I_{sc} = I_N$$

Norton current

$$R_{eq} = R_N$$

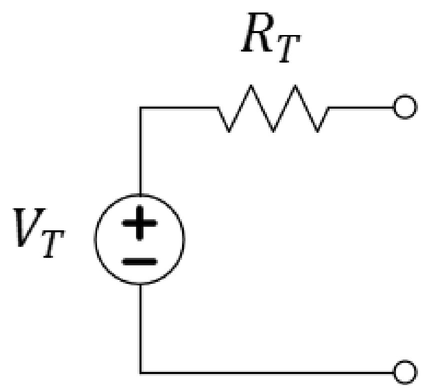
Norton resistance

NOTE:

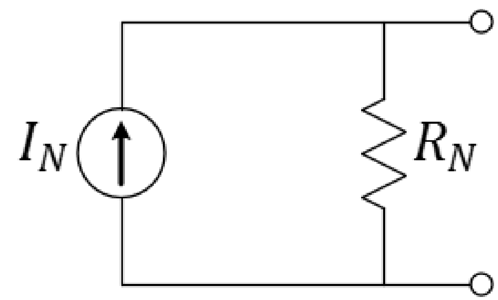
$$R_T = R_N$$

$$I_N = \frac{V_T}{R_T}$$

Thevenin equivalent



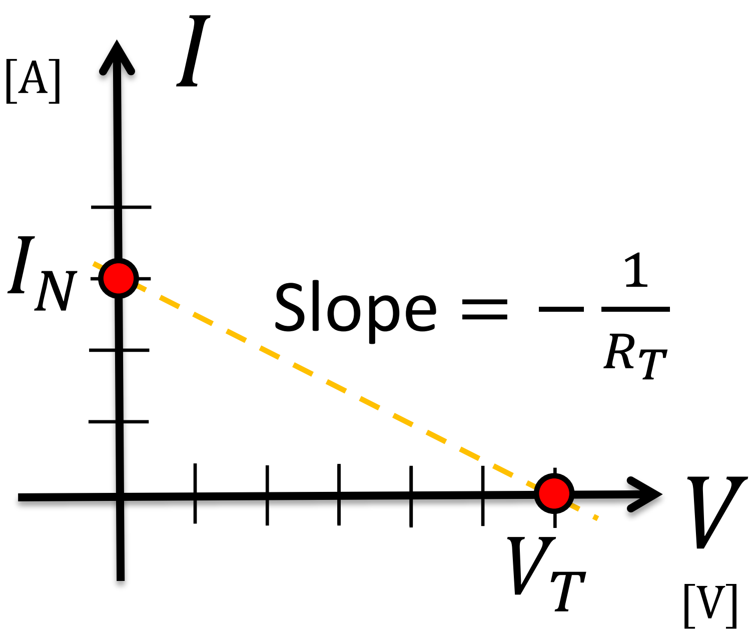
Norton equivalent

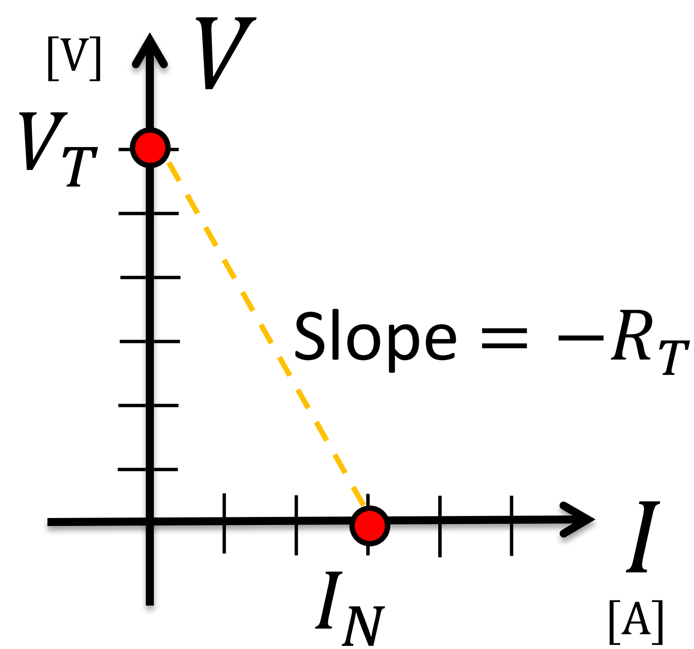


$$R_T = R_N$$

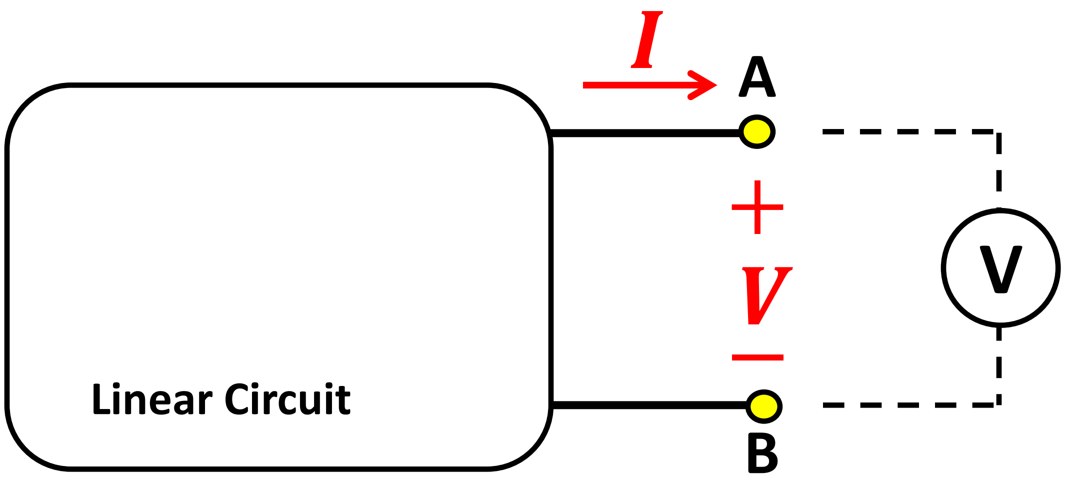
$$I = I_N - \frac{1}{R_T} V$$

$$V = V_T - R_T I$$



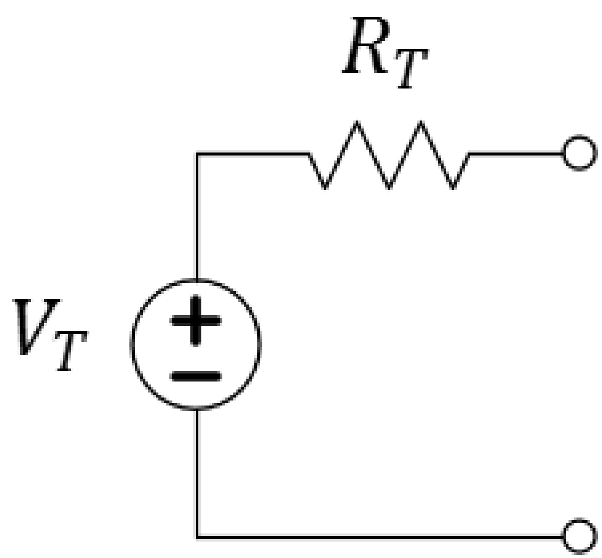


Finding V_T

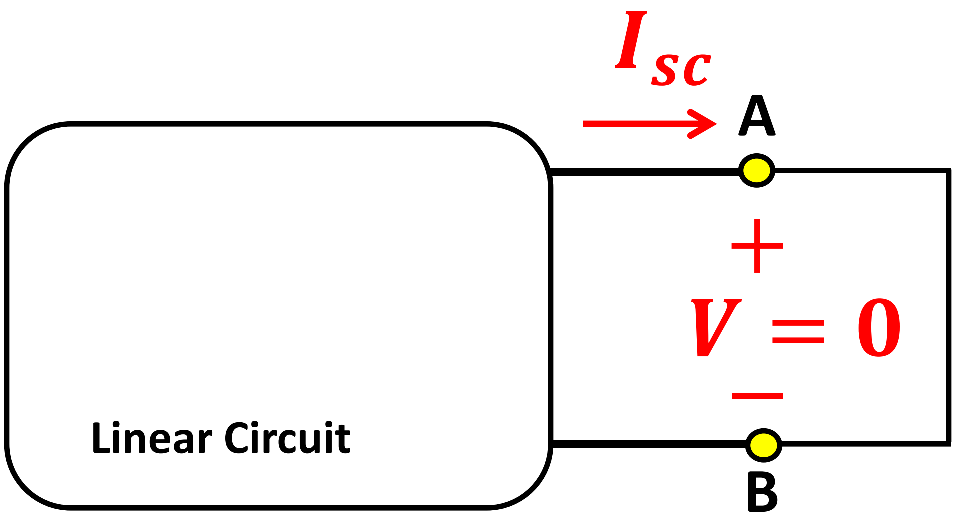


$$V_{oc} = V_T$$

Determine open circuit voltage with no load



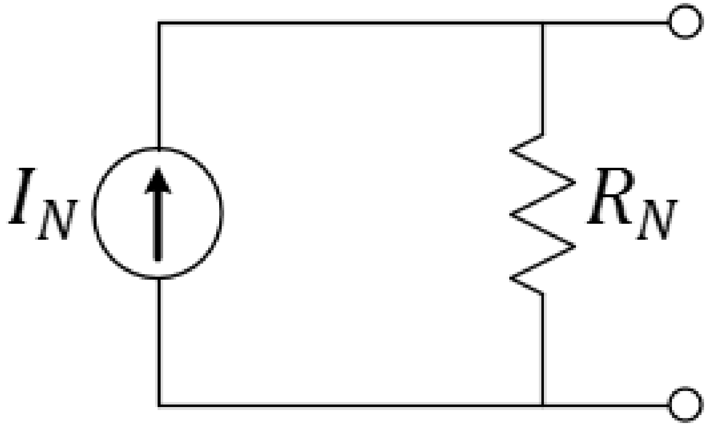
Finding I_N



$$I_{sc} = I_N$$

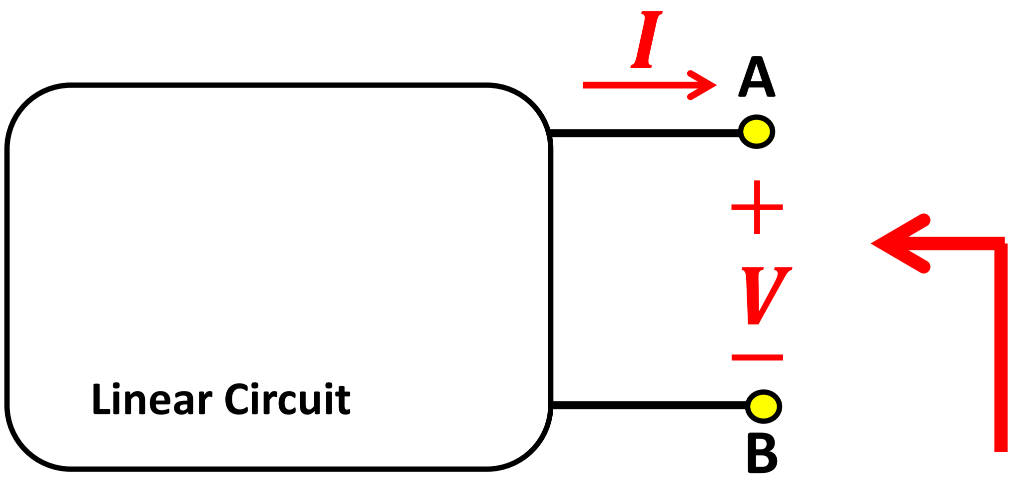
Determine short circuit current

$$I_N = \frac{V_T}{R_T}$$



Finding R_T

$$R_T = \frac{V_T}{I_N}$$



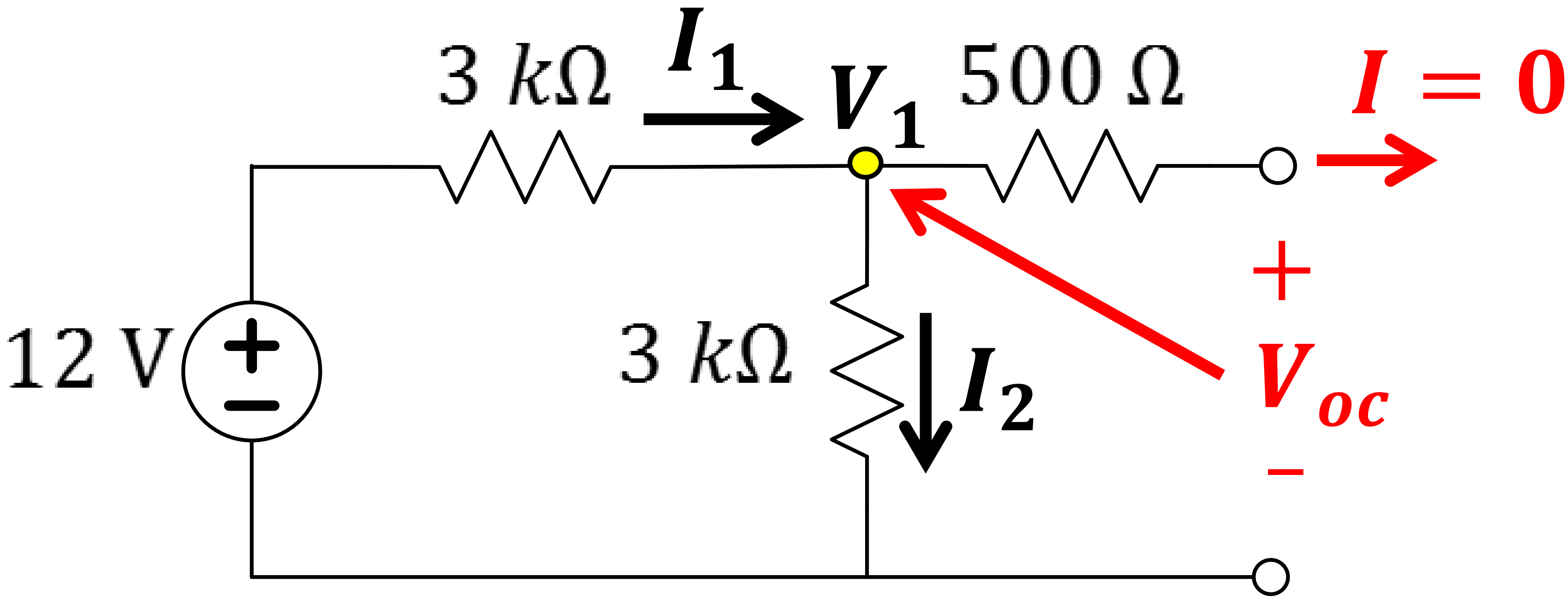
$$R_T = R_{eq}$$

Determine resistance seen at the terminals, after suppressing ideal sources in the circuit (i.e., substitute with the corresponding internal resistance)

Remember:

- **ideal voltage source = zero internal resistance (short circuit)**
- **ideal current source = infinite internal resistance (open circuit)**

Example



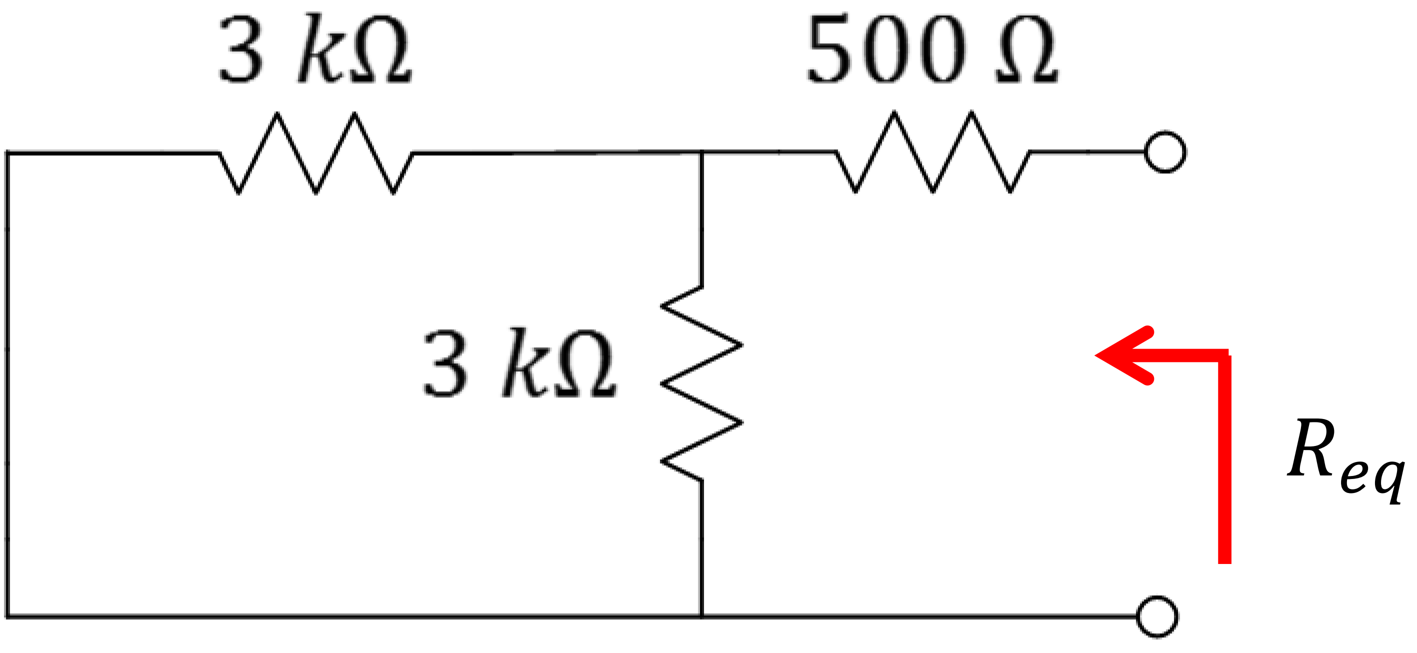
$$I_1 = I_2$$

Voltage Divider

$$V_1 = V_{oc} = 12\text{V}\ \frac{3k\Omega}{3k\Omega + 3k\Omega} = 6\text{V}$$

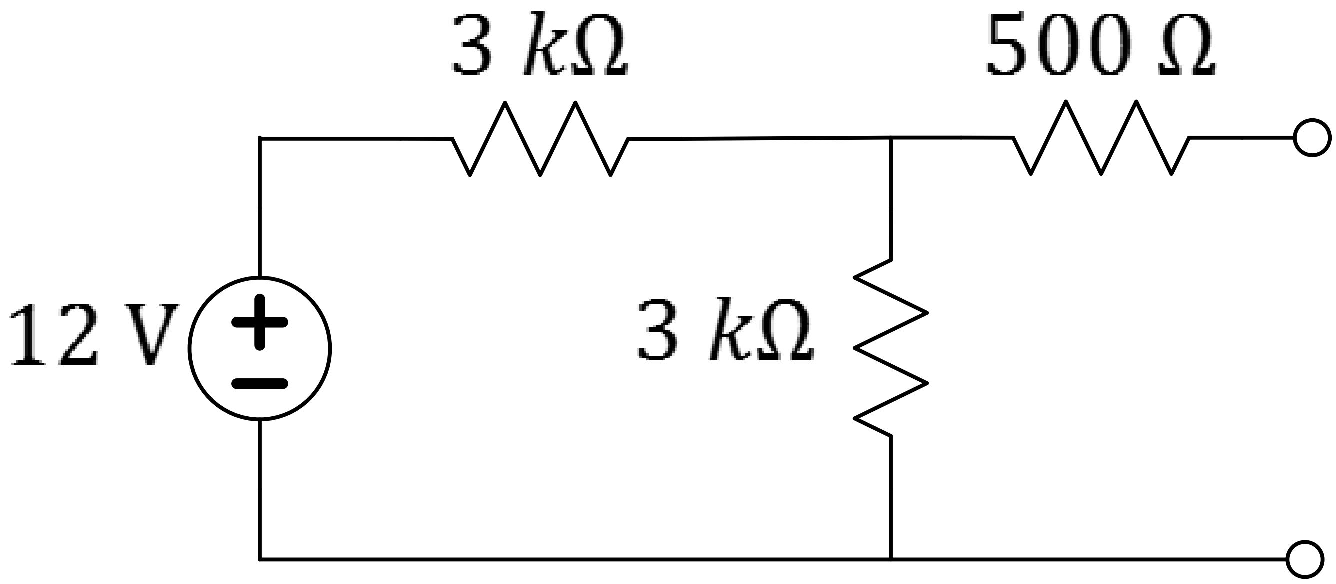
$$\boxed{V_T = 6\text{V}}$$

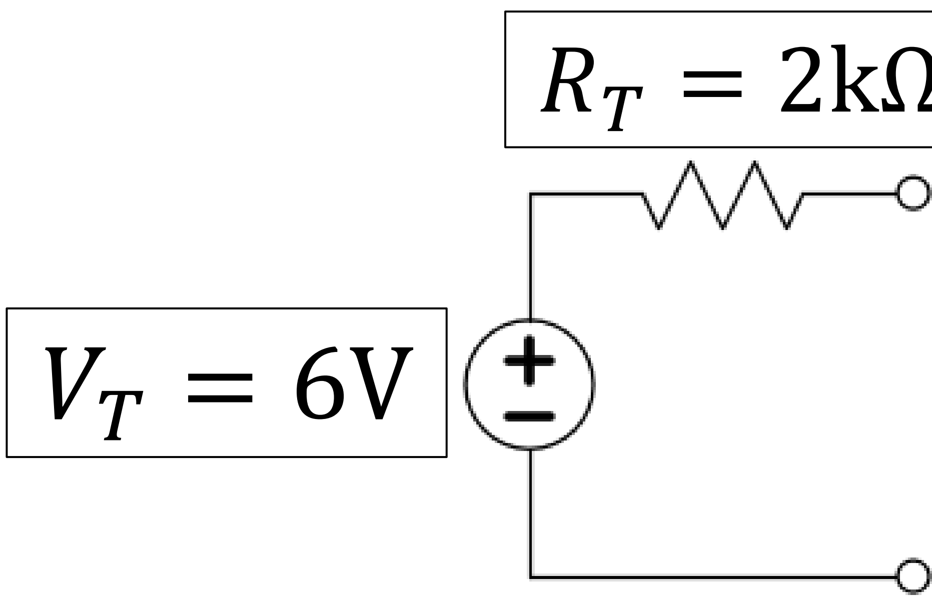
Example



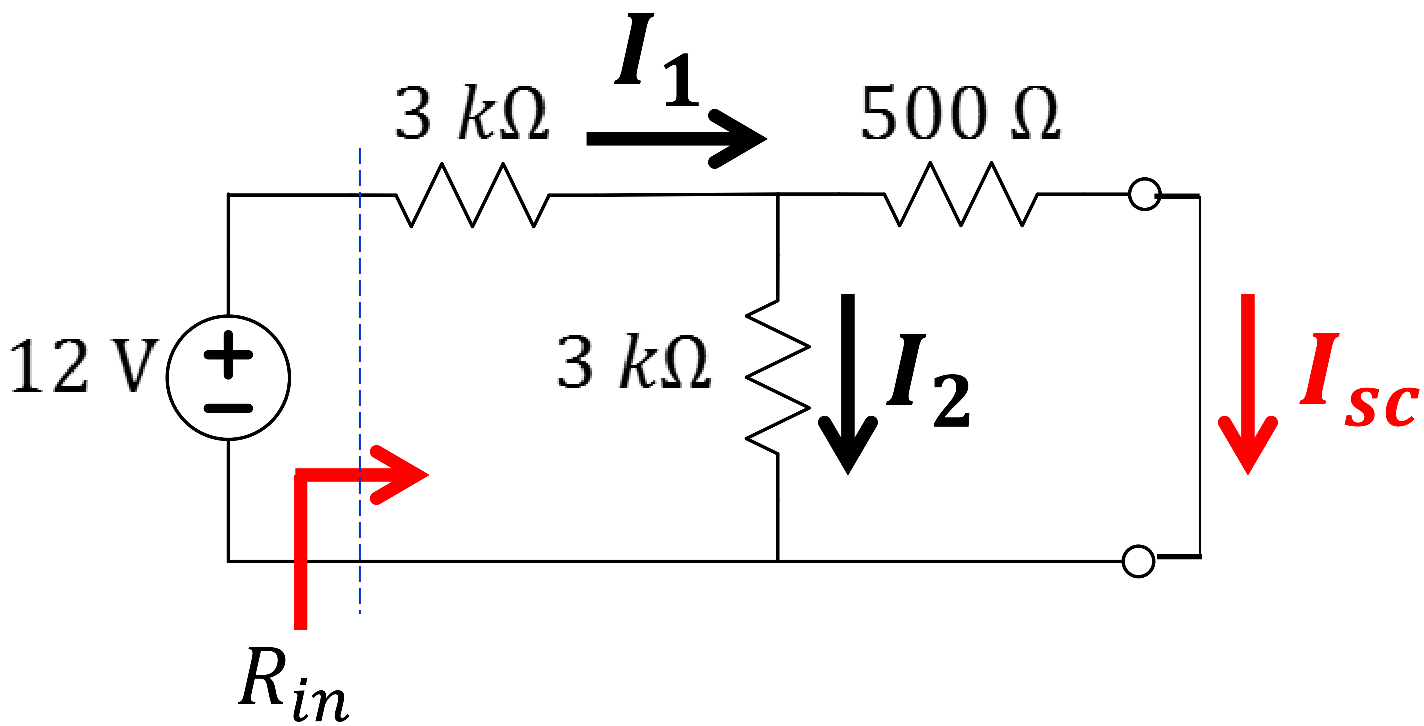
$$R_{eq} = 500\Omega + 3\text{k}\Omega \,//\, 3\text{k}\Omega$$
$$= 500\Omega + 1.5\text{k}\Omega = 2\text{k}\Omega$$

Thevenin Equivalent circuit





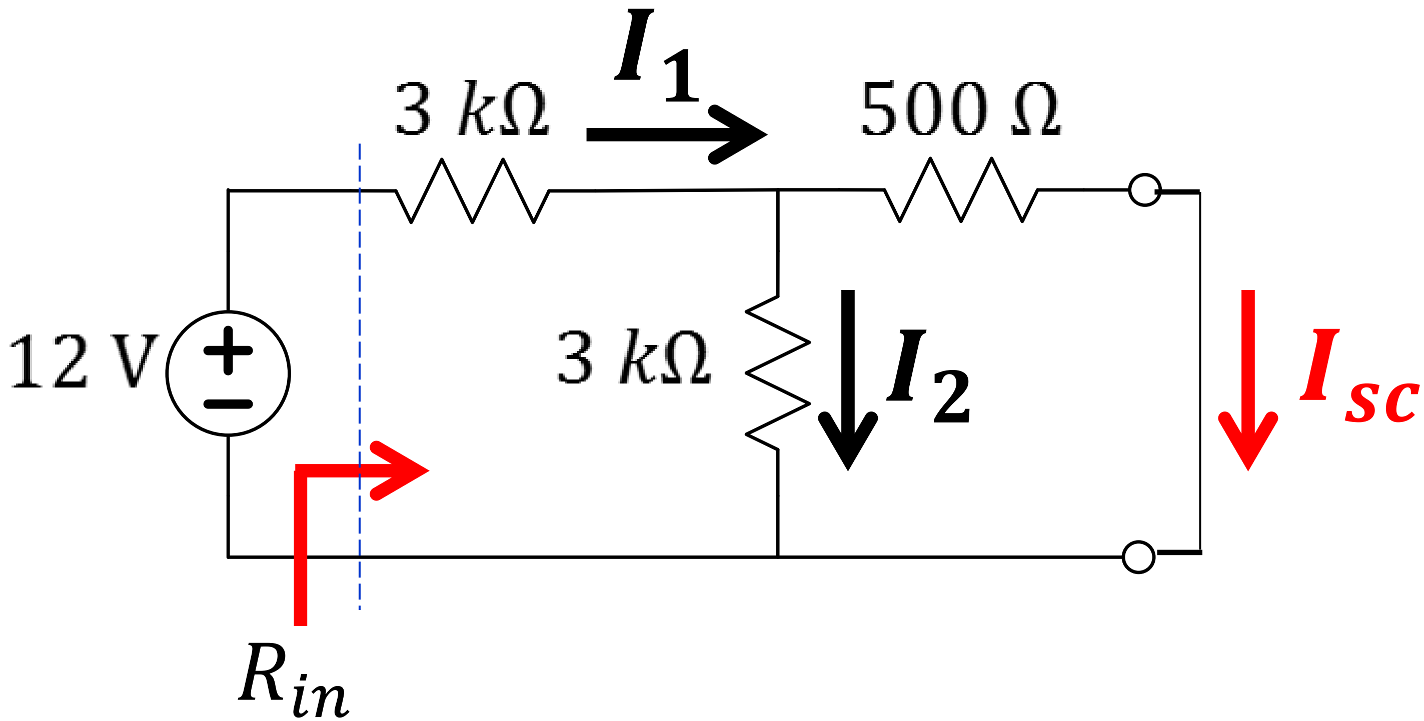
Verify by calculating short circuit current in detail



$$R_{in} = 3\text{k}\Omega + 3\text{k}\Omega \,//\, 500\Omega = 3\text{k}\Omega + 428.6\Omega$$
$$= 3.4286\text{k}\Omega$$

$$I_1 = 12\text{V}/3.4286\text{k}\Omega = 3.5\text{mA}$$

Verify by calculating short circuit current in detail

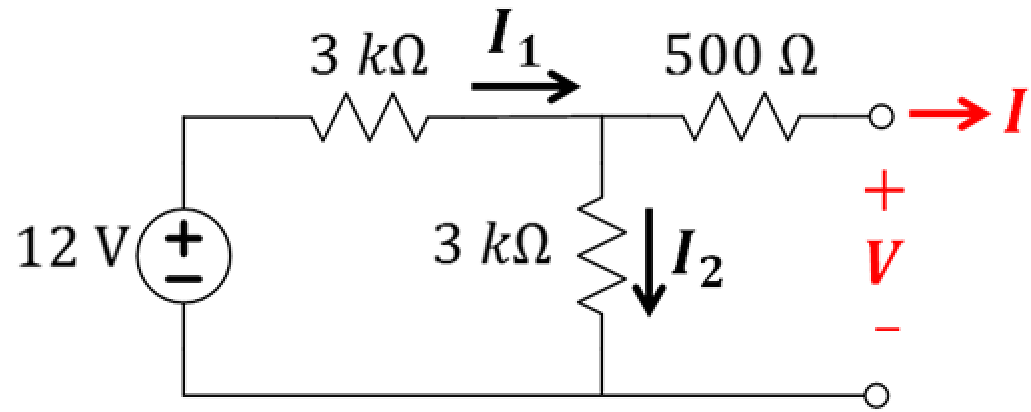


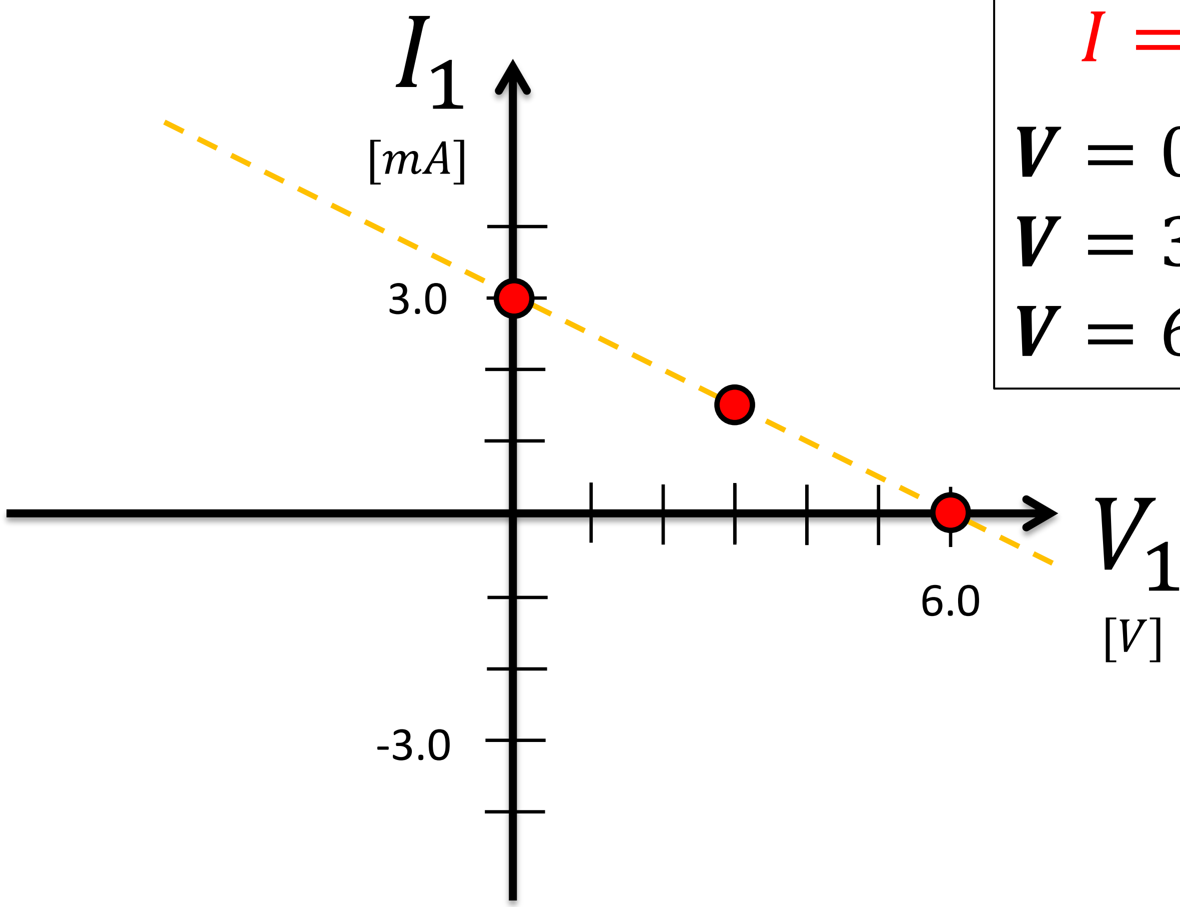
$$I_{sc} = I_N = I_1 \times 3\text{k}\Omega\,/(3\text{k}\Omega + 500\Omega) = 3\text{mA}$$

$$R_{eq} = R_T = V_T/I_{sc} = 6\text{V}/3\text{mA} = 2\text{k}\Omega$$

Same as previous result

Recall Slide #5





$$I = (-V/2 + 3)\ [mA]$$

$$V = 0V \rightarrow I = 3\ mA$$

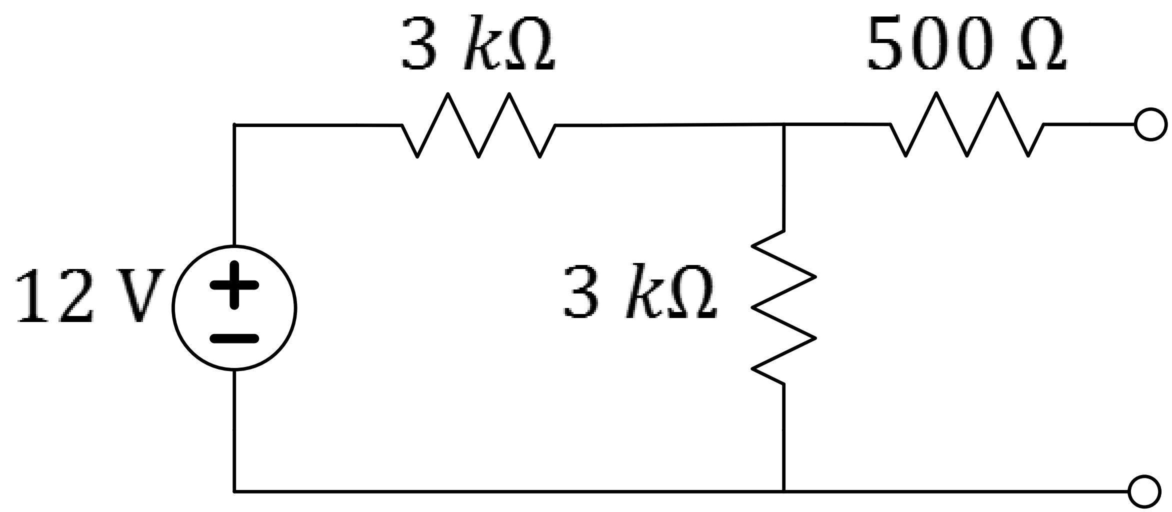
$$V = 3V \rightarrow I = 1.5\ mA$$

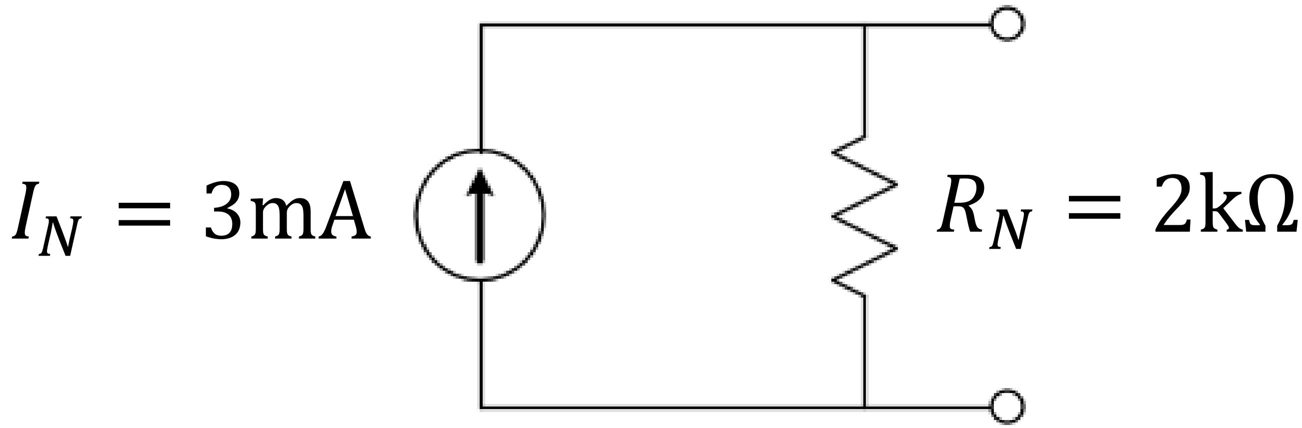
$$V = 6V \rightarrow I = 0$$

Since we have calculated the short circuit current, we can formulate the Norton equivalent circuit

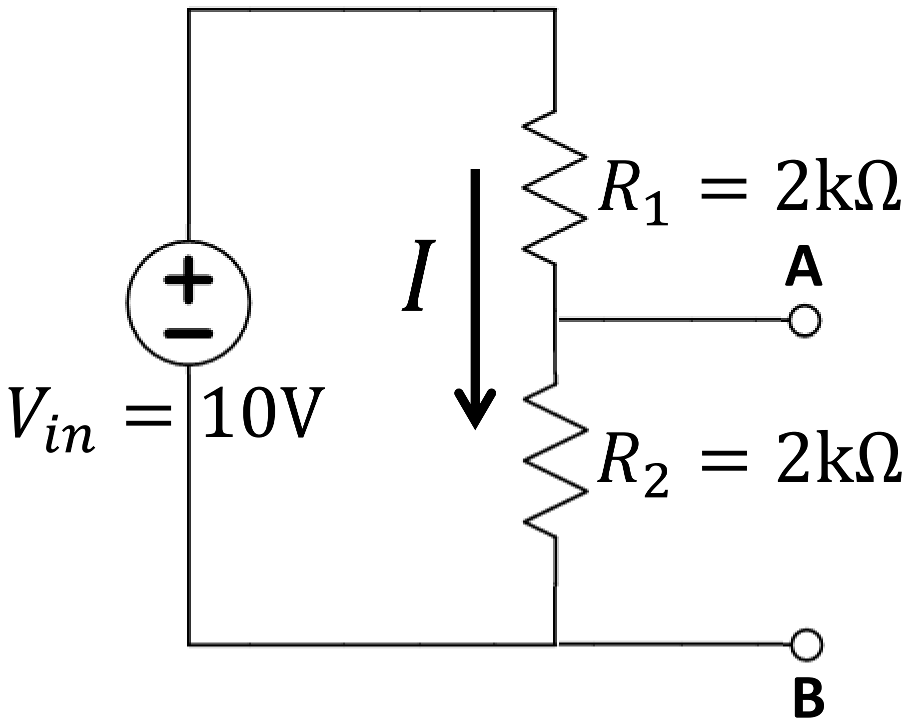
Also, from the Thevenin result:

$$I_N = V_T / R_T = 6\text{V}/2\text{k}\Omega = 3\text{mA}$$





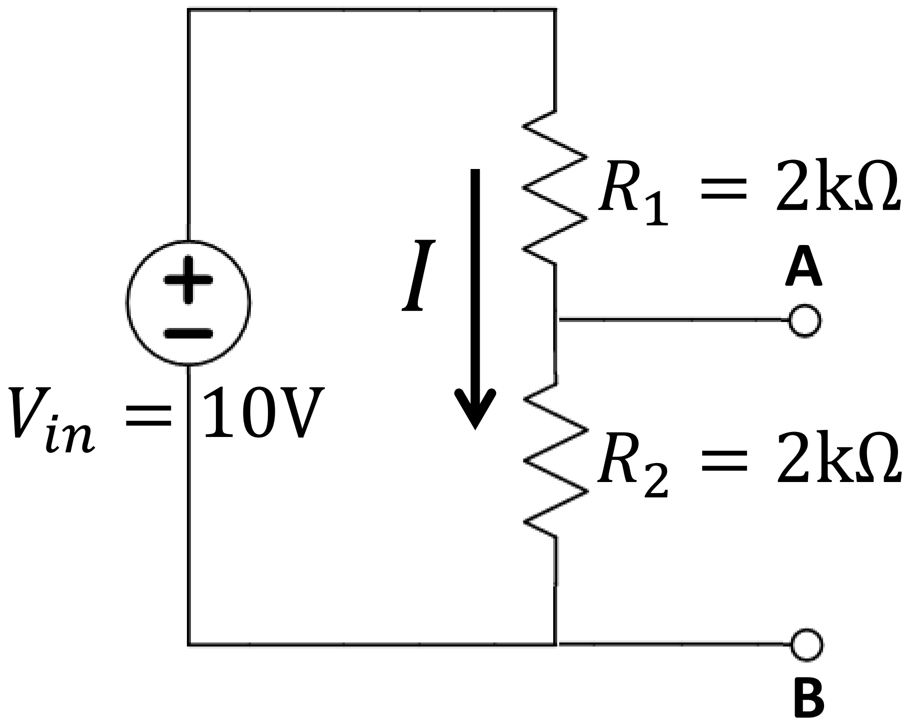
Practice Problem 1 – Find Thevenin equivalent

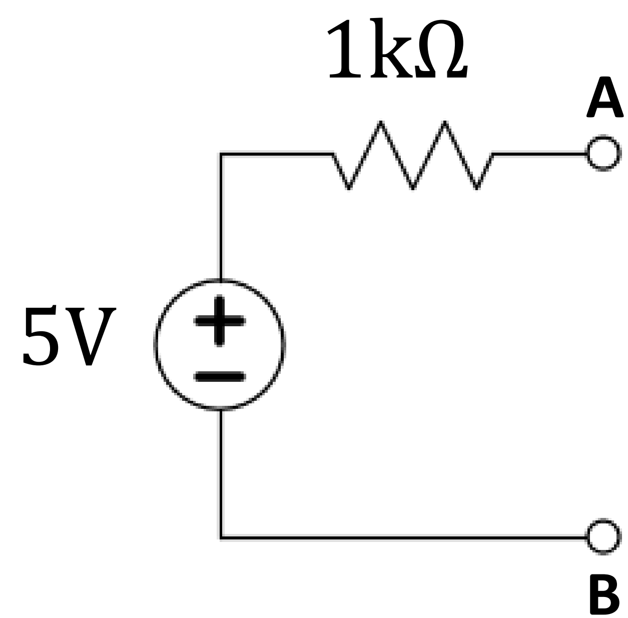


$$V_T = V_{\text{AB}} = IR_2 = V_{in}\frac{R_2}{R_1 + R_2} = \frac{10 \times 2}{4} = 5\text{V}$$

$$R_T = R_{\text{eq}} = R_1 // R_2 = 1\text{k}\Omega$$

Practice Problem 1 – Find Thevenin equivalent

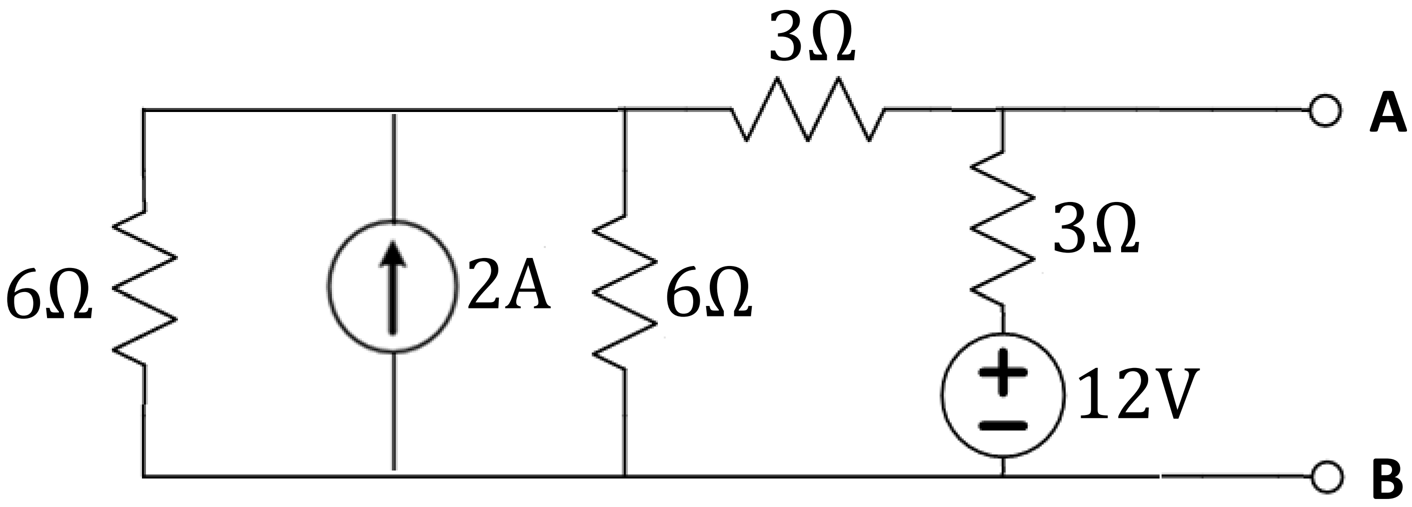




$$V_T = V_{AB} = IR_2 = V_{in}\frac{R_2}{R_1 + R_2} = \frac{10 \times 2}{4} = 5V$$

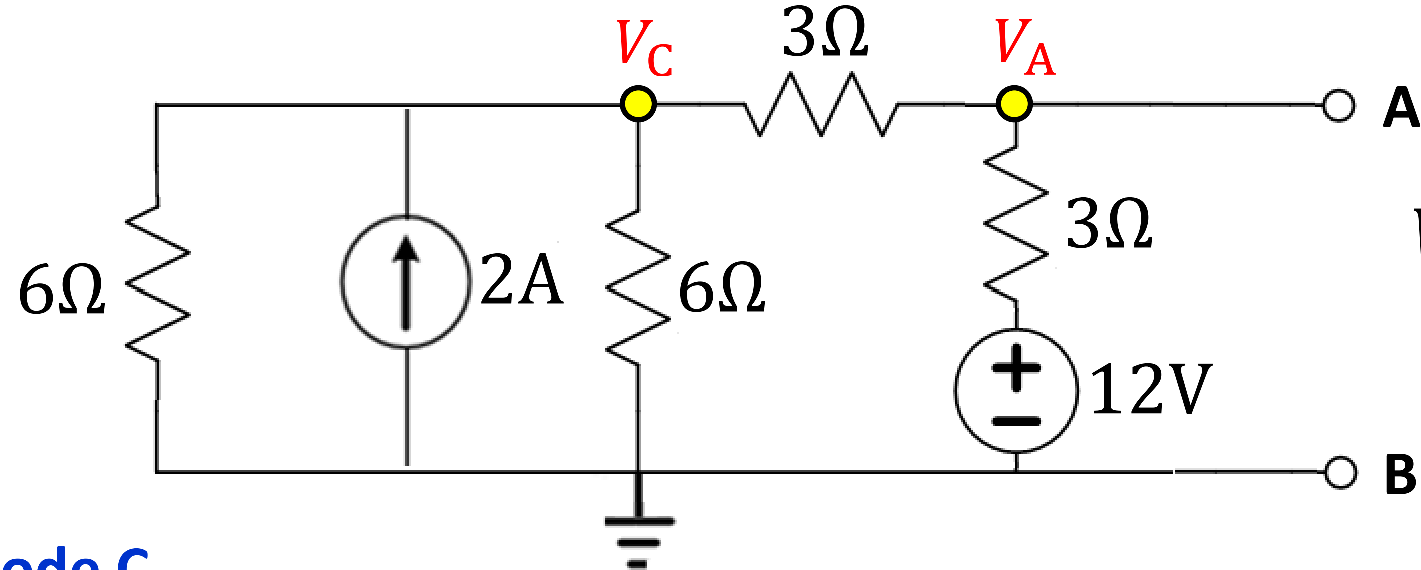
$$R_T = R_{eq} = R_1 // R_2 = 1k\Omega$$

Practice Problem 2 – Find Thevenin equivalent



Let’s use Node Voltage Analysis

Practice Problem 2 – Find Thevenin equivalent



$$V_{AB} = V_A$$

Node C

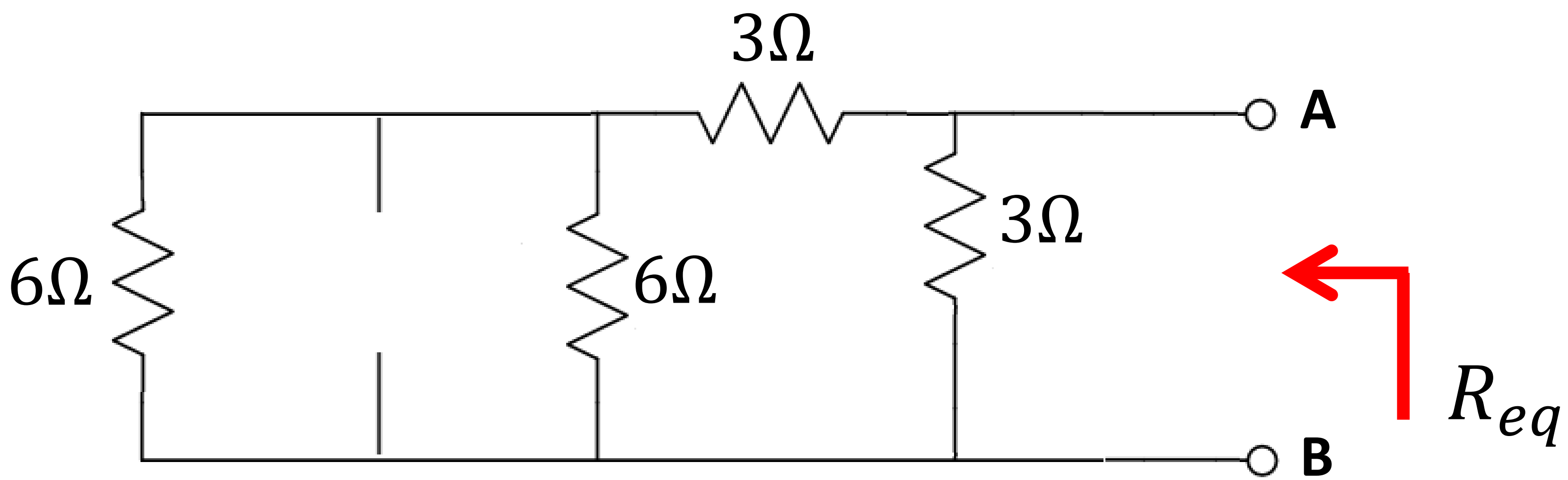
$$\frac{V_C}{6} - 2 + \frac{V_C}{6} + \frac{V_C - V_A}{3} = 0 \Rightarrow 2V_C - V_A = 6$$

Node A

$$\frac{V_A - V_C}{3} + \frac{V_A - 12}{3} = 0 \Rightarrow 2V_A - V_C = 12$$

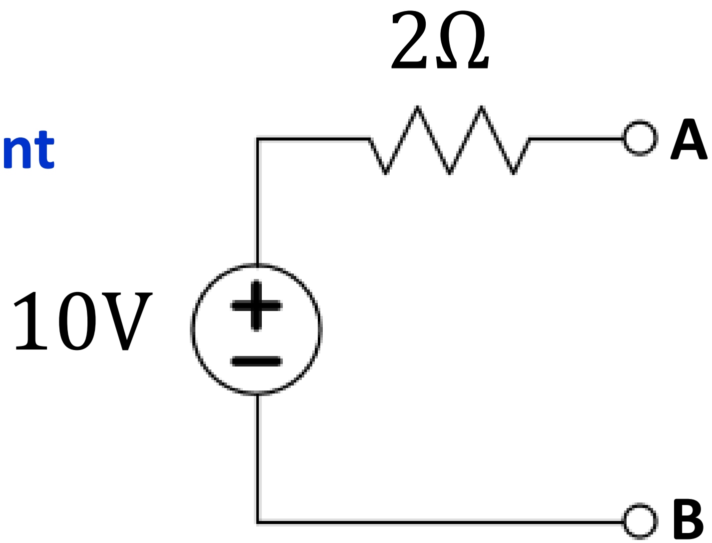
$$\boxed{V_C = 8\text{V}} \quad \boxed{V_A = V_T = 10\text{V}}$$

Practice Problem 2 – Find Thevenin equivalent



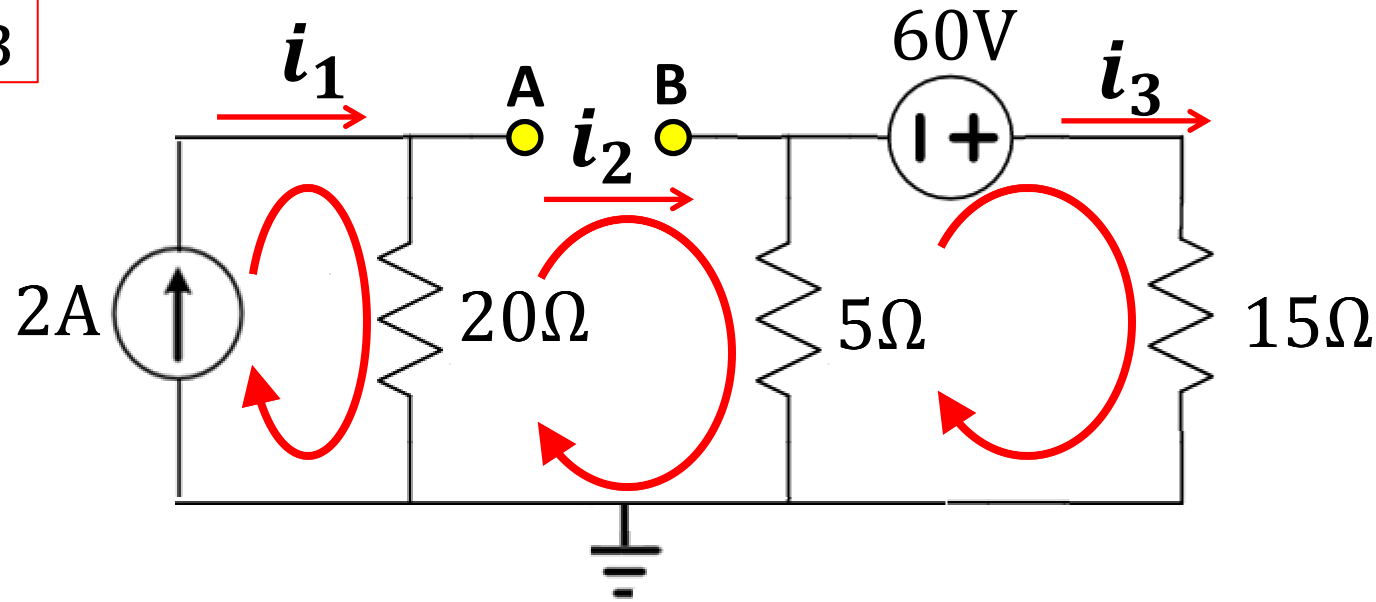
$$R_{eq} = R_T = [(6//6) + 3]//3 = 2\Omega$$

Thevenin equivalent



Practice Problem 3 – Find Thevenin equivalent

$$V_T = V_A - V_B$$



Loop 2

$$i_2 = 0$$

Loop 1

$$i_1 = 2A \Rightarrow V_A = 20\Omega \times 2A = 40V$$

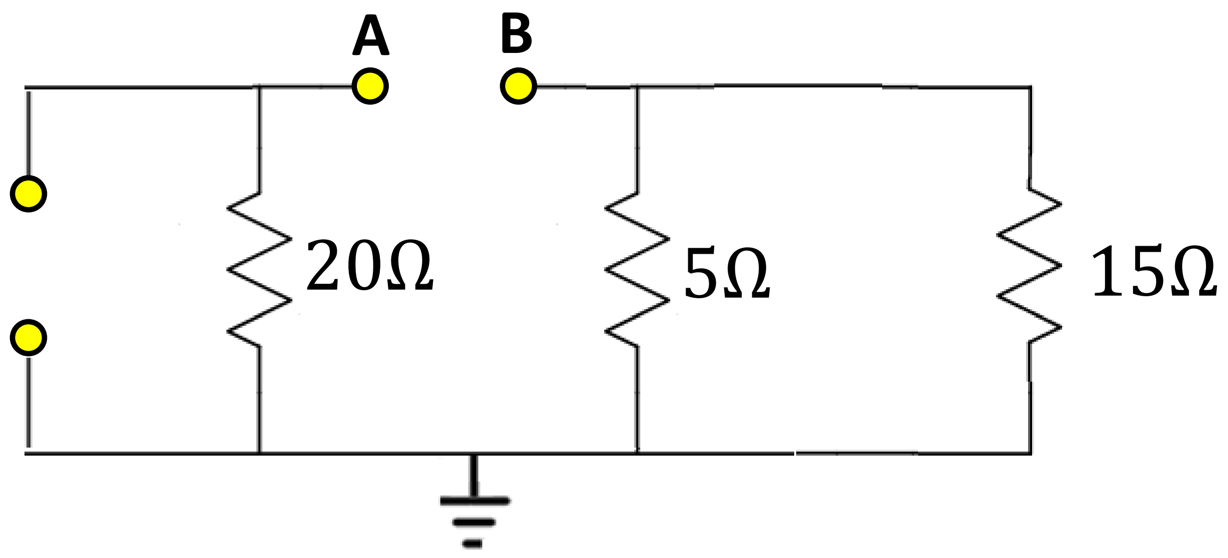
Loop 3

$$60V = 15\Omega\, i_3 + 5\Omega\, i_3 \Rightarrow i_3 = 3A$$

$V_B = -60 + 15\Omega \times 3A = -15V$ **or** $V_B = -5\Omega \times 3A = -15V$

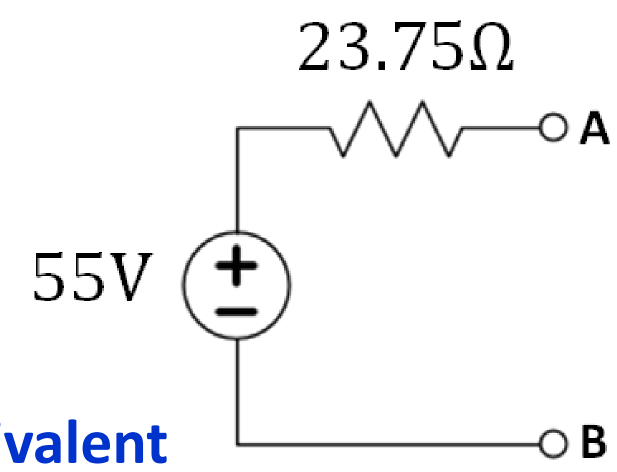
$$V_T = V_A - V_B = 40 - (-15) = 55V$$

Practice Problem 3 – Find Thevenin equivalent



$$R_{eq} = R_T = 20\Omega + 15\Omega // 5\Omega = 20\Omega + 3.75\Omega$$

$$\boxed{R_T = 23.75\Omega}$$



Thevenin equivalent