ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 10

MWF – 12:00pm

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2062 ECE Building

Lecture 10 – Summary

Learning Objectives

1. More practice with equivalent circuits
2. The maximum power transfer theorem

Quiz #1 on February, 12 to 14

To-date: 30 students do not have CBTF reservation!

No Class on Monday 2/12/2024

Practice Problems videos (links on Canvas Modules):

Resistor Circuits Review (Week 2 – Lecture 3)

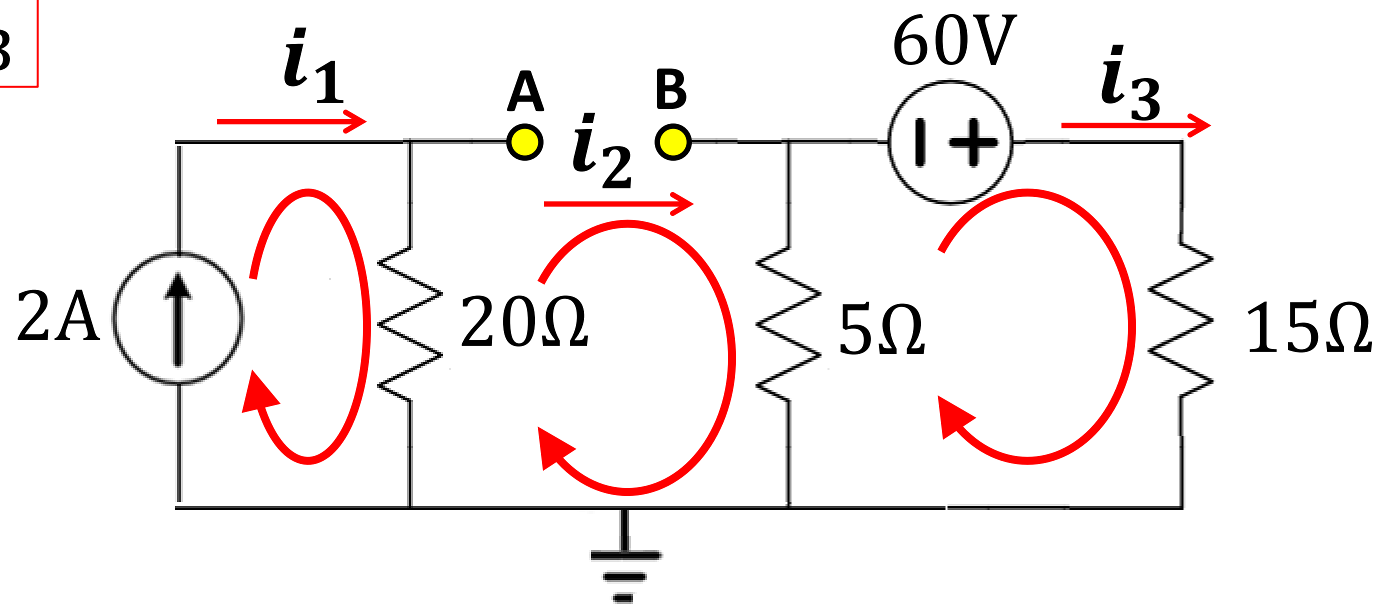
Guided Solution Worksheet 2 (Week 3 – Lecture 6)

Practice Problems on Node method (Week 3 – Lecture 8)

Practice Problems on Equivalent Circuits + WS#3 (Week 4 – Lecture 9)

Practice Problem 3 – Find Thevenin equivalent

$$V_T = V_\mathrm{A} - V_\mathrm{B}$$



Loop 2

$$i_2 = 0$$

Loop 1

$$i_1 = 2\mathrm{A} \Rightarrow V_A = 20\Omega \times 2\mathrm{A} = 40\mathrm{V}$$

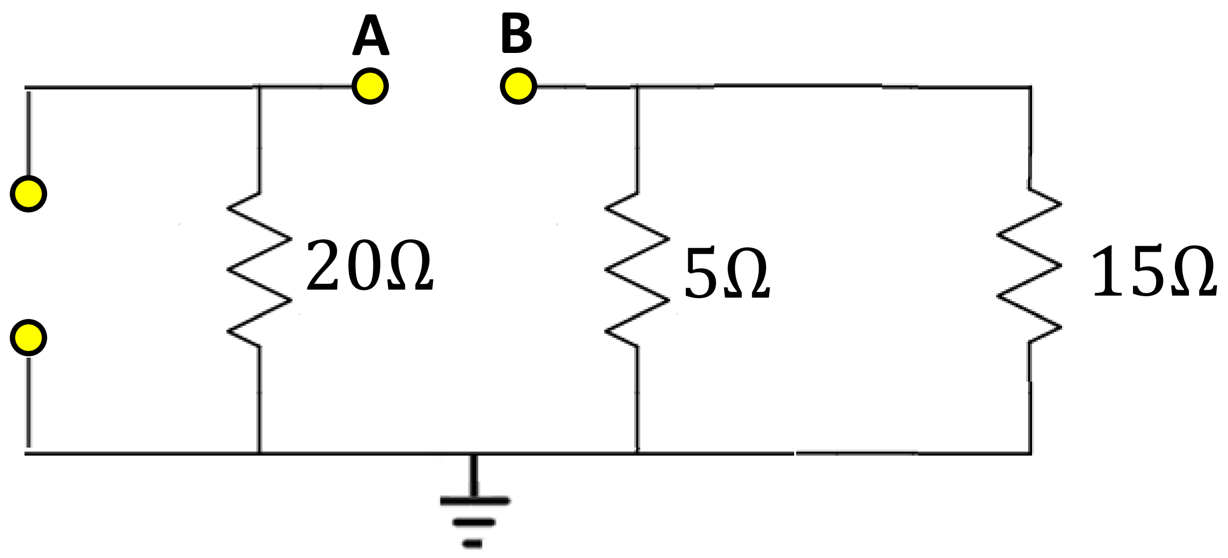
Loop 3

$$60\mathrm{V} = 15\Omega\, i_3 + 5\Omega\, i_3 \Rightarrow i_3 = 3\mathrm{A}$$

$V_B = -60 + 15\Omega \times 3\mathrm{A} = -15\mathrm{V}$ **or** $V_B = -5\Omega \times 3\mathrm{A} = -15\mathrm{V}$

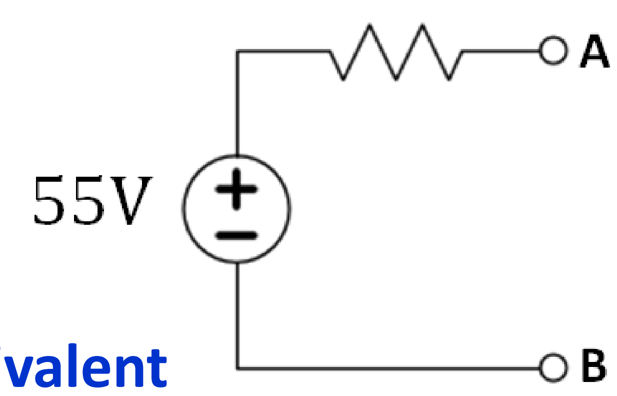
$$V_T = V_\mathrm{A} - V_\mathrm{B} = 40 - (-15) = 55\mathrm{V}$$

Practice Problem 3 – Find Thevenin equivalent



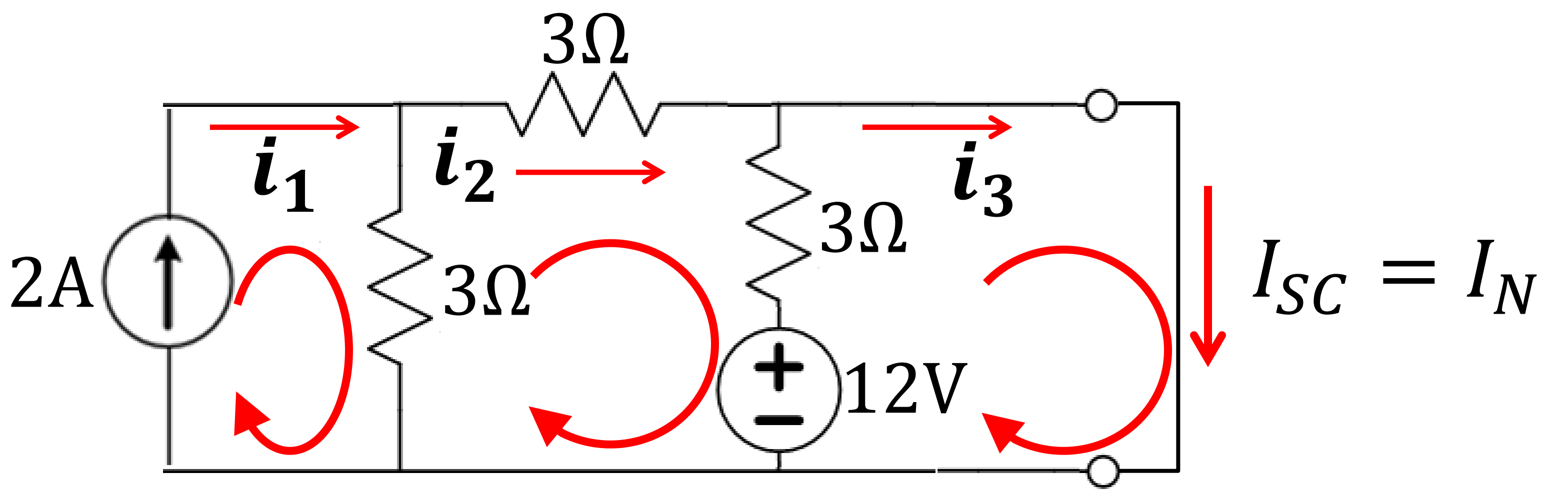
$$R_{eq} = R_T = 20\Omega + 15\Omega//5\Omega = 20\Omega + 3.75\Omega$$

$$\boxed{R_T = 23.75\Omega}$$



Thevenin equivalent

Practice Problem 4 – Repeat #2 for Norton equivalent



Loop 1 $\boxed{i_1 = 2\text{A}}$

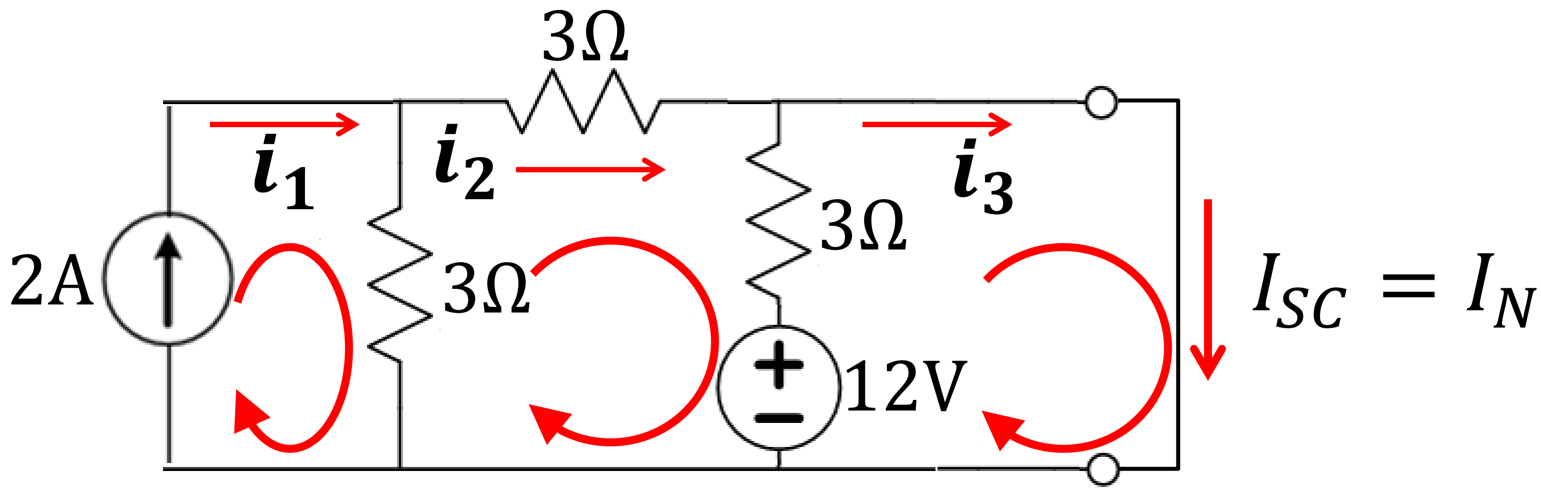
Loop 2 $3\Omega\, i_2 + 3\Omega(i_2 - i_3) + 12\text{V} + 3\Omega(i_2 - 2\text{A}) = 0$

$\Rightarrow \quad 9\Omega\, i_2 - 3\Omega i_3 = -6\text{V} \quad (1)$

Loop 3 $-12\text{V} + 3\Omega(i_3 - i_2) = 0$

$\Rightarrow \quad -3\Omega\, i_2 + 3\Omega i_3 = 12\text{V} \quad (2)$

Practice Problem 4 – Repeat #2 for Norton equivalent

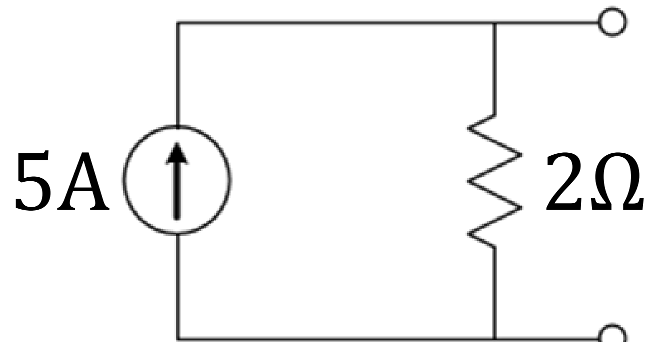


$$9\Omega\, i_2 - 3\Omega i_3 = -6\text{V} \quad (1)$$

$$-3\Omega\, i_2 + 3\Omega i_3 = 12\text{V} \quad (2)$$

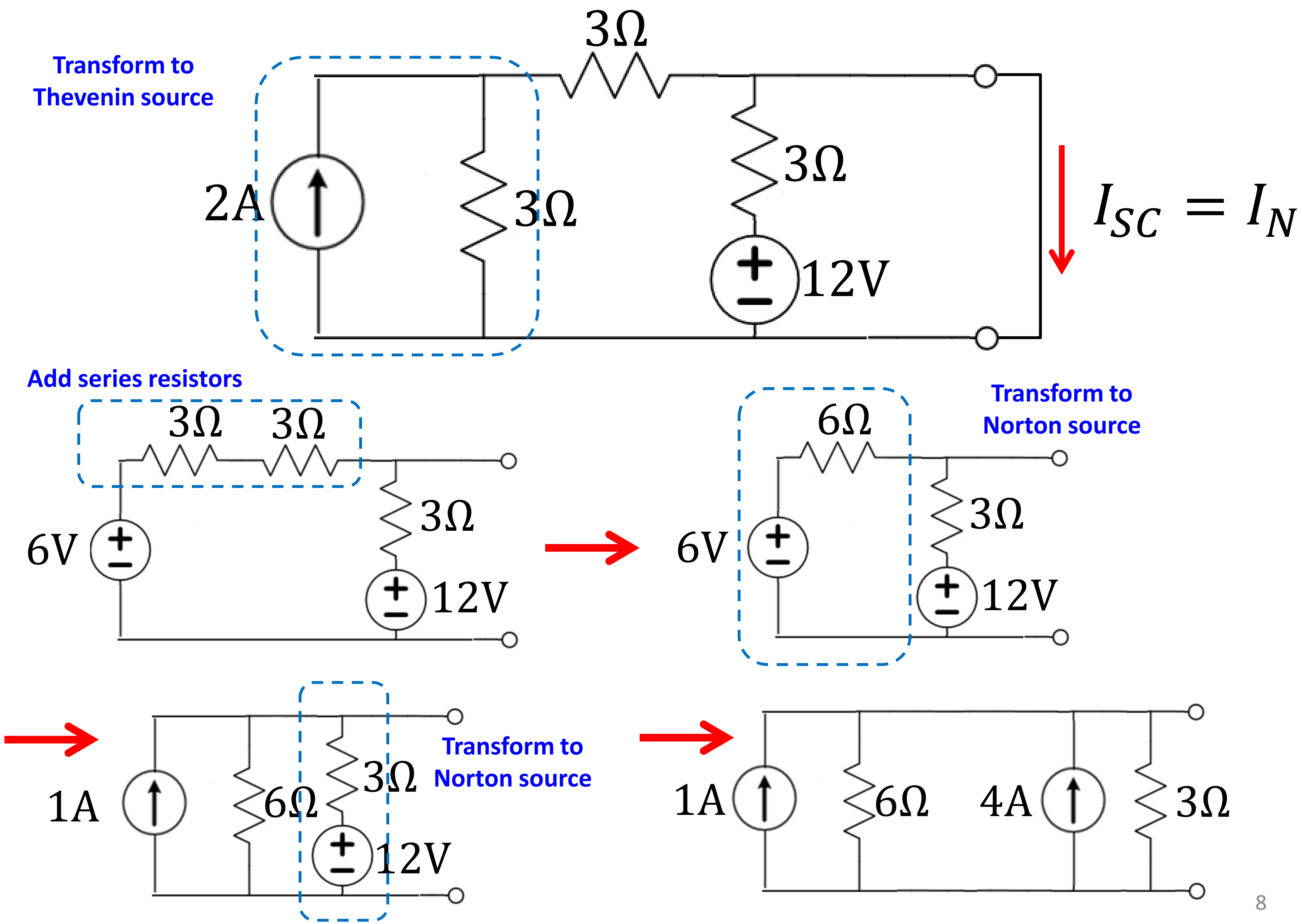
Solving (1) **&** (2) ⇒ $i_2 = 1\text{A}$ $\quad$ $i_3 = I_N = 5\text{A}$

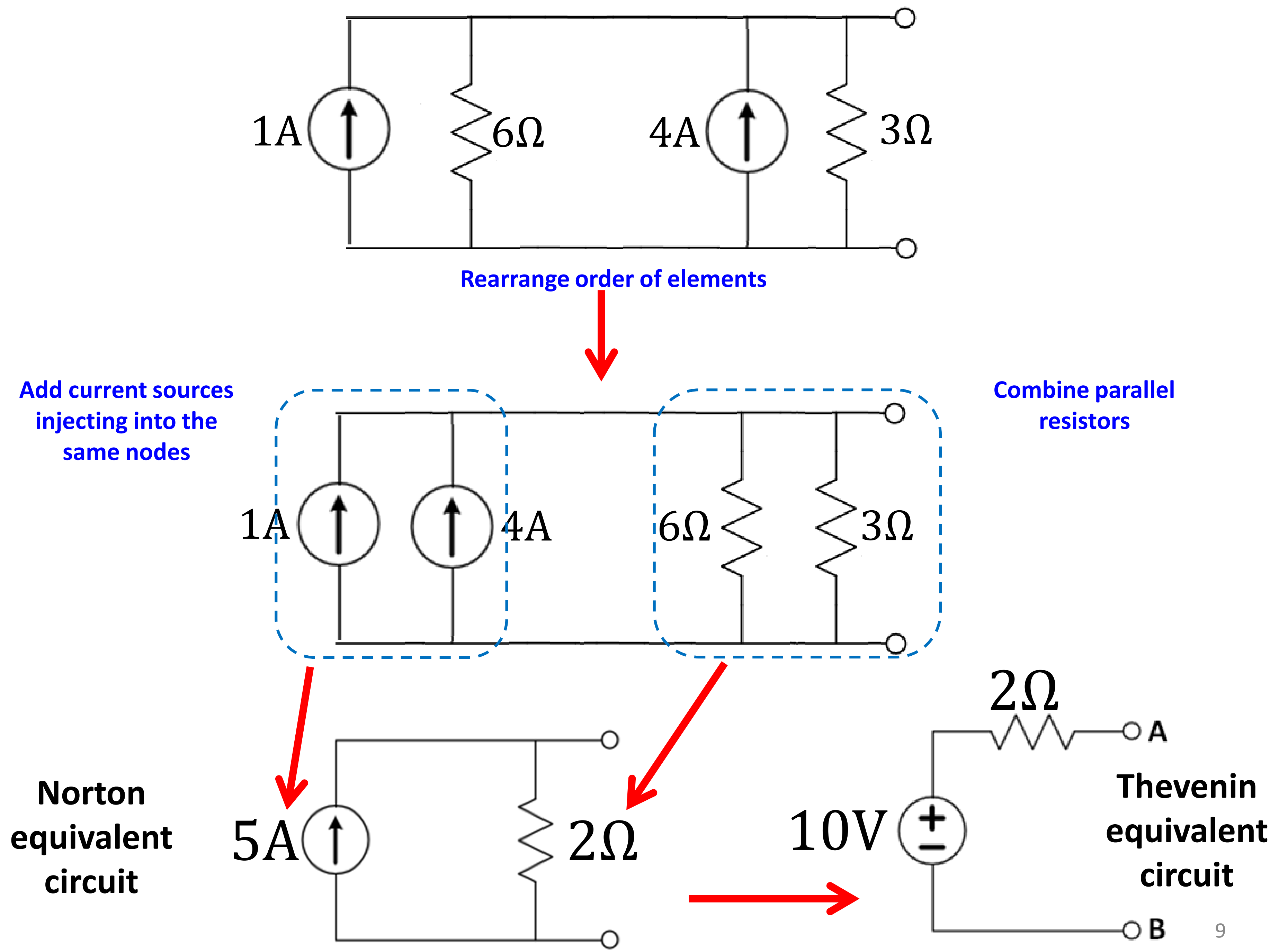
Norton equivalent circuit



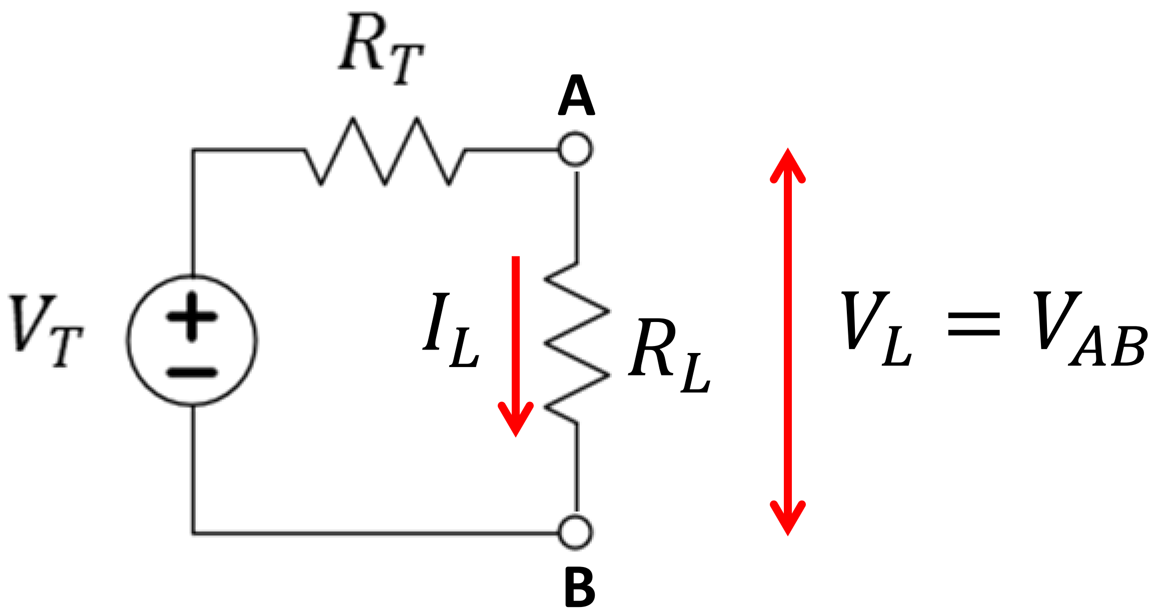
($R_N = R_T$ found earlier)

Practice Problem 4 – Only using source transformations



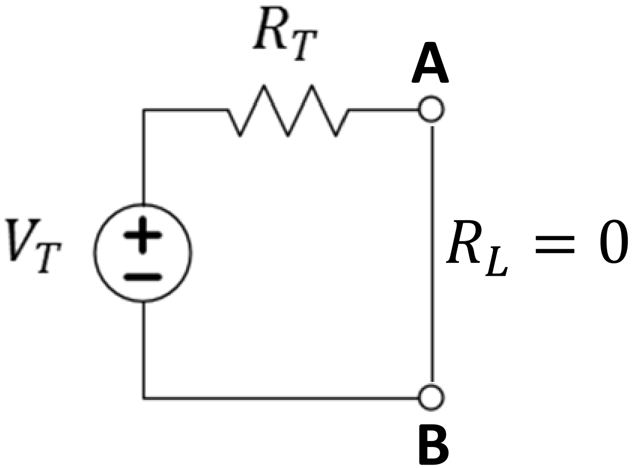

1A
6Ω
4A
3Ω
Rearrange order of elements
Add current sources injecting into the same nodes
1A
4A
6Ω
3Ω
Combine parallel resistors
Norton equivalent circuit
5A
2Ω
10V
2Ω
A
B
Thevenin equivalent circuit

Maximum Power Transfer Theorem



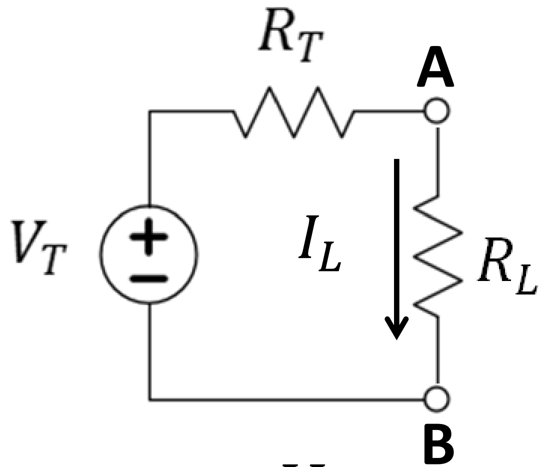
We would like to find for what load resistance R_L the power P_L transferred to R_L is maximum.

Remember: Power is $$P_L = V_L\, I_L$$



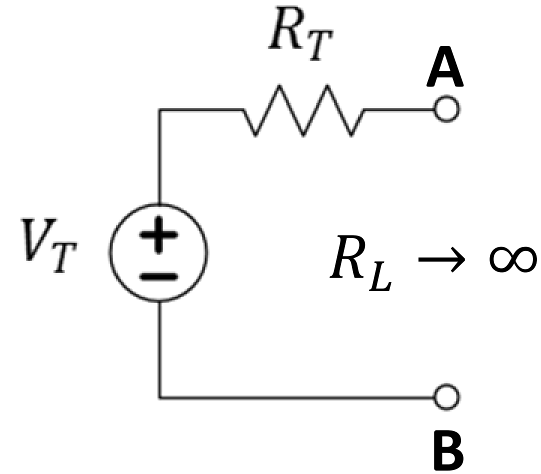
$V_L = 0$

$P_L = 0$



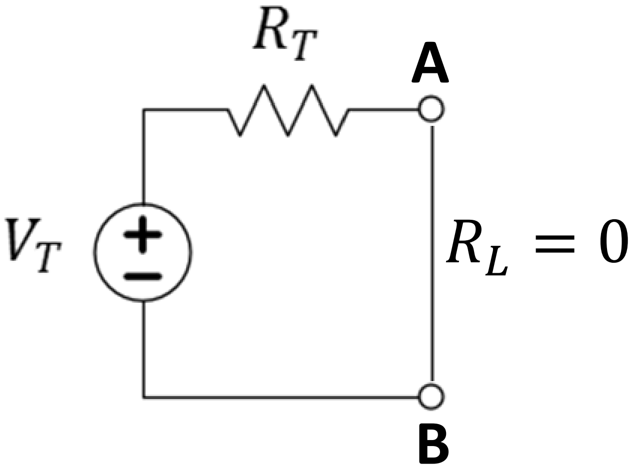
$$I_L = \frac{V_T}{R_T + R_L}$$

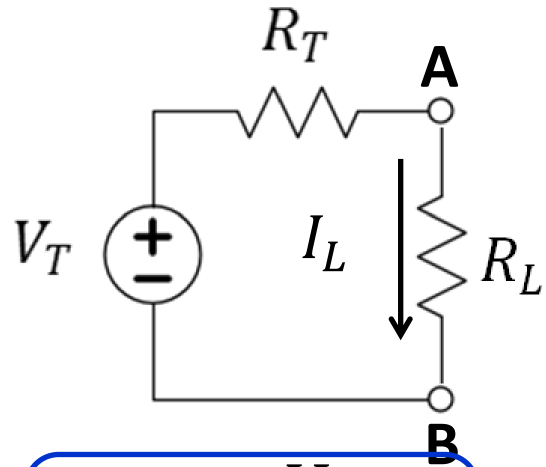
$$P_L = V_L I_L = R_L I_L^2$$

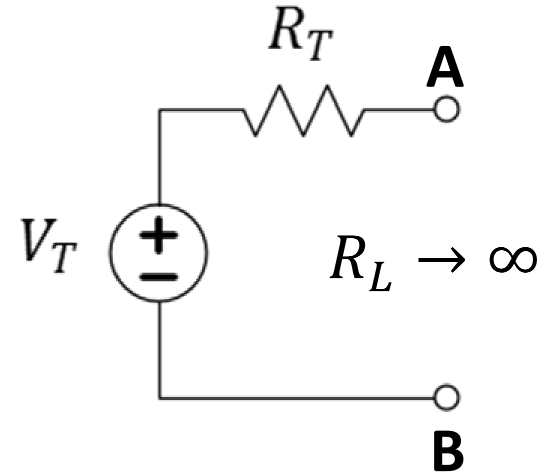


$I_L = 0$

$P_L = 0$







$$V_L = 0$$

$$P_L = 0$$

$$I_L = \frac{V_T}{R_T + R_L}$$

$$P_L = V_L I_L = R_L I_L^2$$

$$I_L = 0$$

$$P_L = 0$$

0 ⟶ ∞

R_L

Maximum Power transfer when $\boldsymbol{R_L = R_T}$ as required by

$$\frac{dP_L}{dR_L} = \frac{d}{dR_L} R_L I_L^2 = V_T^2 \frac{d}{dR_L}\left[\frac{R_L}{(R_T + R_L)^2}\right] = 0$$

Proof

$$\frac{d}{dR_L}\left[\frac{R_L}{(R_T+R_L)^2}\right]=0$$

$$f(R_L)=R_L \qquad g(R_L)=(R_T+R_L)^2$$

$$\frac{d}{dR_L}\left(\frac{f(R_L)}{g(R_L)}\right)=\frac{f'(R_L)g(R_L)-f(R_L)g'(R_L)}{g(R_L)^2}$$

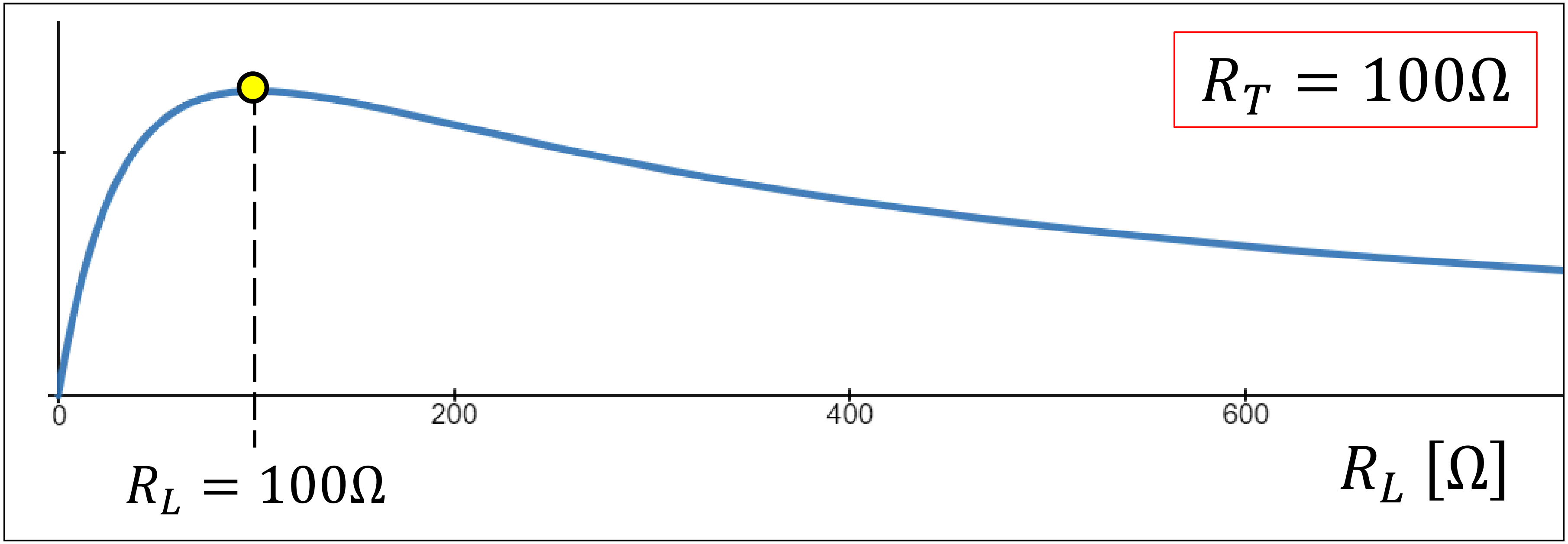
$$\frac{d}{dR_L}\left[\frac{R_L}{(R_T+R_L)^2}\right]=\frac{(R_T+R_L)^2-2R_L(R_T+R_L)}{(R_T+R_L)^4}=0$$

$$(R_T+R_L)^2-2R_L(R_T+R_L)=0$$

$$(R_T+R_L)-2R_L=0$$

$$\Rightarrow \quad R_L=R_T$$

Power transferred to load as a function of load resistance



In conditions of maximum power transfer, 50% of the power generated by the source is dissipated by the source resistance R_T and 50% by the load resistance R_L.