ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 13

MWF – 12:00pm

Prof. Umberto Ravaioli

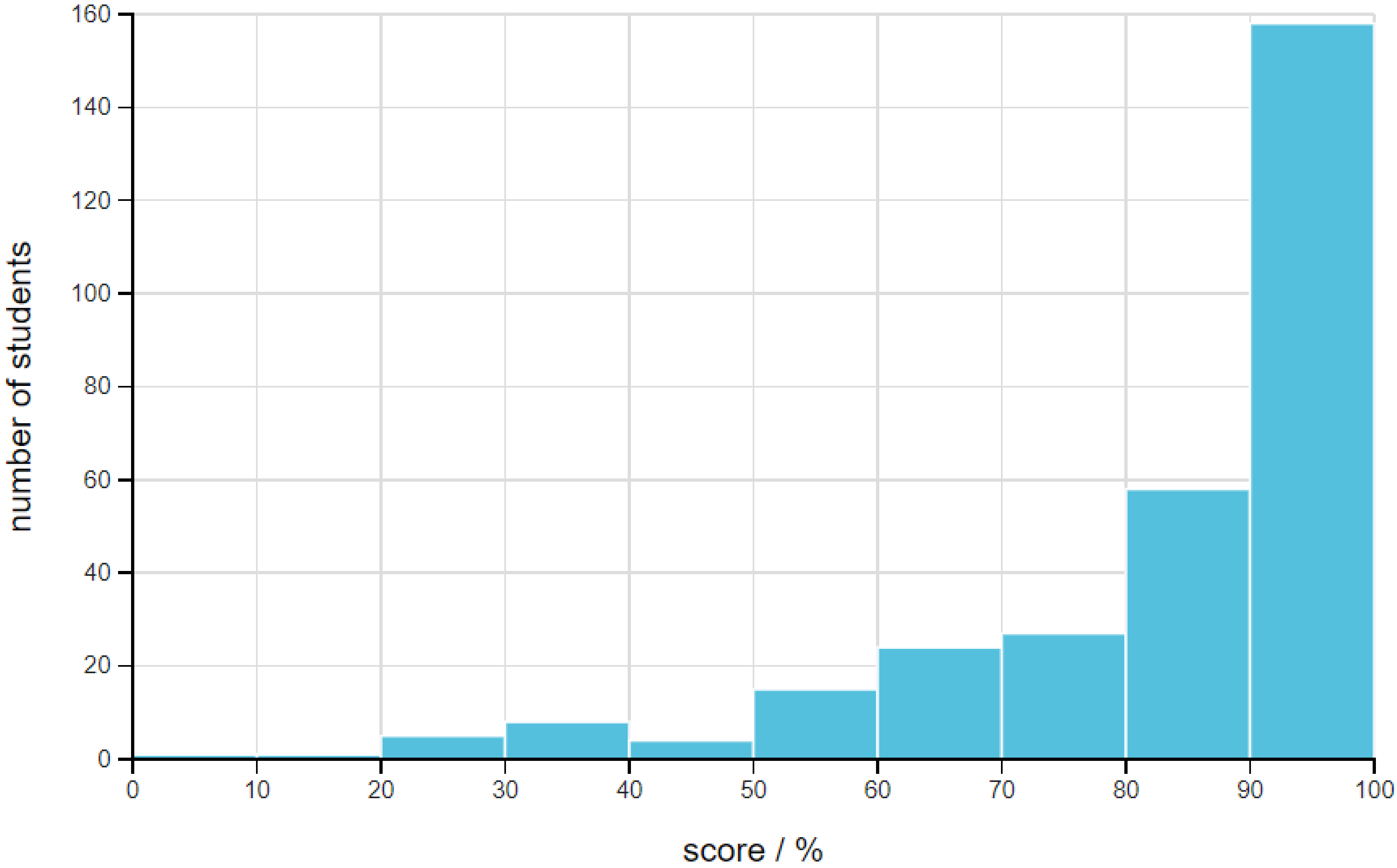
2062 ECE Building

Lecture 13 – Summary

Learning Objectives

1. **Practice with transients in RC circuits**

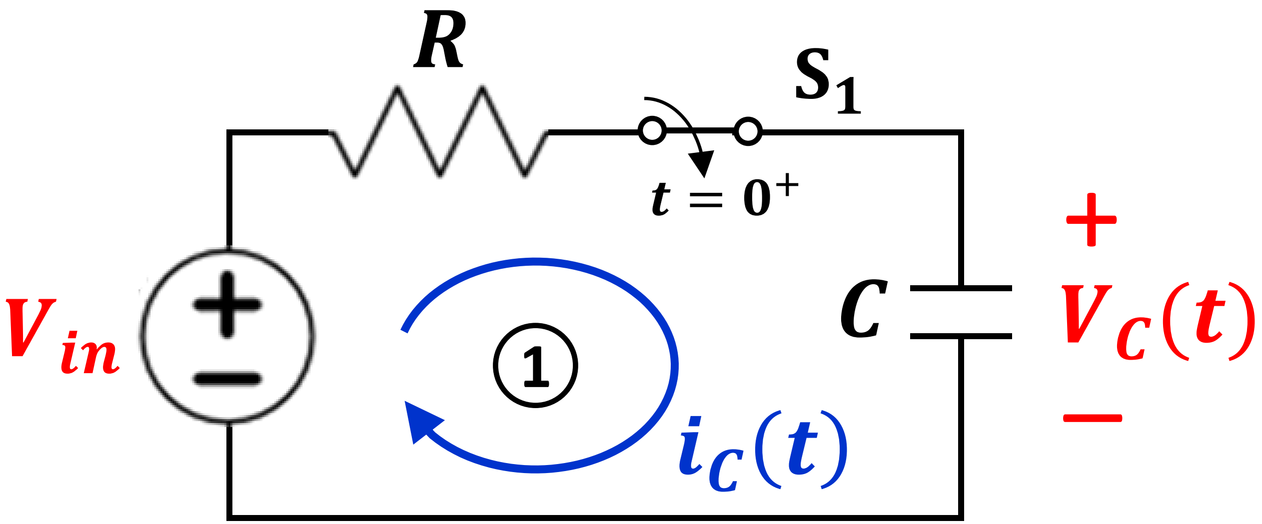
Quiz #1 – Score distribution



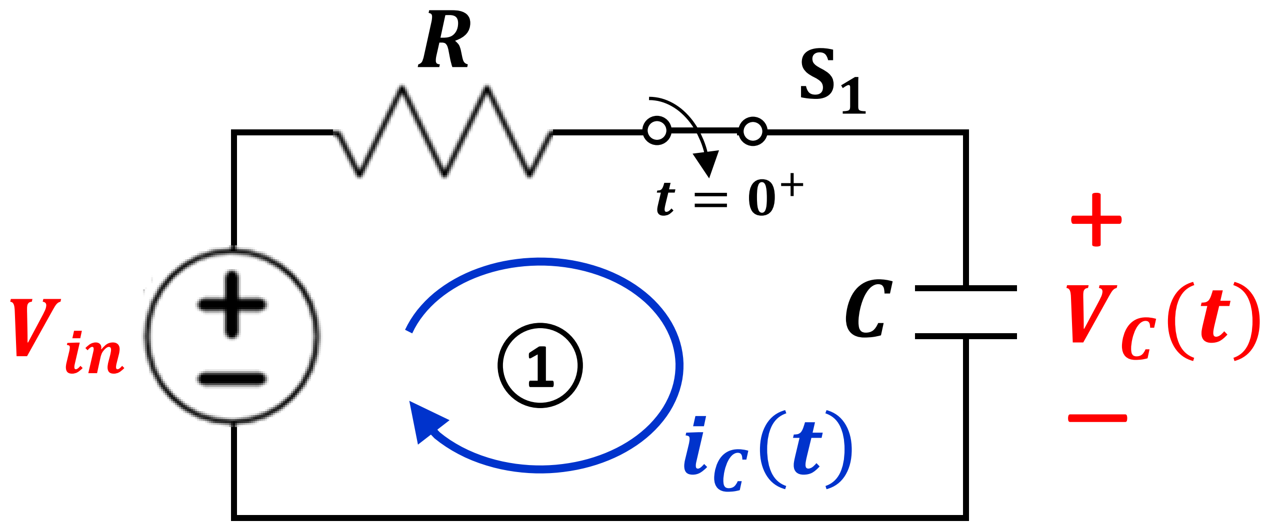
Quiz #1 – Statistics

Number of students	301
Mean score	84%
Standard deviation	19%
Median score	90%
Minimum score	0%
Maximum score	100%
Number of 0%	1 (0% of class)
Number of 100%	41 (14% of class)

RC circuit analysis for constant input voltage

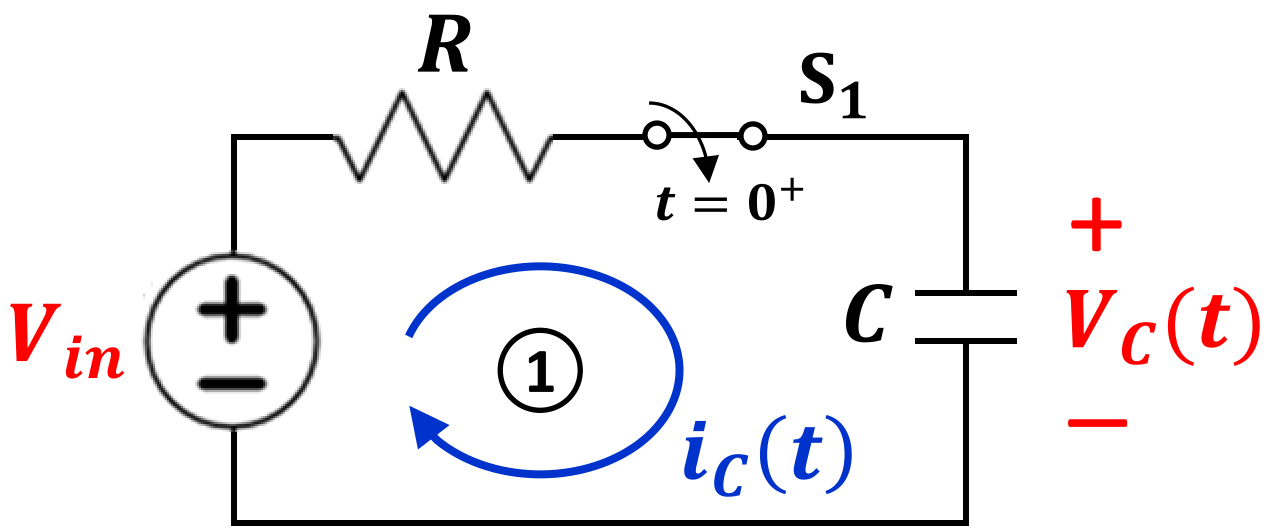


RC circuit analysis for constant input voltage



KVL loop 1 $\boxed{R\, i_C(t) + V_C(t) - V_{in} = 0}$

RC circuit analysis for constant input voltage



KVL loop 1

$$\boxed{R\, i_C(t) + V_C(t) - V_{in} = 0}$$

Since

$$i_C = C\frac{dV_C(t)}{dt}$$

$$\boxed{RC\frac{dV_C(t)}{dt} + V_C(t) - V_{in} = 0}$$

$$\boxed{\frac{dV_C(t)}{dt} + \frac{(V_C(t) - V_{in})}{RC} = 0}$$

First order ordinary differential equation in time, for one energy storage element. N-th order ODE for N energy storage elements.

$$\boxed{\frac{dV_C(t)}{dt} + \frac{(V_C(t) - V_{in})}{RC} = 0}$$

First order ordinary differential equation in time, for one energy storage element. N-th order ODE for N energy storage elements.

General solution has the form

$$\boxed{V_C(t) = K_1 e^{-\alpha t} + K_2 \quad [\mathrm{V}]}$$

$$\boxed{\frac{dV_C(t)}{dt} + \frac{(V_C(t) - V_{in})}{RC} = 0}$$

First order ordinary differential equation in time, for one energy storage element. N-th order ODE for N energy storage elements.

General solution has the form

$$\boxed{V_C(t) = K_1 e^{-\alpha t} + K_2 \qquad [\mathrm{V}]}$$

Constants K_1 and K_2 obtained from initial and final conditions

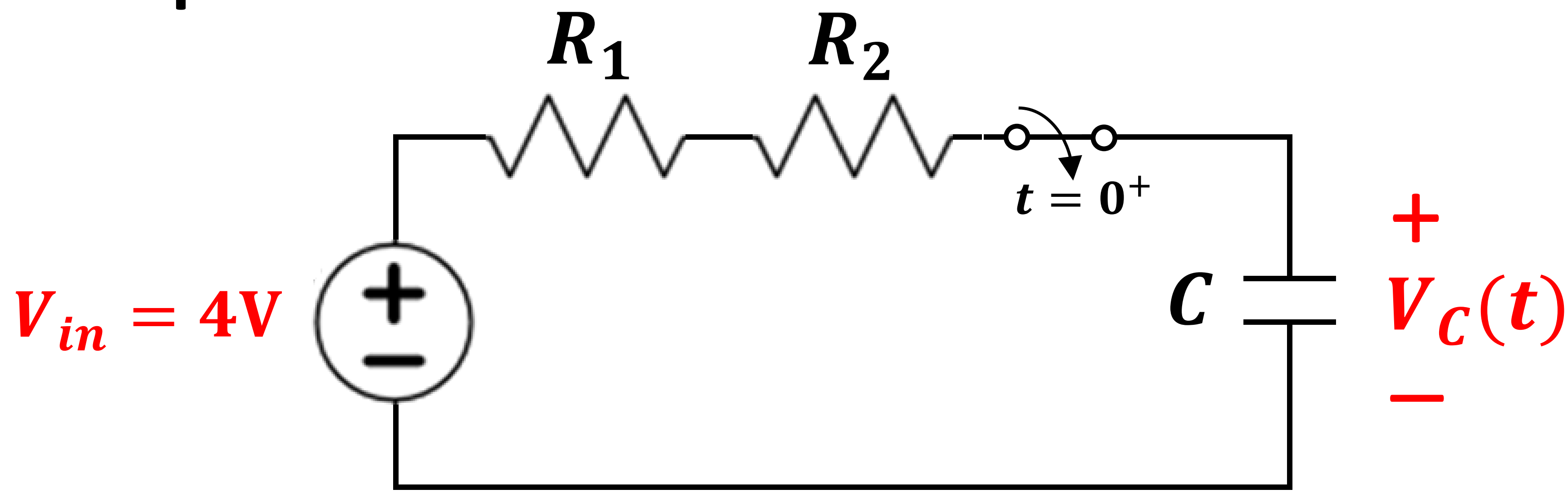
$$V_C(t \to \infty) = K_2$$

$$V_C(t \to 0^+) = K_1 + K_2$$

and

$$\alpha = \frac{1}{\tau} = \frac{1}{R_{eq}C}$$

Example 1

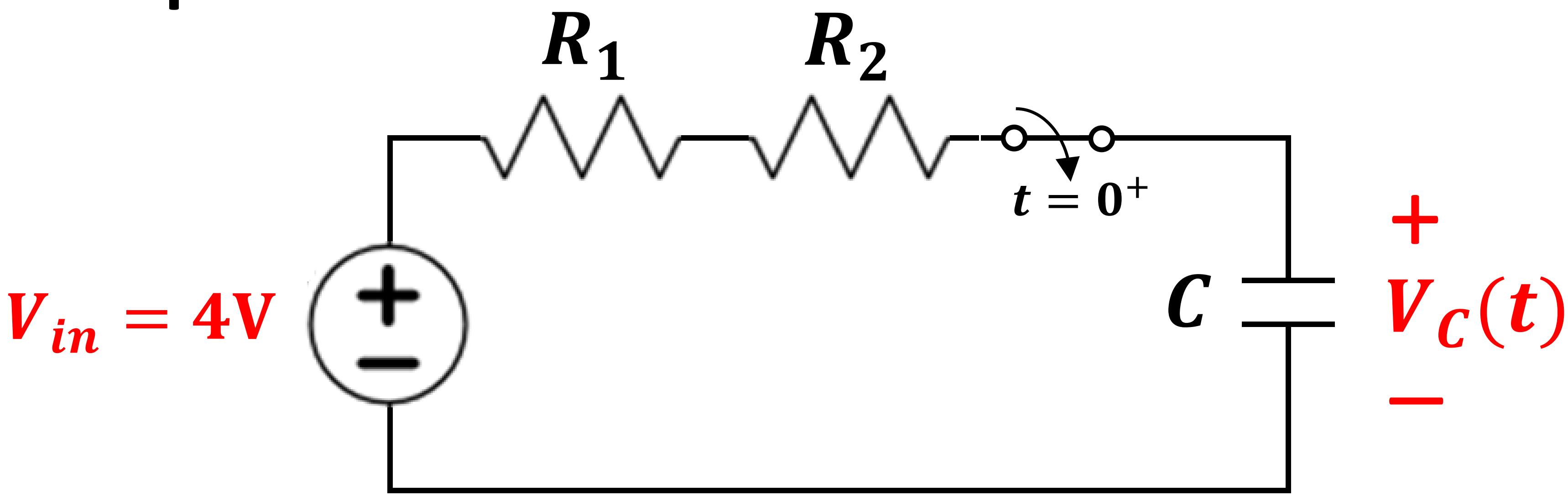


$$R_1 = R_2 = 2k\Omega$$

$$C = 0.25\ \mu F$$

$$V_C(t = 0^-) = 6\mathrm{V}$$

Example 1



$$R_1 = R_2 = 2k\Omega$$

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$$V_C(t = 0^-) = 6\text{V}$$

Switch closes at $t = 0^+$

$$\boxed{V_C(t) = K_1 e^{-\alpha t} + K_2}$$

Example 1

$$\boxed{V_C(t) = K_1 e^{-\alpha t} + K_2}$$

Example 1

$$\boxed{V_C(t) = K_1 e^{-\alpha t} + K_2}$$

Step 1 **Find constants K_1 and K_2**

$$V_C(t = 0^-) = V_C(t = 0^+) = 6\text{V}$$

$$V_C(t = 0^+) = 6\text{V} = K_1 + K_2$$

Example 1

$$\boxed{V_C(t) = K_1 e^{-\alpha t} + K_2}$$

Step 1 Find constants K_1 and K_2

$$V_C(t = 0^-) = V_C(t = 0^+) = 6\text{V}$$

$$V_C(t = 0^+) = 6\text{V} = K_1 + K_2$$

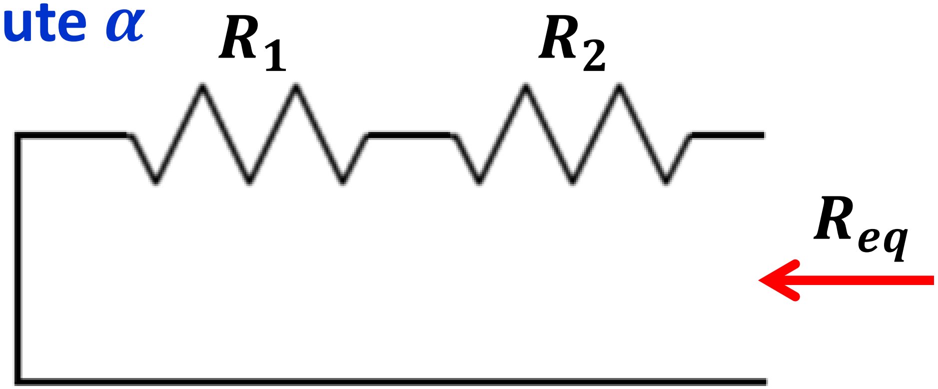
(No current flows) $V_C(t \to \infty) = V_{in} = K_2 = 4\text{V}$

$$\Rightarrow \quad K_1 = 2\text{V}$$

$$K_2 = 4\text{V}$$

Example 1

Step ② Compute α



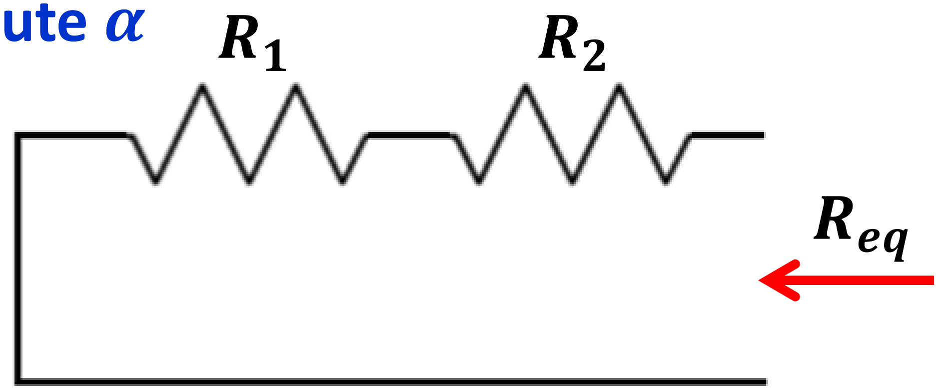
$$R_{eq} = R_1 + R_2 = 4\ \mathrm{k\Omega}$$

$$\tau = R_{eq}C = 4\ \mathrm{k\Omega} \times 0.25\ \mathrm{\mu F} = 10^{-3}\ \mathrm{s}$$

$$\alpha = \frac{1}{\tau} = 10^{3}\ \mathrm{s}^{-1}$$

Example 1

Step 2 Compute α



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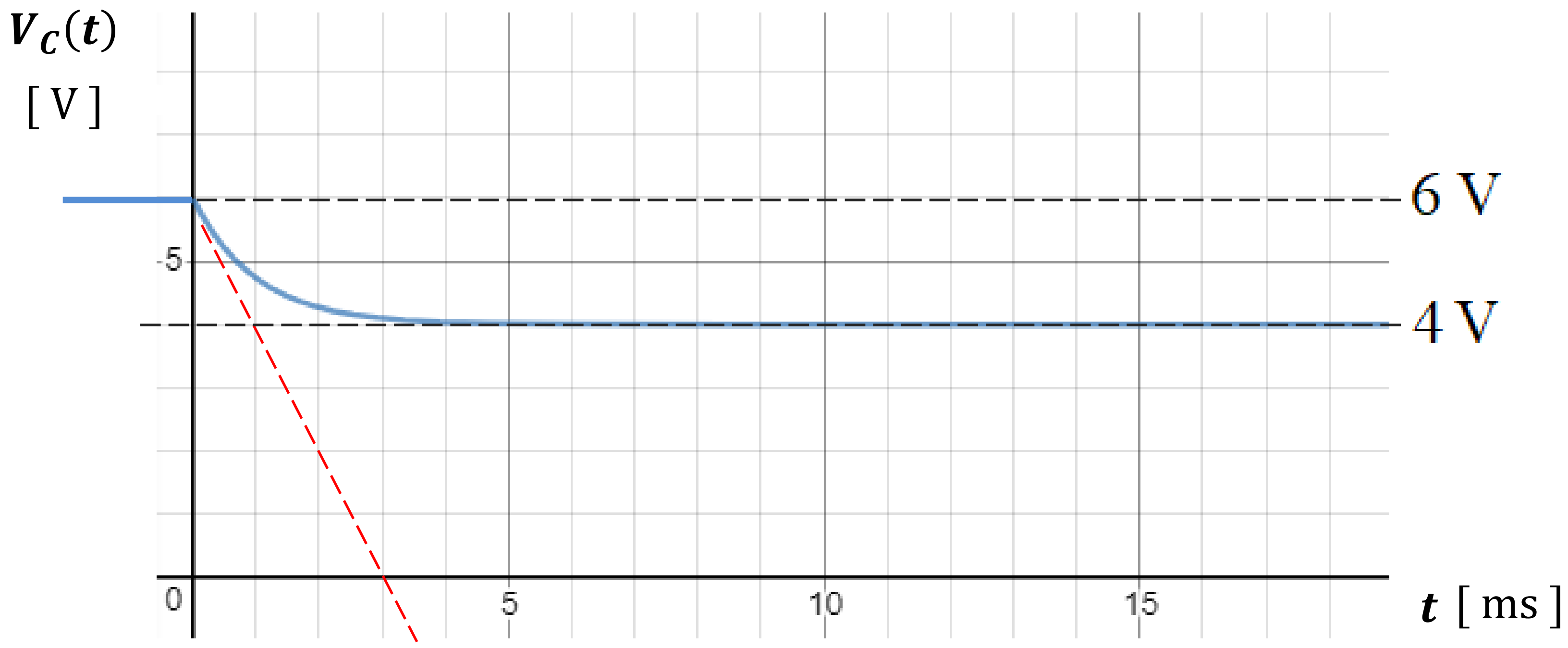
Step 3 Time-dependent voltage solution

$$\boxed{V_C(t) = K_1 e^{-\alpha t} + K_2 = 2\ e^{-10^3 t} + 4\ \ [\mathrm{V}]}$$

Example 1

Time-dependent voltage solution

$$V_C(t) = 2\,e^{-10^3 t} + 4 \;\; [\mathrm{V}]$$

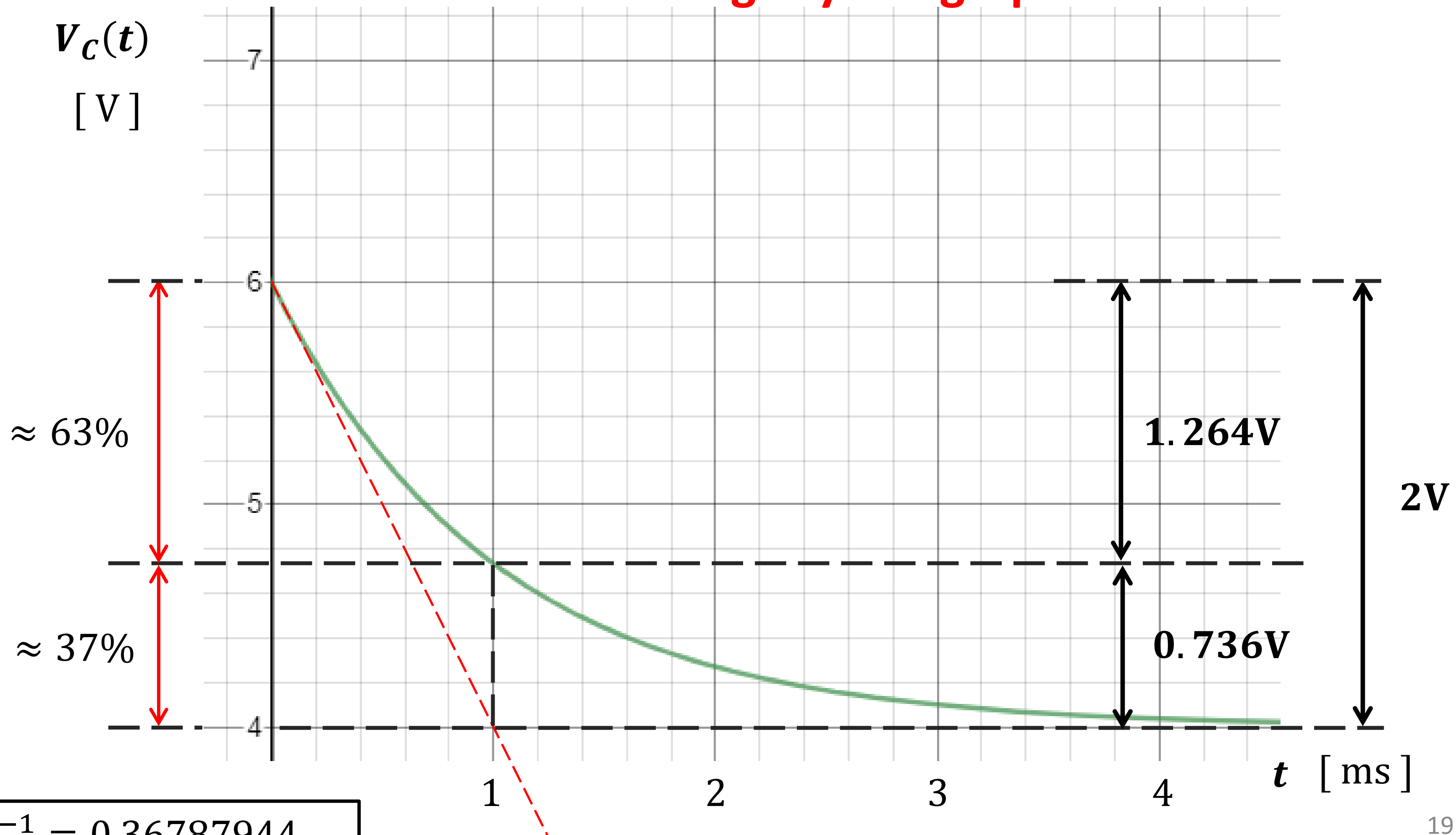


Example 1

Time-dependent voltage solution

$$V_C(t) = 2\, e^{-10^3 t} + 4 \;\; [\mathrm{V}]$$

Let's magnify the graph



$e^{-1} = 0.36787944 \ldots$

Example 1

Time-dependent voltage solution

$$\boxed{V_C(t) = 2\, e^{-10^3 t} + 4 \ \ [\mathrm{V}]}$$

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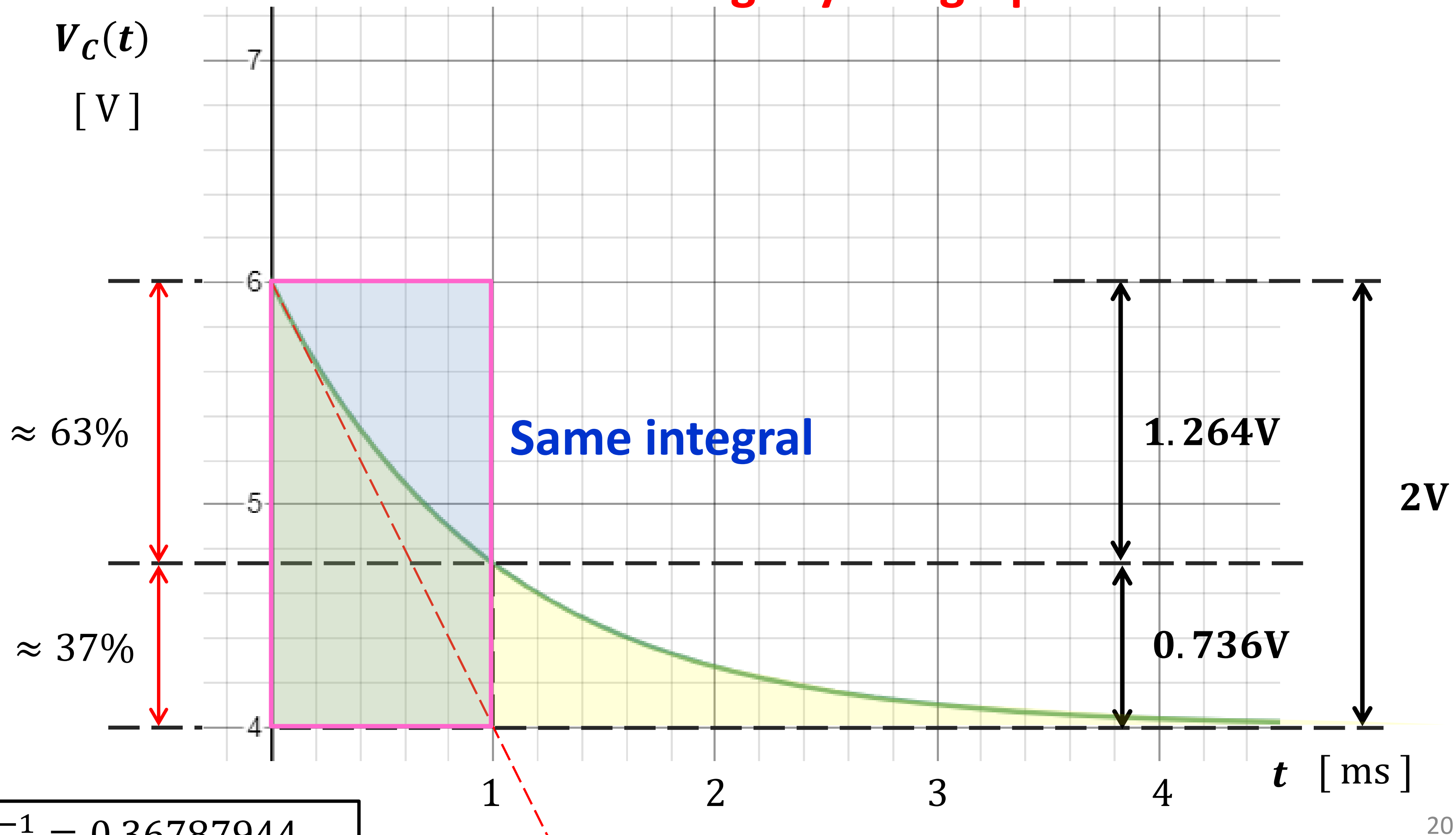


$e^{-1} = 0.36787944\ldots$

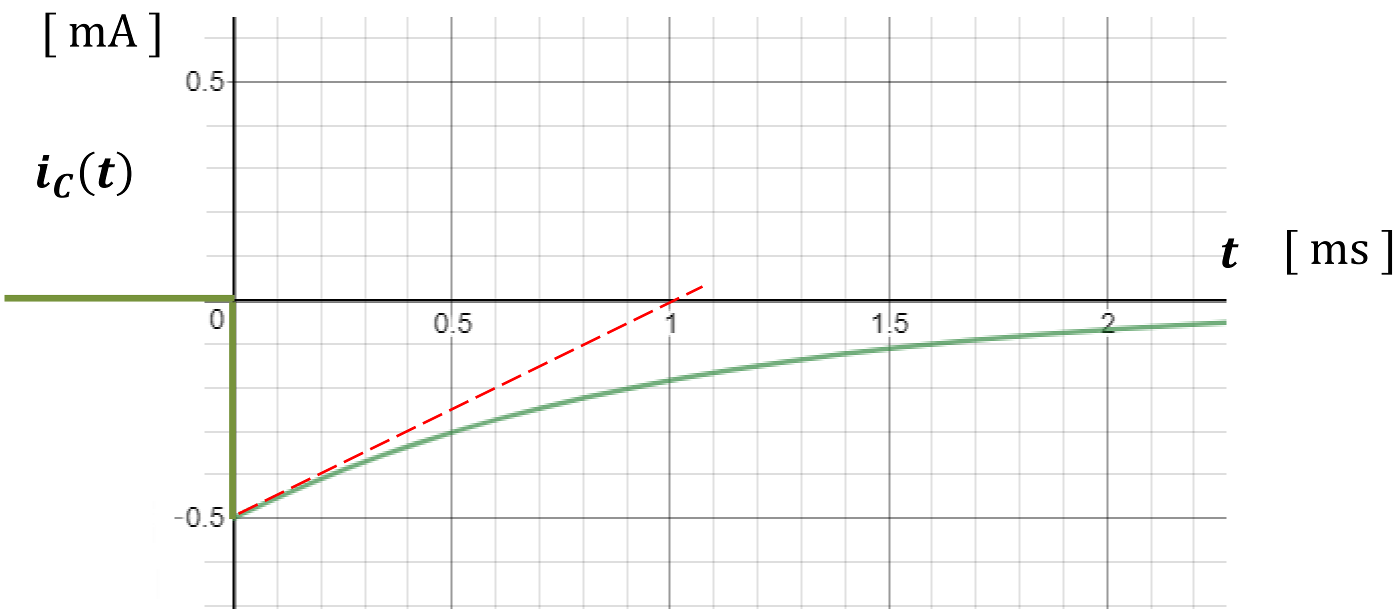
Example 1

Time-dependent current solution

$$i_C(t) = C\frac{dV(t)}{dt} = C\frac{d}{dt}\left[2\,e^{-10^3 t} + 4\right]\ [\mathrm{A}]$$

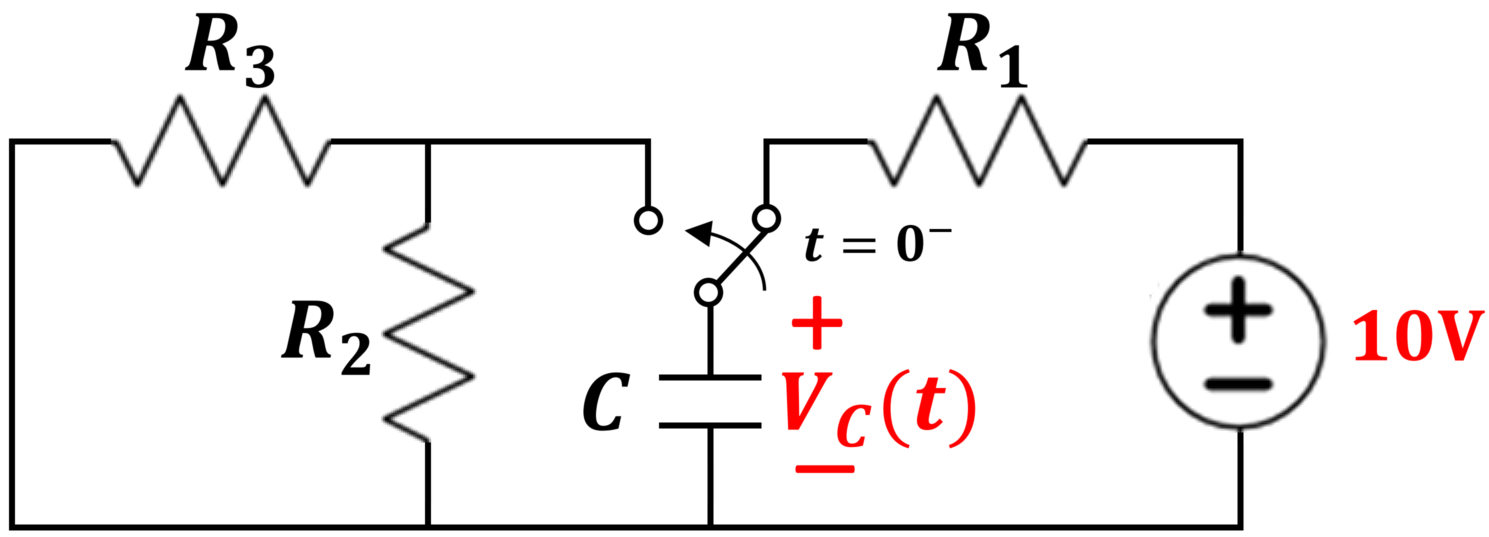
$$= \overbrace{0.25\times10^{-6}}^{C} \times \overbrace{\left(-2\times10^{3}\times e^{-10^3 t}\right)}^{\frac{dV(t)}{dt}}\ [\mathrm{A}]$$

$$= -0.5\,e^{-10^3 t}\ [\mathrm{mA}]$$



Example 2

Switch moves to left terminal at $t = 0^+$

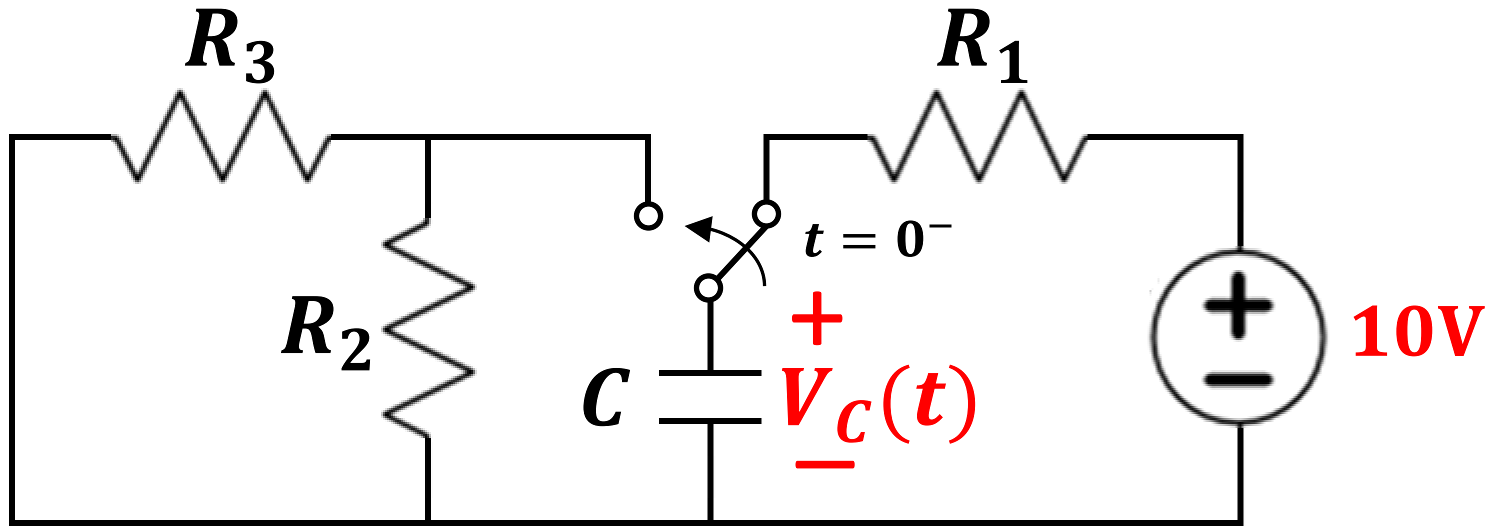


$$R_1 = 1\ \mathrm{k\Omega} \qquad R_2 = R_3 = 2\ \mathrm{k\Omega}$$

$$C = 1\ \mu\mathrm{F}$$

Example 2

Switch moves to left terminal at $t = 0^+$



$R_1 = 1\ \mathrm{k\Omega}$ $\quad$ $R_2 = R_3 = 2\ \mathrm{k\Omega}$

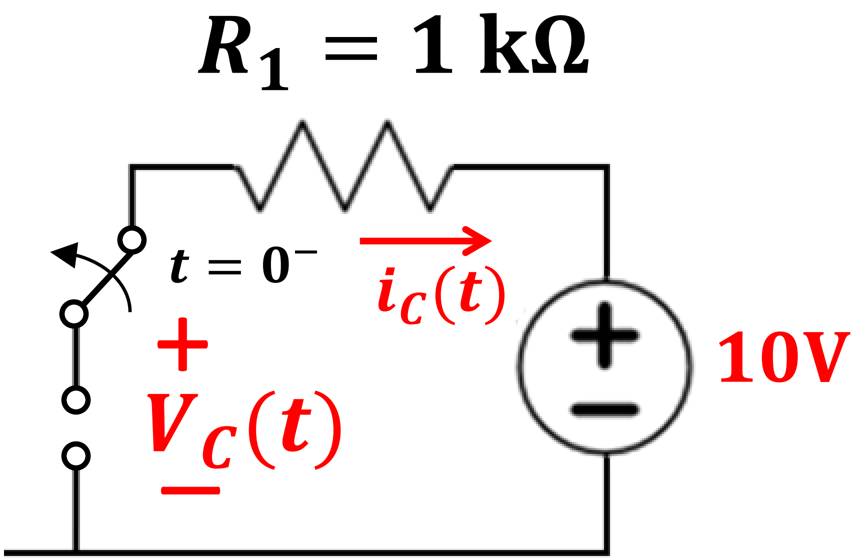
$C = 1\ \mu\mathrm{F}$

Step 1 Find state of capacitor at $t = 0^-$

Assuming that the capacitor is at steady-state before moving the switch, it behaves like an open circuit, and

$$V_C(t = 0^-) = 10\mathrm{V}$$

Example 2



At $t = 0^-$ the capacitor is fully charged and at steady state.

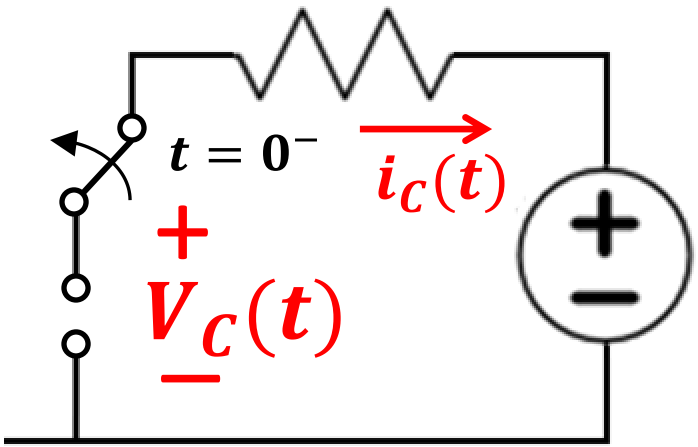
The capacitor behaves like an open circuit:

$$i_C(t = 0^-) = 0\ \mathrm{A}$$

Example 2

At $t = 0^-$ the capacitor is fully charged and at steady state.

The capacitor behaves like an open circuit:



$$i_C(t = 0^-) = 0\ \mathrm{A}$$

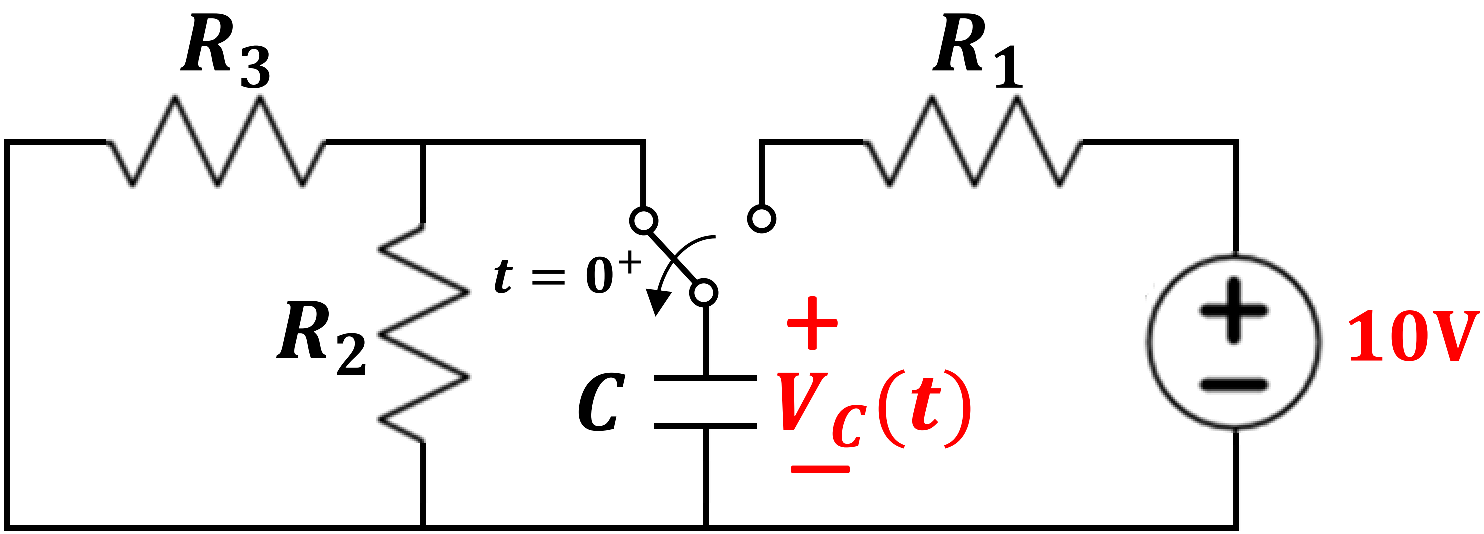
Apply the KVL

$$-V_C(0^-) + \cancel{R_1 i_C}(0^-) + 10\mathrm{V} = 0$$

$$\boxed{V_C(0^-) = 10\mathrm{V}}$$

Example 2

Switch moves to left terminal at $t = 0^+$



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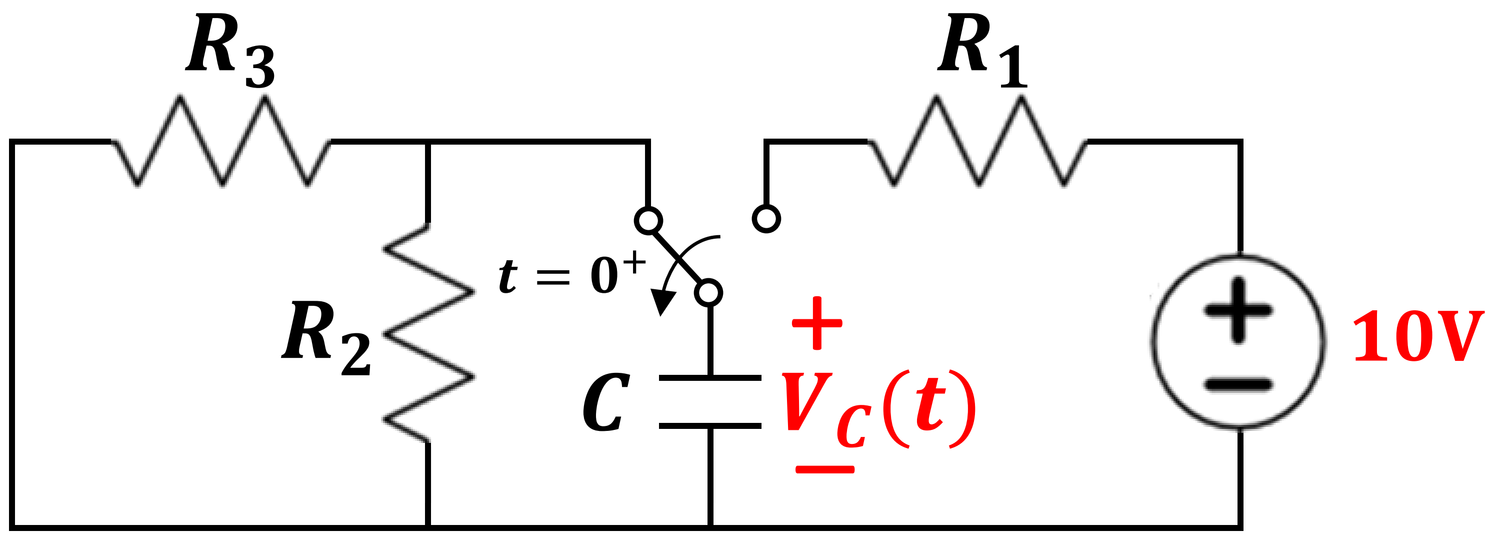
Example 2

Switch moves to left terminal at $t = 0^+$



$$R_1 = 1\ \text{k}\Omega \qquad R_2 = R_3 = 2\ \text{k}\Omega$$

$$C = 1\ \mu\text{F}$$

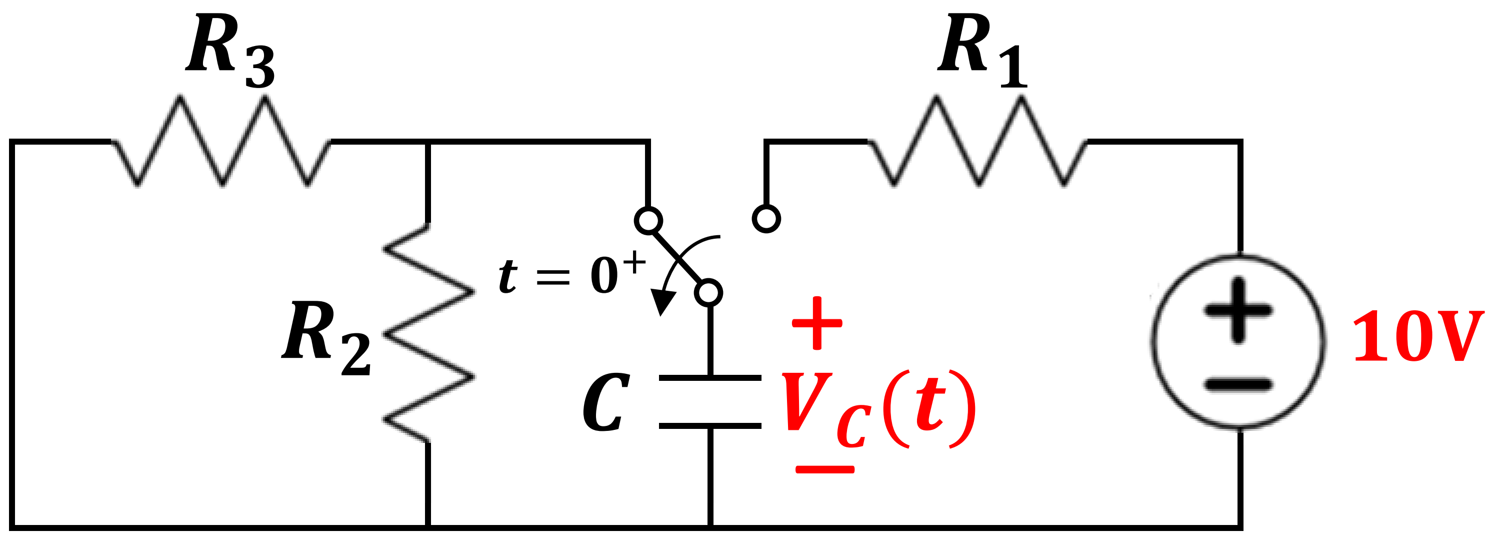
Step 3 Find state of capacitor at $t \to \infty$

There is no source in left circuit. At steady-state the capacitor must be discharged:

$$V_C(t \to \infty) = 0\text{V} \quad \to \quad K_2 = 0$$

Example 2

Switch moves to left terminal at $t = 0^+$



$$R_1 = 1\ \mathrm{k\Omega} \qquad R_2 = R_3 = 2\ \mathrm{k\Omega}$$

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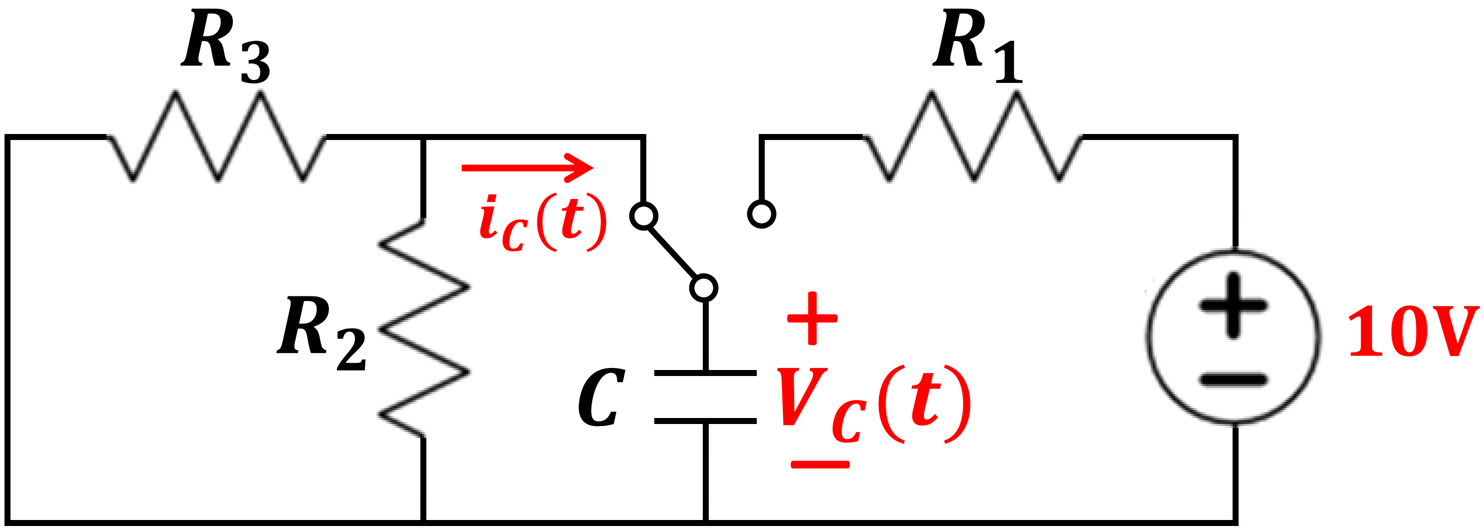
Switch moves to left terminal at $t = 0^+$



$R_1 = 1\ \mathrm{k\Omega}$ $R_2 = R_3 = 2\ \mathrm{k\Omega}$

$C = 1\ \mu\mathrm{F}$

Step 4 Find the current flowing out of the capacitor at $t = 0^+$

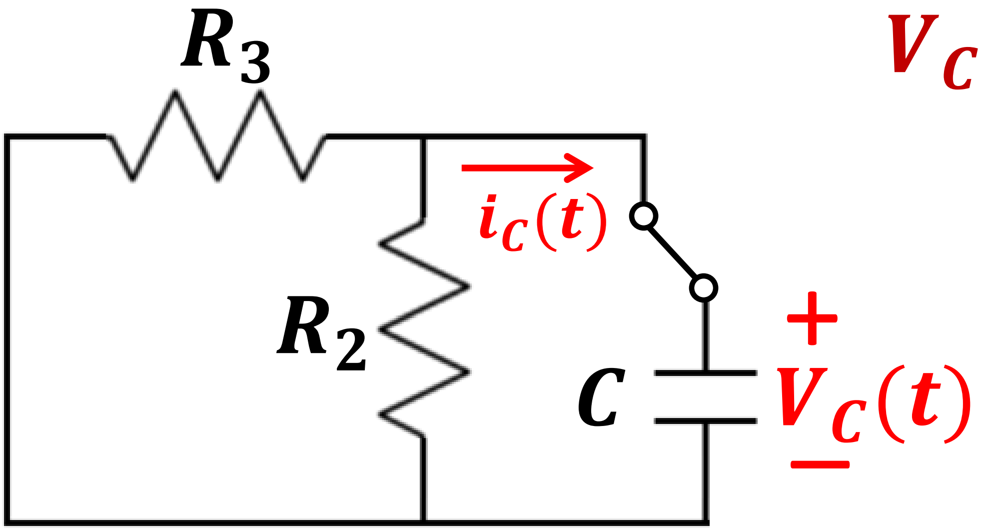
The capacitor starts discharging, behaving like a time dependent source. At $t = 0^+$ it is at 10V and it sees $\mathrm{R}_{eq} = R_2//R_3 = 1\mathrm{k\Omega}$, therefore:

$$i_C(t = 0^+) = -10/1\mathrm{k} = -10\mathrm{mA}$$

Example 2

Switch moves to left terminal at $t = 0^+$

$$V_C(t = 0^+) = 10\text{V}$$



$$C = 1\ \mu\text{F} \qquad R_2 = R_3 = 2\ \text{k}\Omega$$

$$R_2//R_3 = 1\ \text{k}\Omega$$

Apply the KVL at $t = 0^+$

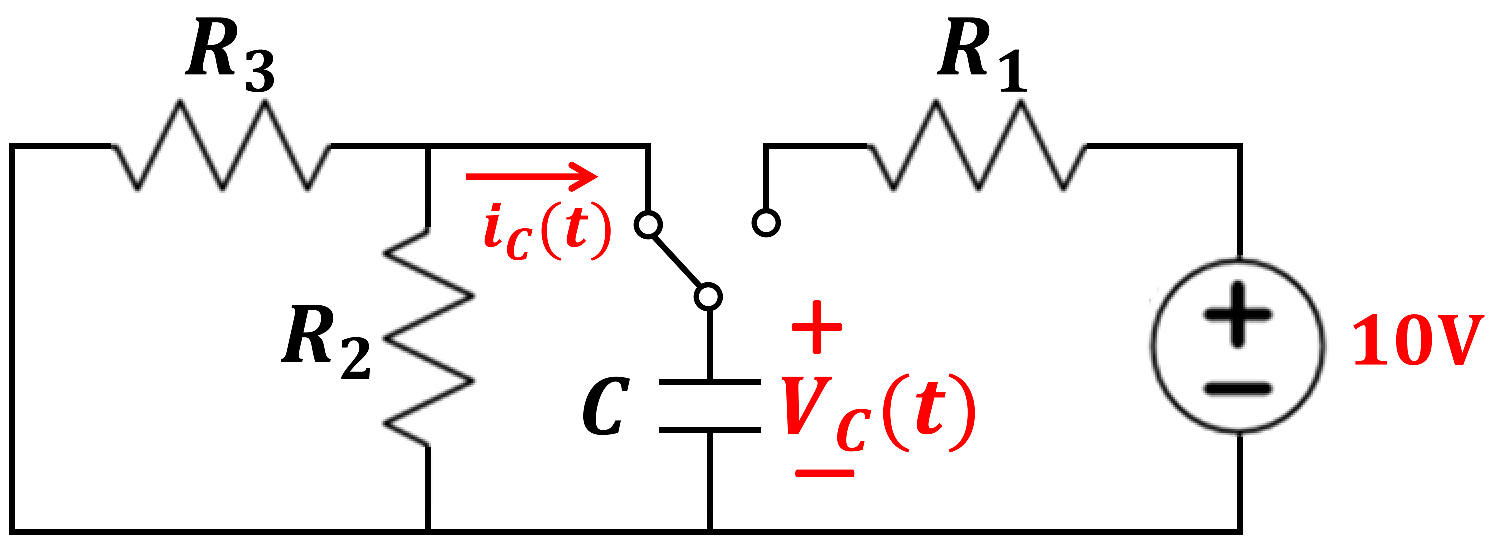
$$[R_2//R_3]\, i_C(0^+) + V_C(0^+) = 0$$

$$i_C(0^+) = -V_C(0^+)/[R_2//R_3]$$

$$\boxed{i_C(t = 0^+) = -10\text{mA}}$$

Example 2

Switch moves to left terminal at $t = 0^+$



$$R_1 = 1\ \mathrm{k\Omega} \qquad R_2 = R_3 = 2\ \mathrm{k\Omega}$$

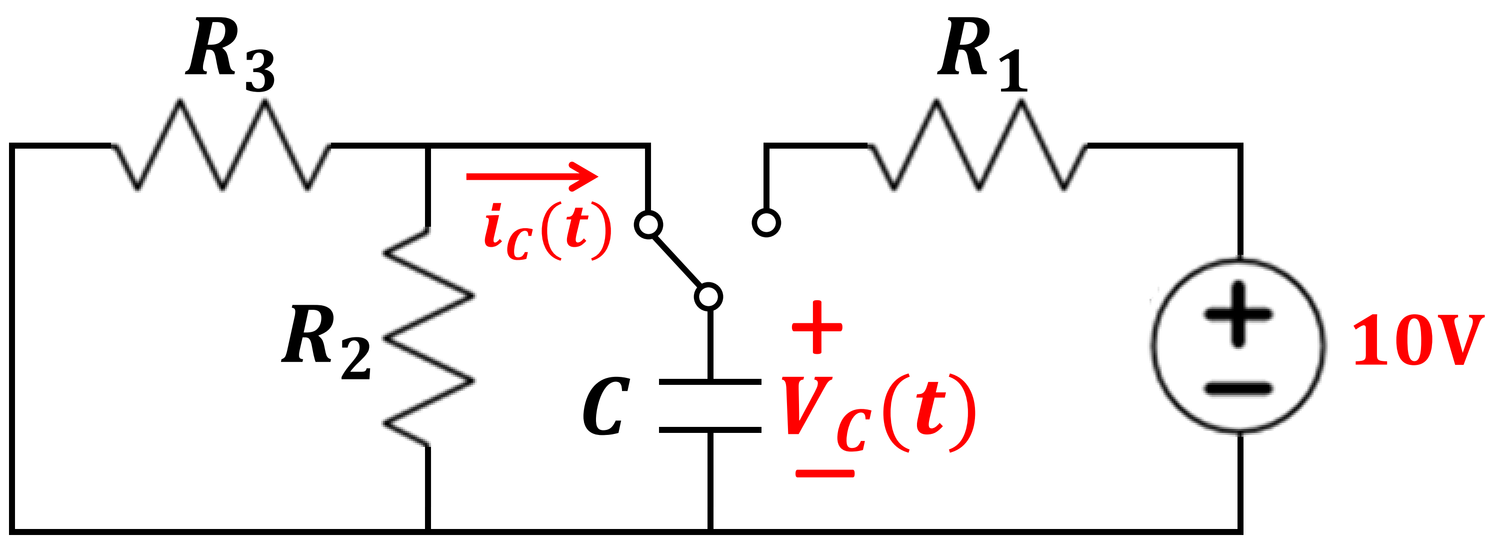
$$C = 1\ \mu\mathrm{F} \qquad R_{eq} = R_2 // R_3 = 1\ \mathrm{k\Omega}$$

Step 5 Find τ and α

$$\tau = R_{eq}C = 1\mathrm{k\Omega} \times 1\mu\mathrm{F} = 10^{-3}s \qquad \alpha = 10^{3}\mathrm{s}^{-1}$$

Example 2

Switch moves to left terminal at $t = 0^+$



$$R_1 = 1\ \mathrm{k\Omega} \qquad R_2 = R_3 = 2\ \mathrm{k\Omega}$$

$$C = 1\ \mathrm{\mu F} \qquad R_{eq} = R_2//R_3 = 1\ \mathrm{k\Omega}$$

Step 5 Find τ and α

$$\tau = R_{eq}C = 1\mathrm{k\Omega}\ \times 1\mathrm{\mu F} = 10^{-3}s \qquad \alpha = 10^3\mathrm{s}^{-1}$$

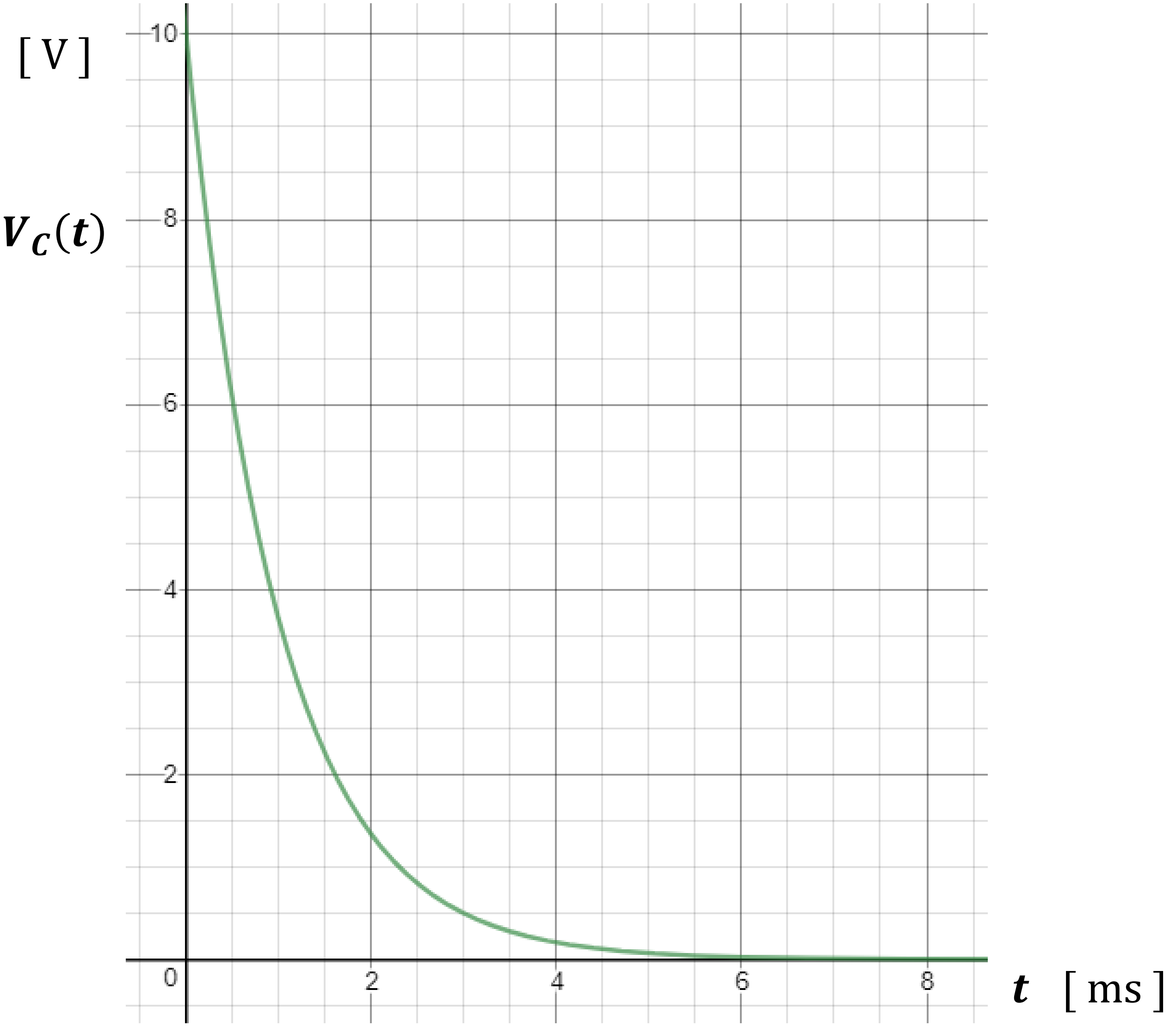
Step 6 Time-dependent voltage

$$\boxed{V_C(t) = K_1 e^{-\alpha t} = 10\ e^{-10^3 t}\ \ [\mathrm{V}]}$$

Example 2

Time-dependent voltage

$$\boxed{V_C(t) = K_1 e^{-\alpha t} = 10\ e^{-10^3 t}\ \ [\mathrm{V}]}$$



Example 2

Time-dependent current

$$\boldsymbol{i_C(t) = C\frac{dV(t)}{dt} = C\frac{d}{dt}\left[10\,e^{-10^3 t}\right]\ [\mathrm{A}]}$$

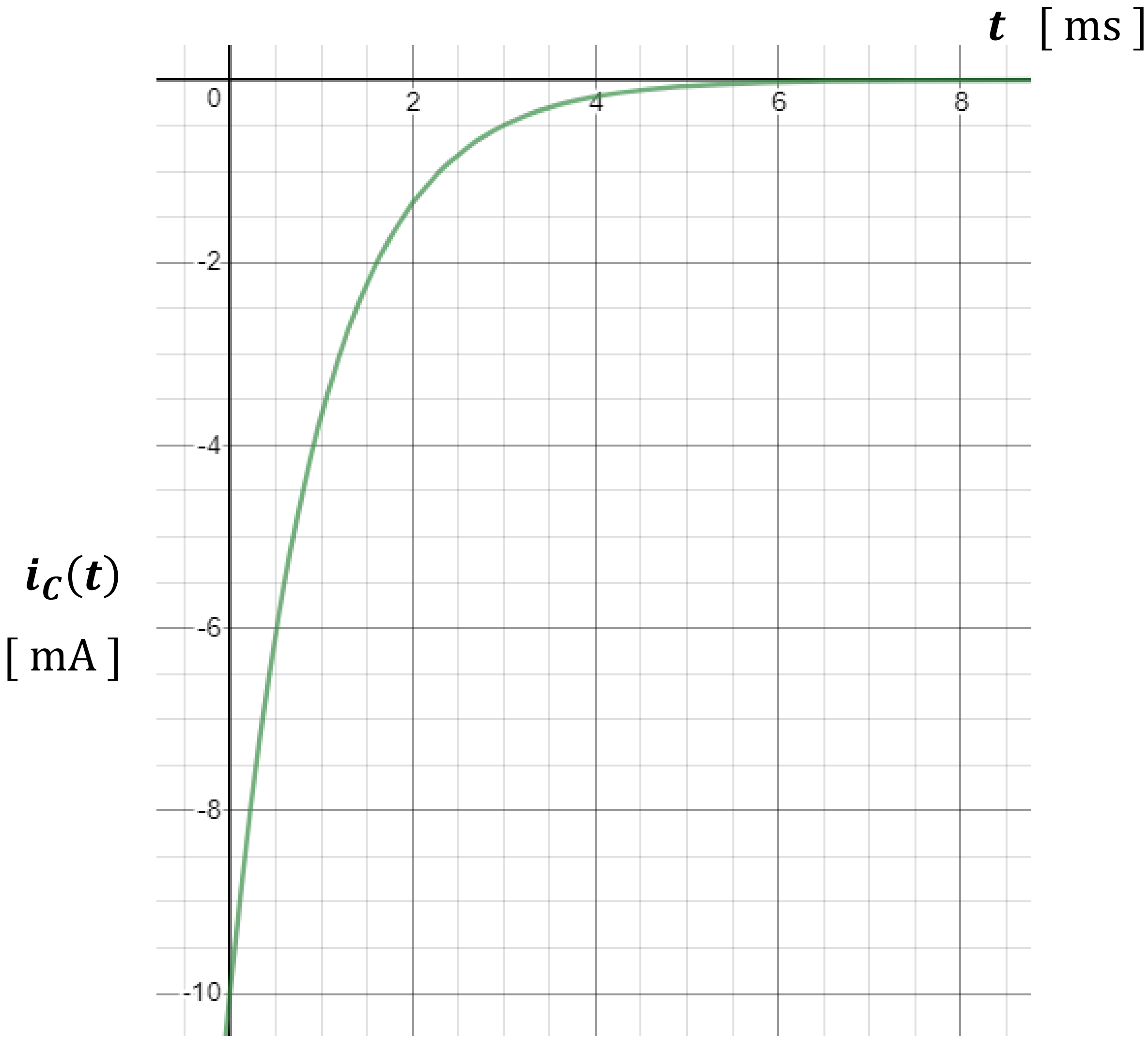
$$\boldsymbol{= 10^{-6} \times \left(-10 \times 10^3 \times e^{-10^3 t}\right)\ [\mathrm{A}]}$$

$$\boldsymbol{= -10\,e^{-10^3 t}\ [\mathrm{mA}]}$$

Example 2

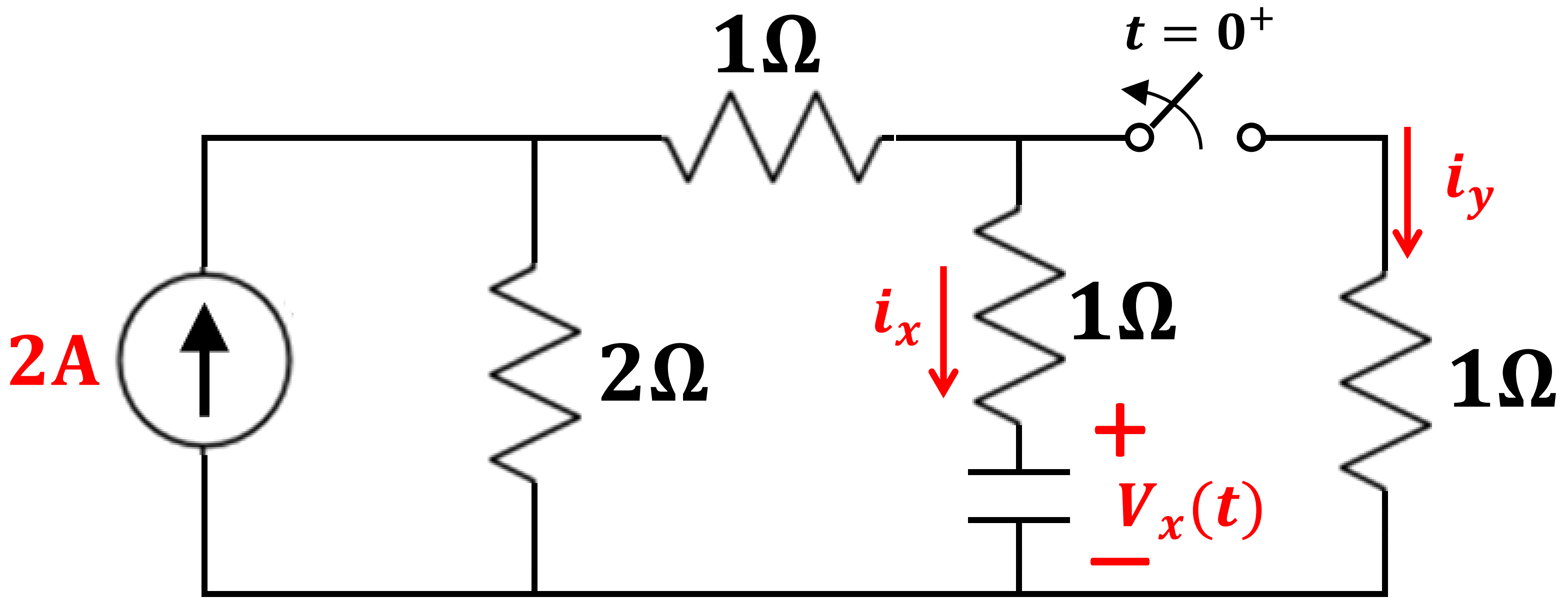
Time-dependent current

$$\boxed{i_C(t) = -10\, e^{-10^3 t}\ [\mathrm{mA}]}$$



Example 3

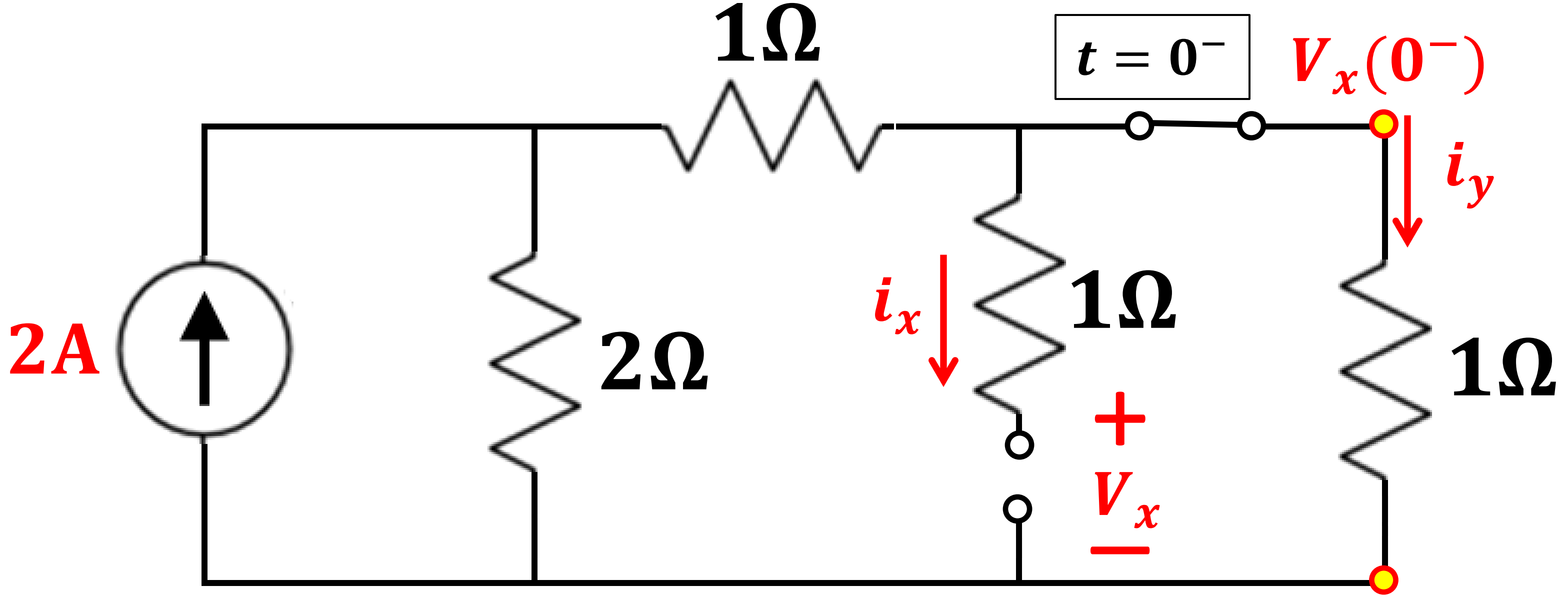
Switch opens at $t = 0^+$



Find ① $i_y(0^-)$ ③ $i_y(0^+)$ ⑤ $V_x(\infty)$

② $V_x(0^-)$ ④ $V_x(0^+)$

Example 3



Find ① $i_y(0^-)$ ② $V_x(0^-)$

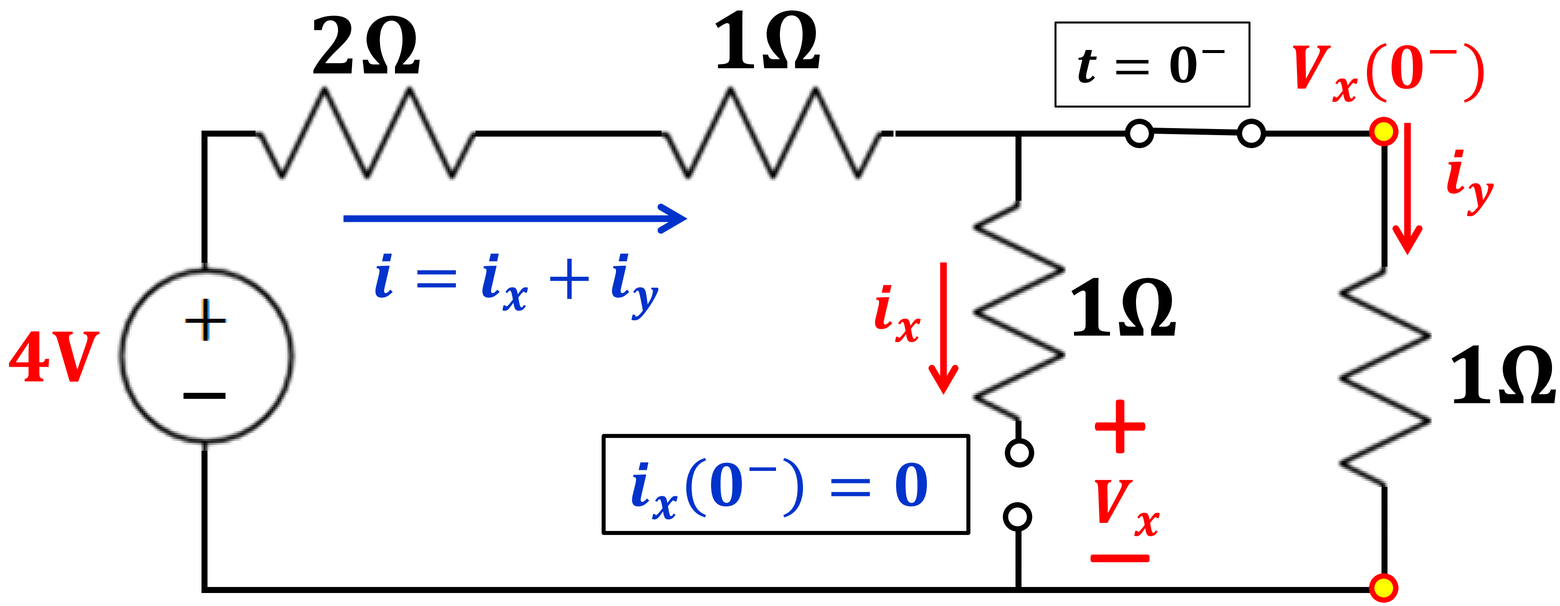
At $t = 0^-$ the capacitor is an open circuit and $i_x(0^-) = 0$.

Current Divider

$$i_y(0^-) = 2A\frac{2\Omega}{2\Omega + 2\Omega} = 1A$$

$$V_x(0^-) = i_y \times 1\Omega = 1V$$

Example 3 For practice, let's do the same with source transformation.



Find ① $i_y(0^-)$ ② $V_x(0^-)$

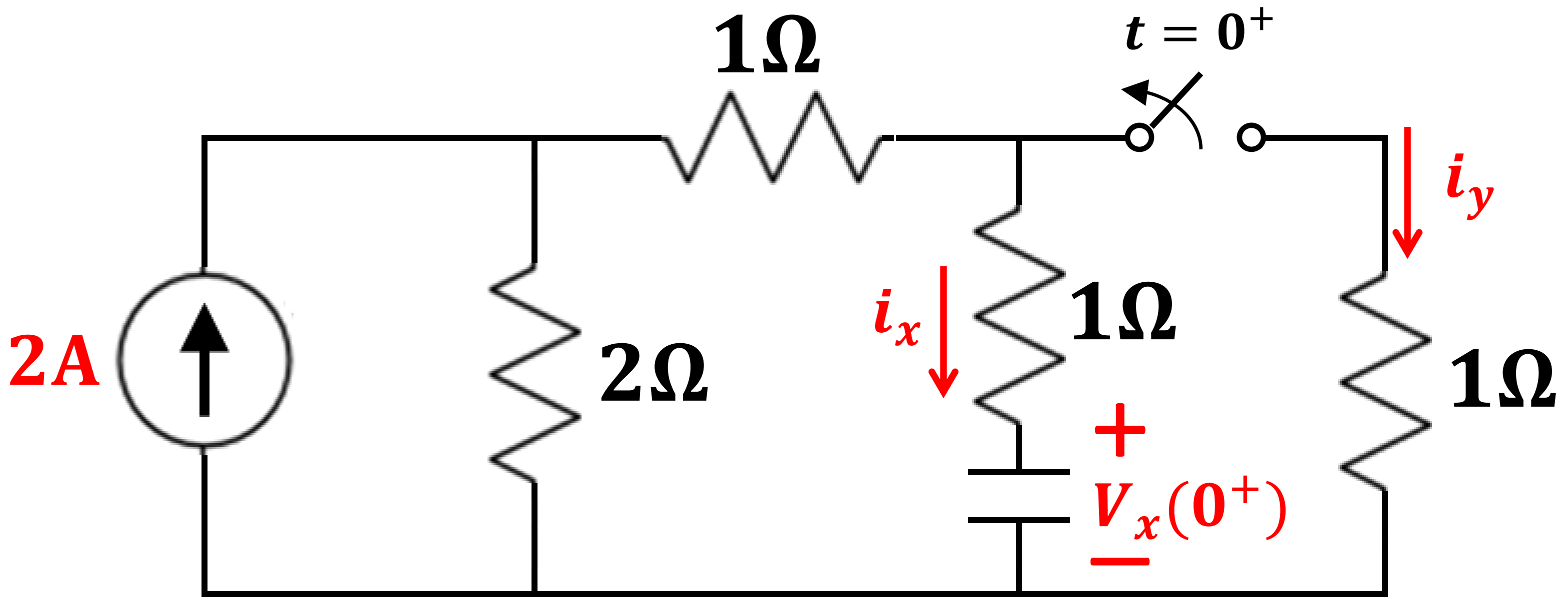
$$i = i_y(0^-) = \frac{4\text{V}}{2\Omega + 1\Omega + 1\Omega} = 1\text{A}$$

Voltage Divider

$$V_x(0^-) = 4 \times \frac{1\Omega}{4\Omega} = 1\text{V}$$

Example 3

Switch opens at $t = 0^+$



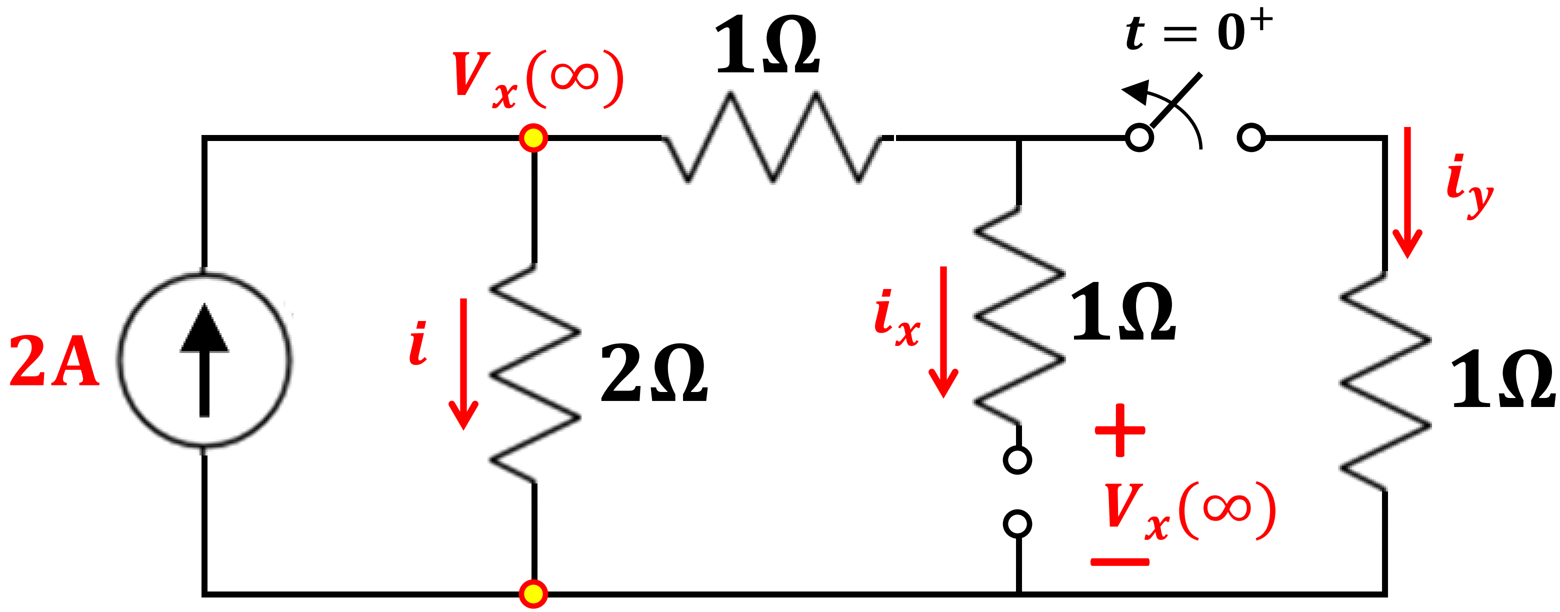
Find ③ $i_y(0^+)$ ④ $V_x(0^+)$

$\boxed{i_y(0^+) = 0}$ since the switch is open

$\boxed{V_x(0^-) = V_x(0^+) = 1\text{V}}$ voltage does not change

Example 3

Switch opens at $t = 0^+$



Find ⑤ $V_x(\infty)$

$$\boxed{i_x(\infty) = 0}$$

Capacitor is an open at $t \rightarrow \infty$

Capacitor has the same voltage as at the terminals of the 2Ω resistor through which flows the only current $i = 2\mathrm{A}$

$$\boxed{V_x(\infty) = 2\mathrm{A} \times 2\Omega = 4\mathrm{V}}$$

Magnetic Inductance

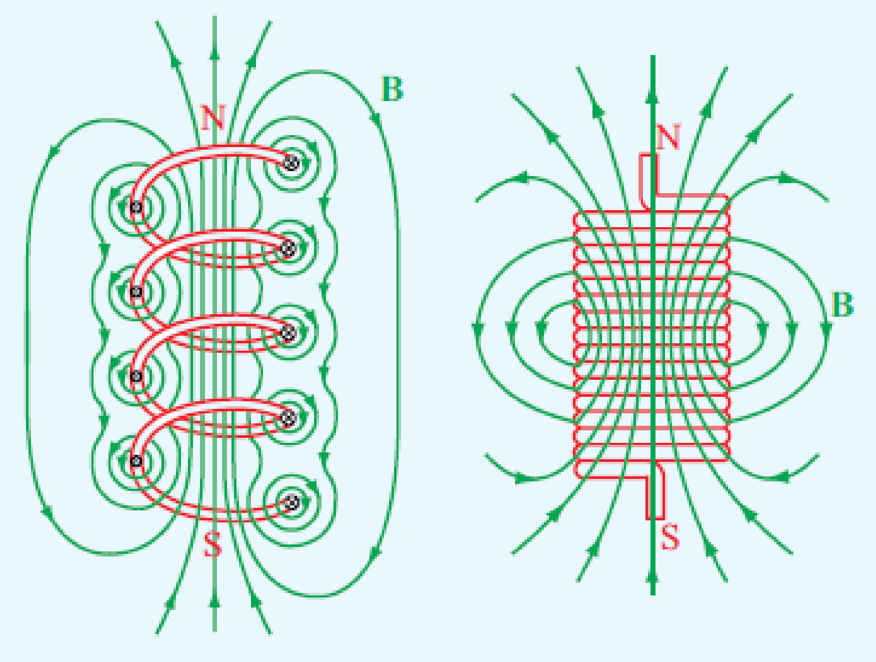
Current flowing in electric wires generates a magnetic field. When a change in current occurs, an "electromotive force" (voltage) is generated as a reaction, due to the change of the magnetic flux concatenated with the wire. The structure is said to "store" magnetic energy.

Inductance (quantified as the ratio between the magnetic field flux and the current) expresses the tendency of a conductor to oppose a change of the current flowing through it.

Inductors

All conductors carrying current exhibit inductance. The devices called inductors are designed to maximize the concatenated magnetic field and the associated storage of magnetic energy.

A coiled wire structure is called a “solenoid” and it is the most common way to realize an inductor.



(a) Loosely wound solenoid

(b) Tightly wound solenoid

Inductance

The inductance value of a solenoid inductor depends on the number of wire loops in the coil, on the cross-sectional area, and on the "magnetic permeability" of the core region. Rods of high relative permeability material are often inserted in solenoids to amplify the local magnetic field and increase the inductance value of the device.

$$L = N^2 \mu_r \, \mu_0 \, A \, \ell$$

total length

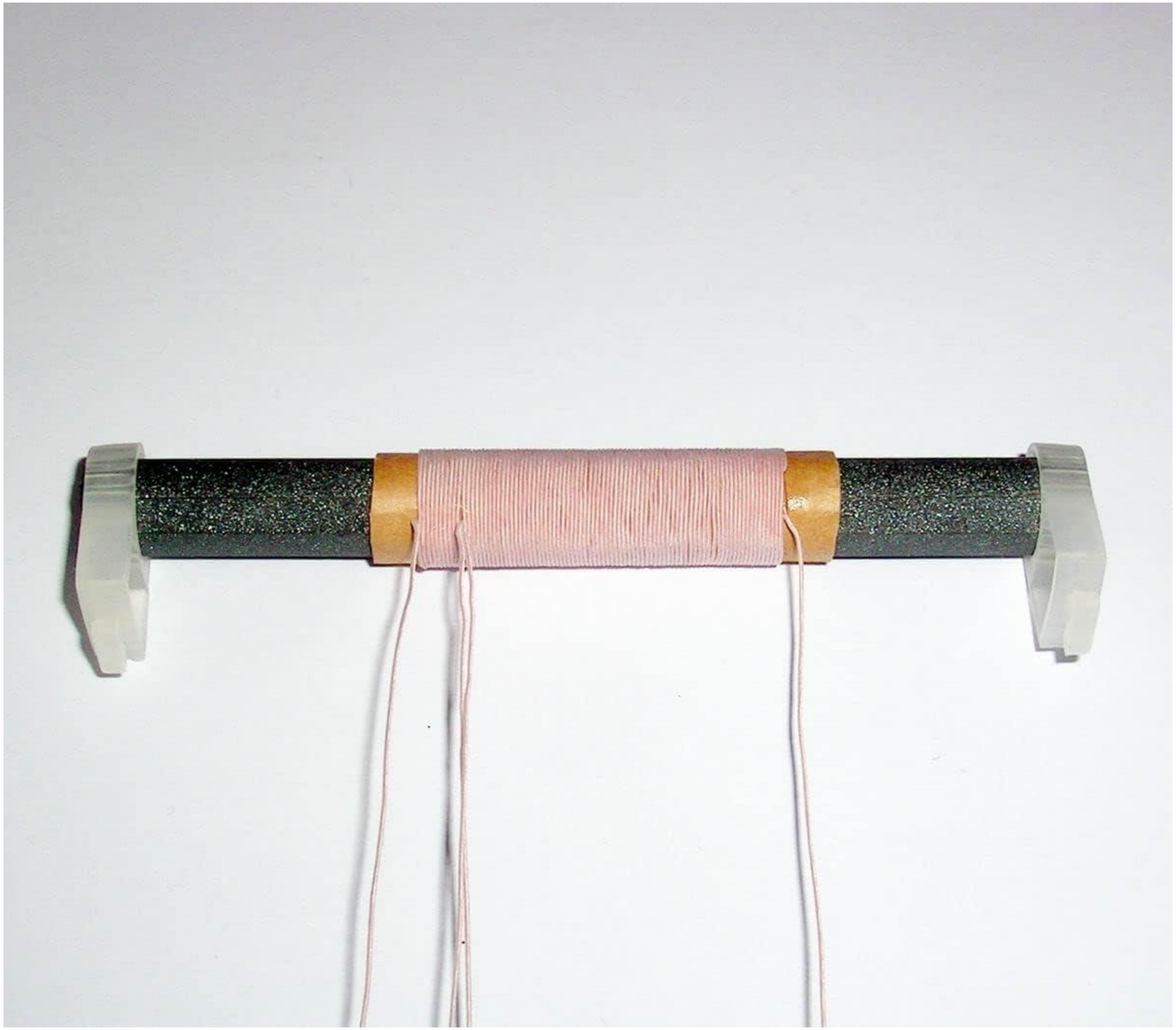
area

vacuum permeability

Relative permeability

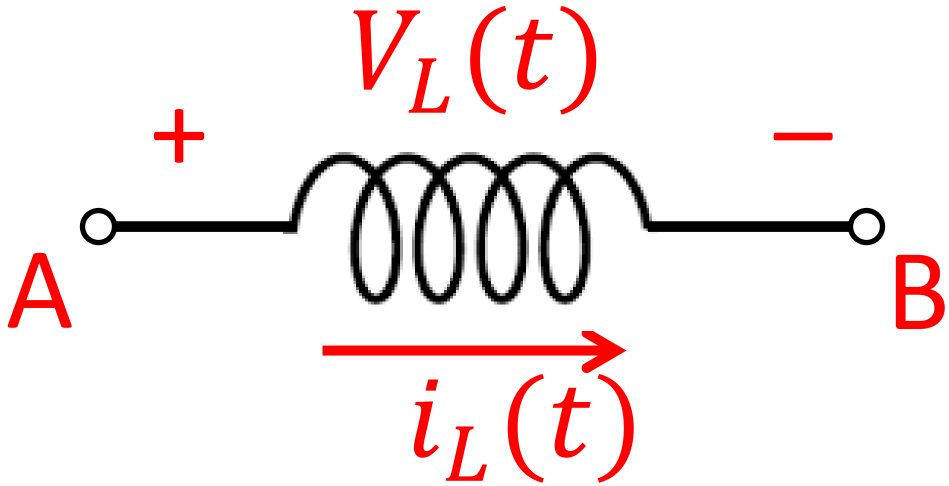
loops per unit length

inductance



$$L = N^2 \mu_r \, \mu_0 \, A \, \ell$$

Circuit symbol of an inductor



Relationship between current and voltage

$$V_L = L\frac{di_L(t)}{dt}$$

The unit of inductance is the *henry* with symbol [H]