ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 14

MWF – 12:00pm

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2062 ECE Building

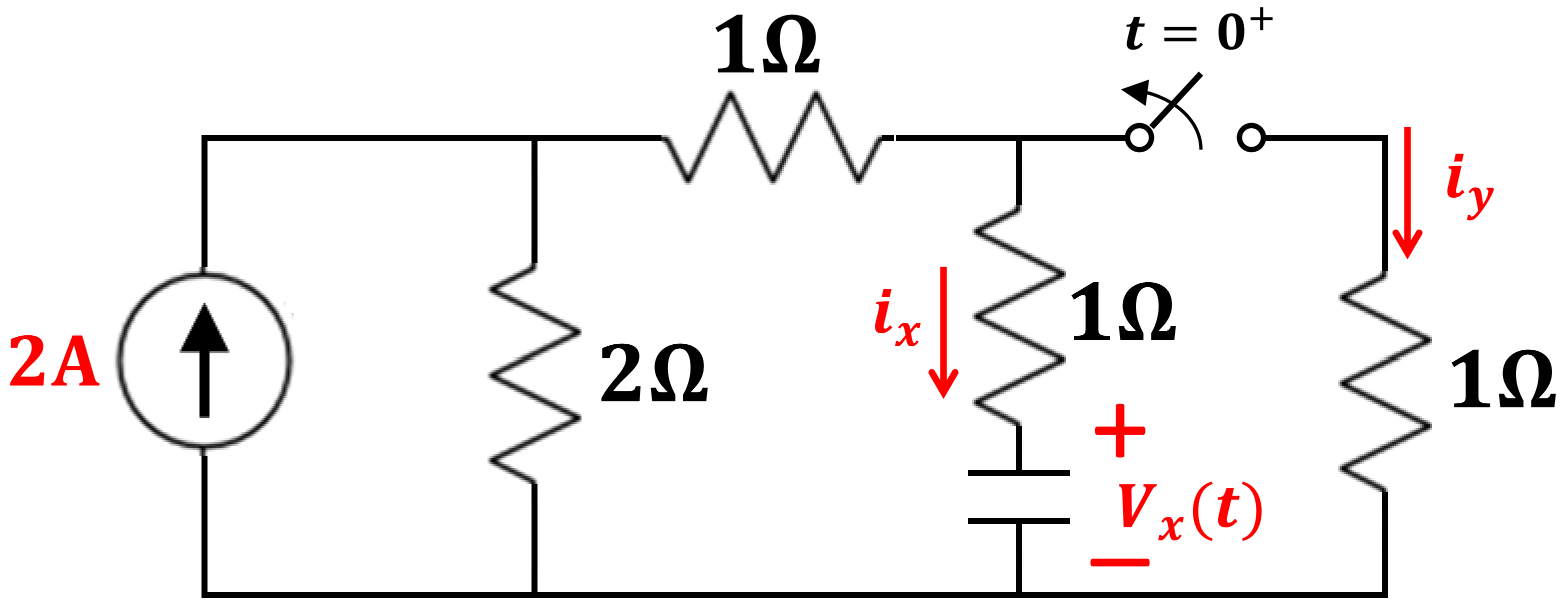
Lecture 14 – Summary

Learning Objectives

1. Inductance and RL circuits

Example 3

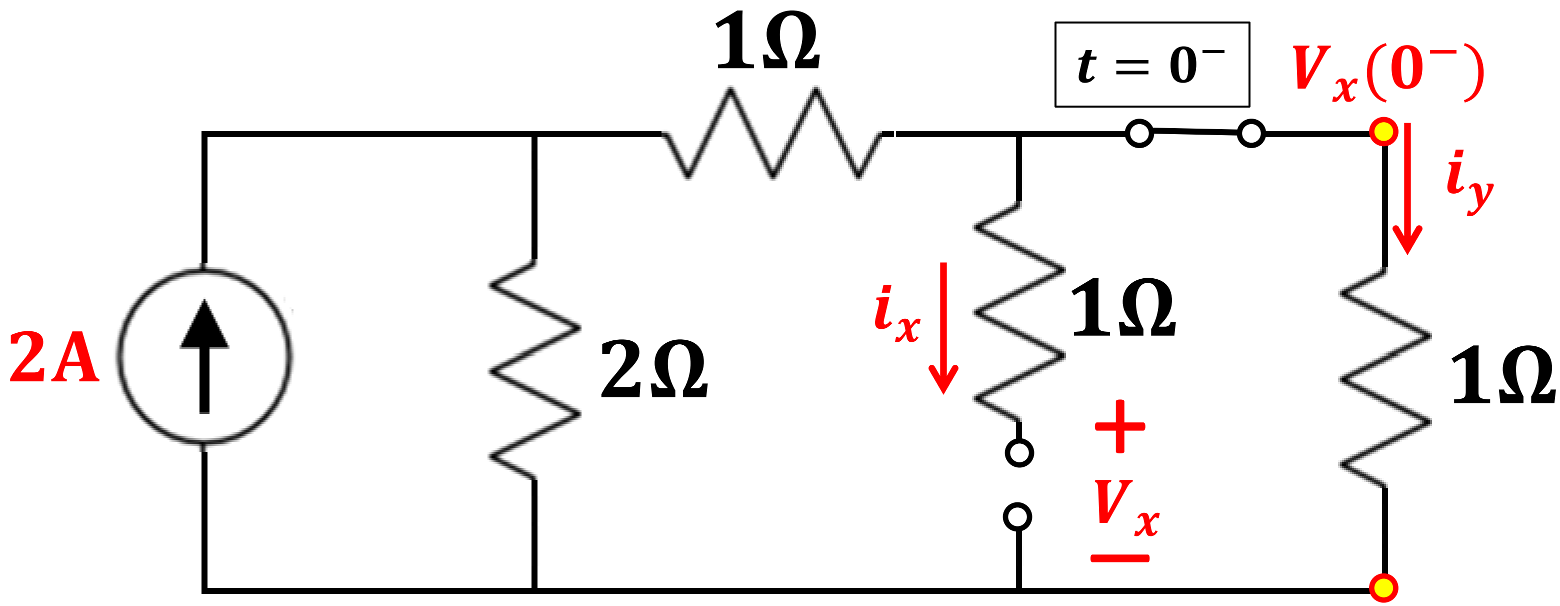
Switch opens at $t = 0^+$



Find ① $i_y(0^-)$ ③ $i_y(0^+)$ ⑤ $V_x(\infty)$

② $V_x(0^-)$ ④ $V_x(0^+)$

Example 3



Find ① $i_y(0^-)$ ② $V_x(0^-)$

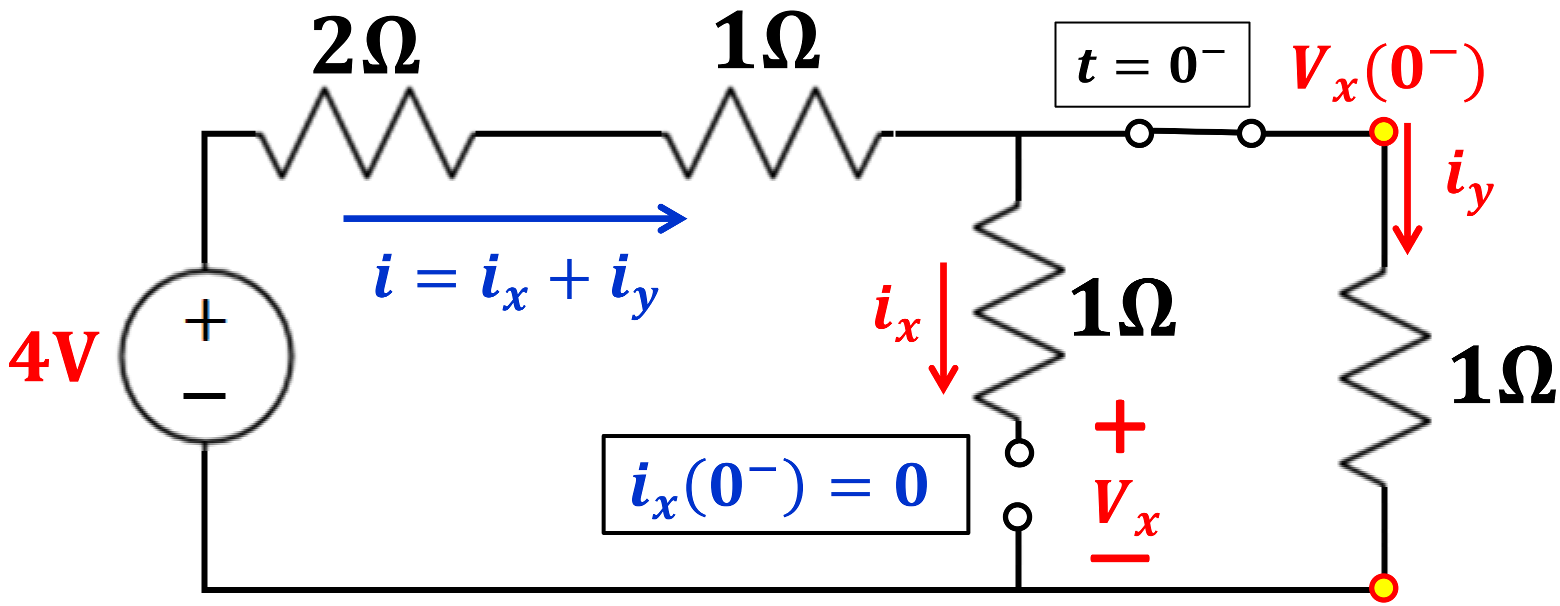
At $t = 0^-$ the capacitor is an open circuit and $i_x(0^-) = 0$.

Current Divider

$$i_y(0^-) = 2\text{A}\frac{2\Omega}{2\Omega + 2\Omega} = 1\text{A}$$

$$V_x(0^-) = i_y \times 1\Omega = 1\text{V}$$

Example 3 For practice, let's do the same with source transformation.



Find ① $i_y(0^-)$ ② $V_x(0^-)$

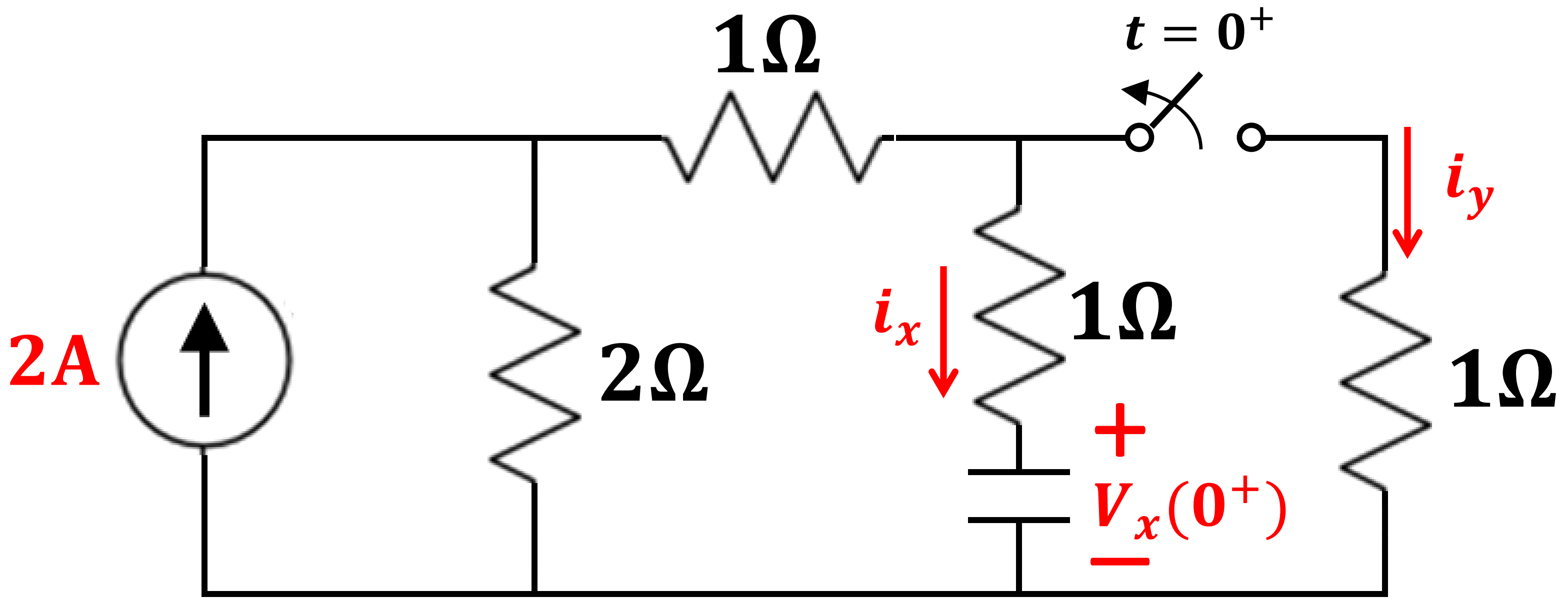
$$i = i_y(0^-) = \frac{4\text{V}}{2\Omega + 1\Omega + 1\Omega} = 1\text{A}$$

Voltage Divider

$$V_x(0^-) = 4 \times \frac{1\Omega}{4\Omega} = 1\text{V}$$

Example 3

Switch opens at $t = 0^+$



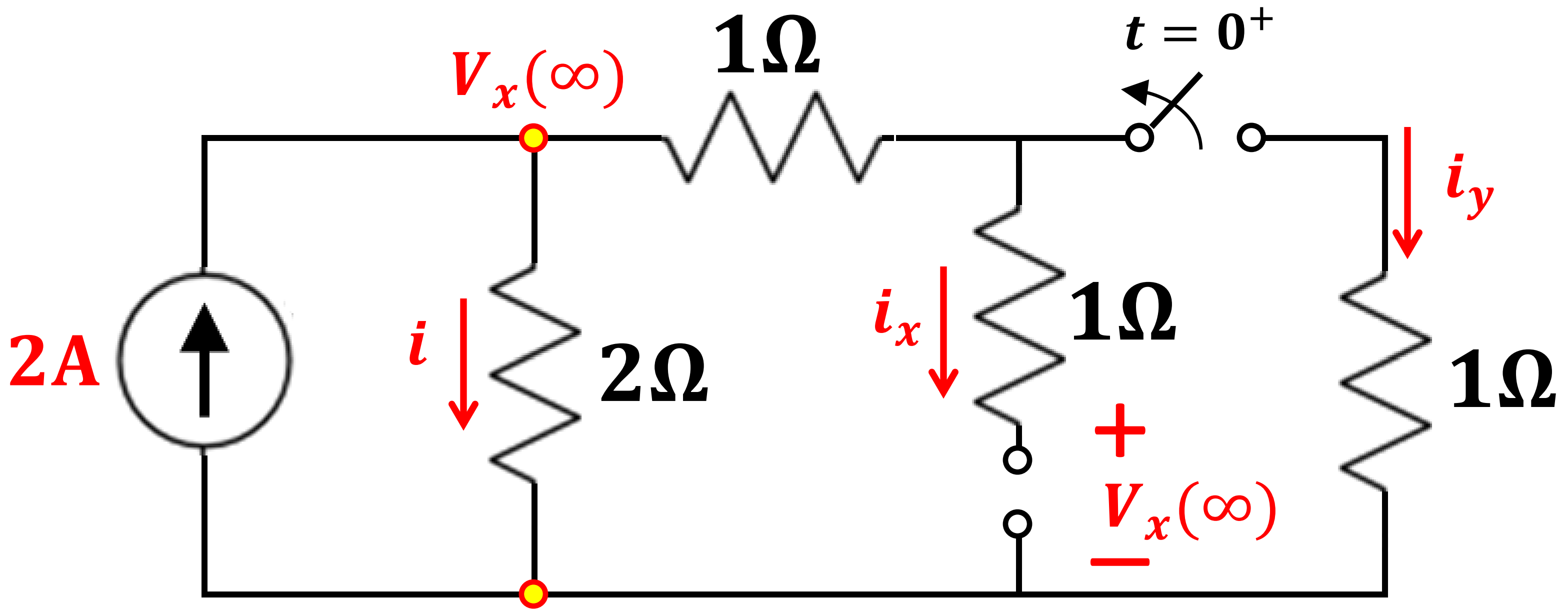
Find ③ $i_y(0^+)$ ④ $V_x(0^+)$

$\boxed{i_y(0^+) = 0}$ since the switch is open

$\boxed{V_x(0^-) = V_x(0^+) = 1V}$ voltage does not change

Example 3

Switch opens at $t = 0^+$



Find ⑤ $V_x(\infty)$

$$\boxed{i_x(\infty) = 0}$$

Capacitor is an open at $t \to \infty$

Capacitor has the same voltage as at the terminals of the 2Ω resistor through which flows the only current $i = 2\text{A}$

$$\boxed{V_x(\infty) = 2\text{A} \times 2\Omega = 4\text{V}}$$

Magnetic Inductance

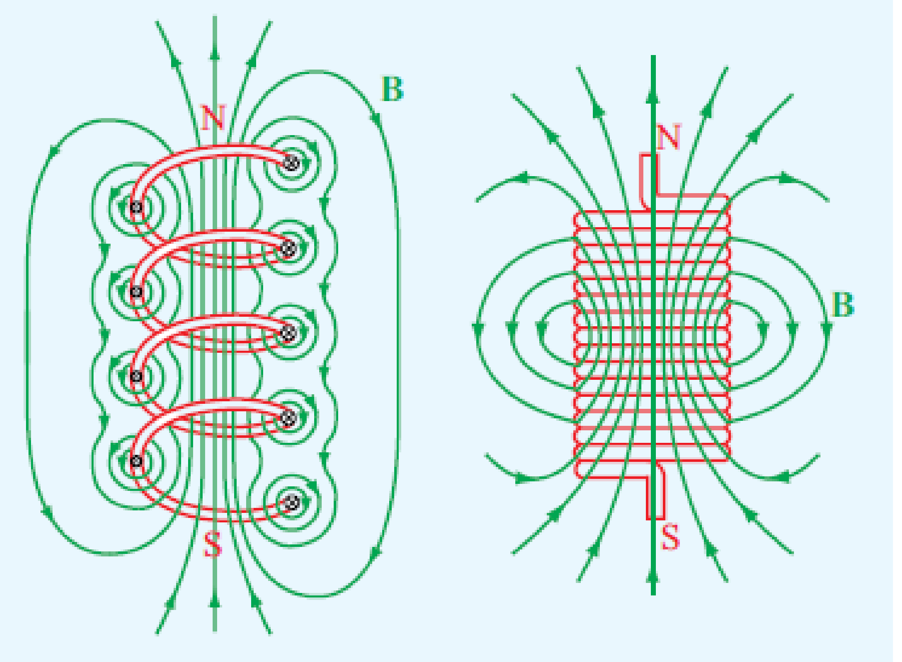
Current flowing in electric wires generates a magnetic field. When a change in current occurs, an "electromotive force" (voltage) is generated as a reaction, due to the change of the magnetic flux concatenated with the wire. The structure is said to "store" magnetic energy.

Inductance (quantified as the ratio between the magnetic field flux and the current) expresses the tendency of a conductor to oppose a change of the current flowing through it.

Inductors

All conductors carrying current exhibit inductance. The devices called inductors are designed to maximize the concatenated magnetic field and the associated storage of magnetic energy.

A coiled wire structure is called a "solenoid" and it is the most common way to realize an inductor.



(a) Loosely wound solenoid (b) Tightly wound solenoid

Inductance

The inductance value of a solenoid inductor depends on the number of wire loops in the coil, on the cross-sectional area, and on the "magnetic permeability" of the core region. Rods of high relative permeability material are often inserted in solenoids to amplify the local magnetic field and increase the inductance value of the device.

$$L = N^2 \mu_r \, \mu_0 \, A \, \ell$$

total length

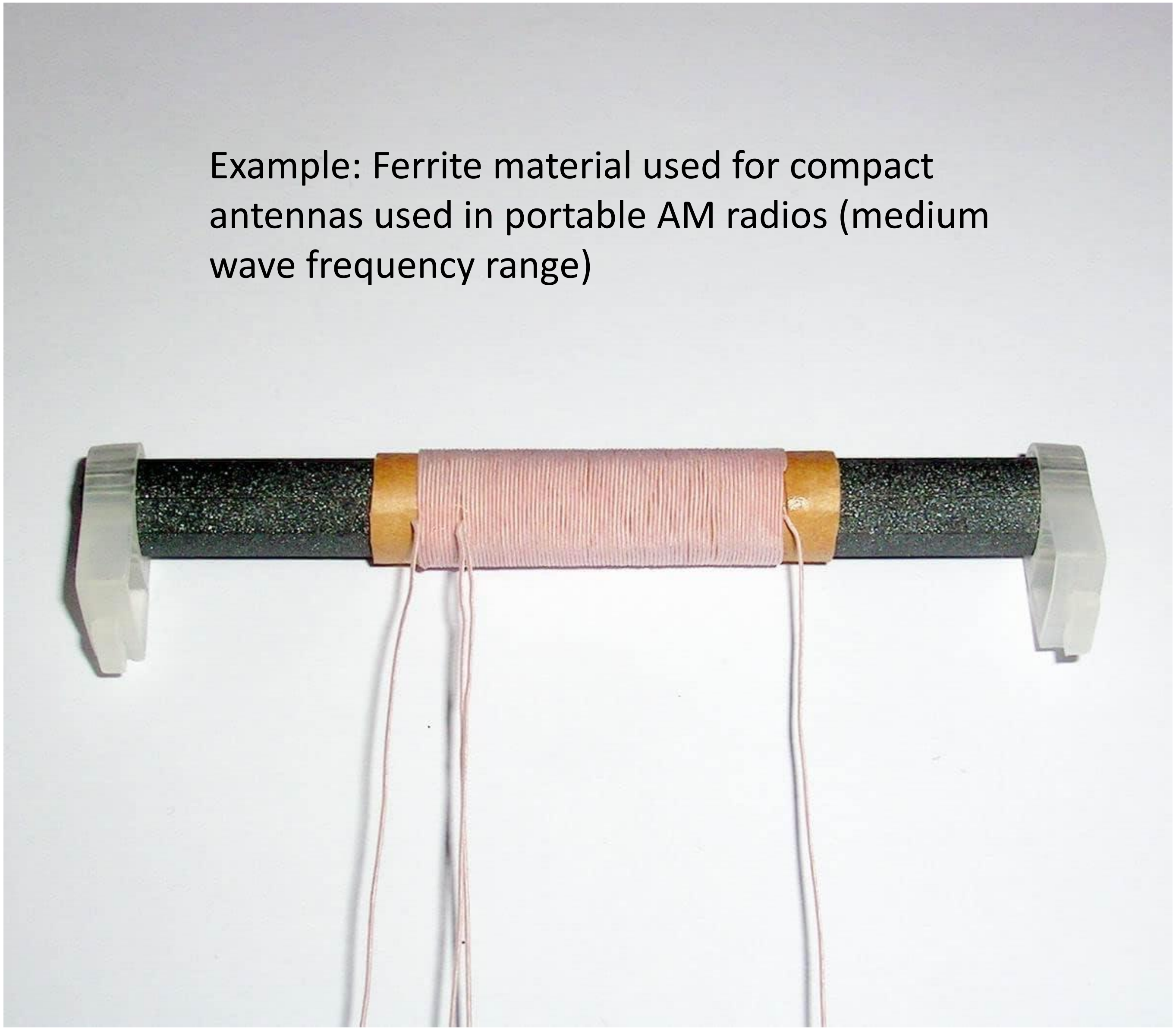
area

vacuum permeability

Relative permeability

loops per unit length

inductance



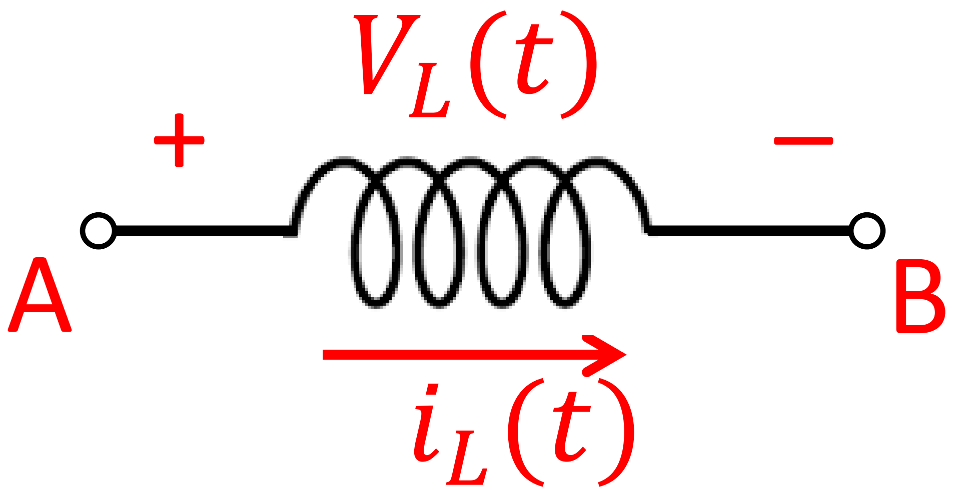
Adjustable inductor

$\approx \mathbf{60}\boldsymbol{nH}$

Knowles Johanson Manufacturing

$$L = N^2 \mu_r \, \mu_0 \, A \, \ell$$

Circuit symbol of an inductor



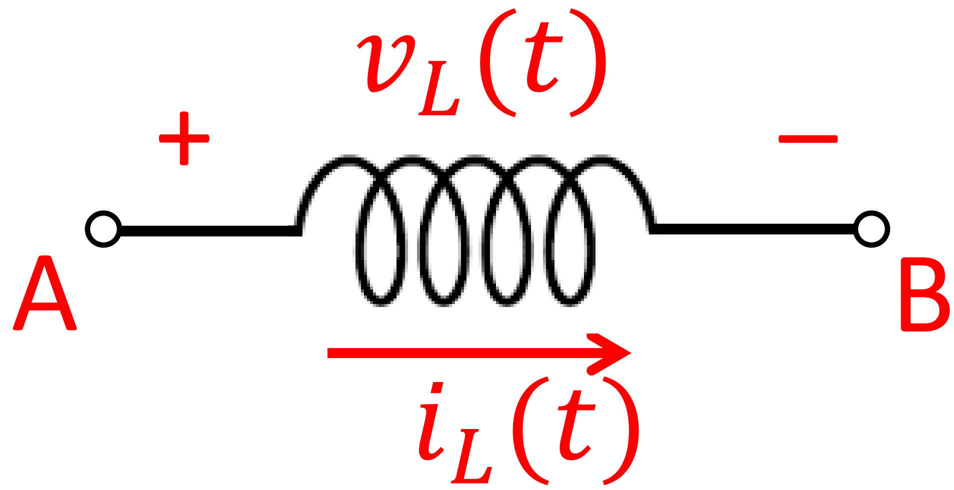
Relationship between current and voltage

$$V_L = L\frac{di_L(t)}{dt}$$

The unit of inductance is the *henry* with symbol [H]

Inductor time-behavior

Faraday's law describes the electrical behavior of an inductor: the emf potential in a current loop equals the time derivative of the loop magnetic flux $\boldsymbol{\Phi}$.



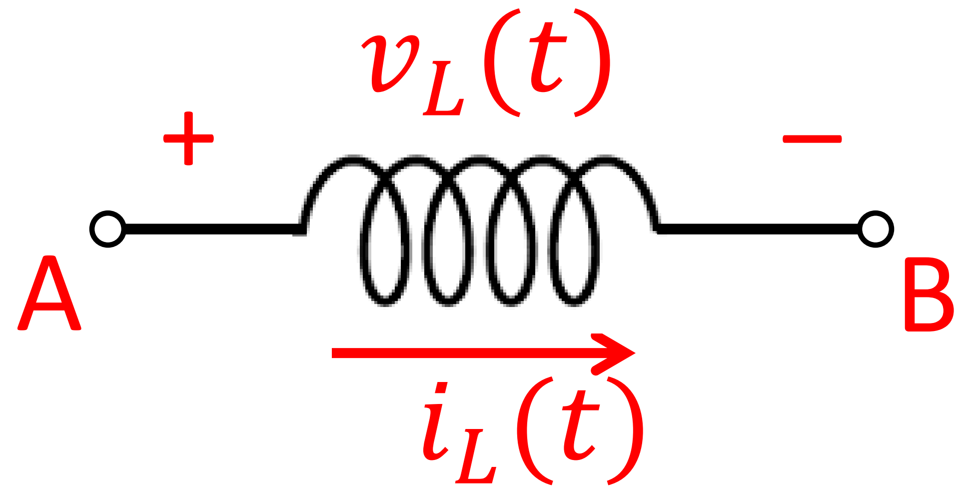
For an inductor

$$\Phi = L\, i$$

$$\boxed{v_L(t) = \frac{d}{dt} L i_L(t) = L \frac{d}{dt} i_L(t)}$$

Inductor time-behavior

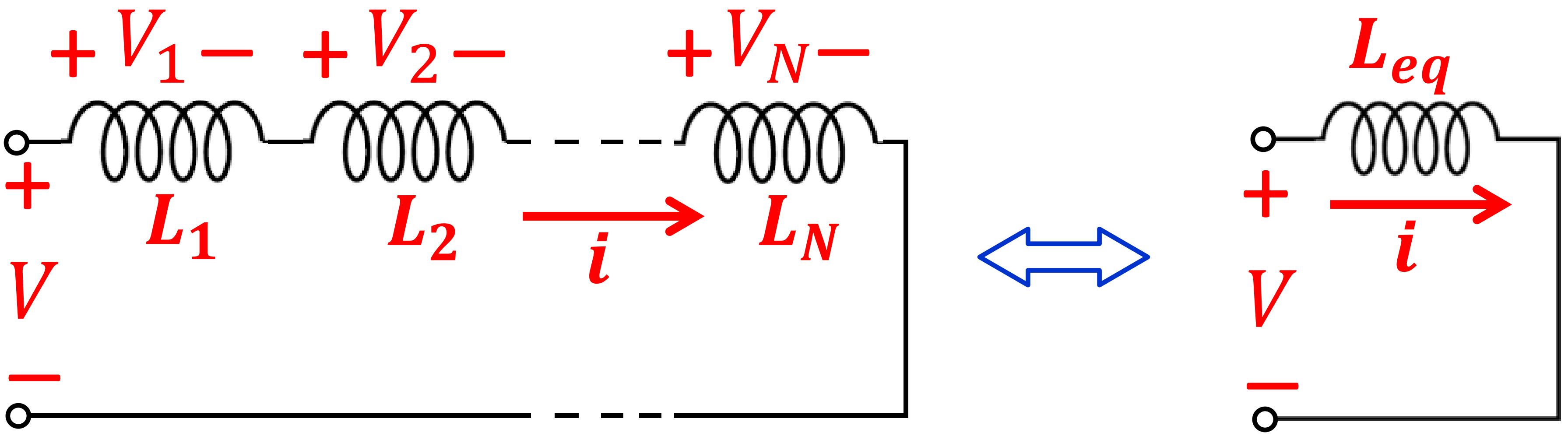
$$v_L(t) = L\frac{d}{dt}i_L(t)$$



Change of inductor current in time interval $[t_0, t]$

$$i_L(t) = i_L(t_0) + \frac{1}{L}\int_{t_0}^{t} v_L(t')\,dt'$$

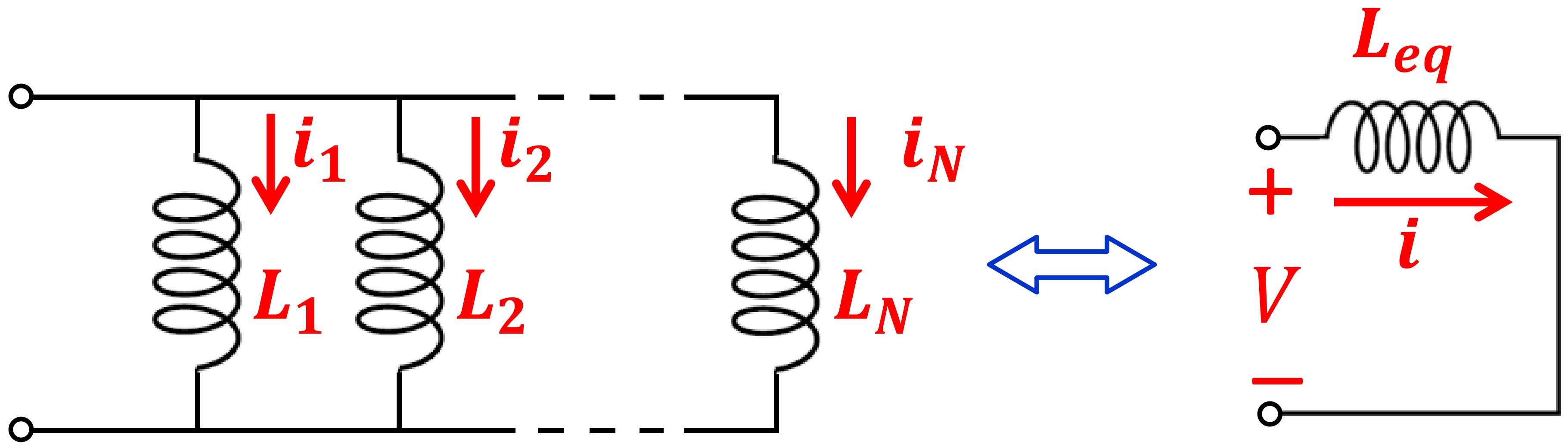
Series of N inductors



Equivalent Inductance

$$L_{eq} = L_1 + L_2 + L_3 + \cdots + L_N$$

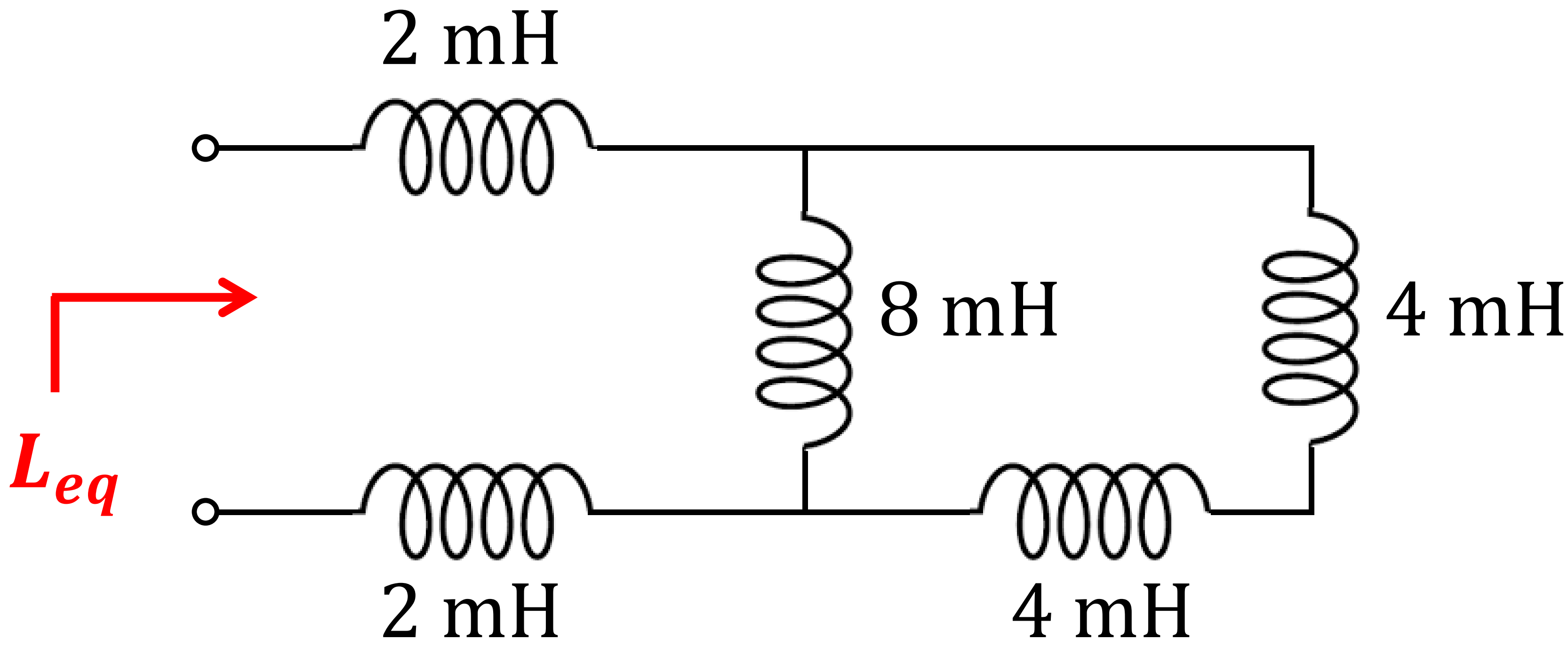
Parallel of N inductors



Equivalent Inductance

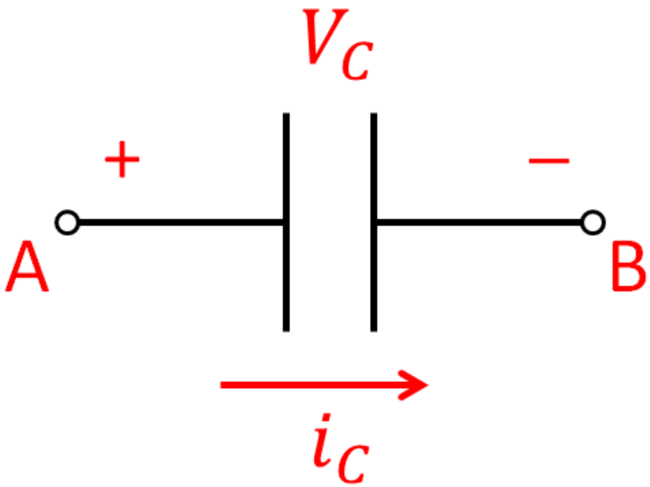
$$L_{eq} = \left[\frac{1}{L_1} + \frac{1}{L_2} + \cdots + \frac{1}{L_N}\right]^{-1}$$

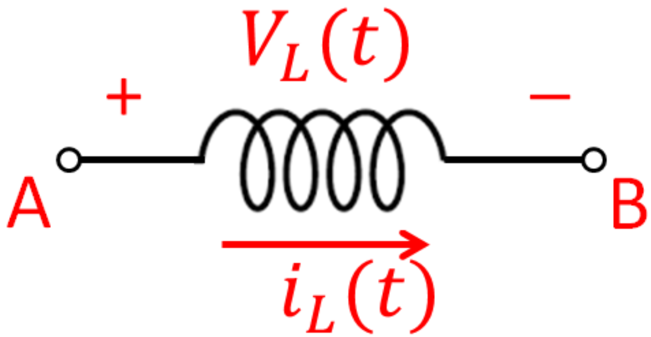
Example



$$L_{eq} = 8\ \mathrm{mH}$$

Duality between capacitor and inductor





$$i_C(t) = C\frac{dV_C(t)}{dt}$$

$$V_L(t) = L\frac{di_L(t)}{dt}$$

$$V_C(t = 0^-) = V_C(t = 0^+)$$

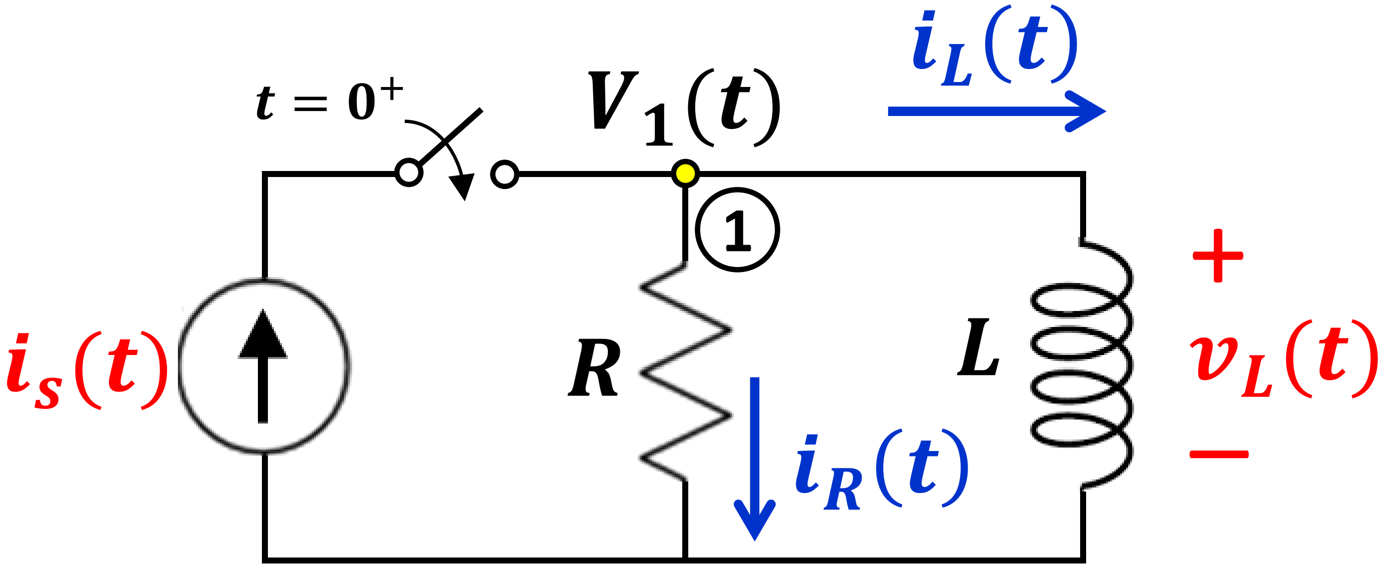
$$i_L(t = 0^-) = i_L(t = 0^+)$$

At steady-state capacitor acts like an open circuit

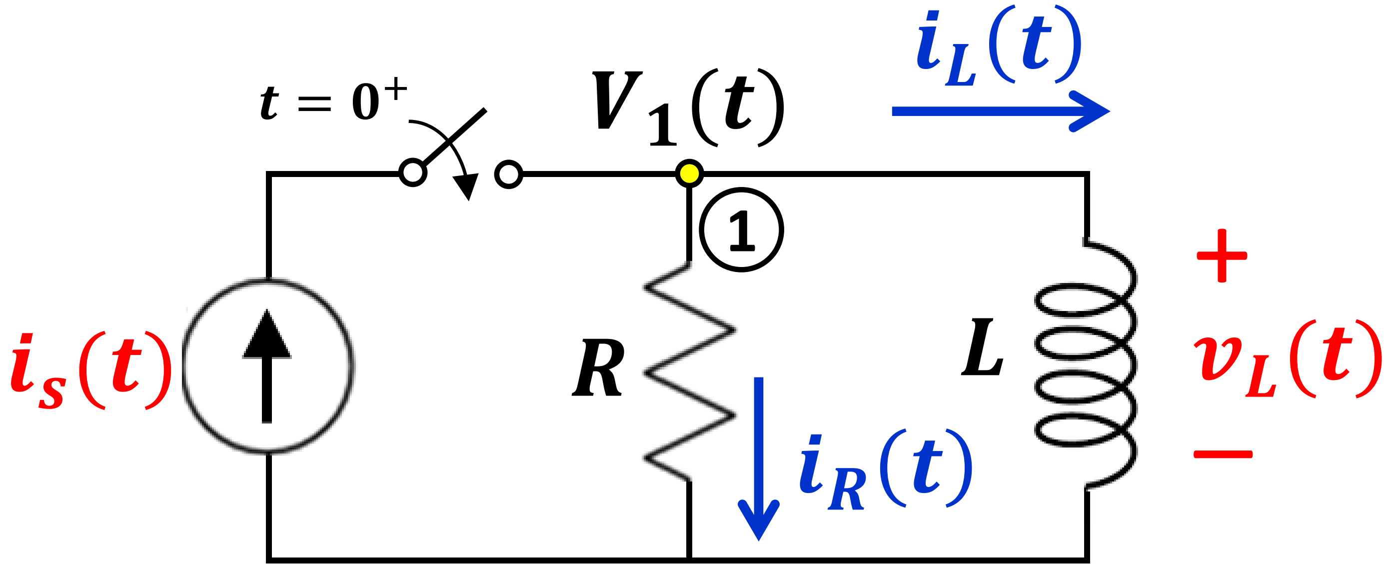
At steady-state inductor acts like a short circuit

RL Circuits – Transient Analysis

Response to “step input”

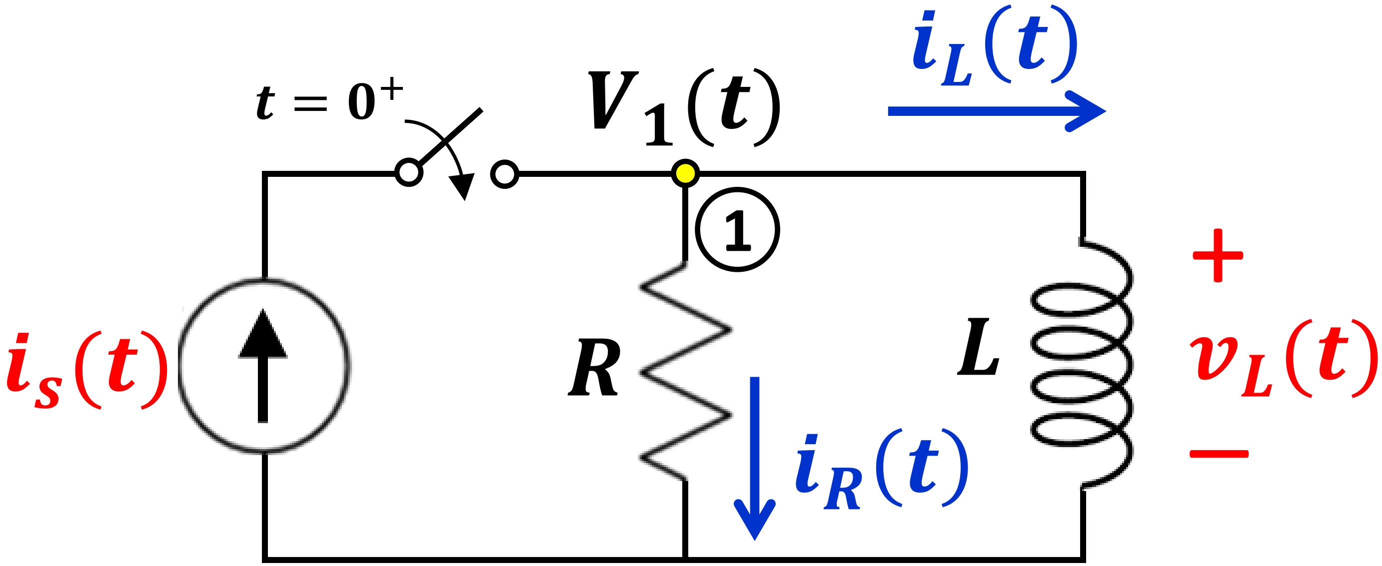


$t = 0^+$
$V_1(t)$
$i_L(t)$
1
$i_s(t)$
R
$i_R(t)$
L
+
$v_L(t)$
−



Write KCL at node 1

$$i_s(t) = i_R(t) + i_L(t) = \frac{V_1(t)}{R} + i_L(t)$$

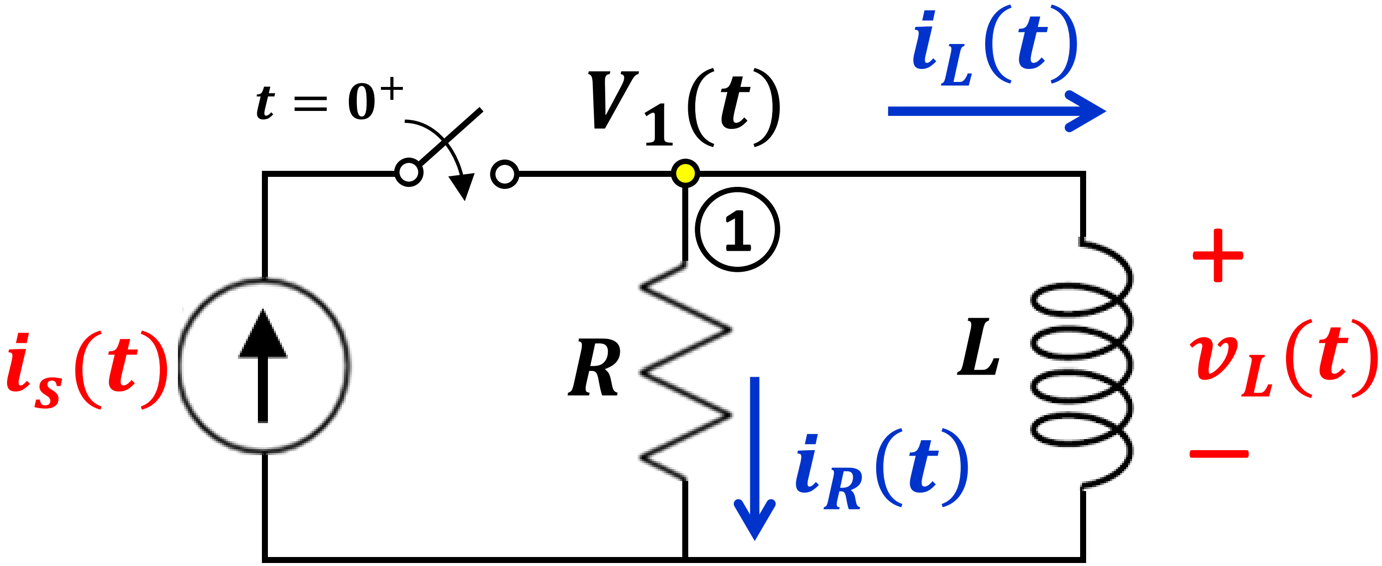


Write KCL at node 1

$$i_s(t) = i_R(t) + i_L(t) = \frac{V_1(t)}{R} + i_L(t)$$

$$V_1(t) = V_L(t) = L\frac{d}{dt}\,i_L(t)$$

$$i_S(t) = \frac{L}{R}\,\frac{d}{dt}i_L(t) + i_L(t)$$



$$\boxed{i_S(t) = \frac{L}{R}\frac{d}{dt}i_L(t) + i_L(t)}$$

$$\frac{d}{dt}i_L(t) + i_L(t)\frac{R}{L} = i_S(t)\frac{R}{L}$$

Equation 1

$$\frac{d}{dt} i_L(t) + i_L(t)\frac{R}{L} = i_S(t)\frac{R}{L}$$

$\boxed{\text{Equation 1}}$

This equation has the same mathematical form of the Ordinary Differential Equation for the voltage in a series RC circuit, considering a constant current source.

$$\frac{d}{dt} i_L(t) + i_L(t)\frac{R}{L} = i_S(t)\frac{R}{L}$$

Equation 1

This equation has the same mathematical form of the Ordinary Differential Equation for the voltage in a series RC circuit, considering a constant current source.

General Solution

$$\boxed{i_L(t) = K_1 e^{-\alpha t} + K_2 \quad [\mathrm{A}]}$$

$$\frac{d}{dt} i_L(t) + i_L(t)\frac{R}{L} = i_S(t)\frac{R}{L}$$

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General Solution

$$\boxed{i_L(t) = K_1 e^{-\alpha t} + K_2 \quad [\text{A}]}$$

$$i_L(t \to \infty) = K_2$$

$$i_L(t \to 0^+) = i_L(t \to 0^-) = K_1 + K_2$$

$$\frac{d}{dt} i_L(t) + i_L(t)\frac{R}{L} = i_S(t)\frac{R}{L}$$

Equation 1

This equation has the same mathematical form of the Ordinary Differential Equation for the voltage in a series RC circuit, considering a constant current source.

General Solution

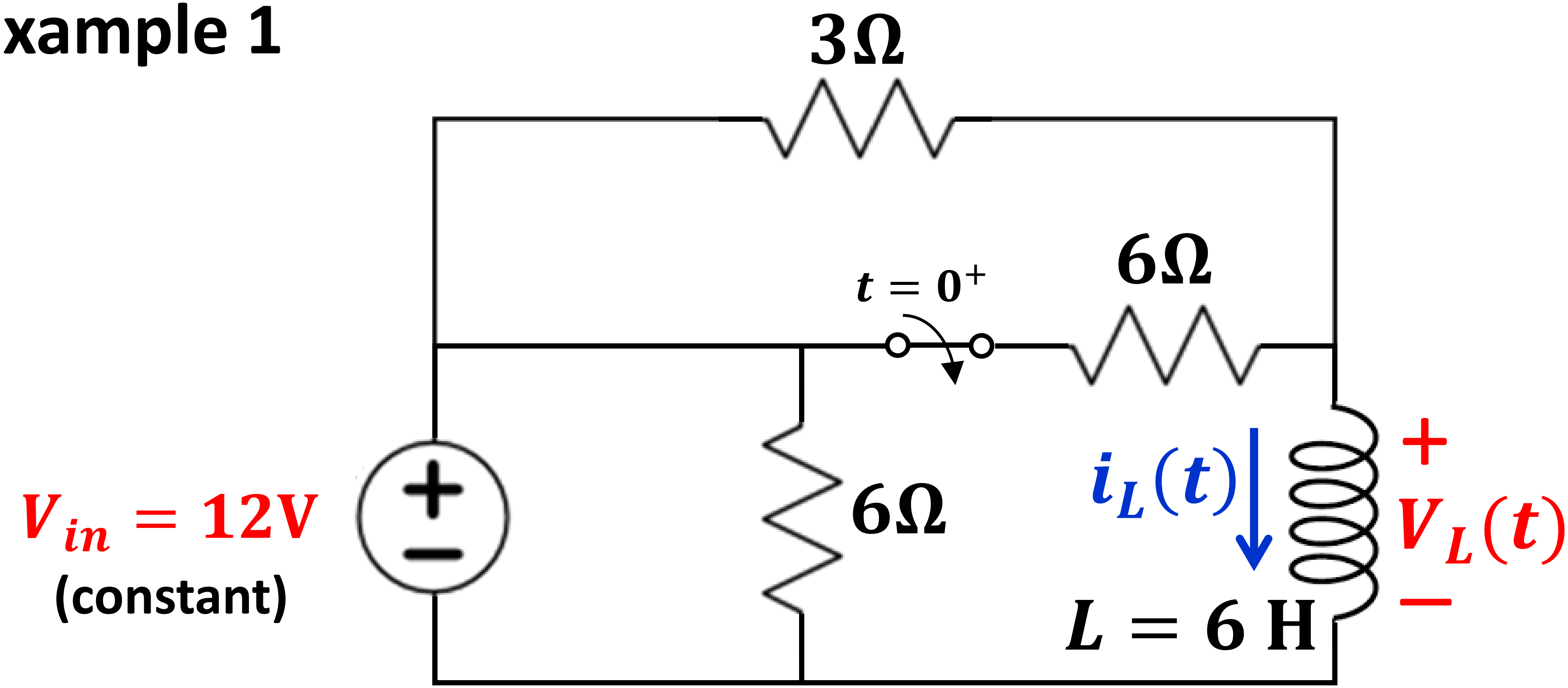
$$\boxed{i_L(t) = K_1 e^{-\alpha t} + K_2 \quad [\mathrm{A}]}$$

$$i_L(t \to \infty) = K_2$$

$$i_L(t \to 0^+) = i_L(t \to 0^-) = K_1 + K_2$$

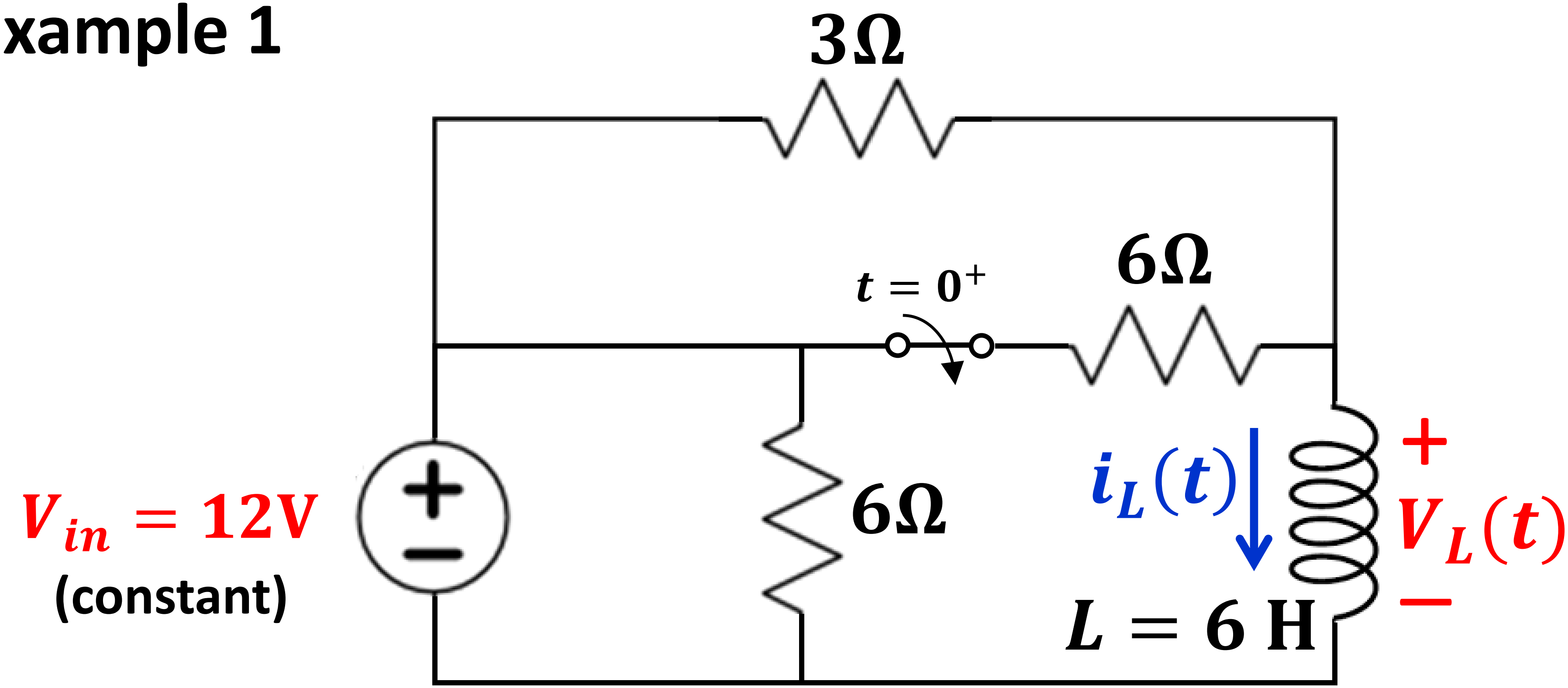
$$\tau = \frac{L}{R_{eq}} \qquad \alpha = \frac{1}{\tau} = \frac{R_{eq}}{L}$$

Example 1



Find $i_L(t)$

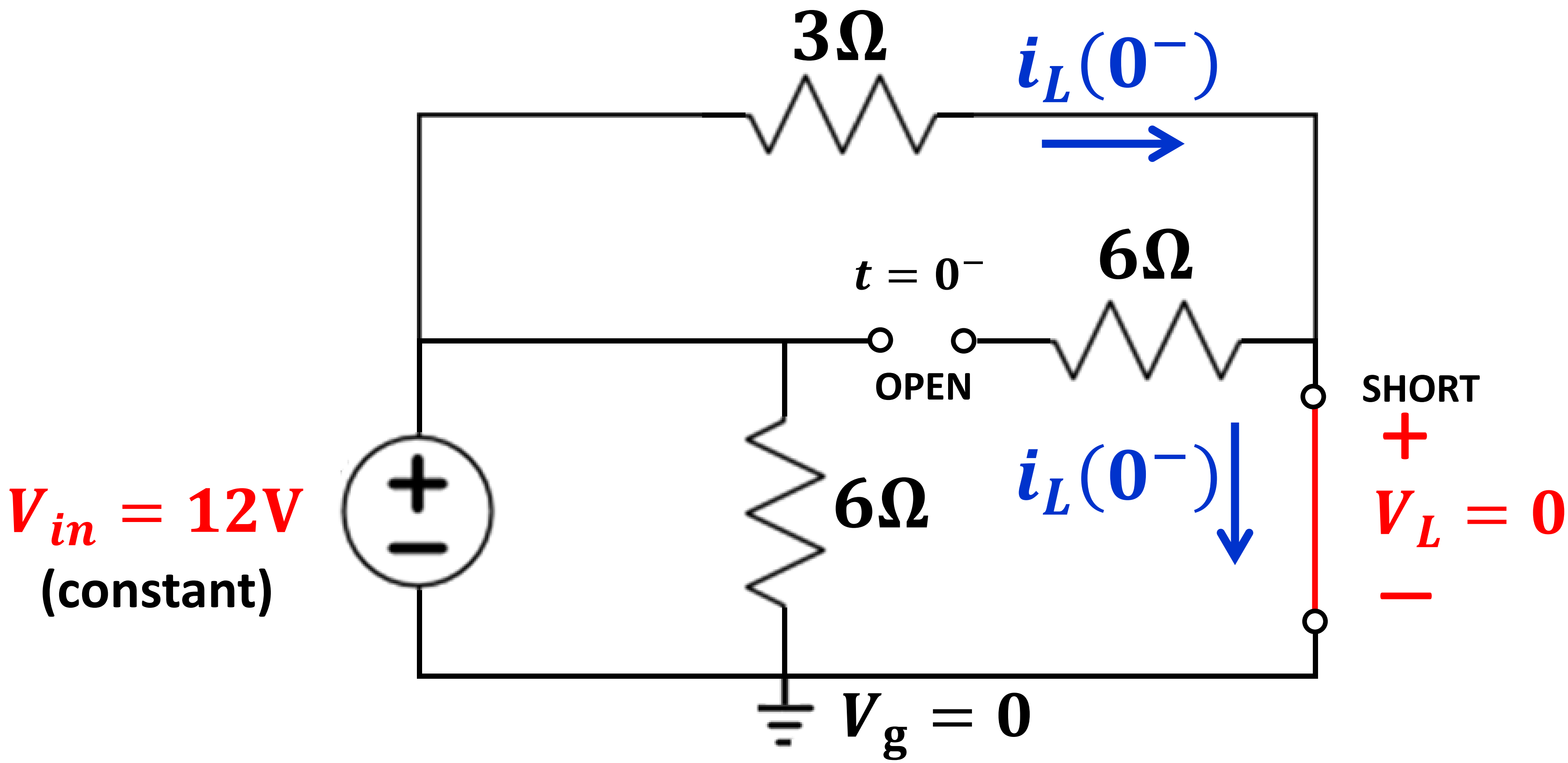
Example 1



Find $i_L(t)$

Switch closes at $t = 0^+$

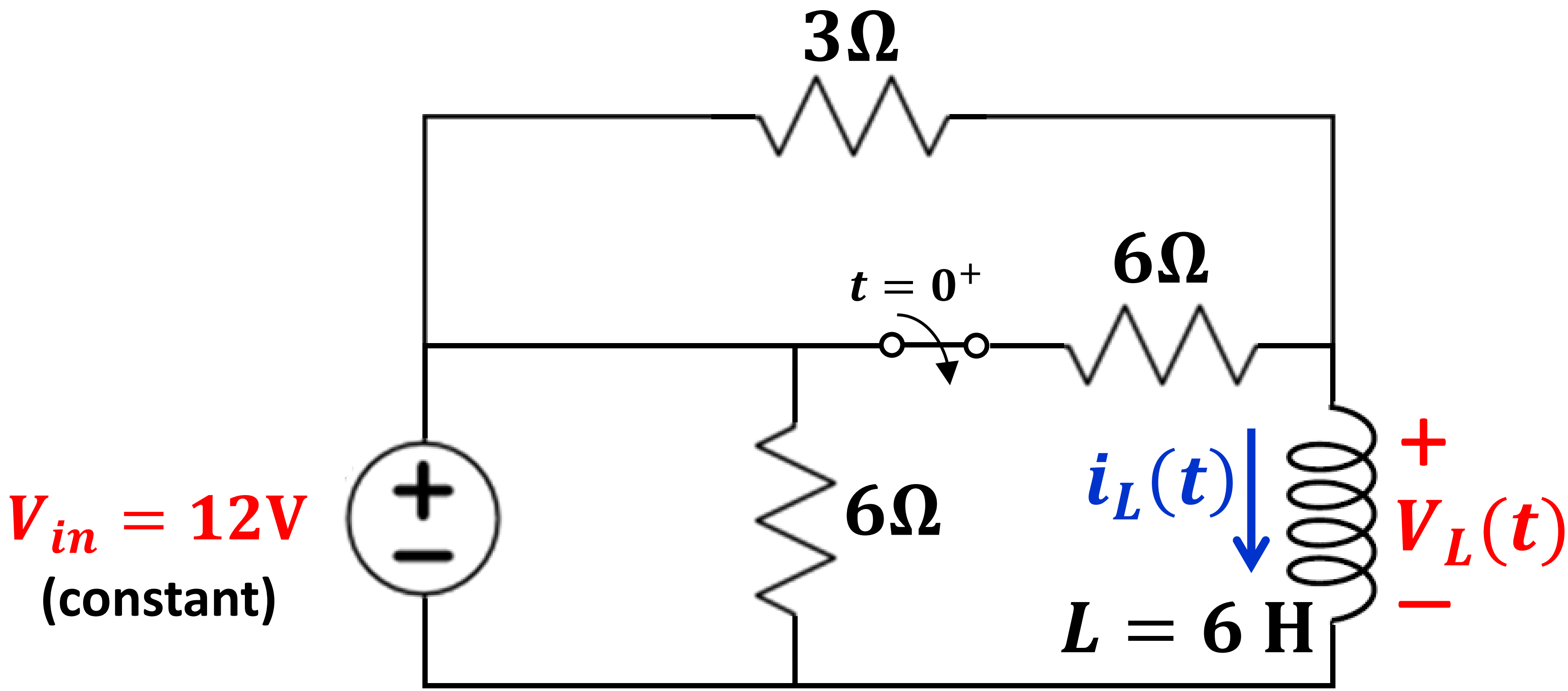
$$\boxed{i_L(t) = K_1 e^{-\alpha t} + K_2 \quad [\text{A}]}$$



Step (1) Find $i_L(0^-)$ and $V_L(0^-)$ before the switch is closed

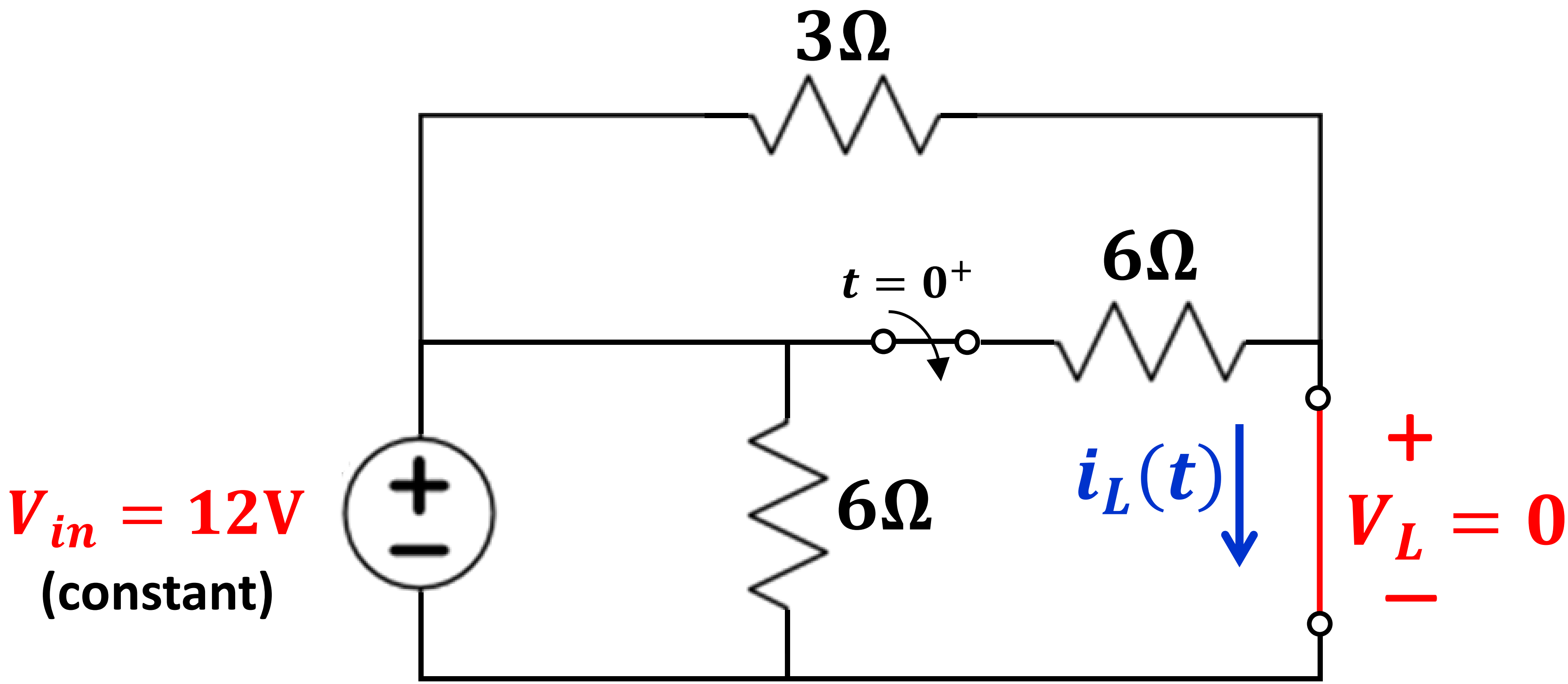
$$i_L(0^-) = \frac{12 - 0}{3} = 4\text{ A}$$

$$V_L(0^-) = 0\text{ V}$$



Step 2 Find $i_L(0^+)$

$$\boxed{i_L(0^+) = i_L(0^-) = K_1 + K_2 = 4\text{ A}}$$



The inductor acts as a short circuit

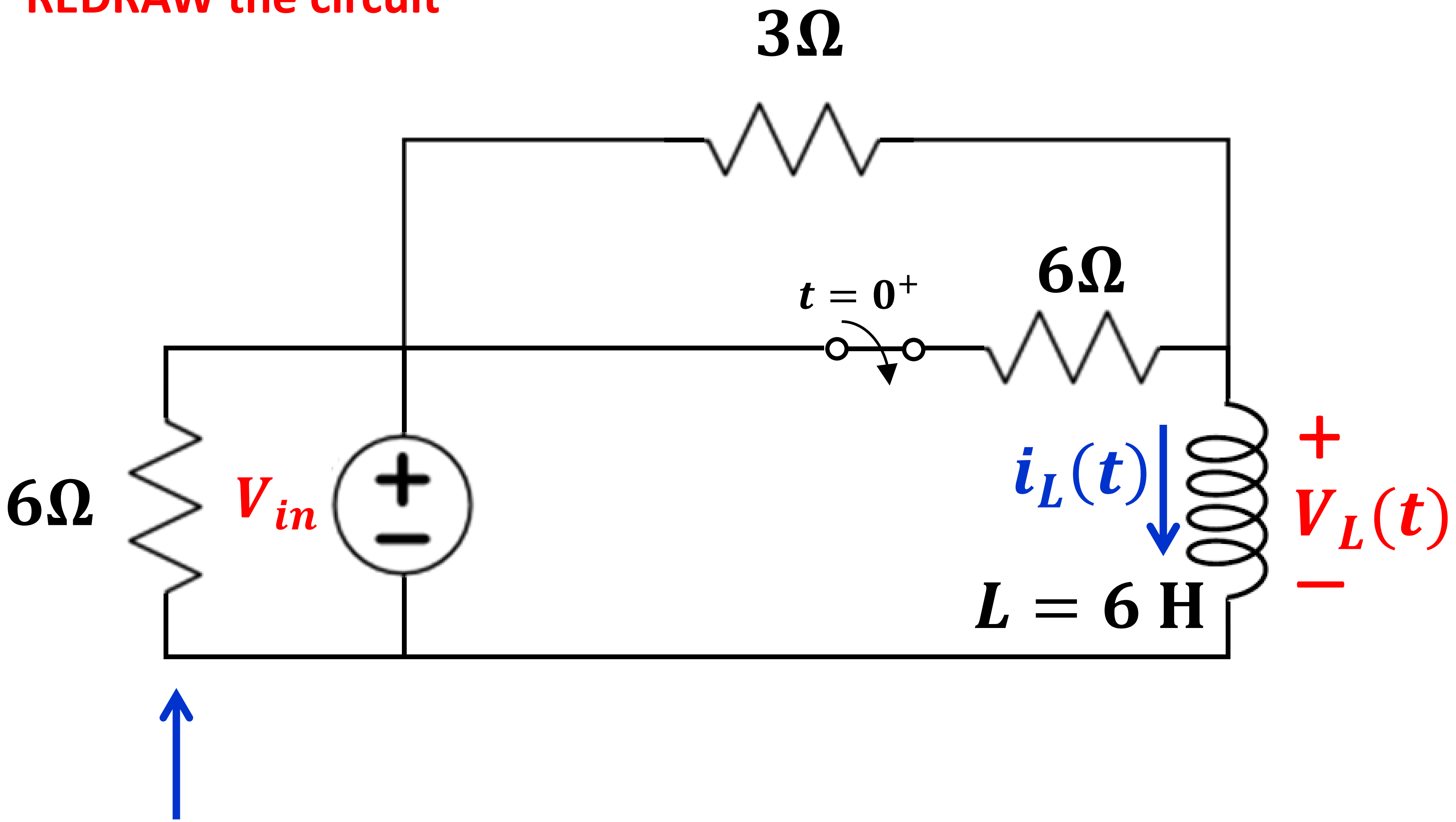
Step 3 Find $i_L(\infty)$

$$\boxed{i_L(\infty) = K_2 = V_{in}/(3\Omega//6\Omega) = 12/2 = 6\text{ A}}$$

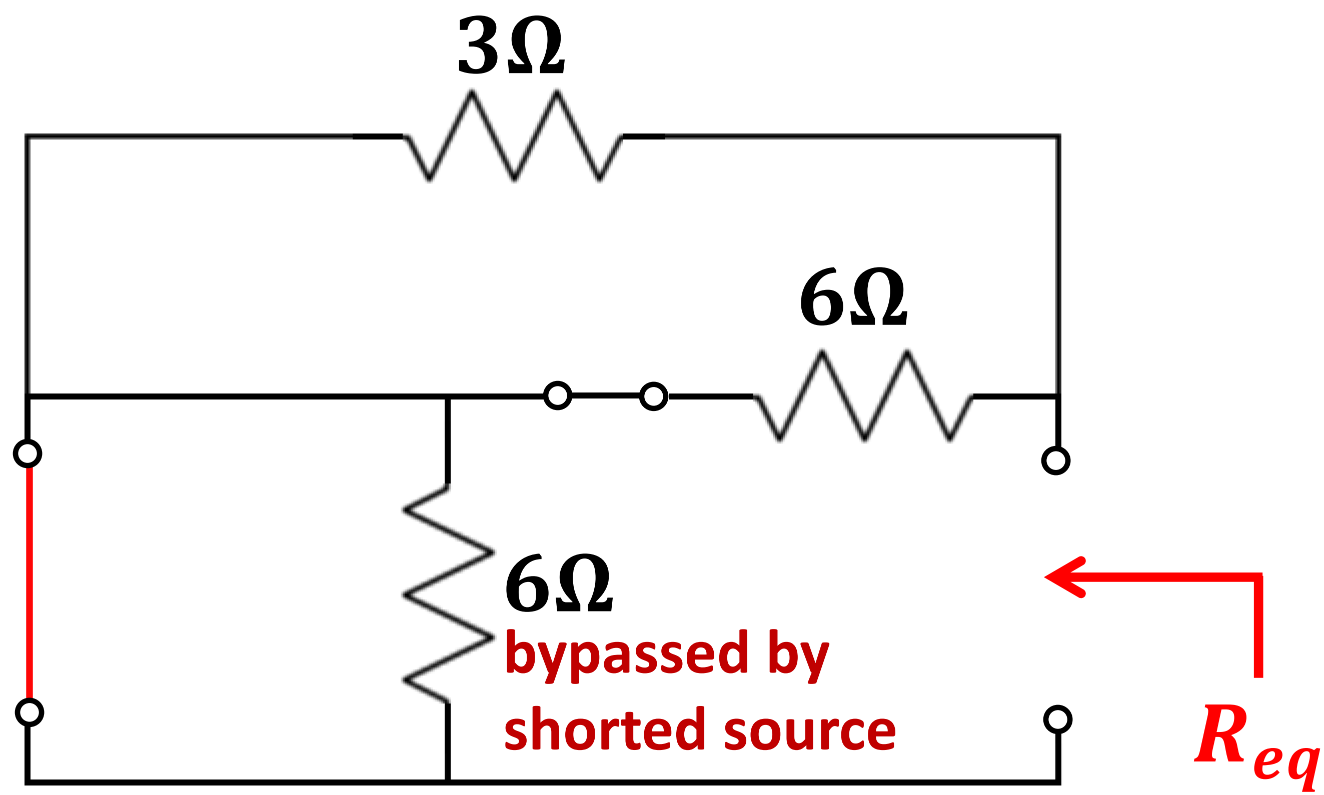
$$\boxed{i_L(0^+) = i_L(0^-) = K_1 + K_2 = 4\text{ A}}$$

→ $\boxed{K_2 = 6\text{ A}}$ $\boxed{K_1 = -2\text{ A}}$

REDRAW the circuit



NOTE: This resistor does not affect the rest of the circuit because it is in parallel with the voltage source.



Step ④ Find α

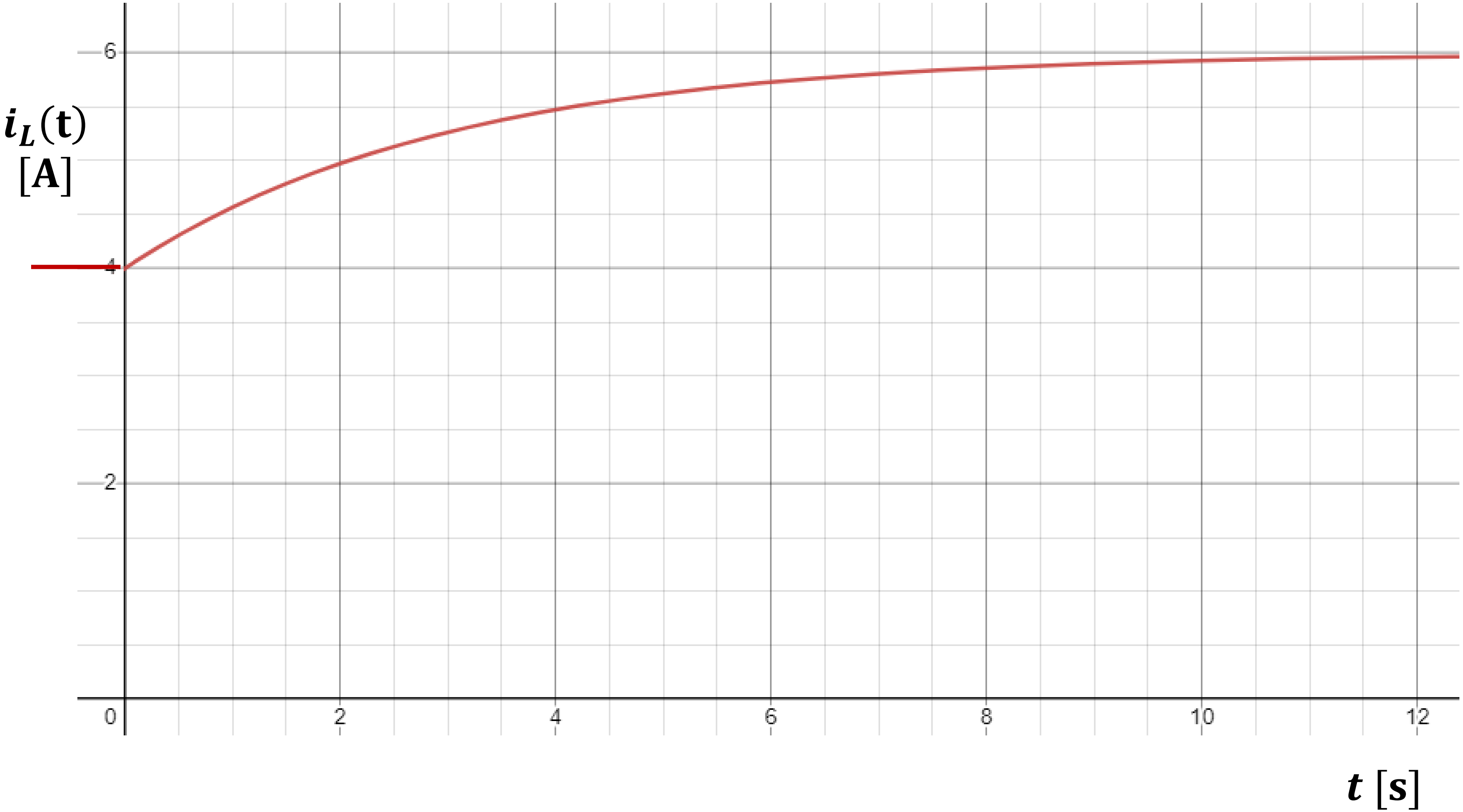
$$\boxed{R_{eq} = 3\Omega // 6\Omega = 2\ \mathrm{A}}$$

$$\boxed{\alpha = \frac{R_{eq}}{L} = \frac{2}{6} = \frac{1}{3}\mathrm{s}^{-1}}$$

Step ⑤ Find $i_L(t)$

$$\boxed{i_L(t) = -2e^{-t/3} + 6\ [\mathrm{A}]}$$

$$\boxed{i_L(t) = -2e^{-t/3} + 6\ [\text{A}]}$$



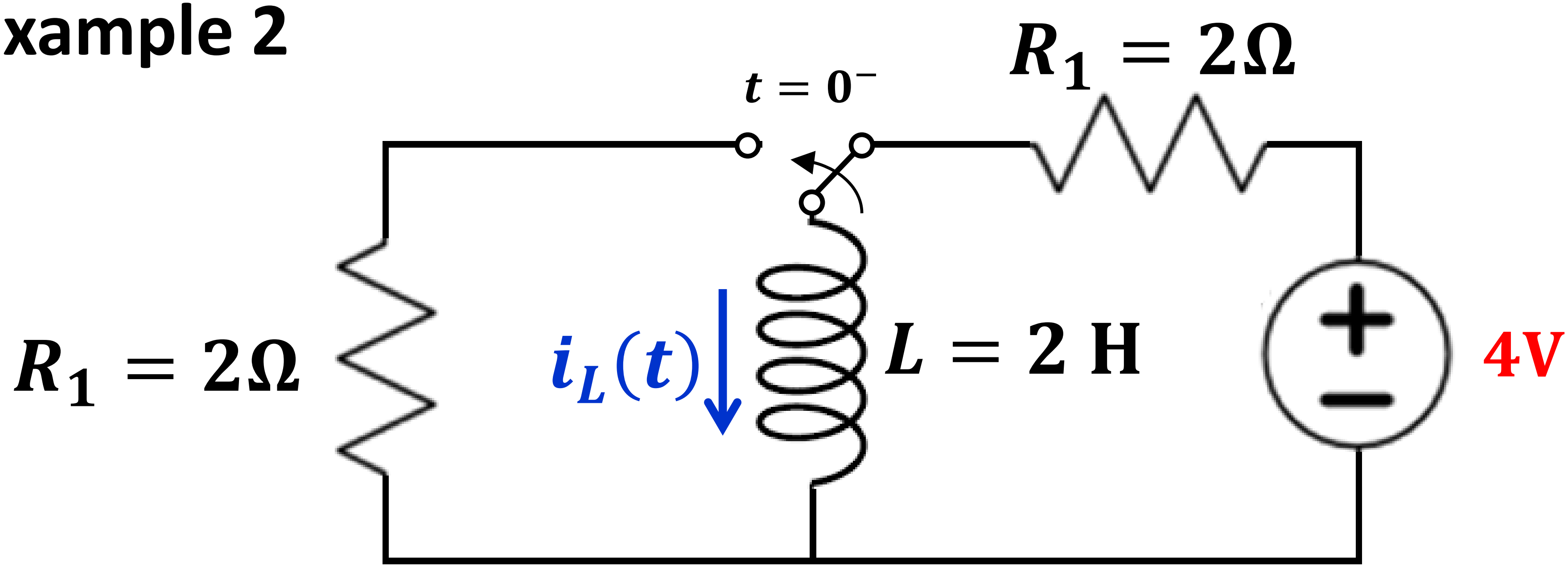
$$\boxed{i_L(t) = -2e^{-t/3} + 6\ [\mathrm{A}]}$$

$$L = 6\ \mathrm{H}$$

Find the inductor voltage

$$\boxed{V_L(t) = L\,\frac{d}{dt}i_L(t) = 6 \times \frac{2}{3}e^{-t/3}\ [\mathrm{V}]}$$

Example 2

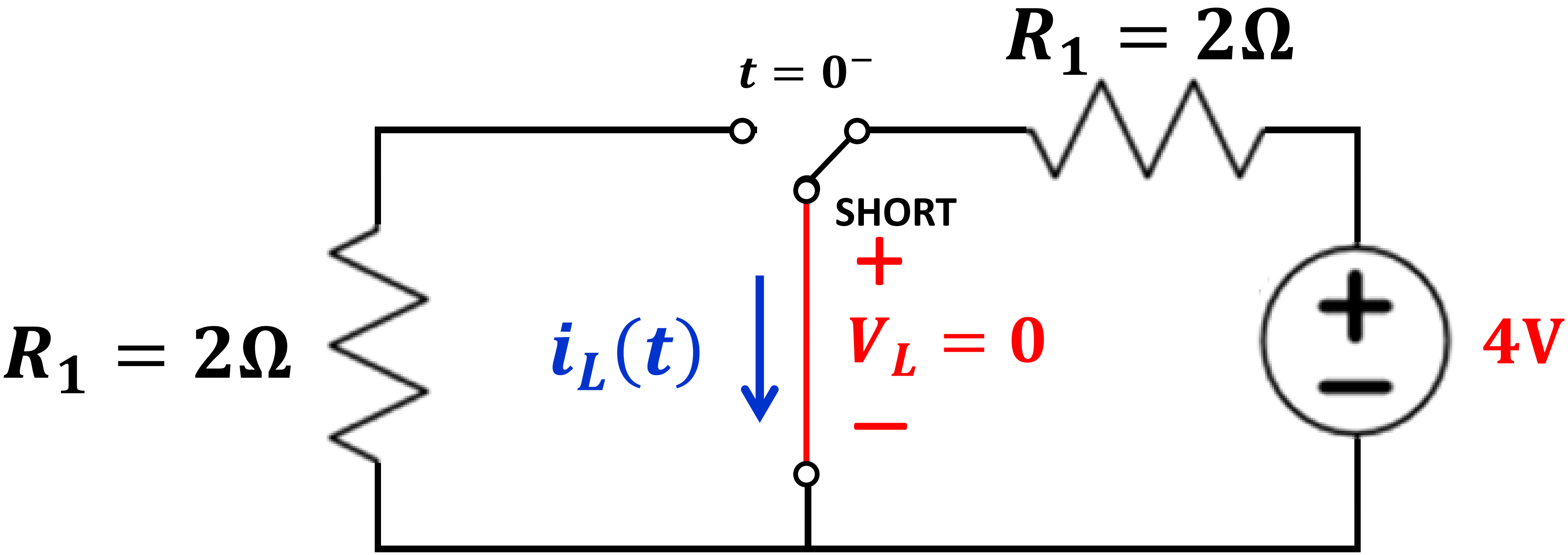


Find $i_L(t)$

Switch moves to the left position at $t = 0^+$

$$\boxed{i_L(t) = K_1 e^{-\alpha t} + K_2 \quad [A]}$$

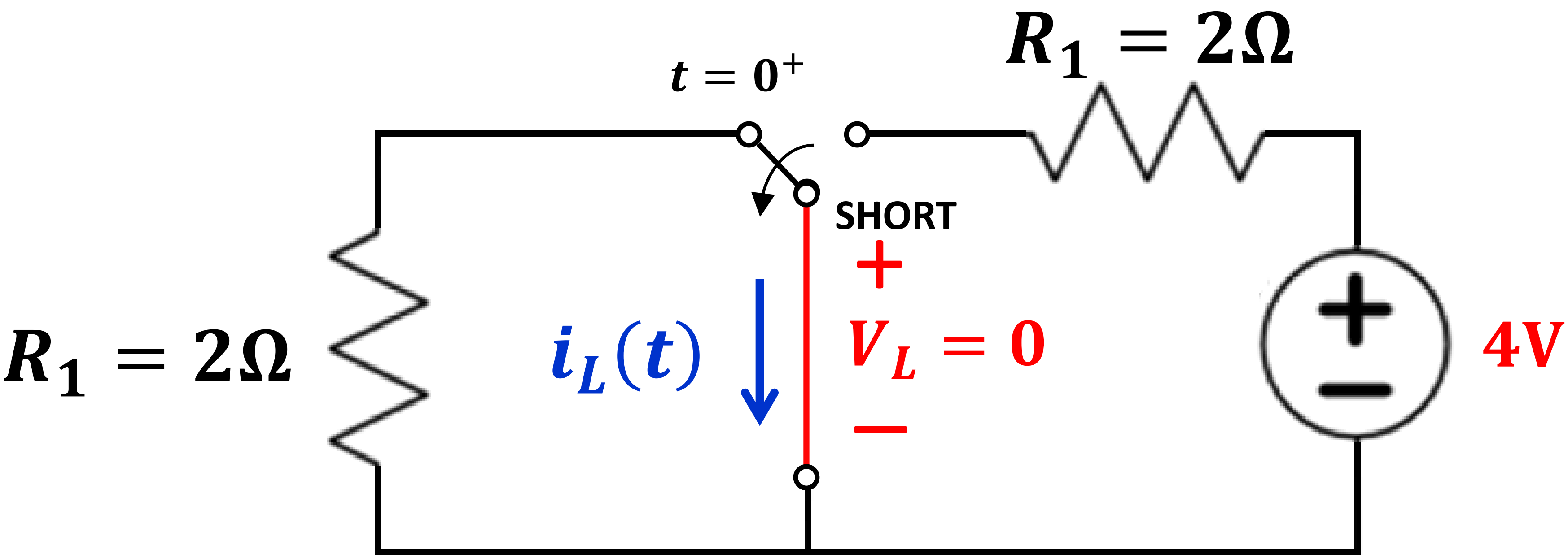
(constant DC source)



Step 1 **Find $i_L(0^-)$ and $V_L(0^-)$ before the switch is closed**

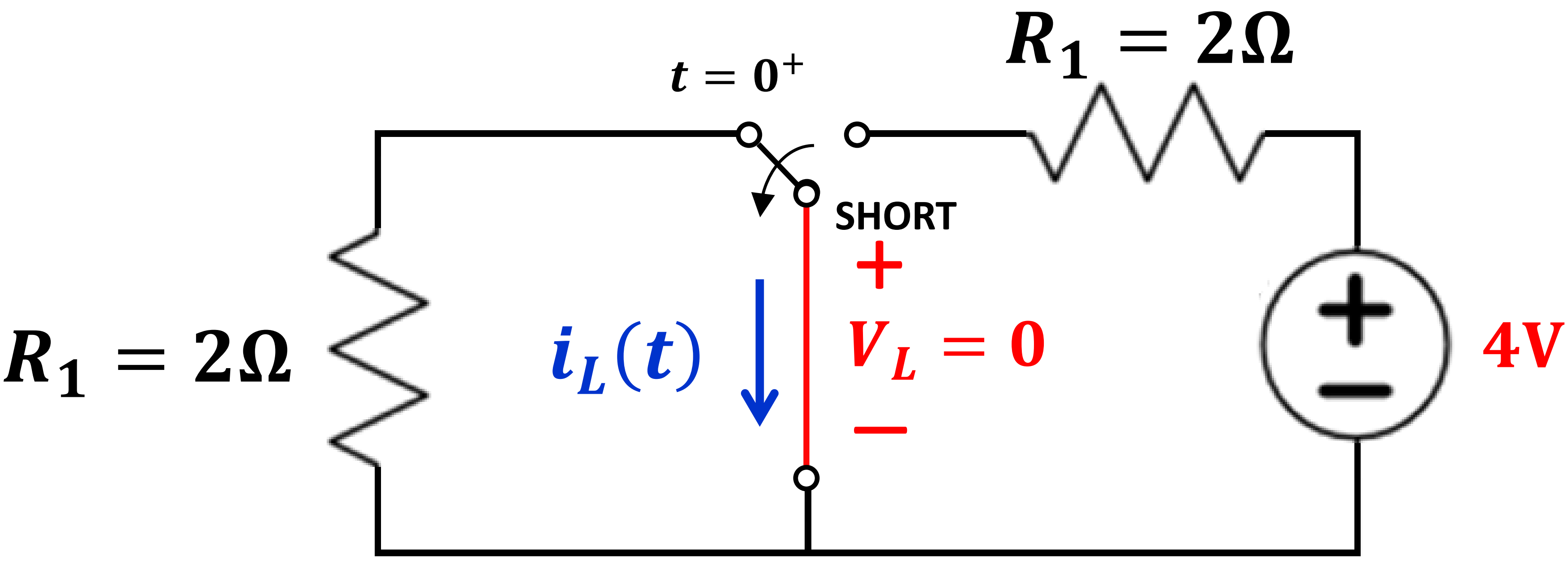
$$i_L(0^-) = \frac{4\text{V}}{2\Omega} = 2\ [\text{A}]$$

$$V_L(0^-) = 0\ \text{V}$$



Step ② Find $i_L(0^+)$

$$\boxed{i_L(0^+) = i_L(0^-) = K_1 + K_2 = 2\text{ A}}$$



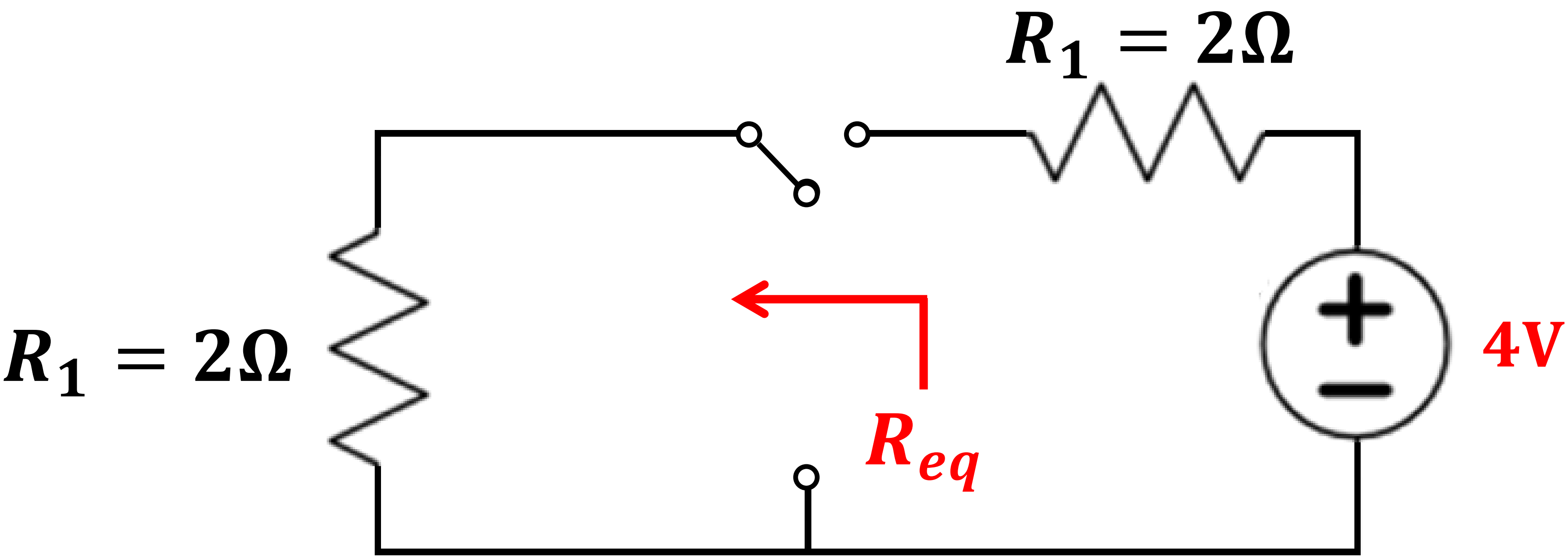
Step ② Find $i_L(0^+)$

$$\boxed{i_L(0^+) = i_L(0^-) = K_1 + K_2 = 2\ \text{A}}$$

Step ③ Find $i_L(\infty)$

$$\boxed{i_L(\infty) = K_2 = 0\ \text{A}}$$

$$\rightarrow \boxed{K_2 = 0\ \text{A}} \quad \boxed{K_1 = 2\ \text{A}}$$



Step ④ Find α

$$\boxed{R_{eq} = 2\ \Omega}$$

$$\boxed{\alpha = \frac{R_{eq}}{L} = \frac{2\Omega}{2\text{H}} = 1\ \text{s}^{-1}}$$

Step ⑤ Find $i_L(t)$

$$\rightarrow \boxed{i_L(t) = K_1 e^{-\alpha t} + K_2 = 2e^{-t}\ [\text{A}]}$$

$$i_L(t) = K_1 e^{-\alpha t} + K_2 = 2e^{-t} \text{ [A]}$$

$$V_L(t) = L\, d(2e^{-t})/dt = -4e^{-t} \text{ [V]}$$

$$L = 2 \text{ H}$$