ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 18

MWF – 12:00pm

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2062 ECE Building

Lecture 18 – Summary

Learning Objectives

1. Phasor representation of harmonic functions
2. Phasor representation of circuit problems in sinusoidal regime

In Canvas Module Week 7

- Trigonometry Identities Table
- Table of Trig function values at special angles

Examples:

4) Express in polar form $$z = \frac{1-j1}{1+j1}$$

Amplitude $$r = |z| = \frac{|1-j1|}{|1+j1|} = \frac{\sqrt{2}}{\sqrt{2}} = 1$$

Phase $$\theta = \angle z = \angle(1 - j1) - \angle(1 + j1) =$$

$$= \left(-\frac{\pi}{4}\right) - \left(\frac{\pi}{4}\right) = -\frac{\pi}{2}$$

Polar form $$z = 1\, e^{-j\frac{\pi}{2}}$$

Ratio of complex numbers

$$z = \frac{a + \mathrm{j}b}{c + \mathrm{j}d} = \frac{r_1 e^{\mathrm{j}\theta_1}}{r_2 e^{\mathrm{j}\theta_2}} = \frac{r_1}{r_2} e^{\mathrm{j}(\theta_1 - \theta_2)}$$

$$|z| = \frac{|z_1|}{|z_2|} \qquad \angle z = \angle z_1 - \angle z_2$$

You can also rationalize to make denominator real

$$z = \frac{a + \mathrm{j}b}{c + \mathrm{j}d} = \frac{(a + \mathrm{j}b)(c - \mathrm{j}d)}{(c + \mathrm{j}d)(c - \mathrm{j}d)}$$

$$= \frac{ac + bd + \mathrm{j}(bc - ad)}{c^2 + d^2}$$

$$= \frac{ac + bd}{c^2 + d^2} + \mathrm{j}\frac{(bc - ad)}{c^2 + d^2}$$

Going back to example 4)

$$z = \frac{1-\mathrm{j}1}{1+\mathrm{j}1} = \frac{1-\mathrm{j}1}{1+\mathrm{j}1} \times \left.\frac{1-\mathrm{j}1}{1-\mathrm{j}1}\right\} = 1$$

complex conjugate $(1+j1)(1-j1) = 2$

$$z = \frac{1-\mathrm{j}2+\overbrace{(\mathrm{j}^2)}^{=-1}}{2} = \frac{1-1-\mathrm{j}2}{2} = -\mathrm{j}$$

Amplitude $\quad r = 1$

Phase $\quad \theta = -\frac{\pi}{2}$

Polar form $\quad z = 1\, e^{-\mathrm{j}\frac{\pi}{2}}$

Examples:

5) Express in Cartesian (rectangular) form: $z = 3\,e^{-j\frac{\pi}{6}}$

$$z = 3\,e^{-j\frac{\pi}{6}} = 3\left[\cos\left(-\frac{\pi}{6}\right) + j\sin\left(-\frac{\pi}{6}\right)\right]$$

$$z = 3\left[\frac{\sqrt{3}}{2} + j\left(-\frac{1}{2}\right)\right]$$

$$z = \frac{3\sqrt{3}}{2} - j\,\frac{3}{2}$$

Examples:

6) Express in Cartesian (rectangular) form:

$$\boldsymbol{z = 3\ e^{\mathrm{j}\frac{\pi}{6}} + 3\ e^{-\mathrm{j}\frac{\pi}{6}}}$$

$$\boldsymbol{z = 3\left[\cos\left(\frac{\pi}{6}\right) + \mathrm{j}\,\sin\left(\frac{\pi}{6}\right)\right] + 3\left[\cos\left(-\frac{\pi}{6}\right) + \mathrm{j}\,\sin\left(-\frac{\pi}{6}\right)\right]}$$

Examples:

6) Express in Cartesian (rectangular) form:

$$z = 3\,e^{j\frac{\pi}{6}} + 3\,e^{-j\frac{\pi}{6}}$$

$$z = 3\left[\cos\left(\frac{\pi}{6}\right) + j\sin\left(\frac{\pi}{6}\right)\right] + 3\left[\cos\left(-\frac{\pi}{6}\right) + j\sin\left(-\frac{\pi}{6}\right)\right]$$

$$z = 3\left[\frac{\sqrt{3}}{2} + j\left(\frac{1}{2}\right)\right] + 3\left[\frac{\sqrt{3}}{2} + j\left(-\frac{1}{2}\right)\right]$$

$$z = \frac{3\sqrt{3}}{2} + \frac{3\sqrt{3}}{2} + j\,\frac{3}{2} - j\,\frac{3}{2} = 3\sqrt{3} - j0$$

Examples:

7) Express in polar form $$z = \frac{(-1+\mathrm{j})^5}{1+\mathrm{j}}$$

As found earlier

$$\boxed{-1+\mathrm{j} = \sqrt{2}\, e^{\,\mathrm{j}\frac{3\pi}{4}}} \qquad \boxed{1+\mathrm{j} = \sqrt{2}\, e^{\,\mathrm{j}\frac{\pi}{4}}}$$

$$(-1+\mathrm{j})^5 = \left(\sqrt{2}\right)^5 e^{\,\mathrm{j}\,5\frac{3\pi}{4}} = 4\sqrt{2}e^{\,\mathrm{j}\frac{15\pi}{4}}$$

Examples:

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$$(-1+\mathrm{j})^5 = \left(\sqrt{2}\right)^5 e^{\,\mathrm{j}\,5\frac{3\pi}{4}} = 4\sqrt{2}e^{\,\mathrm{j}\frac{15\pi}{4}}$$

$$z = \frac{4\sqrt{2}e^{\,\mathrm{j}\frac{15\pi}{4}}}{\sqrt{2}\, e^{\,\mathrm{j}\frac{\pi}{4}}} = 4e^{\,\mathrm{j}\frac{14\pi}{4}} = 4e^{\,\mathrm{j}\frac{7\pi}{2}}$$

$$z = 4e^{\mathrm{j}\left(2\pi+\frac{3\pi}{2}\right)} = 4\underbrace{\boxed{e^{\mathrm{j}2\pi}}}_{=1} e^{\,\mathrm{j}\frac{3\pi}{2}} = 4e^{\,\mathrm{j}\frac{3\pi}{2}}$$

In engineering problems, the following identities are often useful for mathematical manipulations:

$$\mathrm{j} = \exp\left(\mathrm{j}\frac{\pi}{2}\right) \qquad -\mathrm{j} = \exp\left(-\mathrm{j}\frac{\pi}{2}\right)$$

The relations linking exponentials to trigonometric functions of complex variables are also widely used:

$$\cos(z) = \frac{\exp(\mathrm{j}\,z) + \exp(-\mathrm{j}\,z)}{2}$$

$$\sin(z) = \frac{\exp(\mathrm{j}\,z) - \exp(-\mathrm{j}\,z)}{2i}$$

These result from Euler’s identities

$$\exp(\pm\,\mathrm{j}\,z) = \cos(z) \pm \mathrm{j}\sin(z)$$

When time-harmonic functions are considered, it is possible to simplify the analysis of engineering systems by using complex representation.

Example of time-harmonic function:

$$\boldsymbol{A\cos(\omega t+\theta)}$$

Amplitude

Angular frequency
$\boldsymbol{\omega = 2\pi f}$

phase

When **time-harmonic functions** **are considered, it is possible to simplify the analysis of engineering systems by using complex representation.**

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Angular frequency
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By invoking **Euler's identity**, **we can write**

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$$A\cos(\omega t+\theta) =$$

$$= \Re e[A\cos(\omega t+\theta) + \mathrm{j}\,A\sin(\omega t+\theta)]$$

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Angular frequency
$\omega = 2\pi f$

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By invoking Euler's identity, we can write

$$A\cos\left(\omega t + \theta\right) =$$

$$= \Re e\left[A\cos\left(\omega t + \theta\right) + \mathrm{j}\, A\sin\left(\omega t + \theta\right)\right]$$

$$= \Re e\left[A\exp\left(\,\mathrm{j}\omega t + \mathrm{j}\theta\right)\,\right]$$

For unambiguous treatment of phasors, A must be a magnitude (a positive value).

A negative value contains a hidden phase!

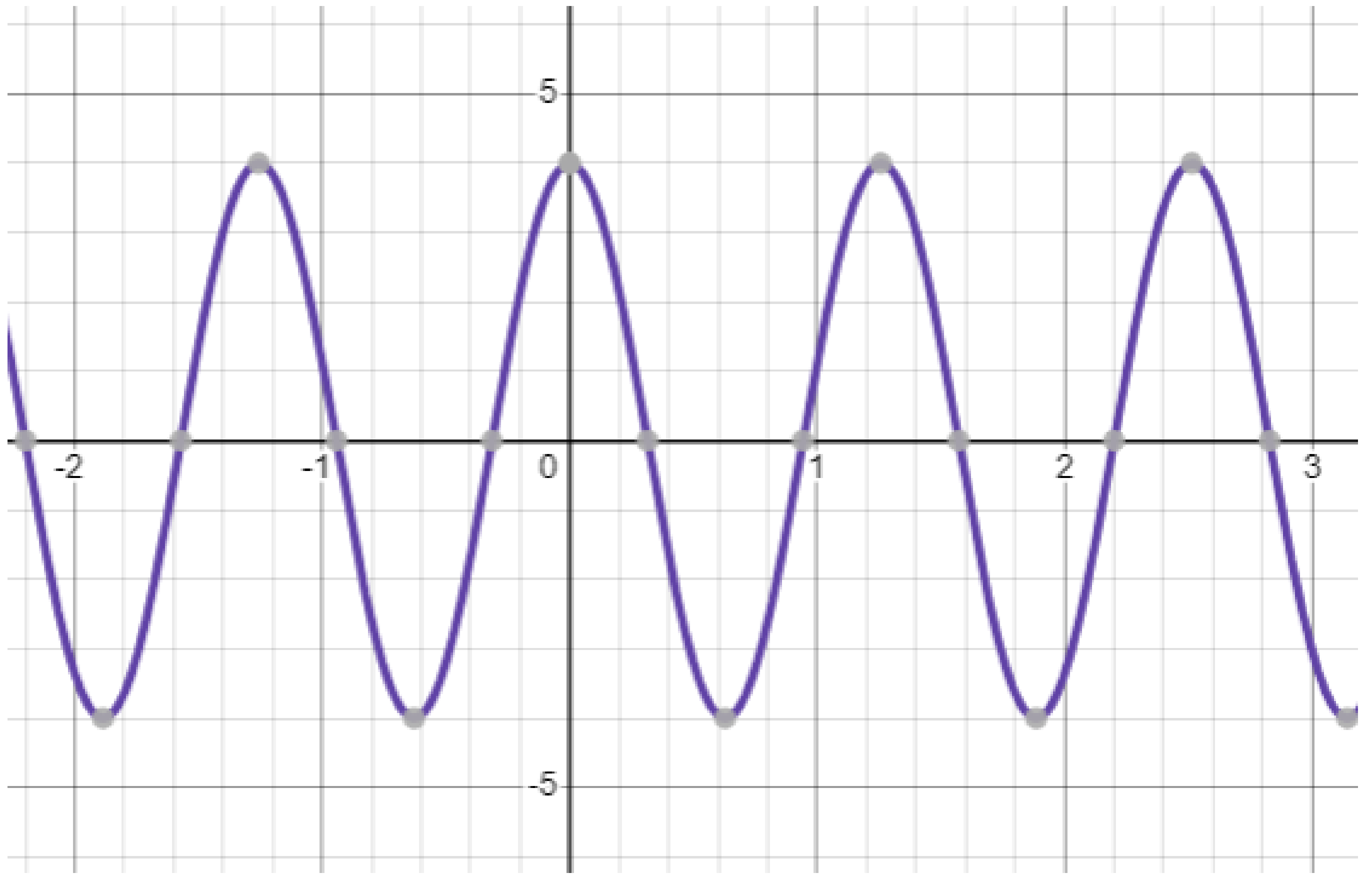
$$-|A|\cos(\omega t+\theta)$$

$\pm\pi$ or $\pm 180°$

PHASE SHIFT

$$\boxed{\begin{aligned}&-|A|\cos(\omega t+\theta)\\&=|A|\cos(\omega t+\theta\pm\pi)\end{aligned}}$$

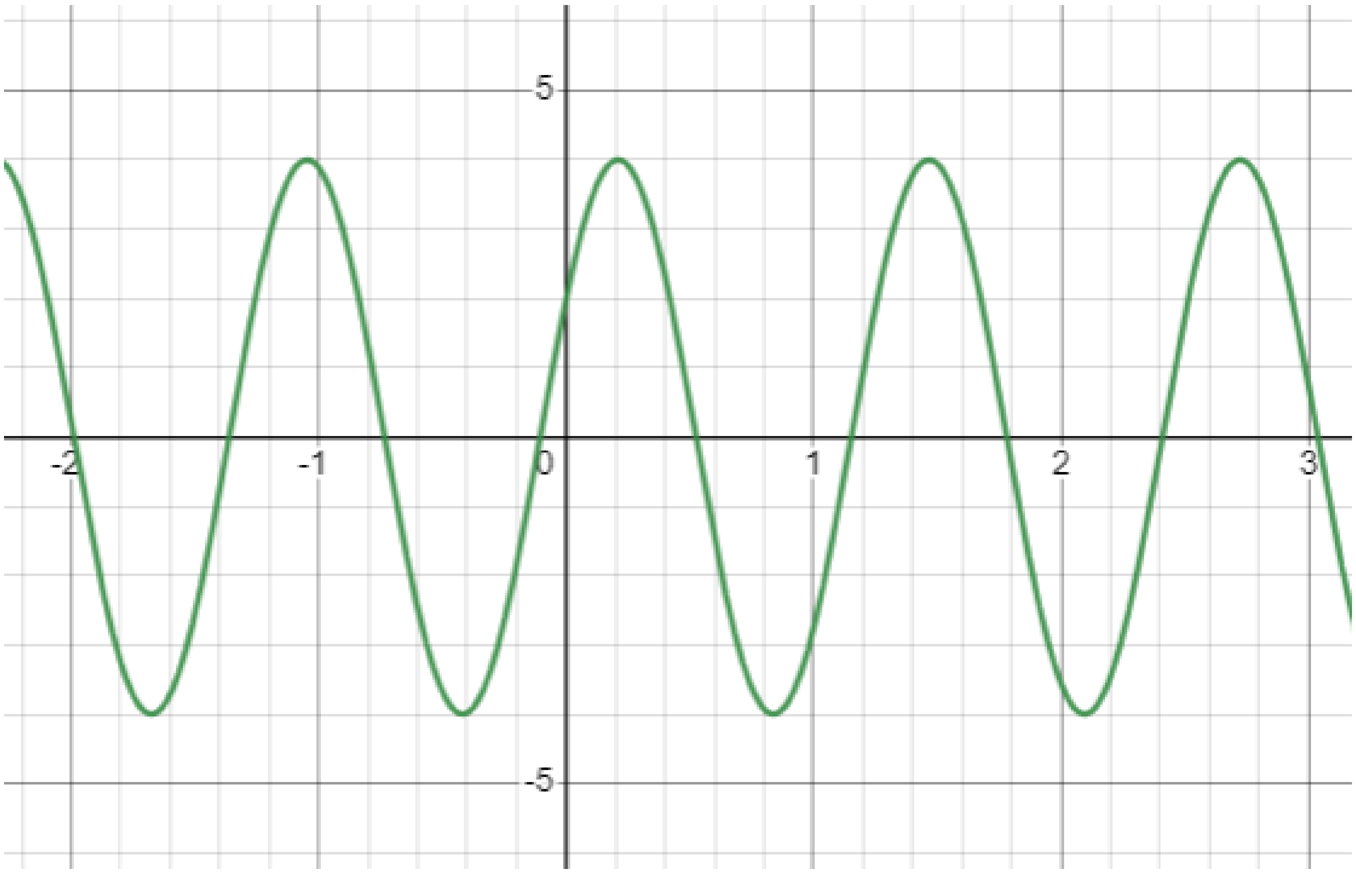
$$A \cos(\omega t)$$



t [s]

$A = 4$ $\theta = 0$ $\omega = 5\ \mathrm{rad/s}$

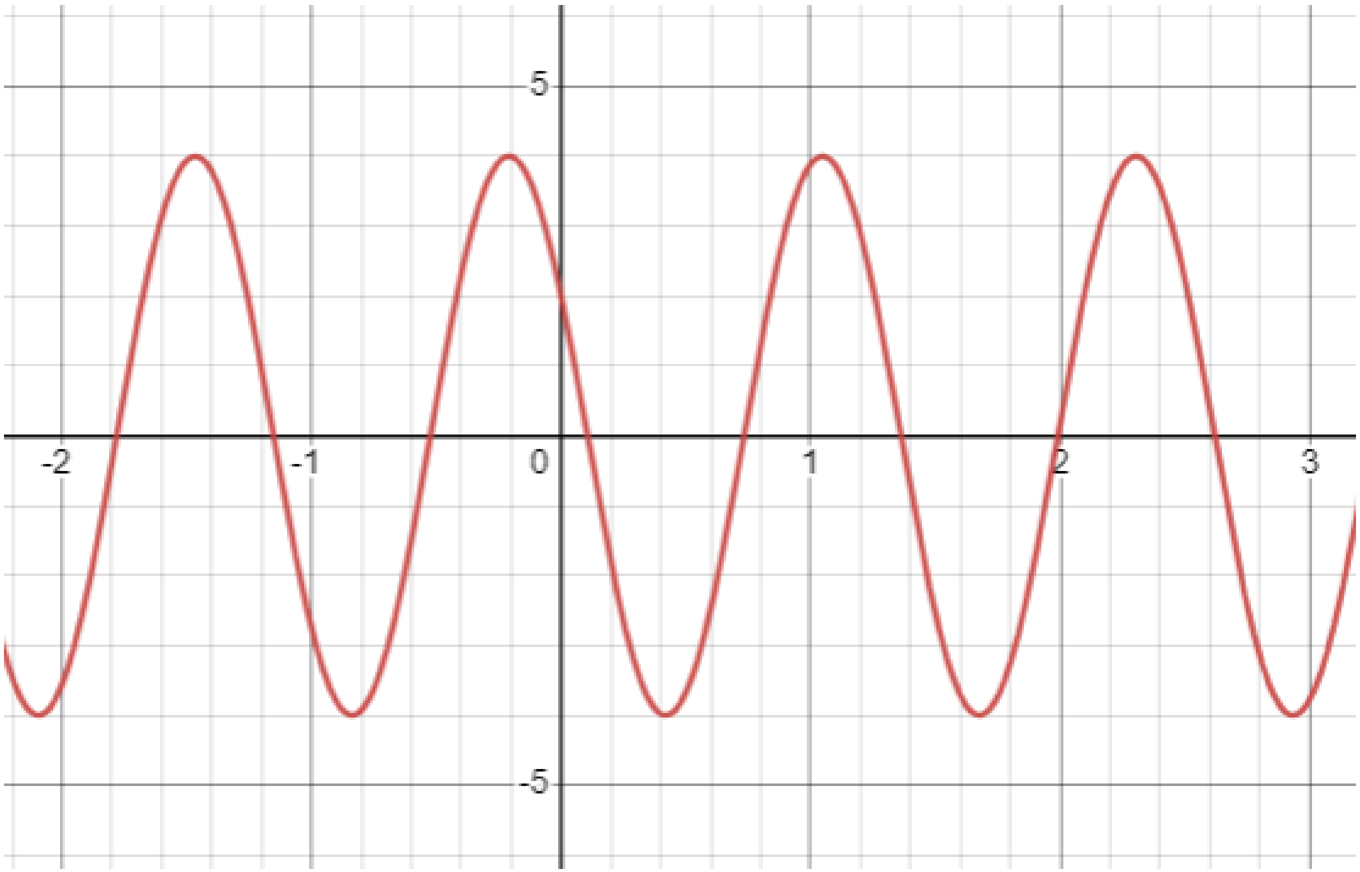
$A\cos\left(\omega t - \pi/3\right)$



$t\ [s]$

$A = 4 \qquad \theta = -\pi/3 \qquad \omega = 5\ \mathrm{rad/s}$

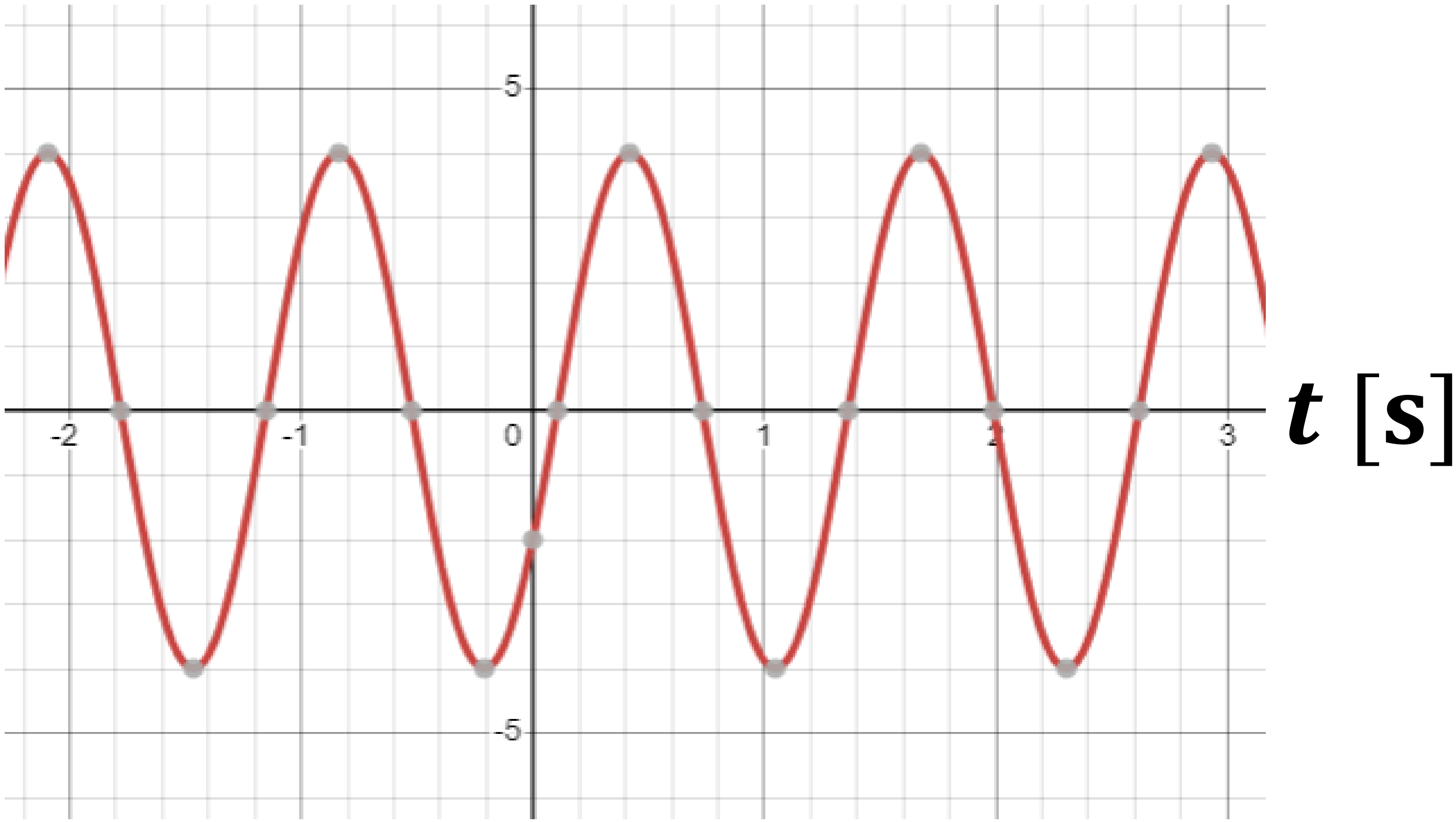
$A \cos(\omega t + \pi/3)$



$t\ [s]$

$A = 4$ $\theta = \pi/3$ $\omega = 5\ \mathrm{rad/s}$

$$-A\cos\left(\omega t + \pi/3\right)$$



$A = 4$ $\theta = \pi/3 \pm \pi$ $\omega = 5$ rad/s

Now we are going to use the properties of the exponentials to split frequency from phase:

$$\Re e[A \exp(\mathrm{j}\omega t + \mathrm{j}\theta)] =$$

$$\Re e[A \exp(\mathrm{j}\omega t)\exp(\mathrm{j}\theta)] =$$

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$$\Re e[\,\underbrace{A\,\exp(\,\mathrm{j}\theta)}_{\boxed{\text{phasor of the time-harmonic function}}}\,\exp(\,\mathrm{j}\omega t)]$$

Now we are going to use the properties of the exponentials to split frequency from phase:

$$\Re e[A\,\exp(\,j\omega t + j\theta)\,] =$$

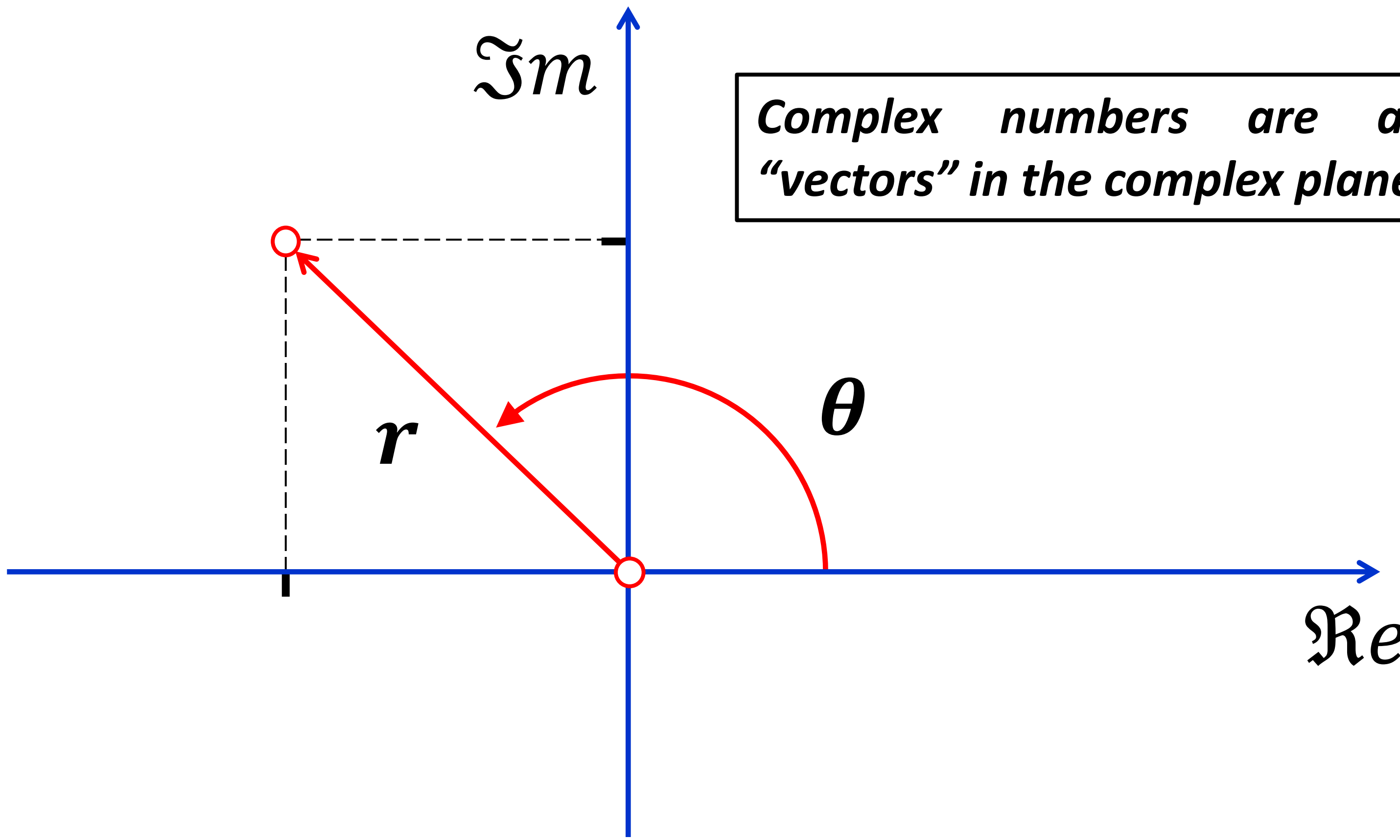
$$\Re e[A\,\exp(\,j\omega t)\exp(\,j\theta)] =$$

$$\Re e[\,\underbrace{A\,\exp(\,j\theta)}_{\boxed{\text{phasor of the time-harmonic function}}}\,\exp(\,j\omega t)]$$

The "phasor" contains the essential information on amplitude and phase.

For a known frequency ω, $A\,\exp(j\theta)$ characterizes completely $A\,\cos(\omega t + \theta)$.

Why do we actually map the (Amplitude, Phase) pair into the complex plane? → Because we can use the powerful vector algebra of complex numbers to perform all kinds of mathematical manipulations.



Complex numbers are actually "vectors" in the complex plane

We have reviewed that complex numbers can be represented in Cartesian (rectangular) form

$$z = a + \mathrm{j}b$$

or in polar form

$$z = r\,\exp(\,\mathrm{j}\,\theta) = \overbrace{r\,\cos(\theta)} + \mathrm{j}\,\overbrace{r\,\sin(\theta)}$$

We wish to use complex functions to represent circuits driven by sinusoidal inputs at specific frequencies.

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$$z = a + \mathrm{j}b$$

or in polar form

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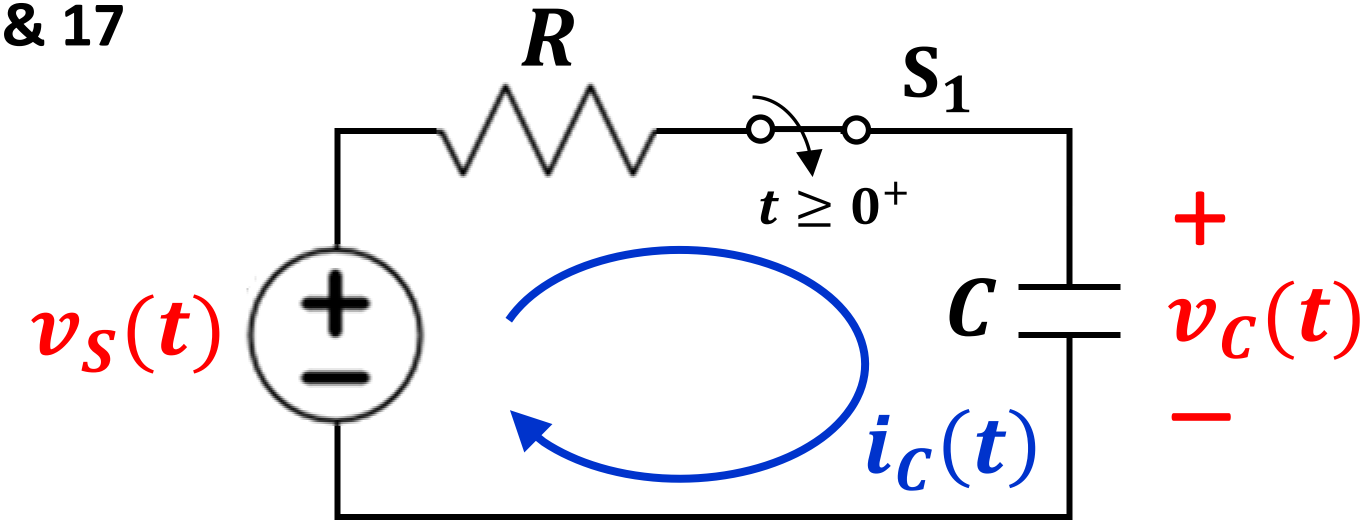
We wish to use complex functions to represent circuits driven by sinusoidal inputs at specific frequencies.

<u>Objection:</u> We already have trigonometry to represent sinusoidal functions. Why should we introduce additional complications?

Let's revisit again the steps needed to solve a simple circuit example shown in Lecture 16.

From Lectures 16 & 17



$$RC = 1\text{s} \qquad v_S(t) = \cos(t)$$

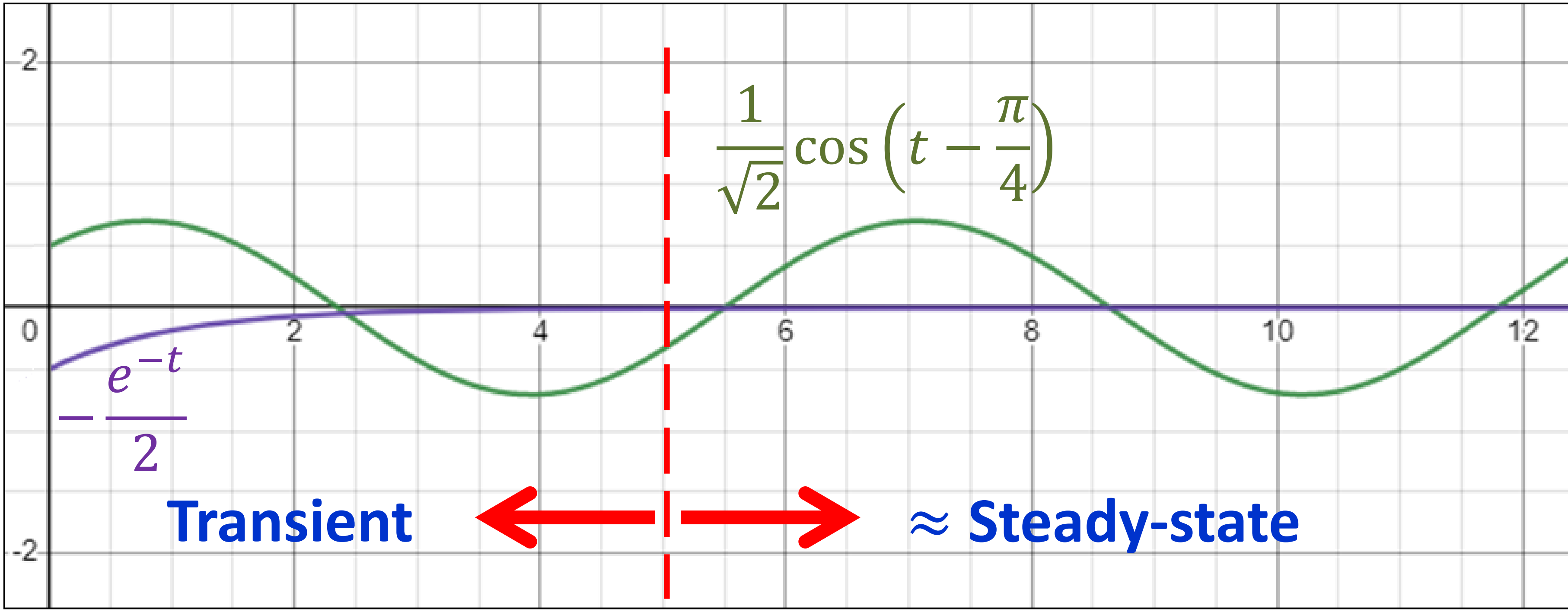
$$v_C(t = 0^-) = 0\text{V}$$

Differential equation:

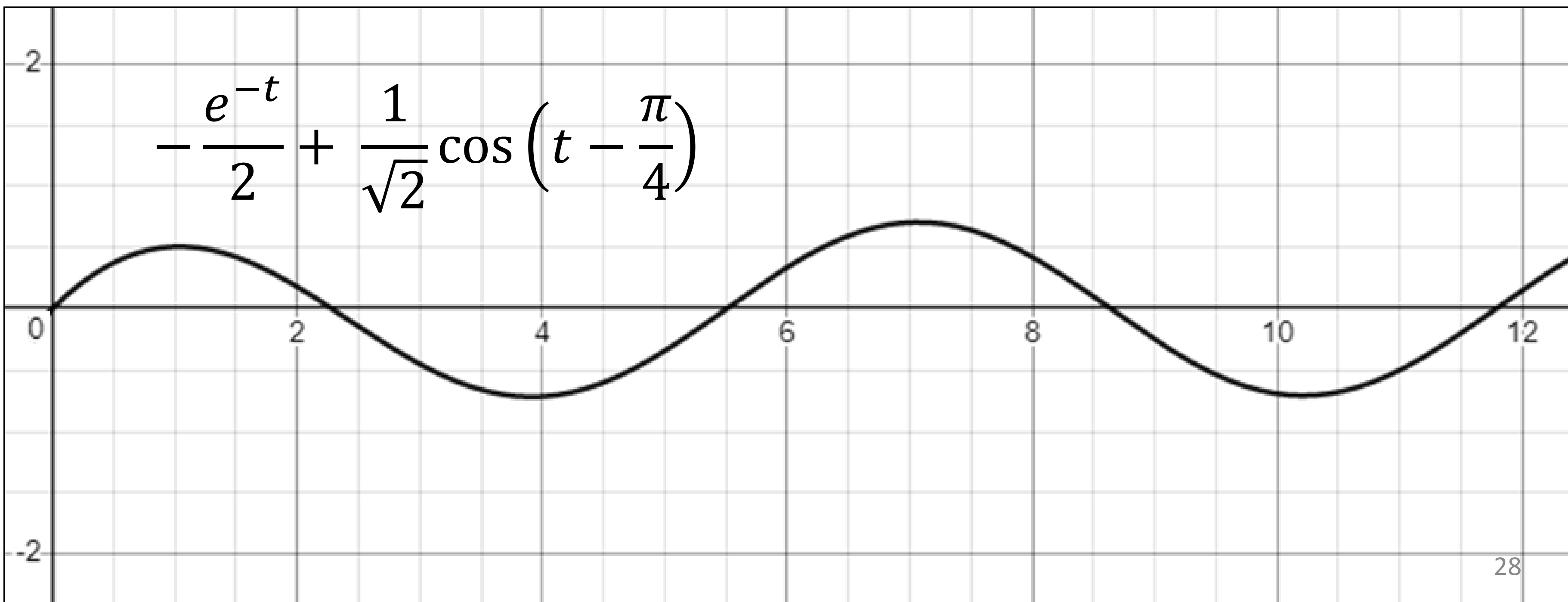
$$\boxed{\frac{d}{dt} v_C(t) + v_C(t) = \cos(t)}$$

Solution:

$$\boxed{V_C(t) = -\frac{e^{-t}}{2} + \frac{1}{\sqrt{2}} \cos\left(t - \frac{\pi}{4}\right)}$$



$\frac{1}{\sqrt{2}}\cos\left(t-\frac{\pi}{4}\right)$
$-\frac{e^{-t}}{2}$
Transient
≈ Steady-state



$-\frac{e^{-t}}{2}+\frac{1}{\sqrt{2}}\cos\left(t-\frac{\pi}{4}\right)$

$$V_C(t) = \underbrace{-\frac{e^{-t}}{2}}_{\substack{\textbf{Transient}\\\textbf{component}}} + \underbrace{\frac{1}{\sqrt{2}}\cos\left(t - \frac{\pi}{4}\right)}_{\substack{\textbf{Steady-state}\\\textbf{component}}}$$

For sufficiently long time, the solution is consisting only of the steady-state component

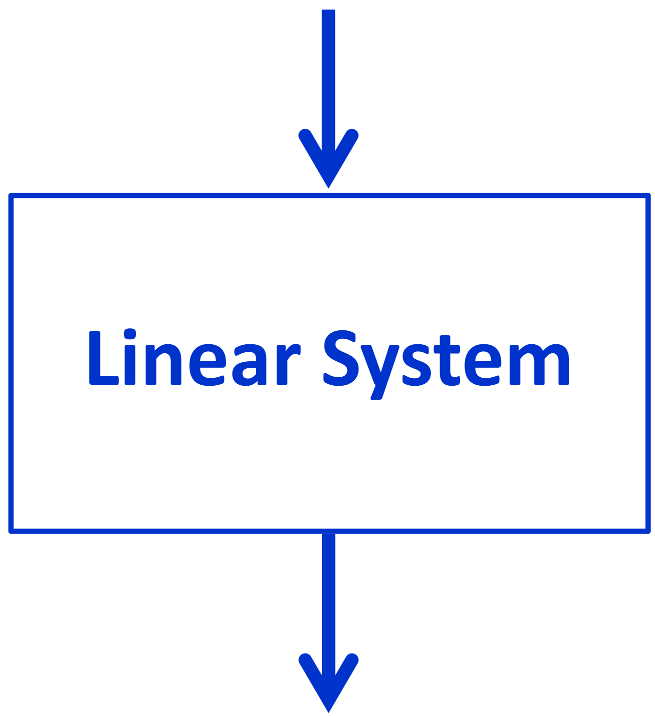
$$V_C(t) \approx \frac{1}{\sqrt{2}}\cos\left(t - \frac{\pi}{4}\right)$$

Recall that the input was:

$$v_S(t) = \cos(t)$$

In many engineering problems of practical importance, we only need to find the steady-state response of the system for sinusoidal (single-frequency) input

$$V_{in} = A_{in}\cos(\omega t - \theta_{in})$$

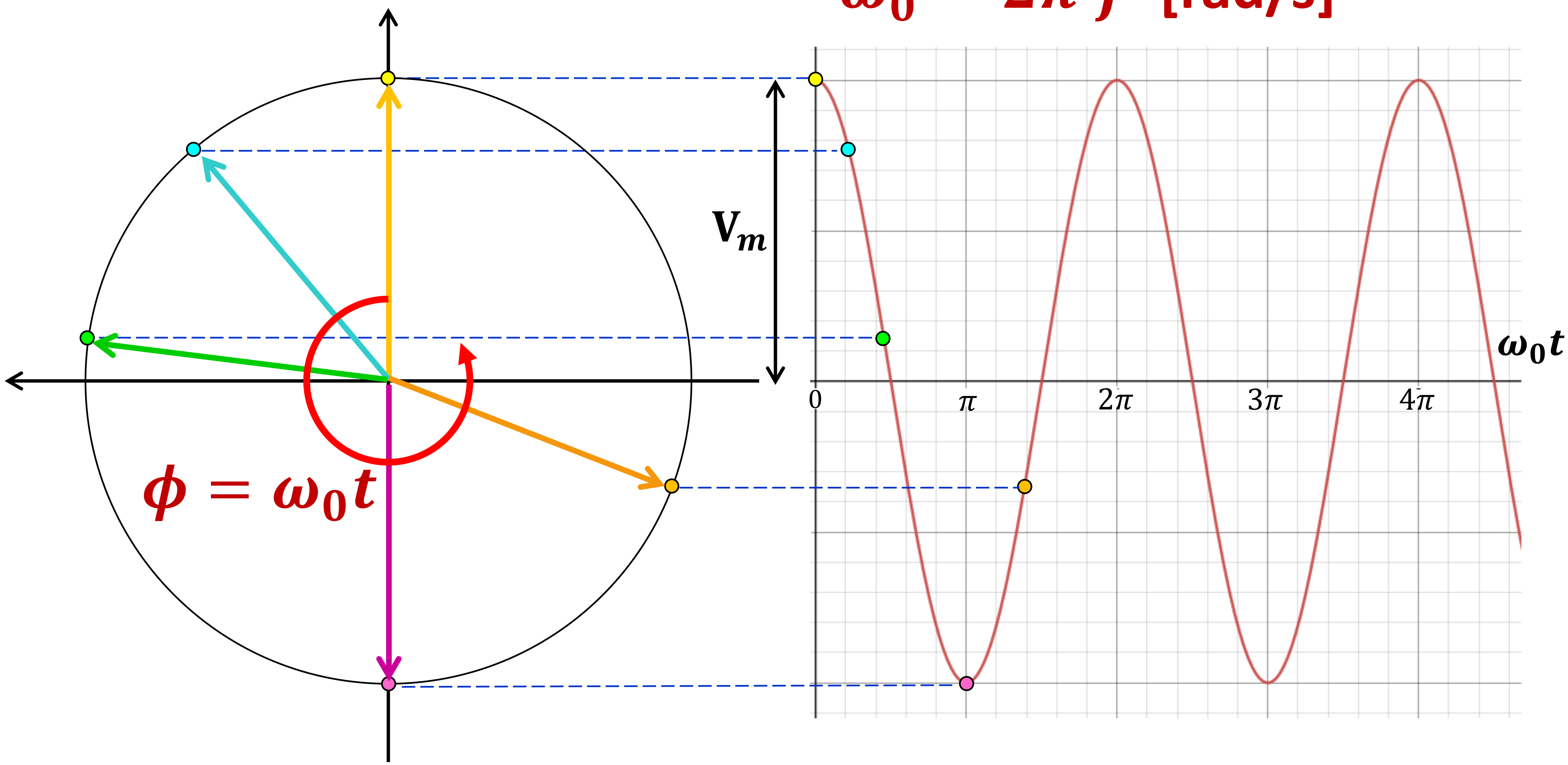


$$V_{out} = A_{out}\cos(\omega t - \theta_{out})$$

Consider a time-harmonic co-sinusoidal function:

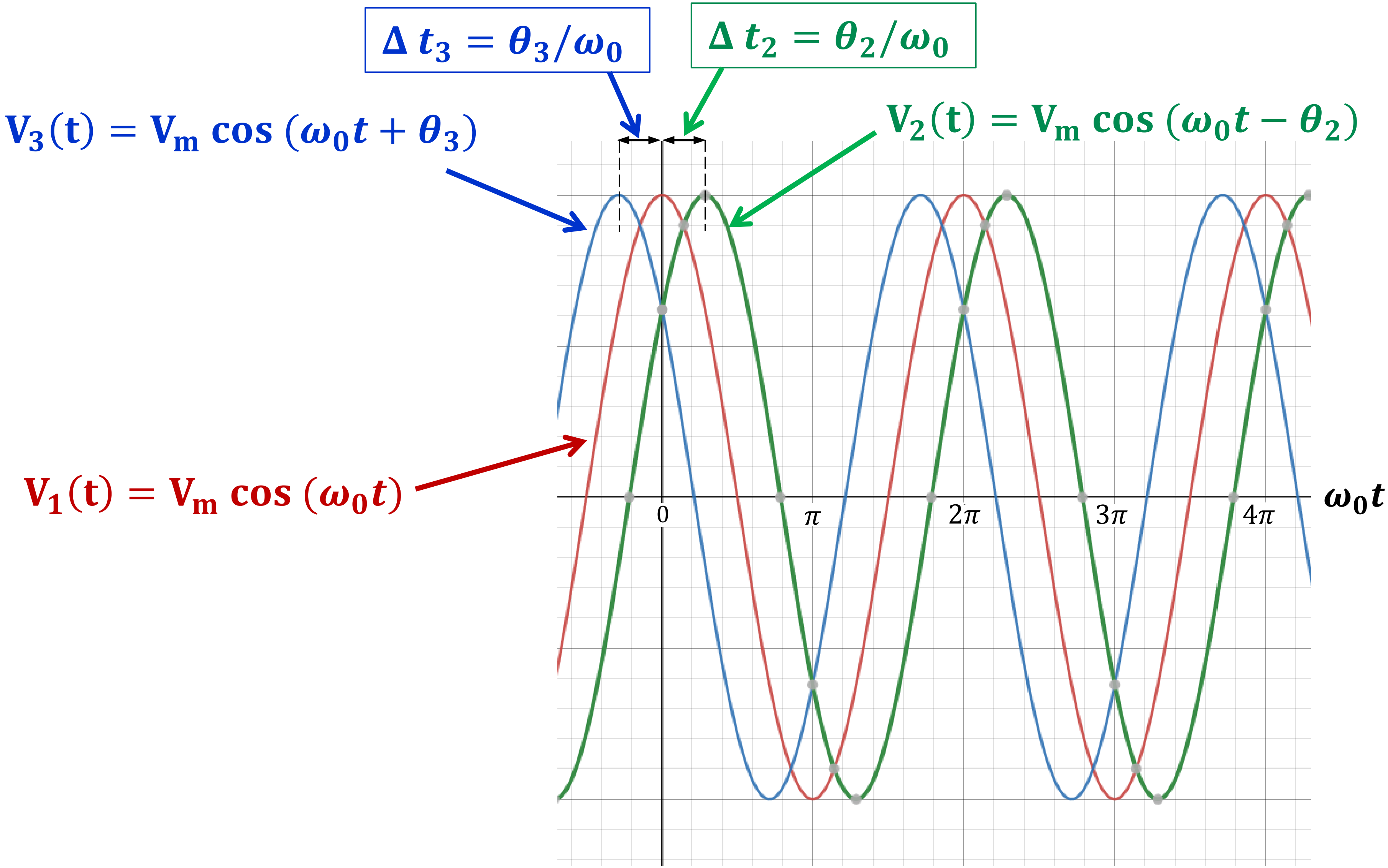
$$\mathrm{V(t)} = \mathrm{V_m} \cos\left(\omega_0 t\right)$$

$$\omega_0 = 2\pi f \;\; [\text{rad/s}]$$



A vector with magnitude V_m and rotating at frequency ω_0 describes the evolution of a harmonic function on a 2D plane.

Phase accounts for anticipation or delay in the cycle



Phasor representation

A time-harmonic co-sinusoidal function of known frequency can simply be represented by a pair of values: Amplitude and Phase, which identify a number in the complex plane

Representation in the time-domain

$$v(t) = V_{\mathrm{m}} \cos\left(\omega_0 t + \theta_{\mathrm{v}}\right)$$

Phasor representation in the frequency domain

$$\mathbf{V} = V_{\mathrm{m}} \angle \theta_{\mathrm{v}}$$

or, equivalently:

$$\mathbf{V} = V_{\mathrm{m}} e^{\mathrm{j}\theta_{\mathrm{v}}}$$

Examples

Find the amplitude and phase values to represent in phasor form:

$$\boldsymbol{i(t) = 5\cos\left(1000t + 30°\right)}$$

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$$i(t) = 5\cos(1000t + 30°)$$

$$I = 5\angle 30°$$ **Phasor form**

In radians: $$30° \times \frac{\pi}{180°} = \frac{\pi}{6}$$

$$I = 5\angle \pi/6$$
(radians)

$$\boxed{I = 5e^{j\pi/6}}$$

Examples

Find the amplitude and phase values to represent in phasor form:

$$i(t) = 5\sin(1000t + 30°)$$

We need to transform sine into cosine: $\sin(\phi) = cos(\phi - 90°)$

$$i(t) = 5\cos(1000t + 30° - 90°)$$
$$= 5\cos(1000t - 60°)$$

$$I = 5\angle - 60°$$ **Phasor form**

$$I = 5\angle - \pi/3$$

(radians)

$$\boxed{I = 5e^{-j\pi/3}}$$

Examples

Find the cosine signal at frequency $\omega_0 = 1000\ \mathrm{rad/s}$ represented by the phasor $\mathrm{V} = \mathrm{j}5$:

$$\mathrm{V} = \mathrm{j}5 = 5\,\mathrm{e}^{\mathrm{j}90°} = 5\,\mathrm{e}^{\mathrm{j}\frac{\pi}{2}}$$

$$= 5\,\angle 90° = 5\angle \pi/2$$

$$v(t) = 5\cos(1000t + 90°)$$
$$= 5\cos(1000t + \pi/2)$$

Suppose you need to add two time-harmonic functions

$$v_1(t) = A_1 \cos(\omega t + \theta_1)$$

$$v_2(t) = A_2 \cos(\omega t + \theta_2)$$

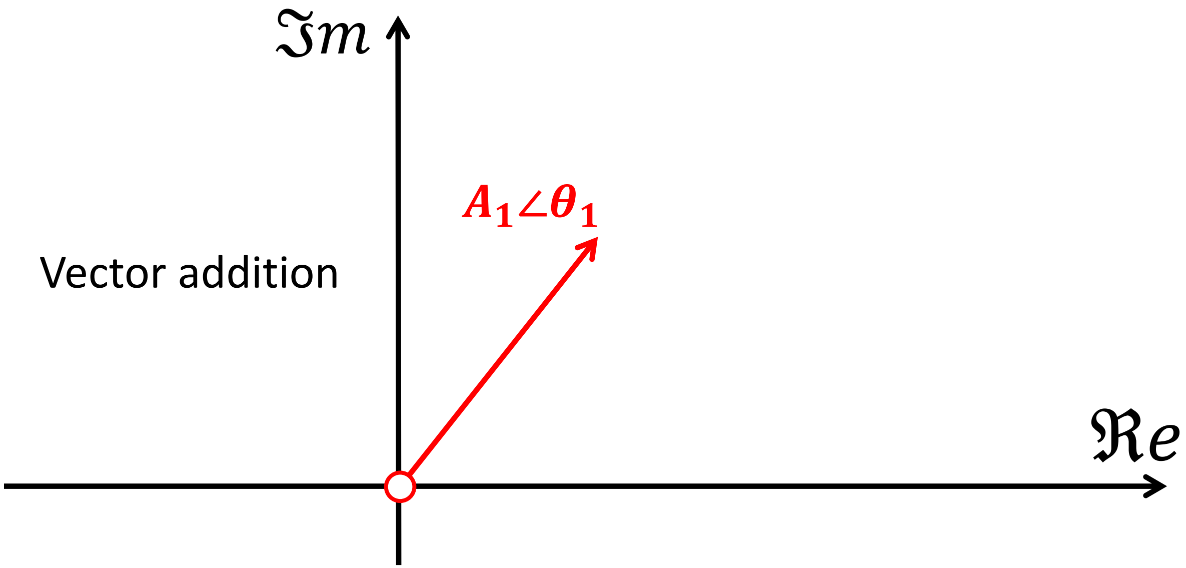
With trigonometry you have to use cumbersome formulas like:

$$\cos \mathrm{X} + \cos \mathrm{Y} = 2 \cos\left(\frac{X+Y}{2}\right) \cos\left(\frac{X-Y}{2}\right)$$

In phasor form:

$$v_1(t) = A_1 \cos(\omega t + \theta_1) = \Re e[A_1 \exp(\,j\omega t + j\theta_1)\,]$$

$$\Leftrightarrow \ \mathbf{V}_1 = A_1 \exp(\,j\theta_1) = A_1 \angle \theta_1$$



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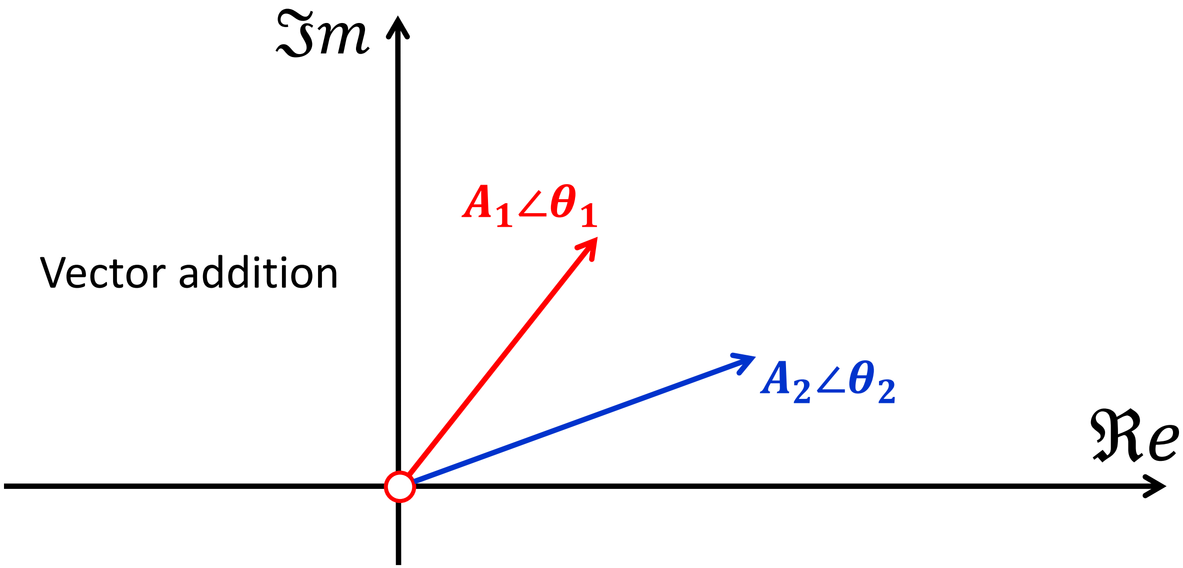
$$v_1(t) = A_1 \cos\left(\omega t + \theta_1\right) = \Re e\left[A_1 \exp(\, j\omega t + j\theta_1)\,\right]$$

$$\Leftrightarrow \quad \mathbf{V}_1 = A_1 \exp(\, j\theta_1) = A_1 \angle \theta_1$$

$$v_2(t) = A_2 \cos\left(\omega t + \theta_2\right) = \Re e\left[A_2 \exp(\, j\omega t + j\theta_2)\,\right]$$

$$\Leftrightarrow \quad \mathbf{V}_2 = A_2 \exp(\, j\theta_2) = A_2 \angle \theta_2$$

Vector addition



In phasor form:

$$v_1(t) = A_1\cos(\omega t + \theta_1) = \Re e[A_1 \exp(\,j\omega t + j\theta_1)\,]$$

$$\Leftrightarrow\ \mathbf{V}_1 = A_1 \exp(\,j\theta_1) = A_1\angle\theta_1$$

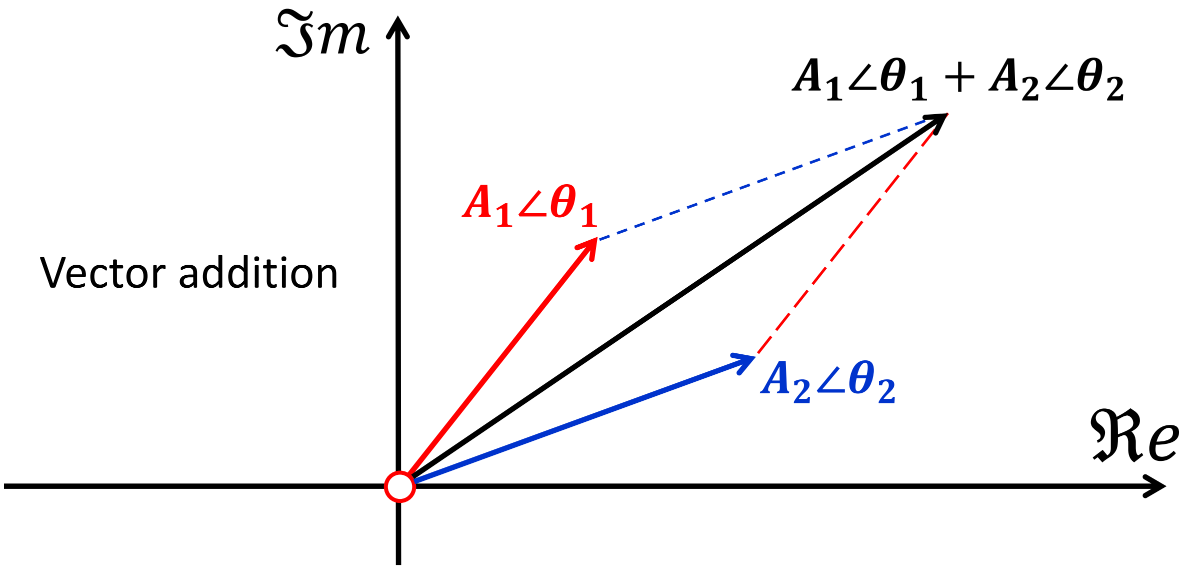
$$v_2(t) = A_2\cos(\omega t + \theta_2) = \Re e[A_2 \exp(\,j\omega t + j\theta_2)\,]$$

$$\Leftrightarrow\ \mathbf{V}_2 = A_2 \exp(\,j\theta_2) = A_2\angle\theta_2$$

$$v_1(t) + v_2(t) \Leftrightarrow \mathbf{V}_1 + \mathbf{V}_2 = A_1 \exp(\,j\theta_1) + A_2 \exp(\,j\theta_2)$$

CAUTION: $\Leftrightarrow$ is a "transformation" NOT an "equality"!

Vector addition



Example – Express the following in its phasor form:

$$v(t) = 2\sqrt{2}\sin\left(1000t + \frac{\pi}{4}\right) + 2\sqrt{2}\cos\left(1000t + \frac{\pi}{4}\right)$$

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$$v(t) = 2\sqrt{2}\sin\left(1000t + \frac{\pi}{4}\right) + 2\sqrt{2}\cos\left(1000t + \frac{\pi}{4}\right)$$

First term:

$$v_1(t) = 2\sqrt{2}\cos\left(1000t + \frac{\pi}{4} - \frac{\pi}{2}\right)$$

$$v_1(t) = 2\sqrt{2}\cos\left(1000t - \frac{\pi}{4}\right)$$

$$\mathbf{V_1} = 2\sqrt{2}\angle -\frac{\pi}{4} = 2\sqrt{2}\, e^{-j\frac{\pi}{4}}$$

$$= 2\sqrt{2}\left(\cos\frac{\pi}{4} - \mathrm{j}\sin\frac{\pi}{4}\right)$$

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$$v(t) = 2\sqrt{2}\sin\left(1000t + \frac{\pi}{4}\right) + 2\sqrt{2}\cos\left(1000t + \frac{\pi}{4}\right)$$

Second term:

$$v_2(t) = 2\sqrt{2}\cos\left(1000t + \frac{\pi}{4}\right)$$

$$\mathbf{V_1} = 2\sqrt{2}\angle\frac{\pi}{4} = 2\sqrt{2}\, e^{\mathrm{j}\frac{\pi}{4}}$$

$$= 2\sqrt{2}\left(\cos\frac{\pi}{4} + \mathrm{j}\sin\frac{\pi}{4}\right)$$

Example – Express the following in its phasor form:

$$v(t) = 2\sqrt{2}\sin\left(1000t + \frac{\pi}{4}\right) + 2\sqrt{2}\cos\left(1000t + \frac{\pi}{4}\right)$$

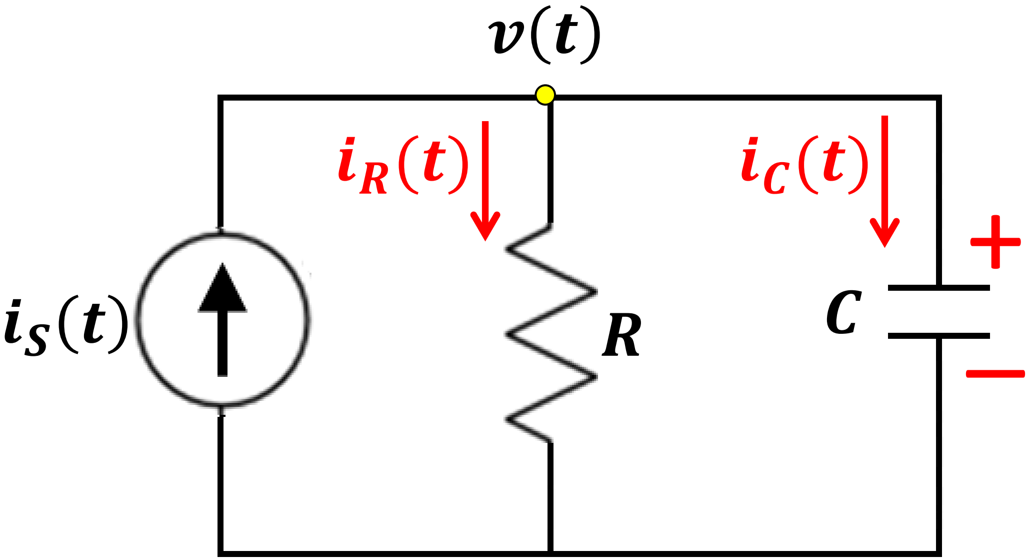
Combine the results:

$$\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2 = 2\sqrt{2}\left(\cos\frac{\pi}{4} - \cancel{\mathrm{j}\sin\frac{\pi}{4}}\right) + 2\sqrt{2}\left(\cos\frac{\pi}{4} + \cancel{\mathrm{j}\sin\frac{\pi}{4}}\right)$$

$$= 2\sqrt{2}\left(2\cos\frac{\pi}{4}\right) = 2\sqrt{2}\left(2\sqrt{2}/2\right) = 4\angle 0°$$

$$\Rightarrow \boxed{v(t) = 4\cos(1000t)\ [\mathrm{V}]}$$

RC Circuit Example with time-harmonic forcing term



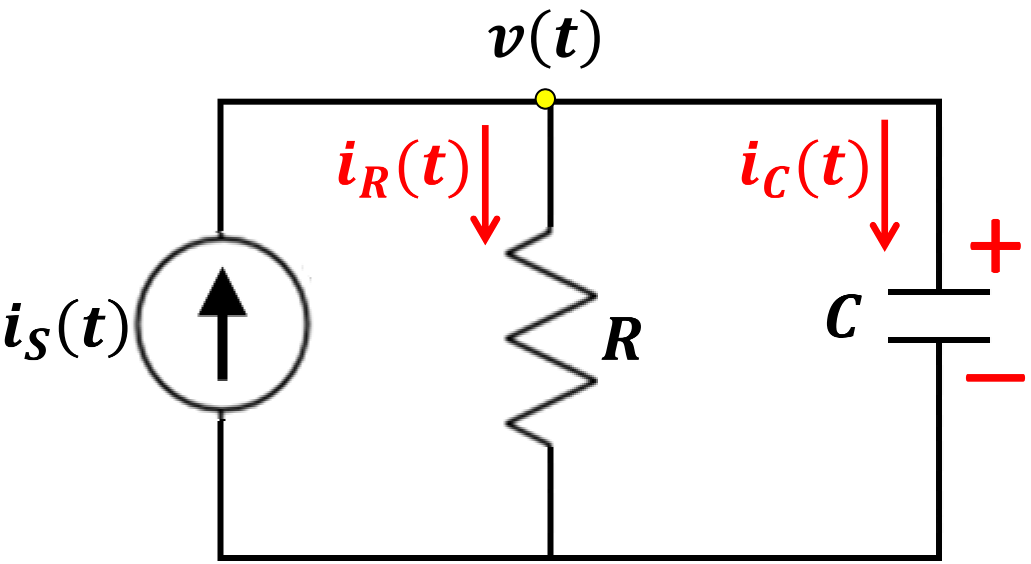
Find $i_S(t)$ when $v(t)$ is measured as:

$$v(t) = \frac{10}{\sqrt{2}} \cos\left(1000t - \frac{\pi}{4}\right)$$

$R = 1\Omega$

$C = 1\text{mF}$

RC Circuit Example with time-harmonic forcing term



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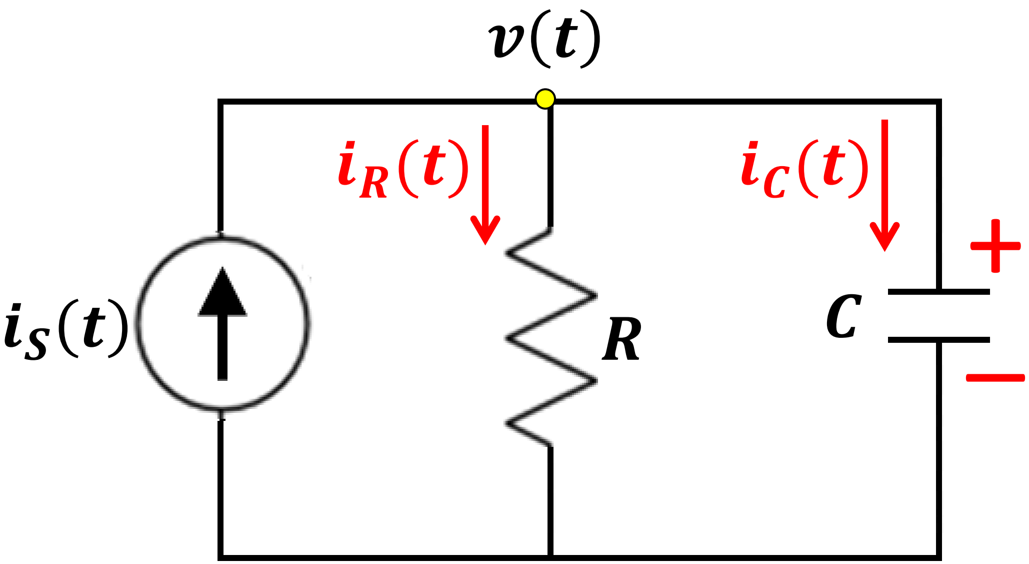
KCL $\quad i_S = i_R(t) + i_C(t) = \frac{v(t)}{R} + C\frac{d\,v(t)}{dt}$

RC Circuit Example with time-harmonic forcing term



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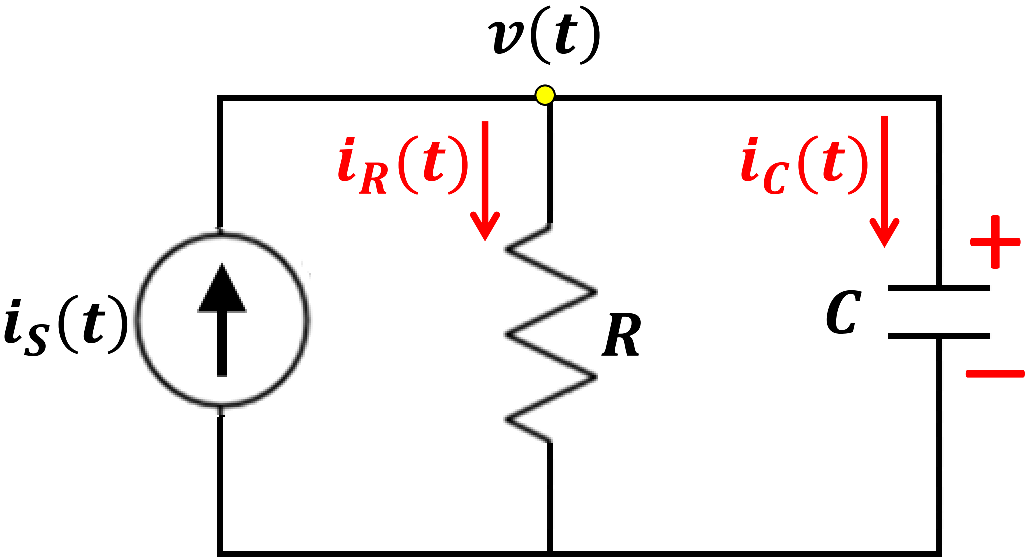
$R = 1\Omega$ $C = 1\text{mF}$

KCL

$$i_S = i_R(t) + i_C(t) = \frac{v(t)}{R} + C\frac{d\,v(t)}{dt}$$

$$= \frac{10}{1 \times \sqrt{2}} \cos\left(1000t - \frac{\pi}{4}\right) - 10^{-3}\frac{10^4}{\sqrt{2}} \sin\left(1000t - \frac{\pi}{4}\right)$$

RC Circuit Example with time-harmonic forcing term



Find $i_S(t)$ when $v(t)$ is measured as:

$$v(t) = \frac{10}{\sqrt{2}}\cos\left(1000t - \frac{\pi}{4}\right)$$

$R = 1\Omega$ $\quad$ $C = 1\text{mF}$

KCL

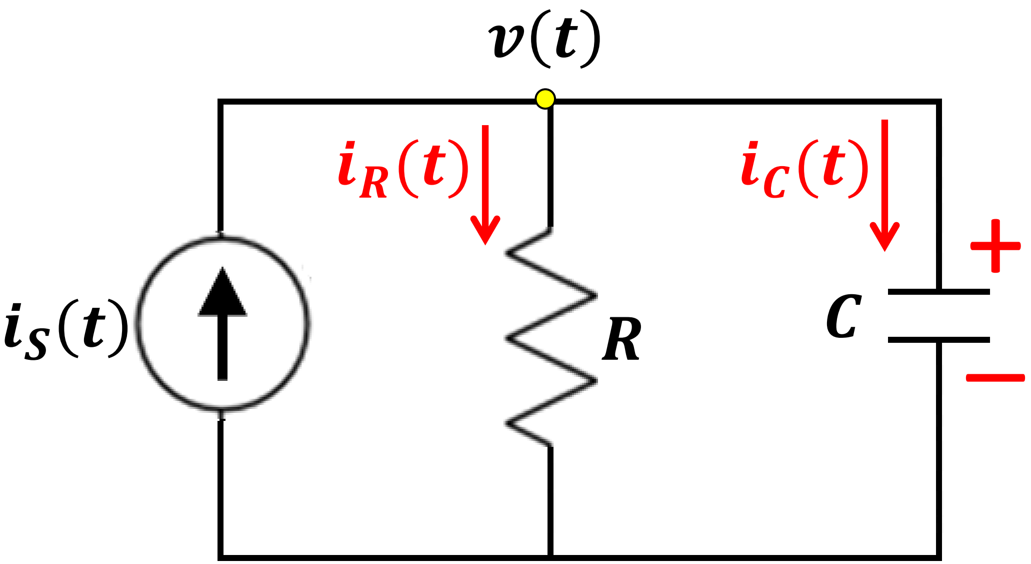
$$i_S = i_R(t) + i_C(t) = \frac{v(t)}{R} + C\frac{d\,v(t)}{dt}$$

$$= \frac{10}{1 \times \sqrt{2}}\cos\left(1000t - \frac{\pi}{4}\right) - 10^{-3}\frac{10^4}{\sqrt{2}}\sin\left(1000t - \frac{\pi}{4}\right)$$

$$= \frac{10}{\sqrt{2}}\cos\left(1000t - \frac{\pi}{4}\right) - \frac{10}{\sqrt{2}}\sin\left(1000t - \frac{\pi}{4}\right)$$

$$= \frac{10}{\sqrt{2}}\cos\left(1000t - \frac{\pi}{4}\right) + \frac{10}{\sqrt{2}}\cos\left(1000t + \frac{\pi}{4}\right)$$

RC Circuit Example with time-harmonic forcing term



Find $i_S(t)$ when $v(t)$ is measured as:

$$v(t) = \frac{10}{\sqrt{2}} \cos\left(1000t - \frac{\pi}{4}\right)$$

$R = 1\Omega$ $C = 1\text{mF}$

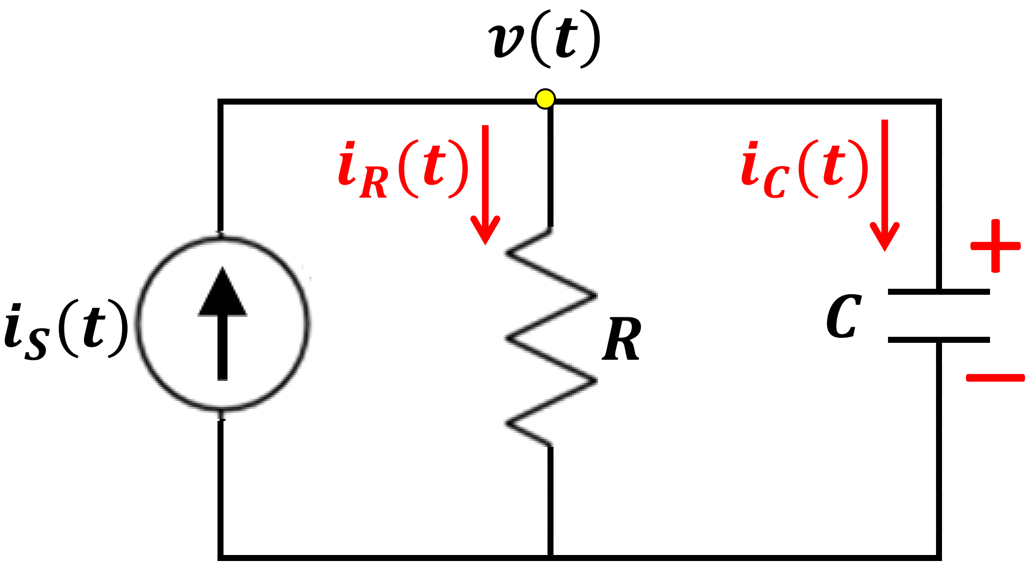
$$i_S(t) = \frac{10}{\sqrt{2}} \cos\left(1000t - \frac{\pi}{4}\right) + \frac{10}{\sqrt{2}} \cos\left(1000t + \frac{\pi}{4}\right)$$

Phasor form

$$\mathbf{I}_S = \frac{10}{\sqrt{2}} \angle -\frac{\pi}{4} + \frac{10}{\sqrt{2}} \angle \frac{\pi}{4}$$

$$= \frac{10}{\sqrt{2}} \exp\left(-j\frac{\pi}{4}\right) + \frac{10}{\sqrt{2}} \exp\left(j\frac{\pi}{4}\right)$$

RC Circuit Example with time-harmonic forcing term



Find $i_S(t)$ when $v(t)$ is measured as:

$$v(t) = \frac{10}{\sqrt{2}} \cos\left(1000t - \frac{\pi}{4}\right)$$

$\boxed{R = 1\Omega}$ $\boxed{C = 1\text{mF}}$

$$\mathbf{I}_S = \frac{10}{\sqrt{2}} \exp\left(-j\frac{\pi}{4}\right) + \frac{10}{\sqrt{2}} \exp\left(j\frac{\pi}{4}\right)$$

$$= \frac{10}{\sqrt{2}}\left(\cos\frac{\pi}{4} - \cancel{\text{j}\sin\frac{\pi}{4}} + \cos\frac{\pi}{4} + \cancel{\text{j}\sin\frac{\pi}{4}}\right)$$

$$\mathbf{I}_S = \frac{10}{\sqrt{2}}\left(2\frac{\sqrt{2}}{2}\right) = 10\angle 0^\circ \quad \Leftrightarrow \quad i_S(t) = 10\cos(1000t)\ [\text{A}]$$