ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 22

MWF – 12:00pm

Prof. Umberto Ravaioli

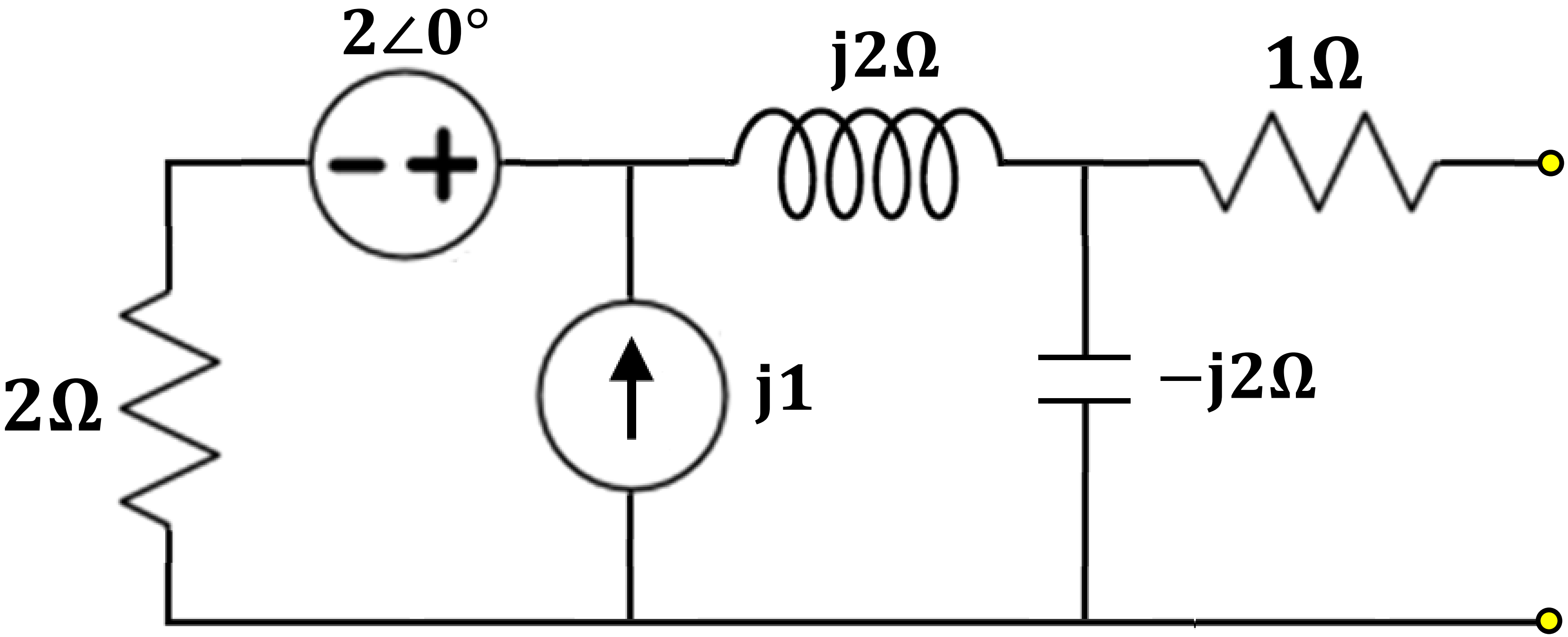
2062 ECE Building

Lecture 22 – Summary

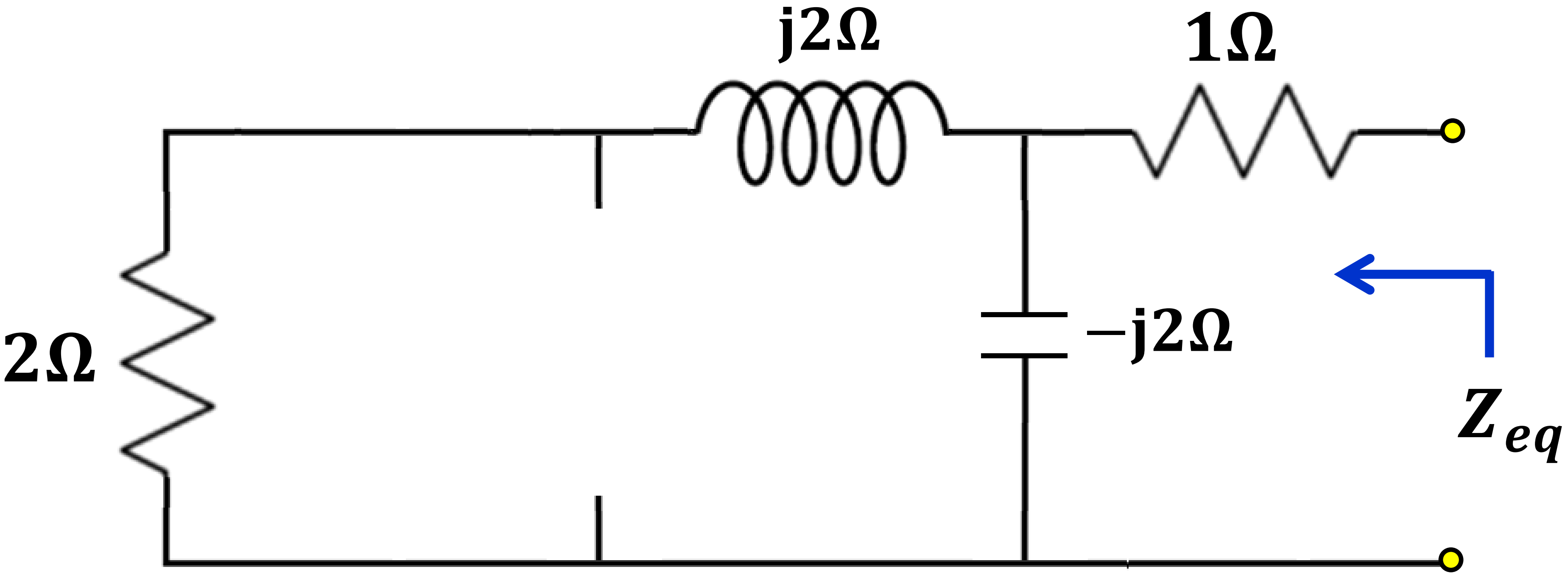
Learning Objectives

1. Introduction to semiconductor diodes
2. Qualitative theory of semiconductors
3. *p*-*n* junction

Find the Thevenin equivalent circuit



Equivalent impedance

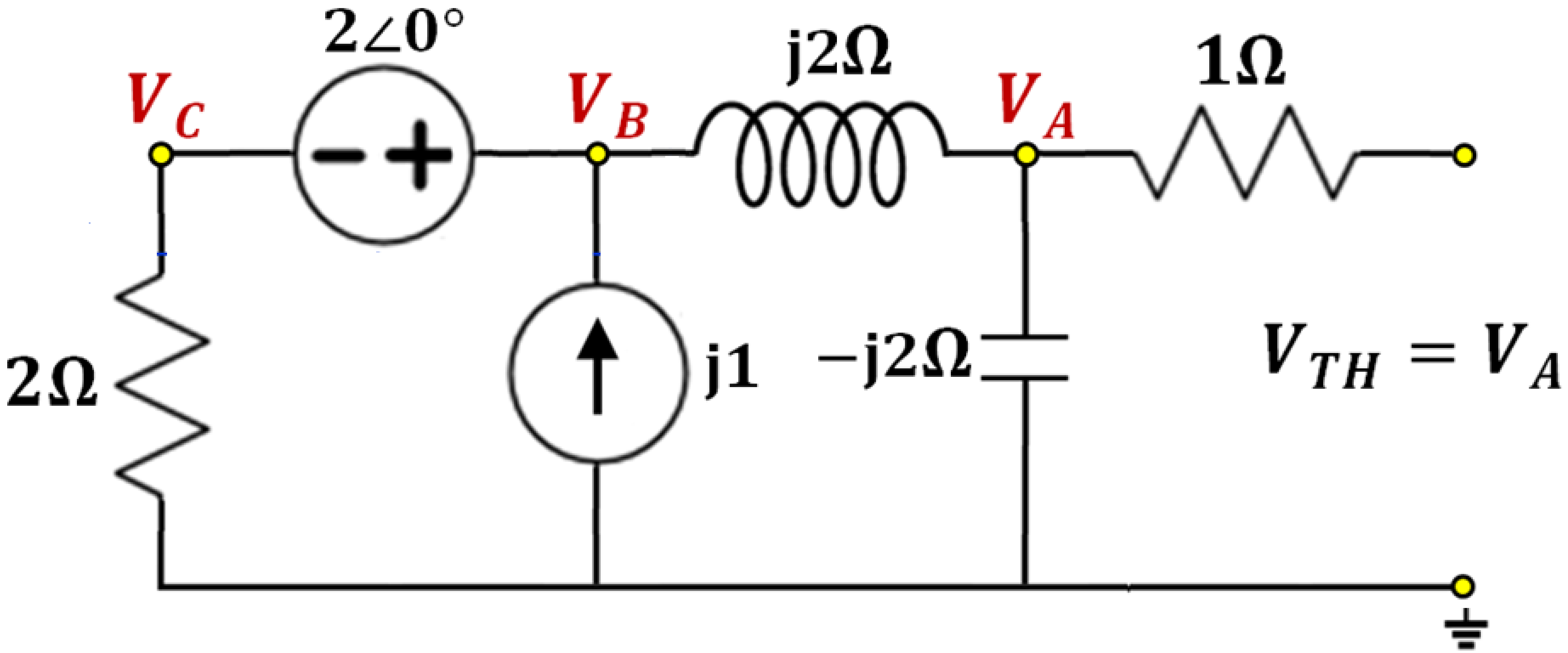


$$Z_{eq} = 1\Omega + (2\Omega + j2\Omega)//(-j2\Omega)$$

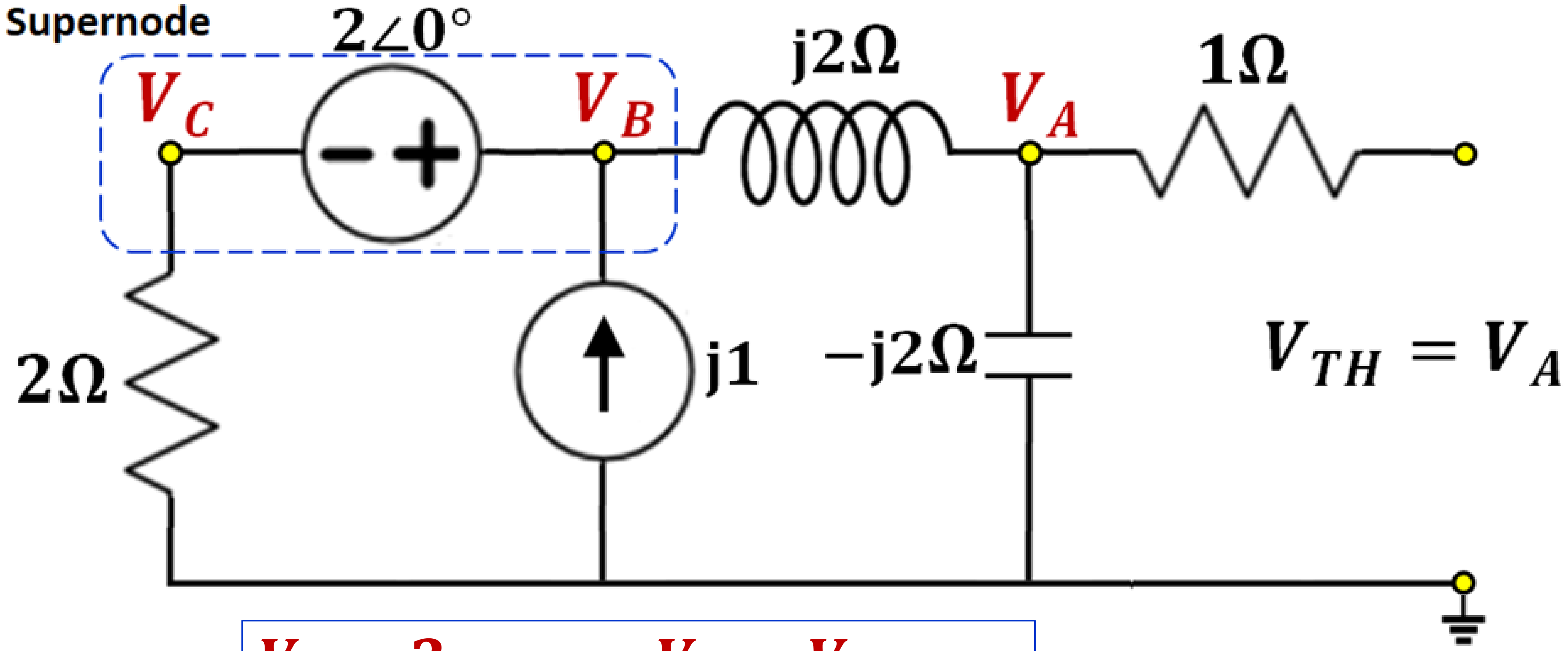
$$Z_{eq} = 1 + \left[\frac{1}{(2 + j2)} + \frac{1}{(-j2)}\right]^{-1} = 1 + \frac{(2 + j2)(-j2)}{2 + j2 - j2}$$

$$Z_{eq} = 1 + \frac{4 - j4}{2} = 3 - j2\ \Omega$$

Find open circuit voltage V_{TH}



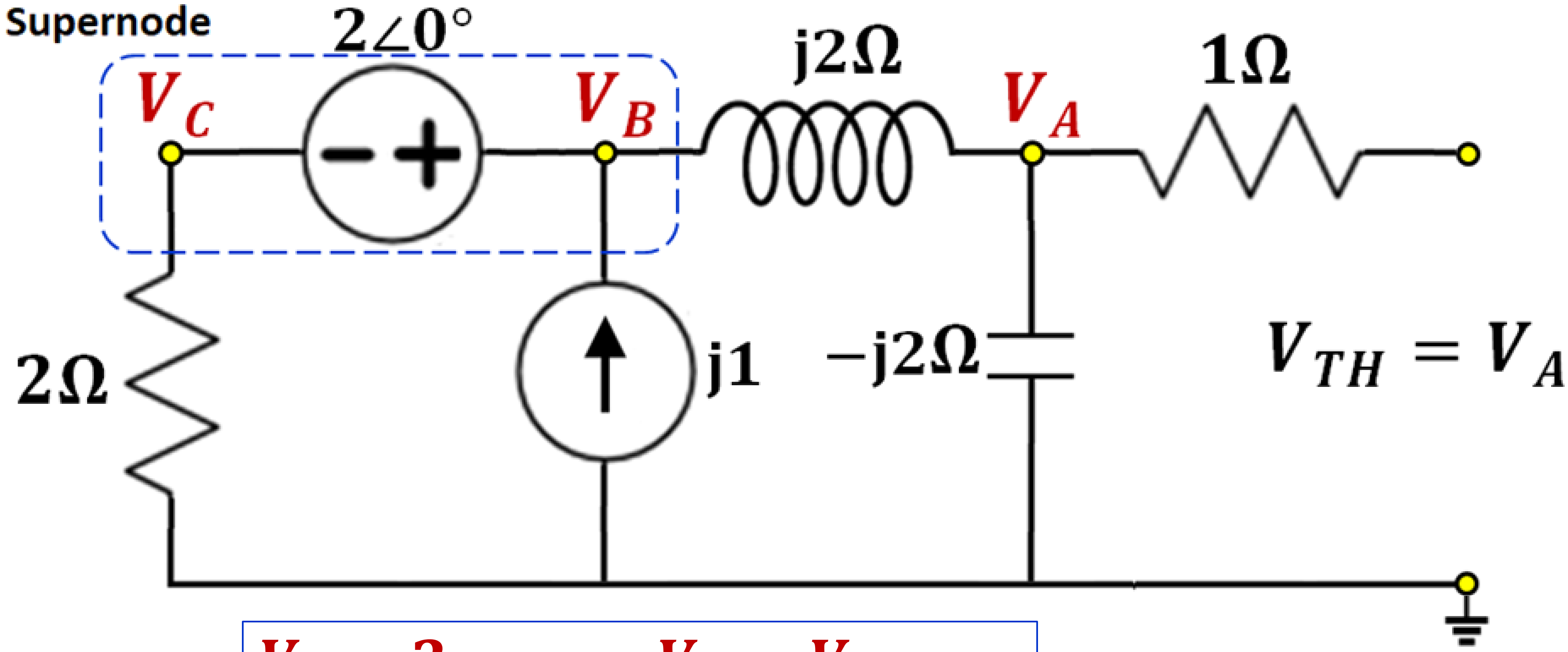
Find open circuit voltage V_{TH}



(Supernode) $$\boxed{\frac{V_B - 2}{2\Omega} - j1 + \frac{V_B - V_A}{j2\Omega} = 0}$$

$$\boxed{V_C = V_B - 2\text{V}}$$

Find open circuit voltage V_{TH}



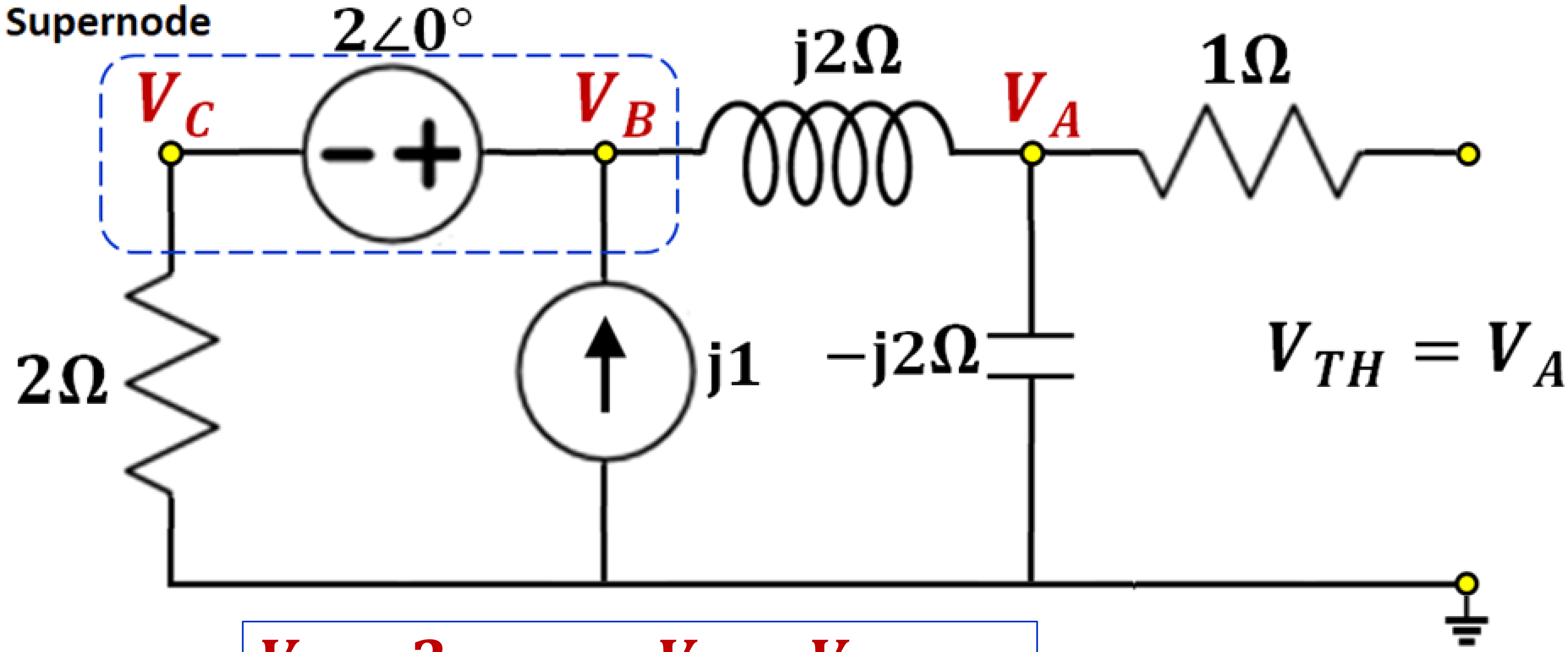
$$V_{TH} = V_A$$

(Supernode) $$\boxed{\frac{V_B - 2}{2\Omega} - \mathrm{j}1 + \frac{V_B - V_A}{\mathrm{j}2\Omega} = 0}$$

$$\boxed{V_C = V_B - 2\mathrm{V}}$$

(Node A) $$\boxed{\frac{V_A - V_B}{\mathrm{j}2\Omega} + \frac{V_A}{-\mathrm{j}2\Omega} = 0} \Rightarrow \boxed{V_B = 0}$$

Find open circuit voltage V_{TH}



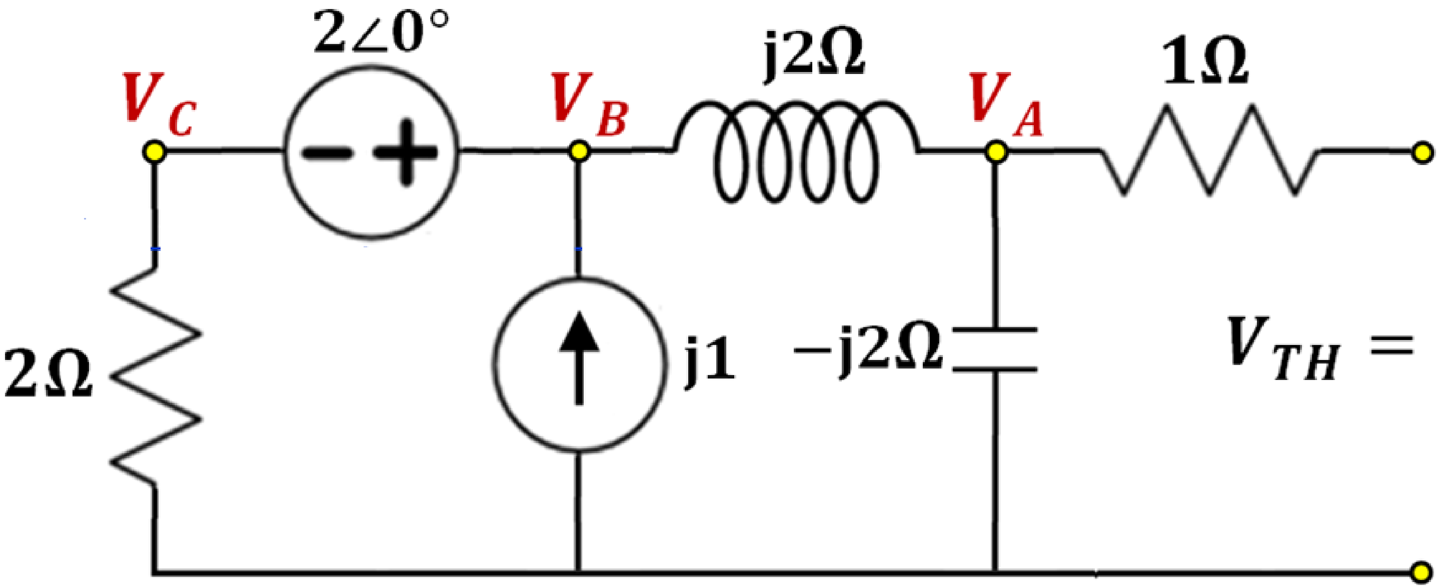
(Supernode) $$\boxed{\frac{V_B - 2}{2\Omega} - j1 + \frac{V_B - V_A}{j2\Omega} = 0}$$

$$\boxed{V_C = V_B - 2\text{V}}$$

(Node A) $$\boxed{\frac{V_A - V_B}{j2\Omega} + \frac{V_A}{-j2\Omega} = 0} \Rightarrow \boxed{V_B = 0}$$

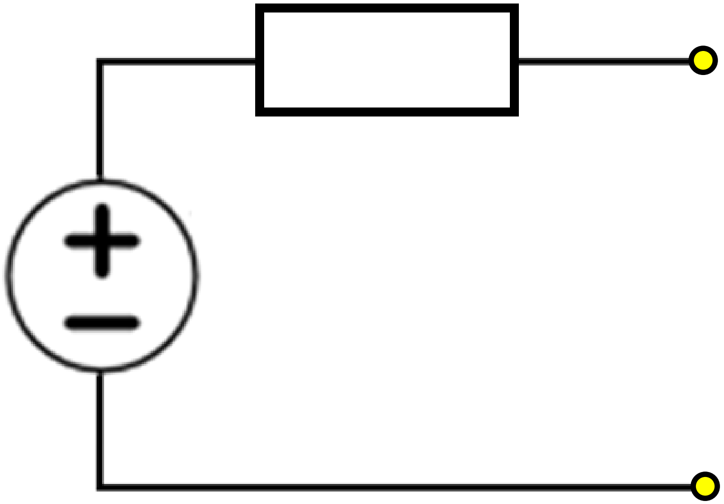
Substituting $V_B = 0$ in Supernode KCL: $$\boxed{V_A = 2 - j2\ \text{V}}$$

Thevenin equivalent circuit



$$V_{TH} = V_A$$

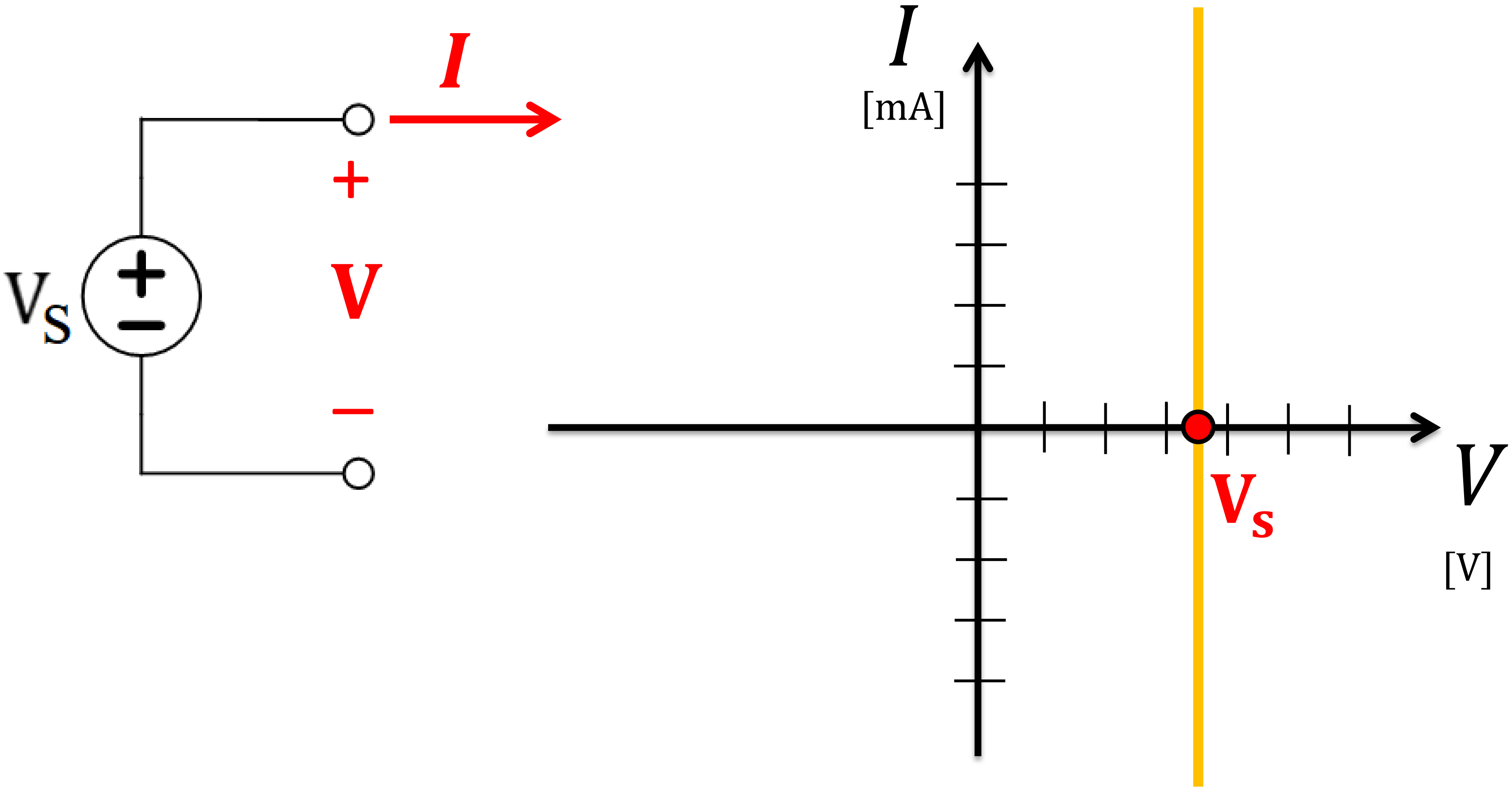
$$Z_{TH} = 3 - j2\ \Omega$$



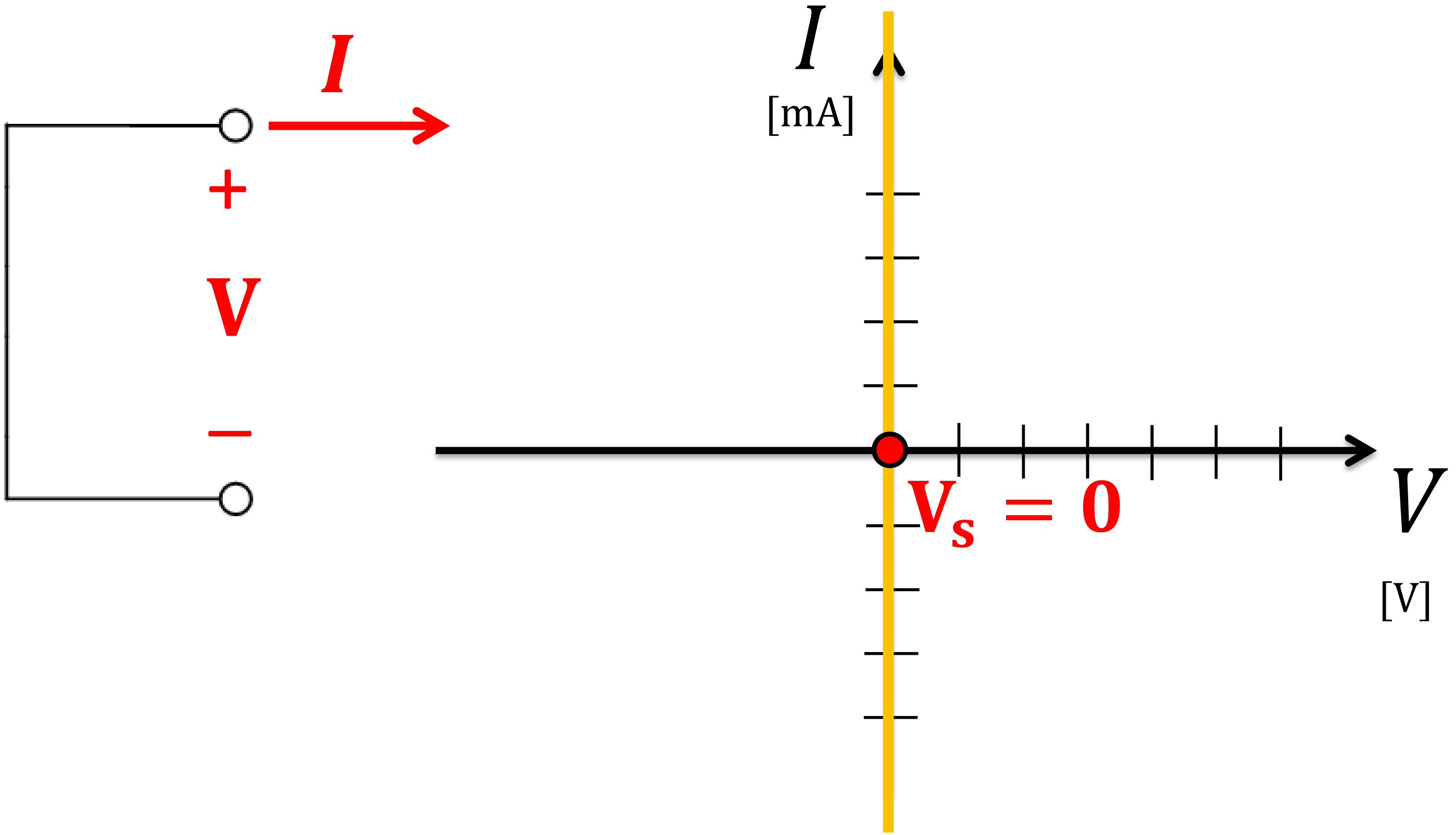
$$V_{TH} = 2 - j2\ \text{V}$$

The Rectifying Diode

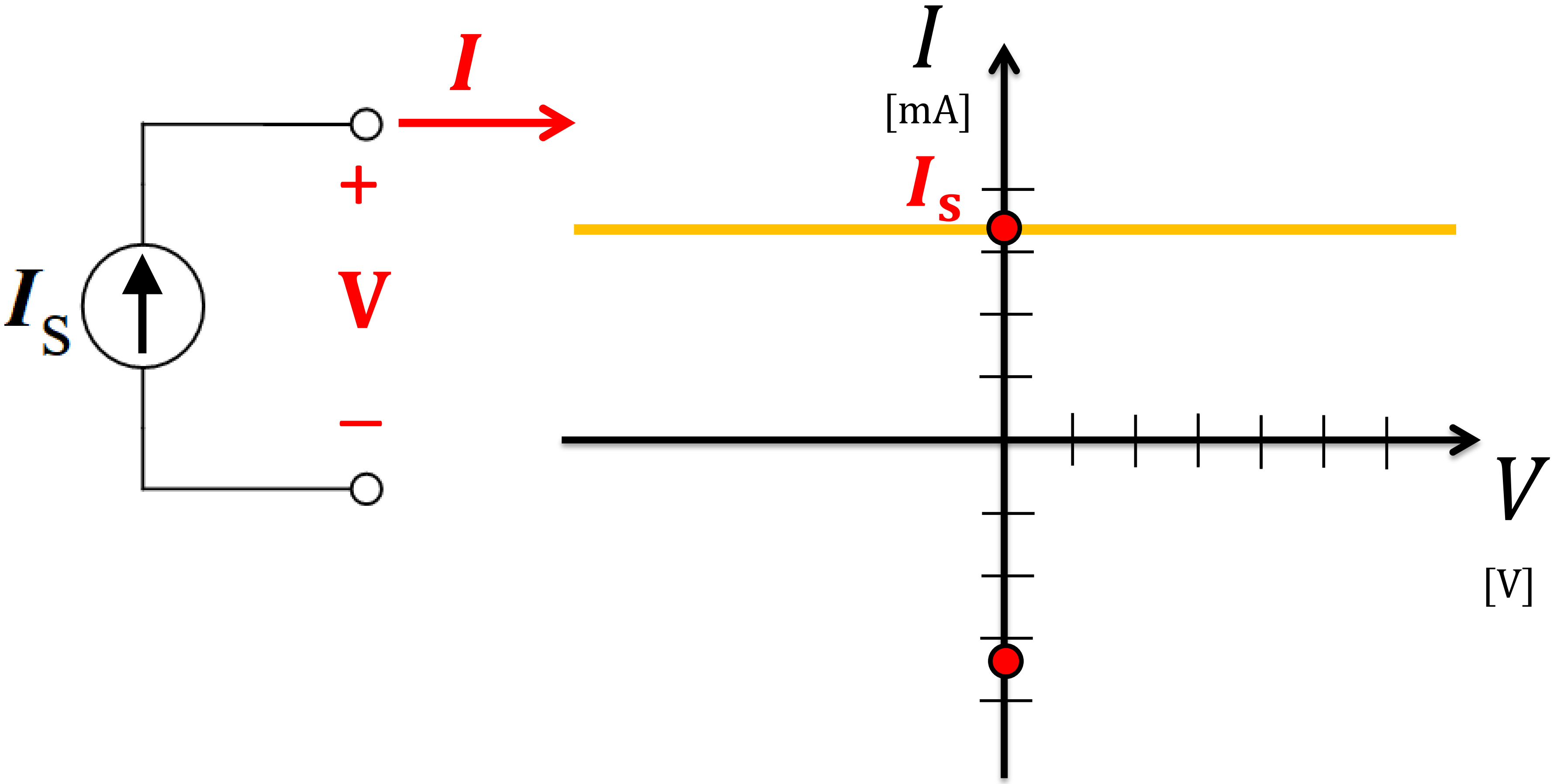
I-V Curve of ideal voltage sources



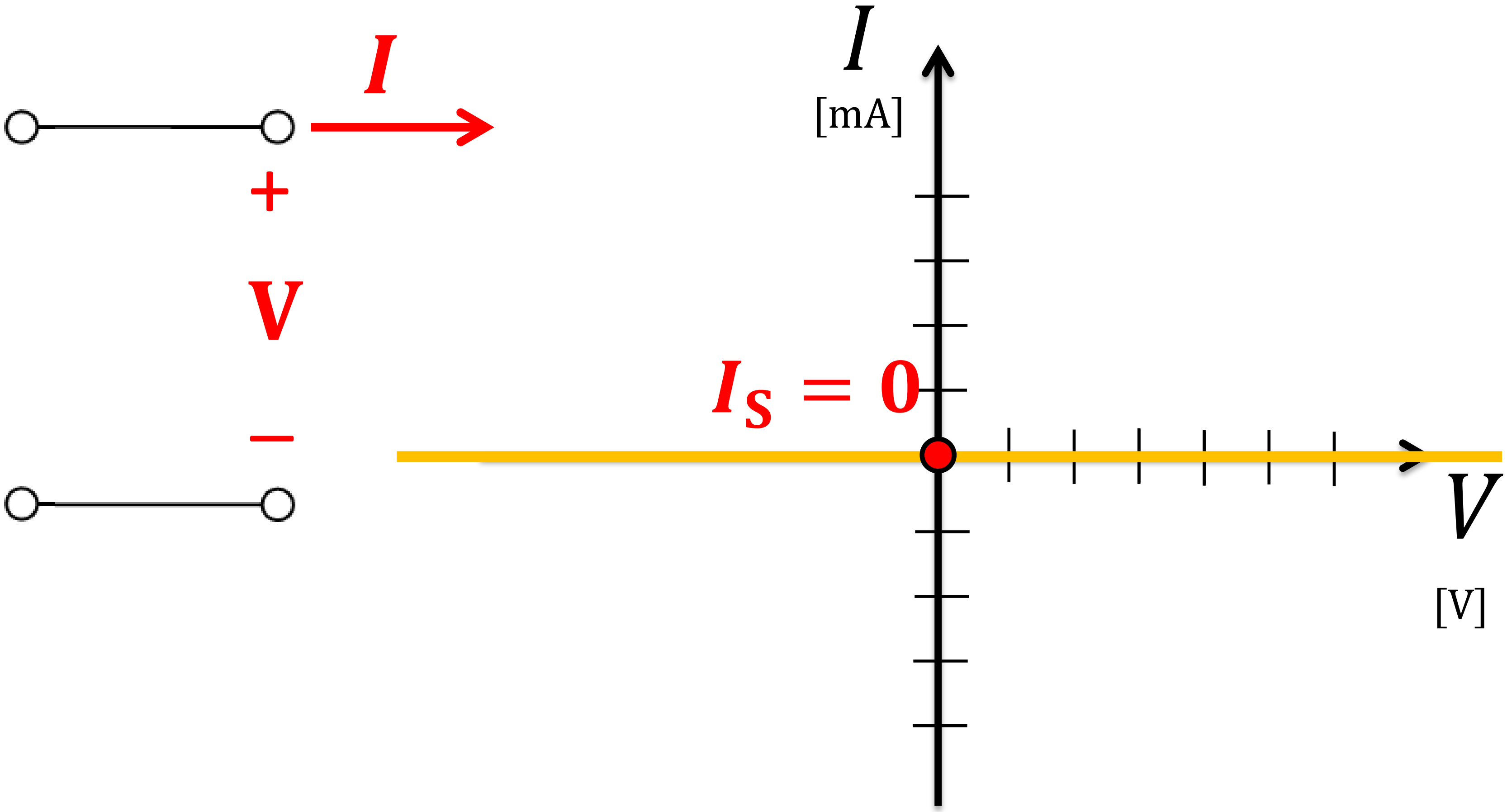
I-V Curve of a short circuit



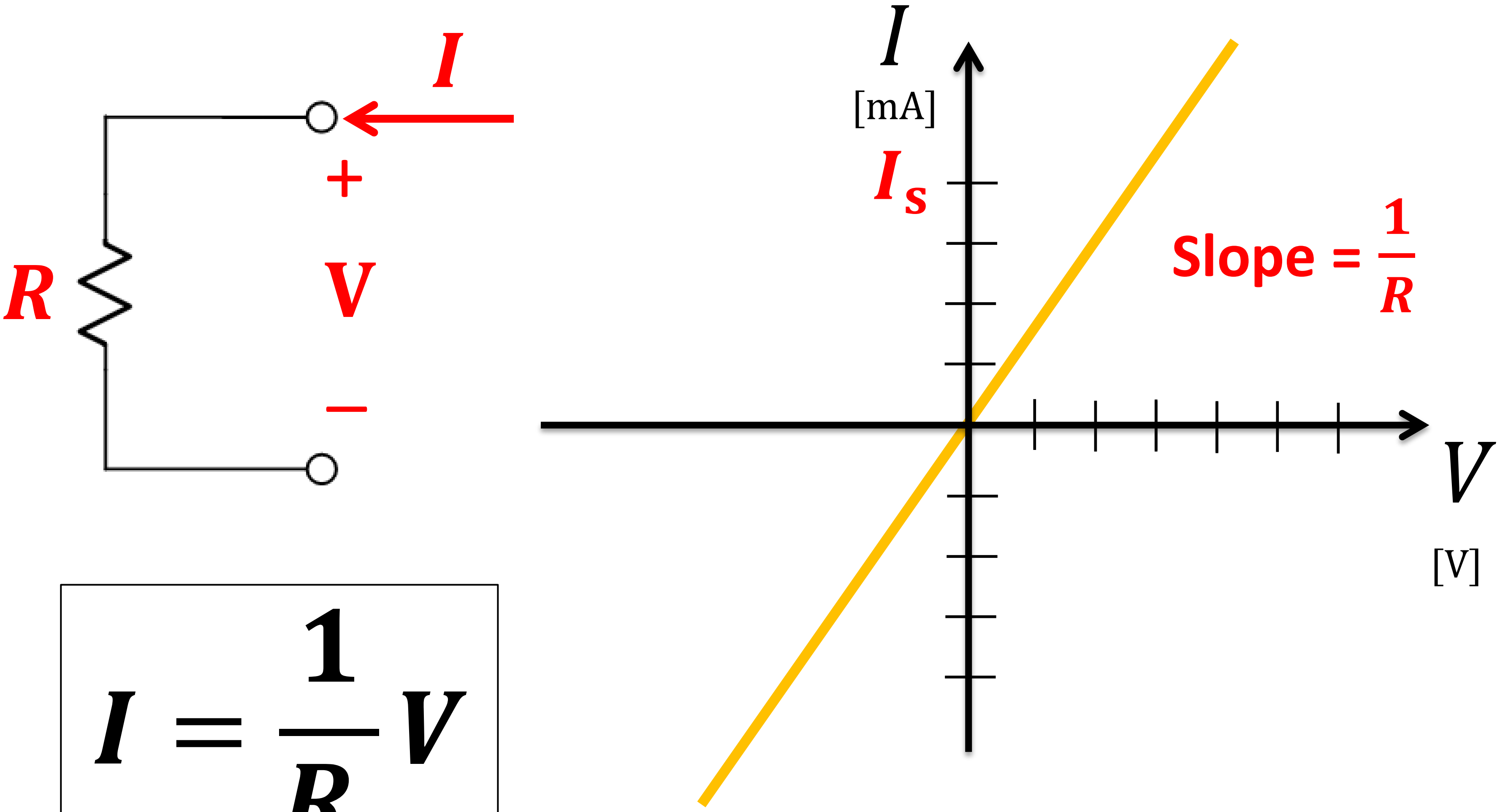
I-V Curve of ideal current sources



I-V Curve of an open circuit

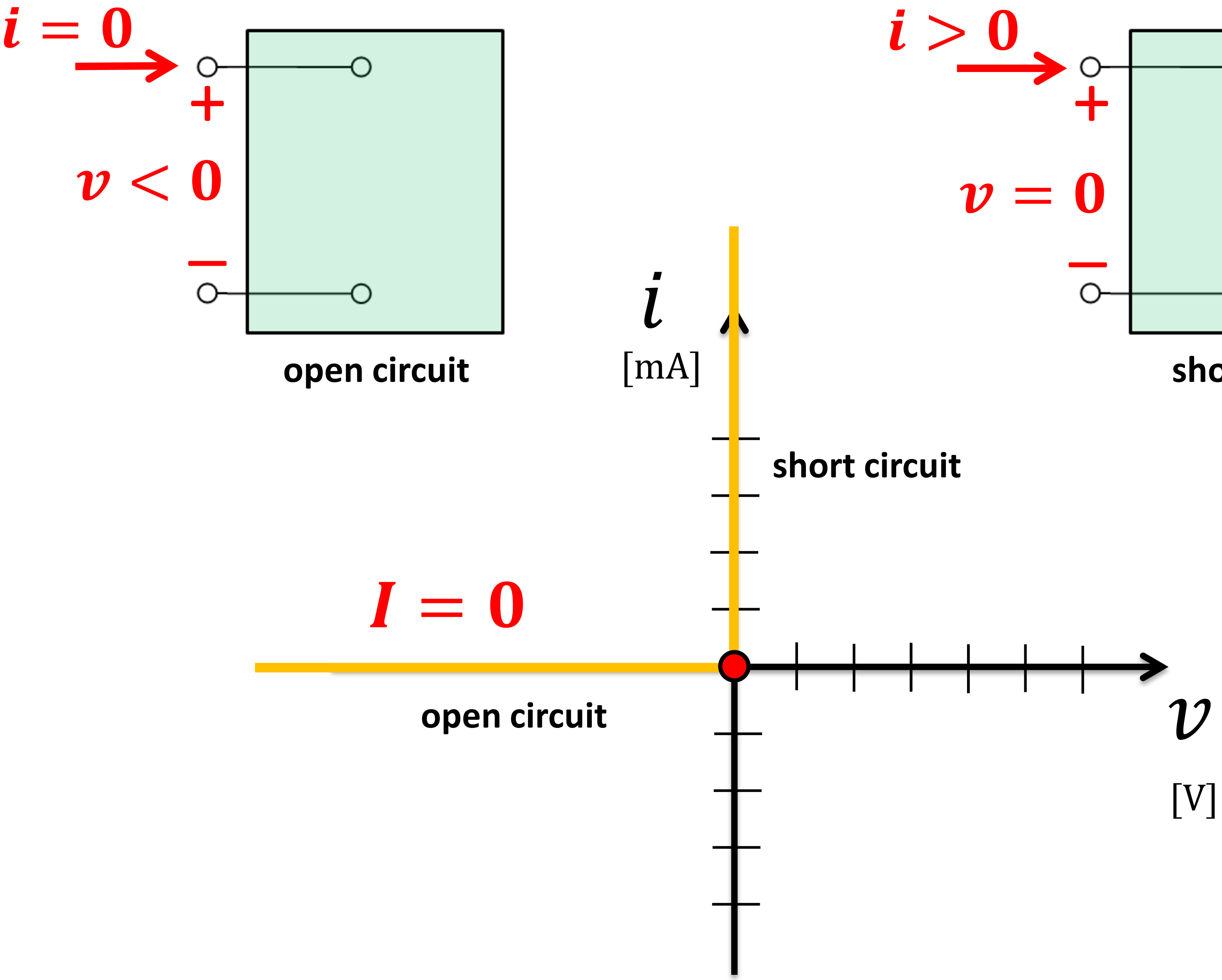


I-V Curve of a resistor



$$\boxed{I = \frac{1}{R} V}$$

Ideal voltage-controlled valve



$i = 0$

$v < 0$

open circuit

$i > 0$

$v = 0$

i

short circuit

i [mA]

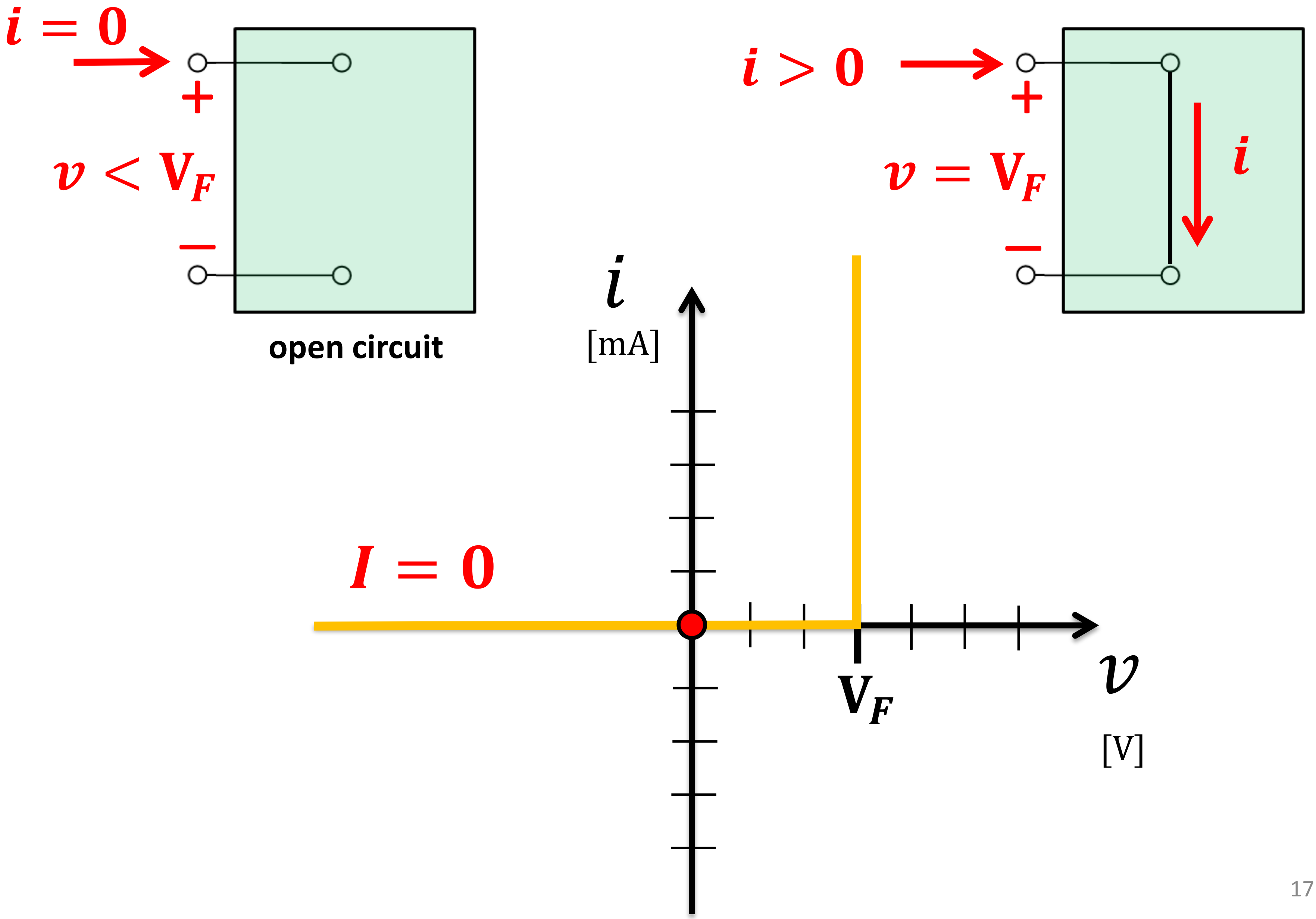
short circuit

$I = 0$

open circuit

v [V]

Voltage-controlled valve with threshold



$i = 0$

$+$

$v < \mathbf{V}_F$

$-$

open circuit

$i > 0$

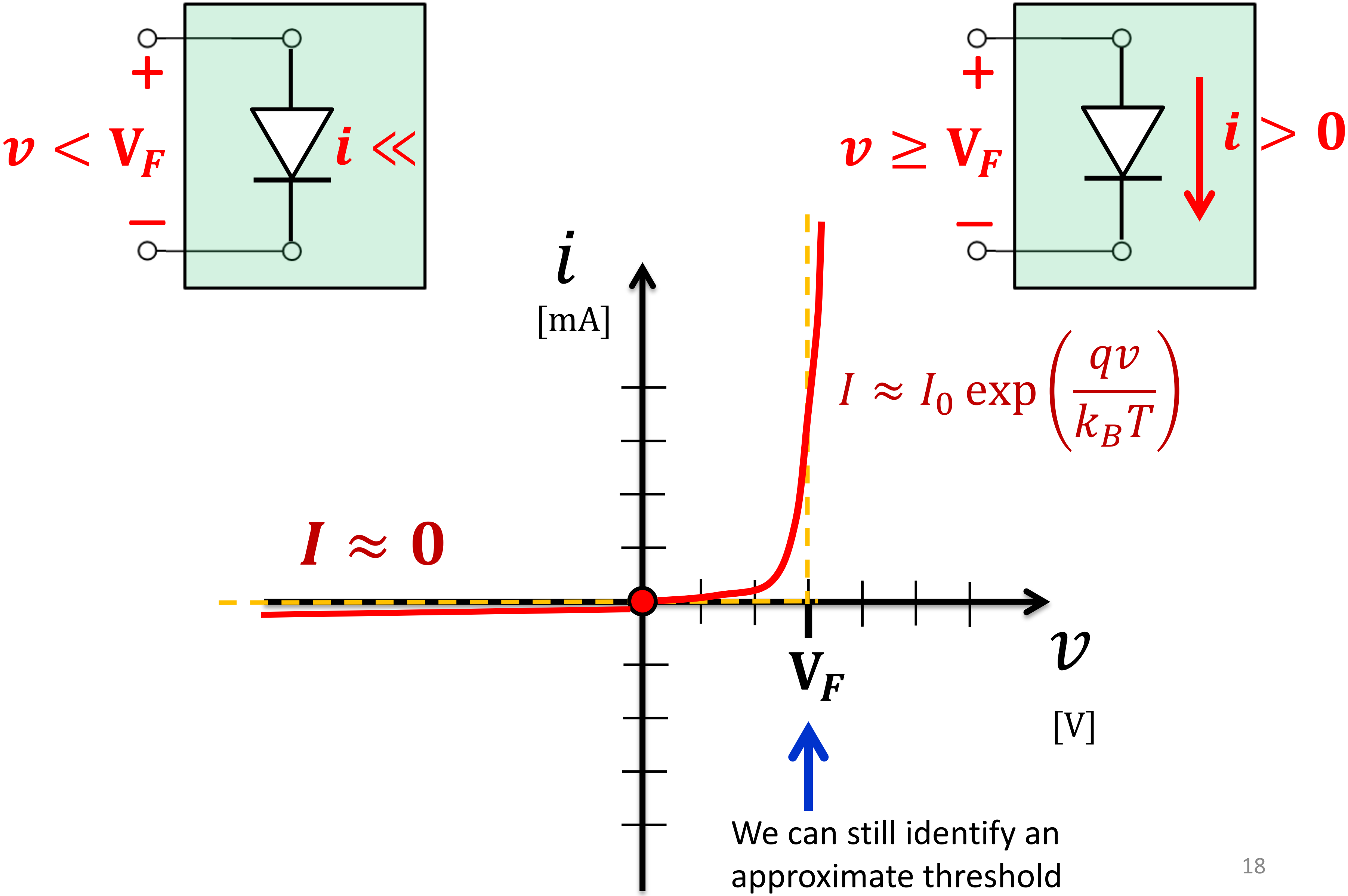
$+$

$v = \mathbf{V}_F$

i

$-$

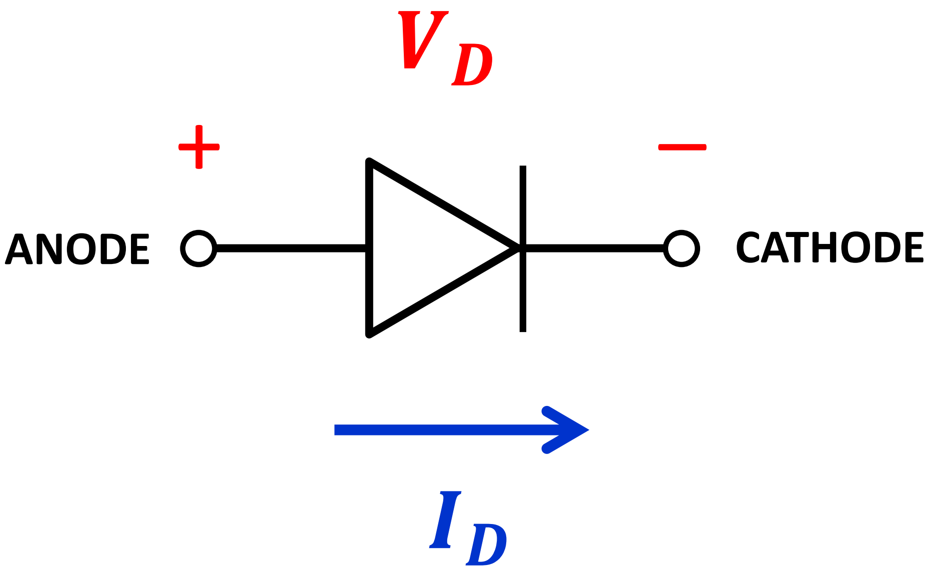
Realistic diode valve



We can still identify an approximate threshold

Diode = a two-terminal semiconductor device which allows current to flow only in one direction

Circuit symbol of a Diode



Circuit model

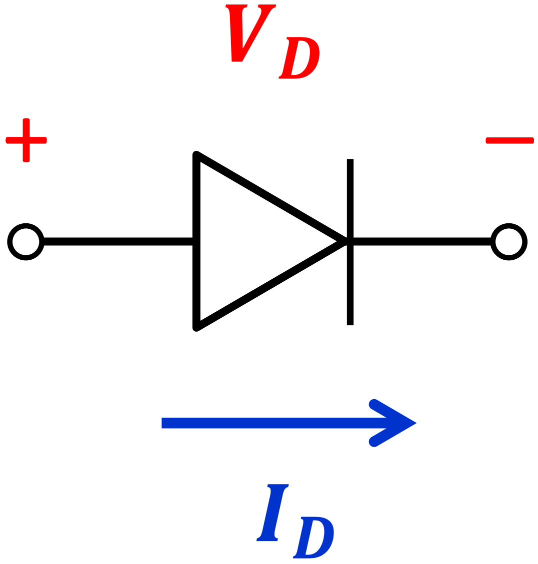
k_B = Boltzmann constant

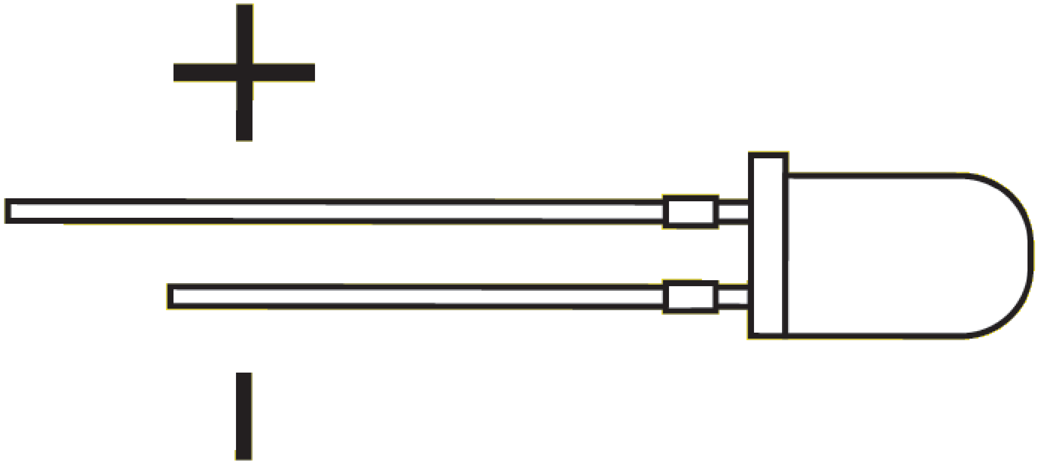
T = temperature [K]

I_0 = reverse current

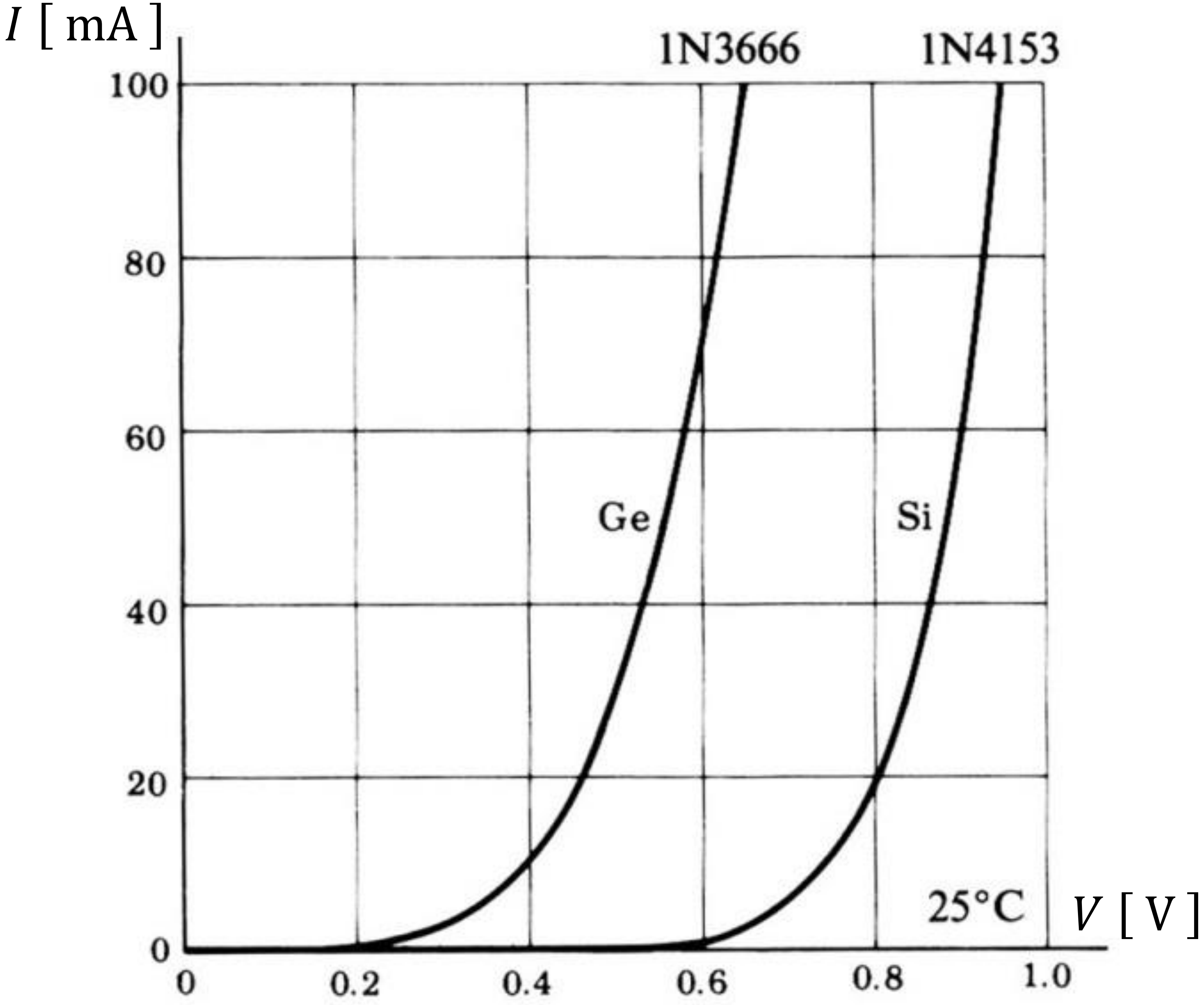
n = non-ideality factor (experimental)

$$I = I_0\left[\exp\left(\frac{qv}{nk_BT}\right) - 1\right]$$

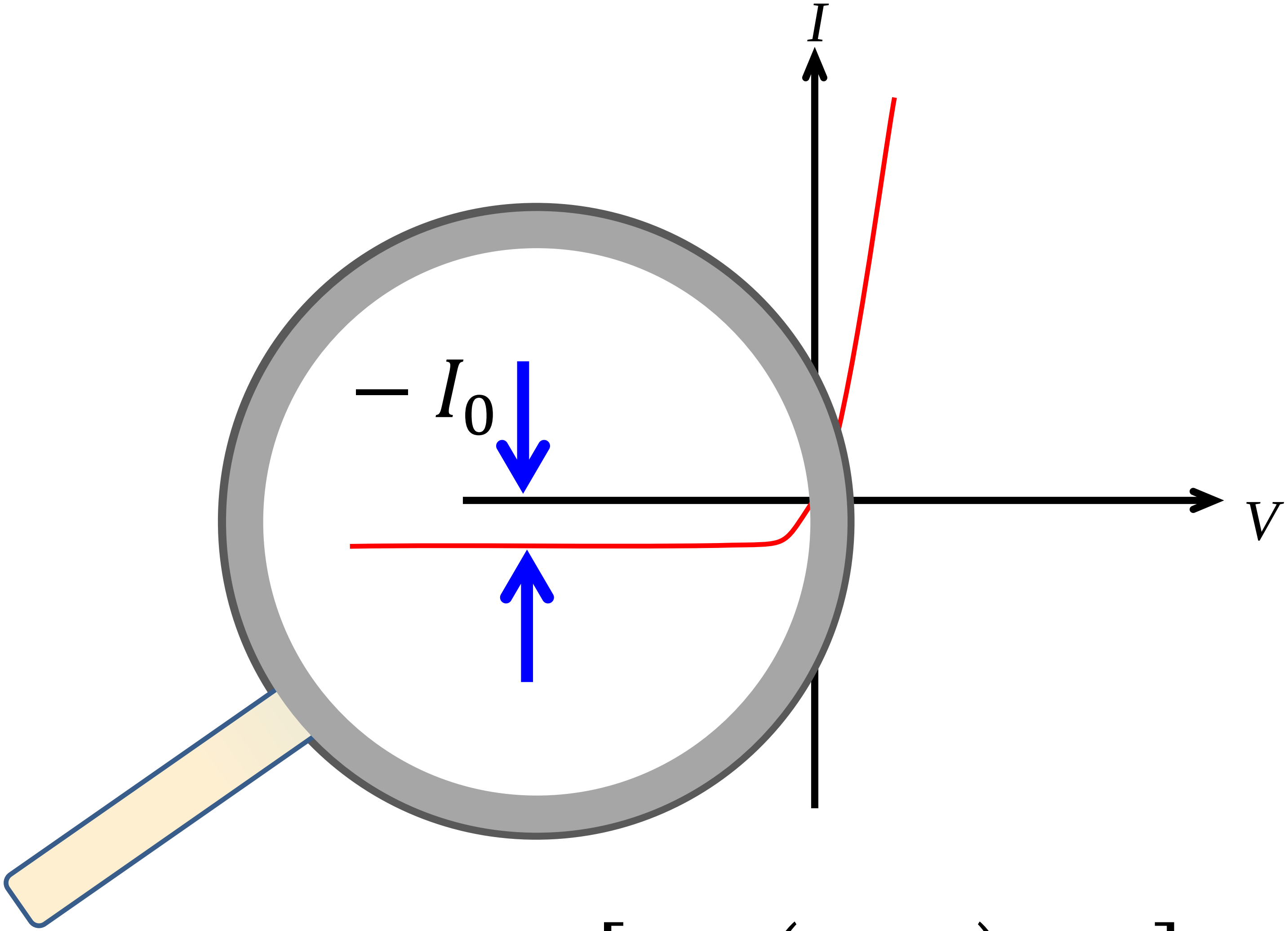

V_D
+
−
I_D


+
−

Different semiconductor materials have different thresholds



I_0 **is the reverse saturation current and it is very small**

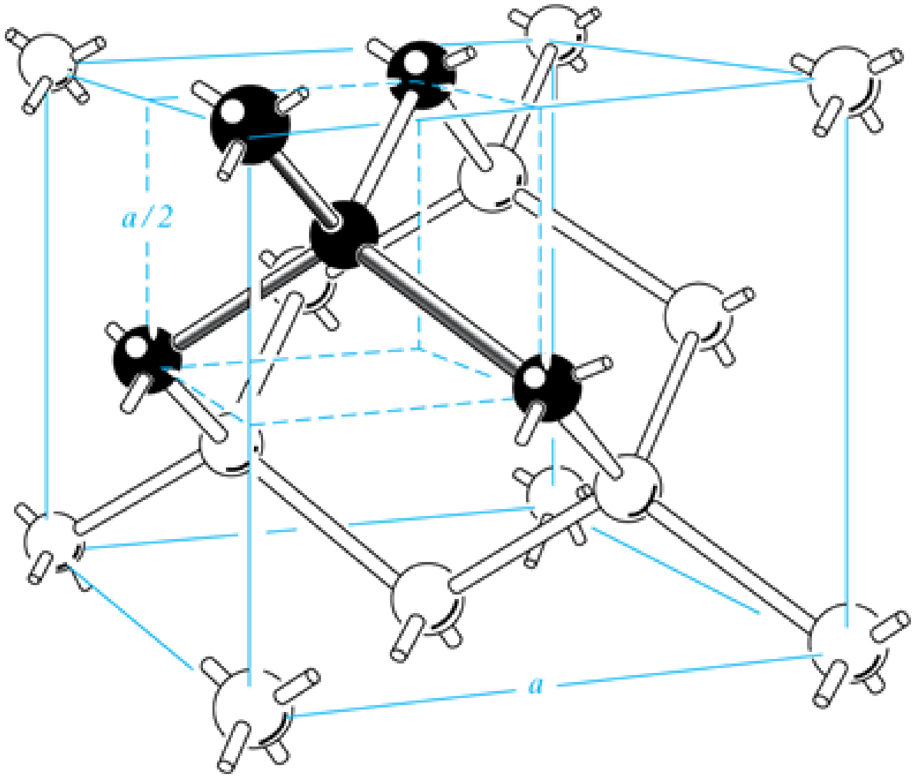


$\boxed{v \ll 0}$

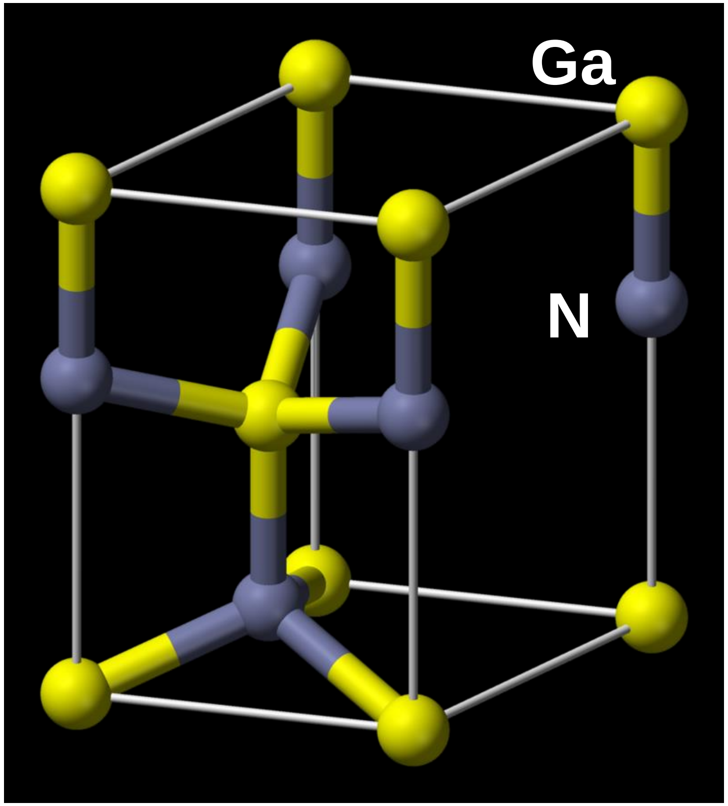
$$I = I_0\left[\exp\left(\frac{qv}{nk_BT}\right) - 1\right] \approx -I_0$$

Brief introduction to semiconductors

Semiconductors are materials with regular periodic crystalline structure, more commonly the so-called diamond structure for elements in column IV of the periodic table (e.g., Si, Ge) or pairs of elements in columns III and V (or II and VI) of the table, in the similar Zinc-blende structure (e.g., GaAs, InP). Another important crystal structure is Wurtzite (e.g., GaN)

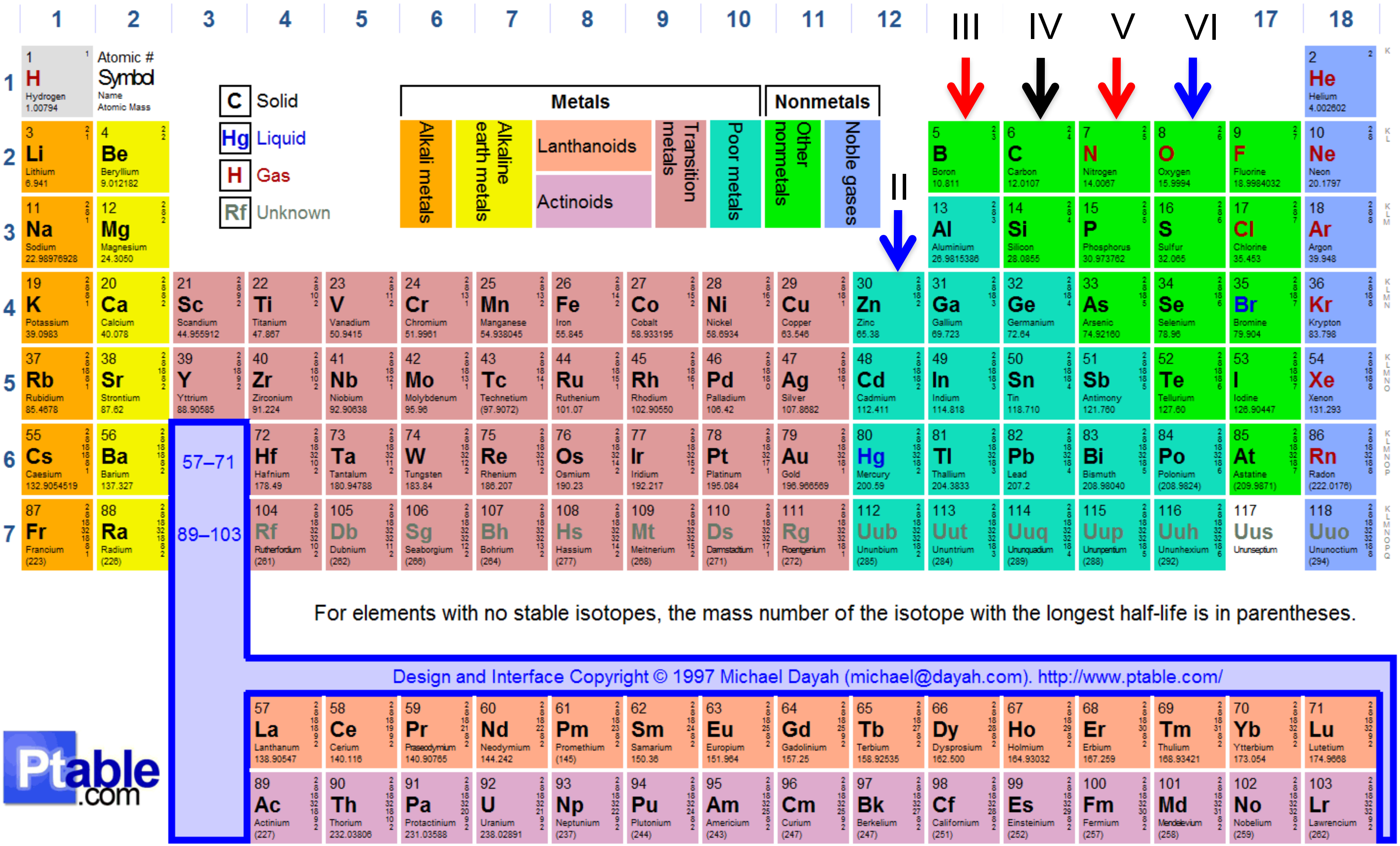


Crystal model for diamond or Zinc-blende



Crystal model for Wurtzite

Periodic Table of Elements



	1	2	3	4	5	6	7	8	9	10	11	12	III	IV	V	VI	17	18	
1	1 **H** Hydrogen 1.00794	Atomic # Symbol Name Atomic Mass																2 **He** Helium 4.002602	K
2	3 **Li** Lithium 6.941	4 **Be** Beryllium 9.012182											5 **B** Boron 10.811	6 **C** Carbon 12.0107	7 **N** Nitrogen 14.0067	8 **O** Oxygen 15.9994	9 **F** Fluorine 18.9984032	10 **Ne** Neon 20.1797	K L
3	11 **Na** Sodium 22.98976928	12 **Mg** Magnesium 24.3050										II	13 **Al** Aluminium 26.9815386	14 **Si** Silicon 28.0855	15 **P** Phosphorus 30.973762	16 **S** Sulfur 32.065	17 **Cl** Chlorine 35.453	18 **Ar** Argon 39.948	K L M
4	19 **K** Potassium 39.0983	20 **Ca** Calcium 40.078	21 **Sc** Scandium 44.955912	22 **Ti** Titanium 47.867	23 **V** Vanadium 50.9415	24 **Cr** Chromium 51.9961	25 **Mn** Manganese 54.938045	26 **Fe** Iron 55.845	27 **Co** Cobalt 58.933195	28 **Ni** Nickel 58.6934	29 **Cu** Copper 63.546	30 **Zn** Zinc 65.38	31 **Ga** Gallium 69.723	32 **Ge** Germanium 72.64	33 **As** Arsenic 74.92160	34 **Se** Selenium 78.96	35 **Br** Bromine 79.904	36 **Kr** Krypton 83.798	K L M N
5	37 **Rb** Rubidium 85.4678	38 **Sr** Strontium 87.62	39 **Y** Yttrium 88.90585	40 **Zr** Zirconium 91.224	41 **Nb** Niobium 92.90638	42 **Mo** Molybdenum 95.96	43 **Tc** Technetium (97.9072)	44 **Ru** Ruthenium 101.07	45 **Rh** Rhodium 102.90550	46 **Pd** Palladium 106.42	47 **Ag** Silver 107.8682	48 **Cd** Cadmium 112.411	49 **In** Indium 114.818	50 **Sn** Tin 118.710	51 **Sb** Antimony 121.760	52 **Te** Tellurium 127.60	53 **I** Iodine 126.90447	54 **Xe** Xenon 131.293	K L M N O
6	55 **Cs** Caesium 132.9054519	56 **Ba** Barium 137.327	57–71	72 **Hf** Hafnium 178.49	73 **Ta** Tantalum 180.94788	74 **W** Tungsten 183.84	75 **Re** Rhenium 186.207	76 **Os** Osmium 190.23	77 **Ir** Iridium 192.217	78 **Pt** Platinum 195.084	79 **Au** Gold 196.966569	80 **Hg** Mercury 200.59	81 **Tl** Thallium 204.3833	82 **Pb** Lead 207.2	83 **Bi** Bismuth 208.98040	84 **Po** Polonium (208.9824)	85 **At** Astatine (209.9871)	86 **Rn** Radon (222.0176)	K L M N O P
7	87 **Fr** Francium (223)	88 **Ra** Radium (226)	89–103	104 **Rf** Rutherfordium (261)	105 **Db** Dubnium (262)	106 **Sg** Seaborgium (266)	107 **Bh** Bohrium (264)	108 **Hs** Hassium (277)	109 **Mt** Meitnerium (268)	110 **Ds** Darmstadtium (271)	111 **Rg** Roentgenium (272)	112 **Uub** Ununbium (285)	113 **Uut** Ununtrium (284)	114 **Uuq** Ununquadium (289)	115 **Uup** Ununpentium (288)	116 **Uuh** Ununhexium (292)	117 **Uus** Ununseptium	118 **Uuo** Ununoctium (294)	K L M N O P Q
				57 **La** Lanthanum 138.90547	58 **Ce** Cerium 140.116	59 **Pr** Praseodymium 140.90765	60 **Nd** Neodymium 144.242	61 **Pm** Promethium (145)	62 **Sm** Samarium 150.36	63 **Eu** Europium 151.964	64 **Gd** Gadolinium 157.25	65 **Tb** Terbium 158.92535	66 **Dy** Dysprosium 162.500	67 **Ho** Holmium 164.93032	68 **Er** Erbium 167.259	69 **Tm** Thulium 168.93421	70 **Yb** Ytterbium 173.054	71 **Lu** Lutetium 174.9668	
				89 **Ac** Actinium (227)	90 **Th** Thorium 232.03806	91 **Pa** Protactinium 231.03588	92 **U** Uranium 238.02891	93 **Np** Neptunium (237)	94 **Pu** Plutonium (244)	95 **Am** Americium (243)	96 **Cm** Curium (247)	97 **Bk** Berkelium (247)	98 **Cf** Californium (251)	99 **Es** Einsteinium (252)	100 **Fm** Fermium (257)	101 **Md** Mendelevium (258)	102 **No** Nobelium (259)	103 **Lr** Lawrencium (262)	

C Solid
Hg Liquid
H Gas
Rf Unknown

Metals: Alkali metals, Alkaline earth metals, Lanthanoids, Actinoids, Transition metals, Poor metals
Nonmetals: Other nonmetals, Noble gases

For elements with no stable isotopes, the mass number of the isotope with the longest half-life is in parentheses.

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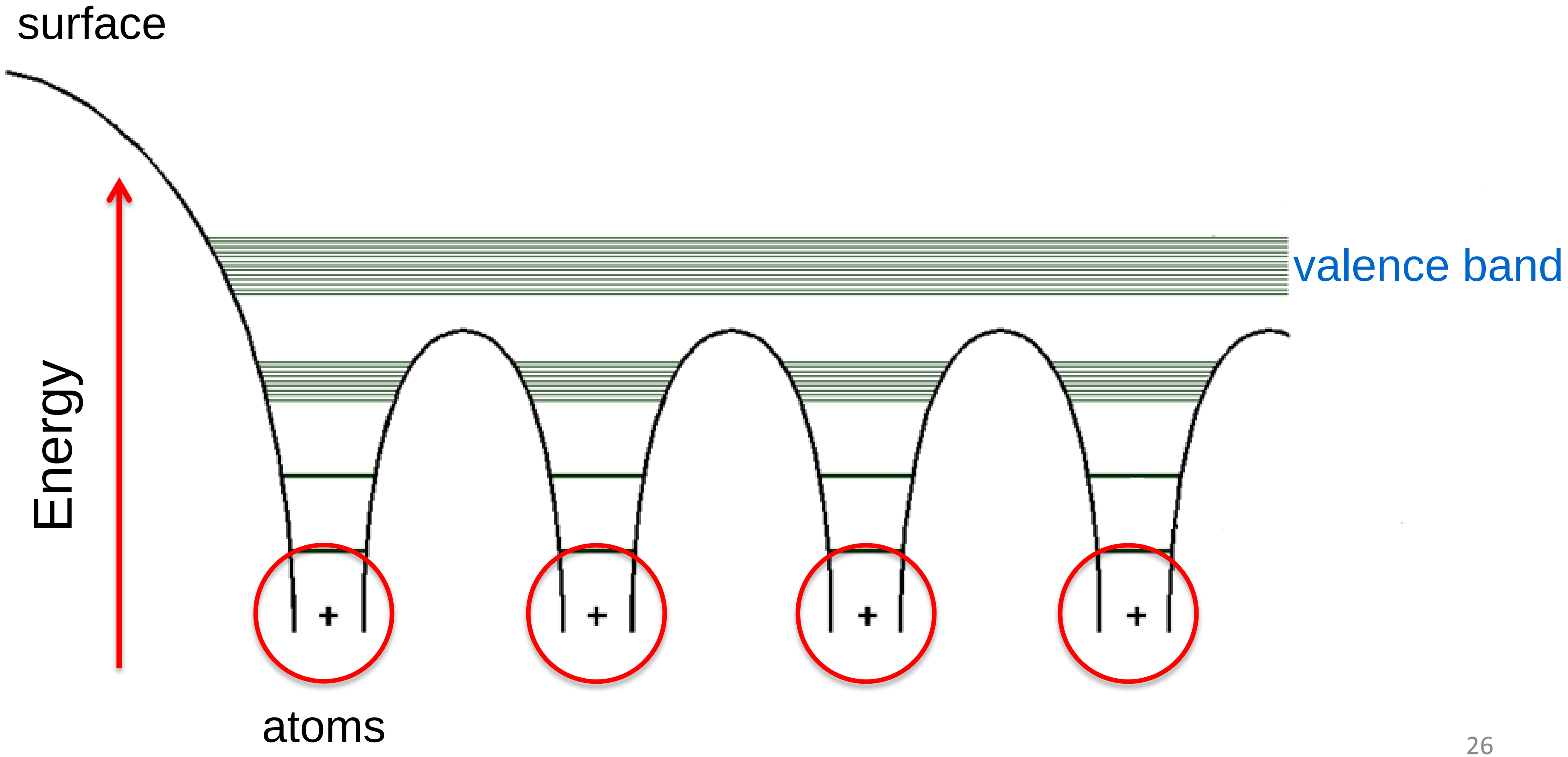
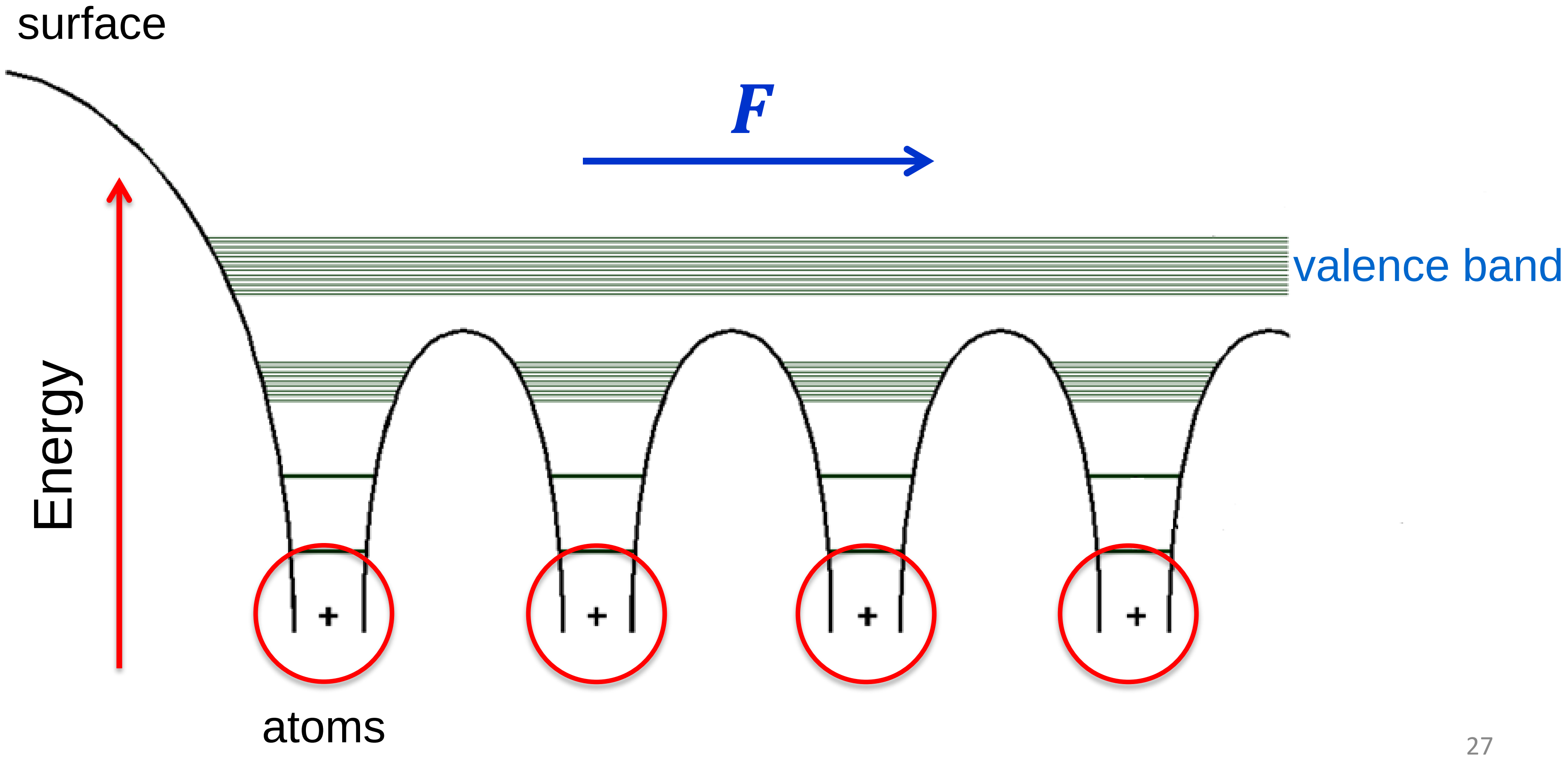
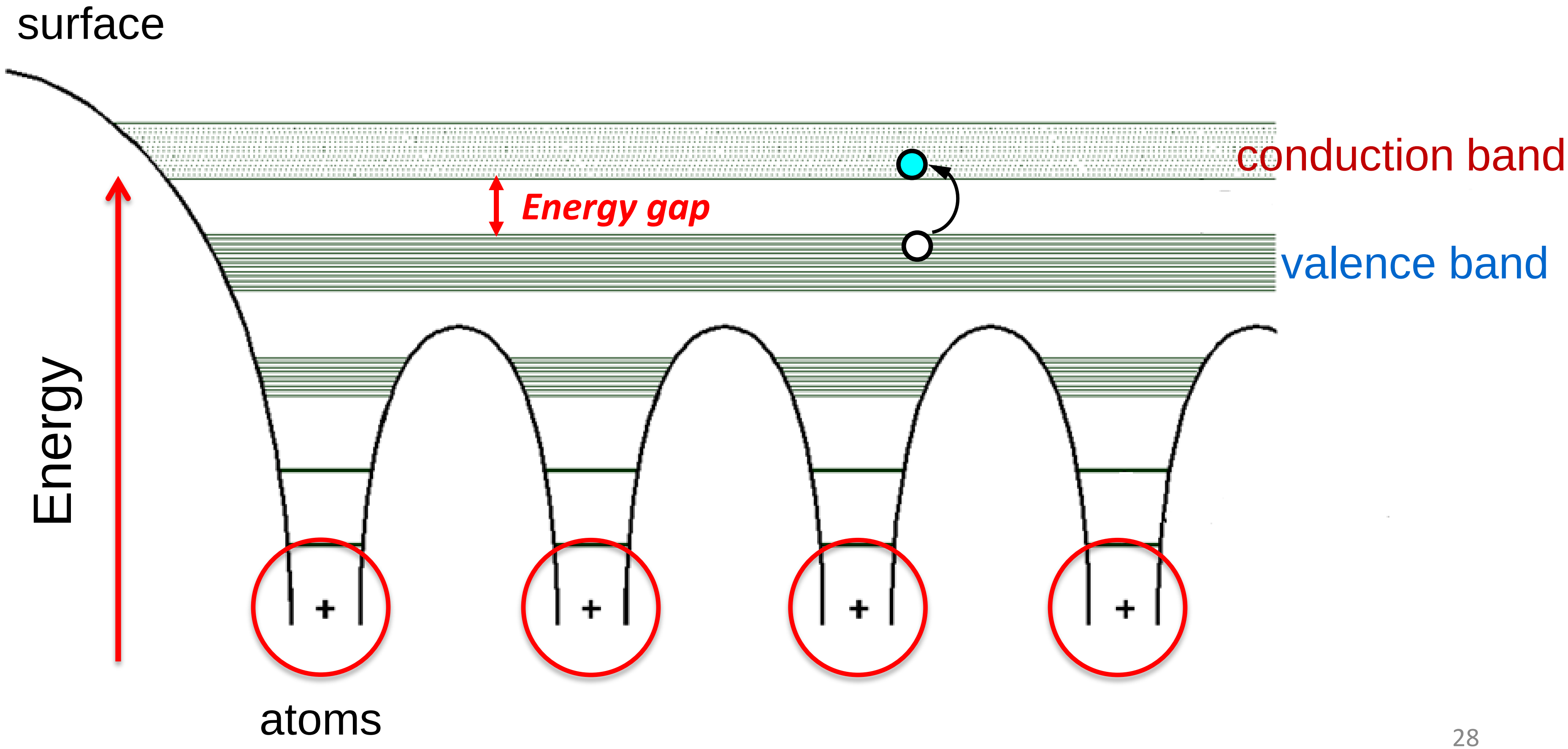
In these crystals, bonding is very strong and it is due to electron orbitals in the higher atomic states, which are shared by neighboring atoms. These states form an "energy band" called the *valence band*, which extends to the whole crystal structure.



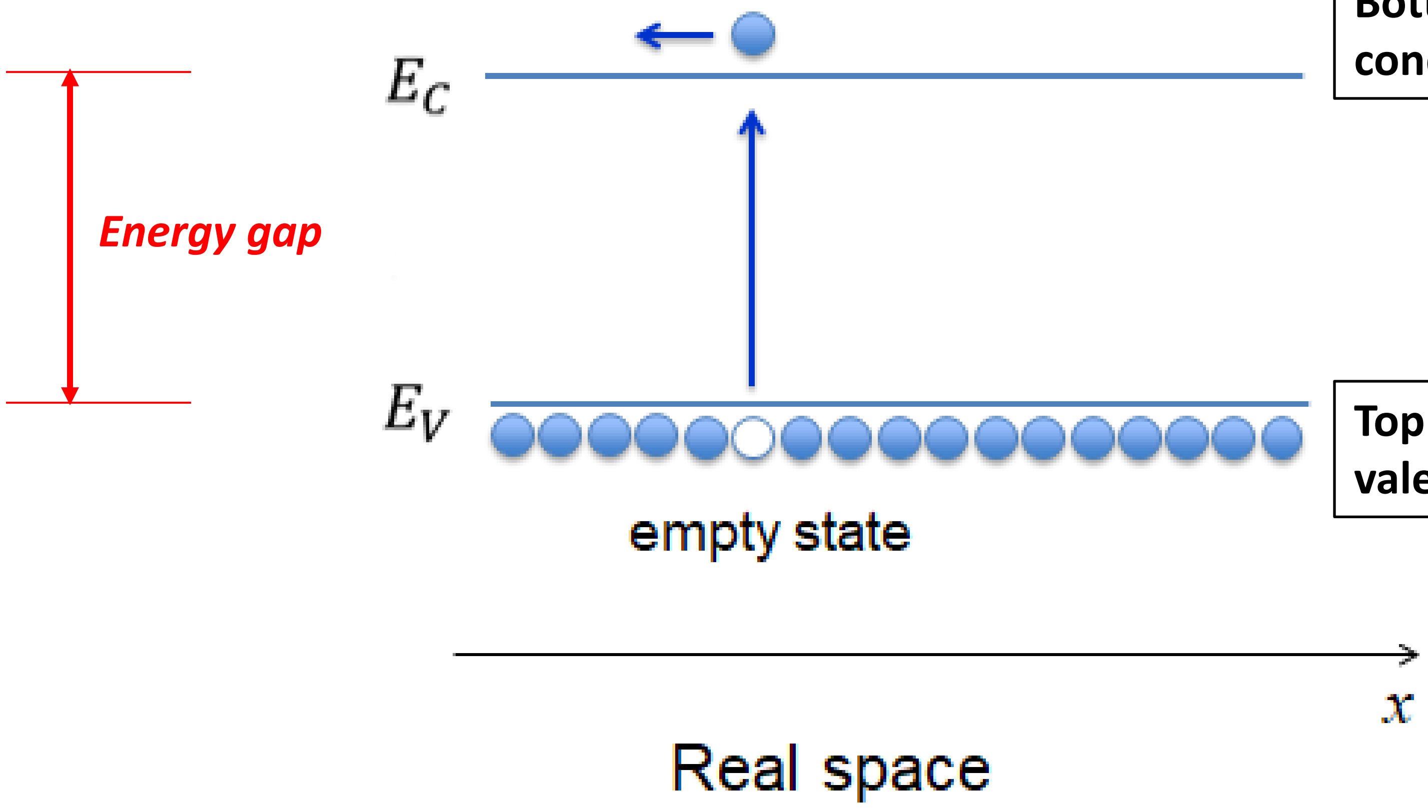
Electrons may exchange places freely in the valence band, but if all energy/momentum states are occupied, no net current can flow if an electric field is applied: for any electron with an allowed momentum (velocity) state, in a full band there is always another electron with opposite momentum.



The atomic excited states also form an extended energy band, called the *conduction band*. Electrons may “jump” to the conduction band from the valence band, if they collect sufficient thermal energy to overcome the energy gap.



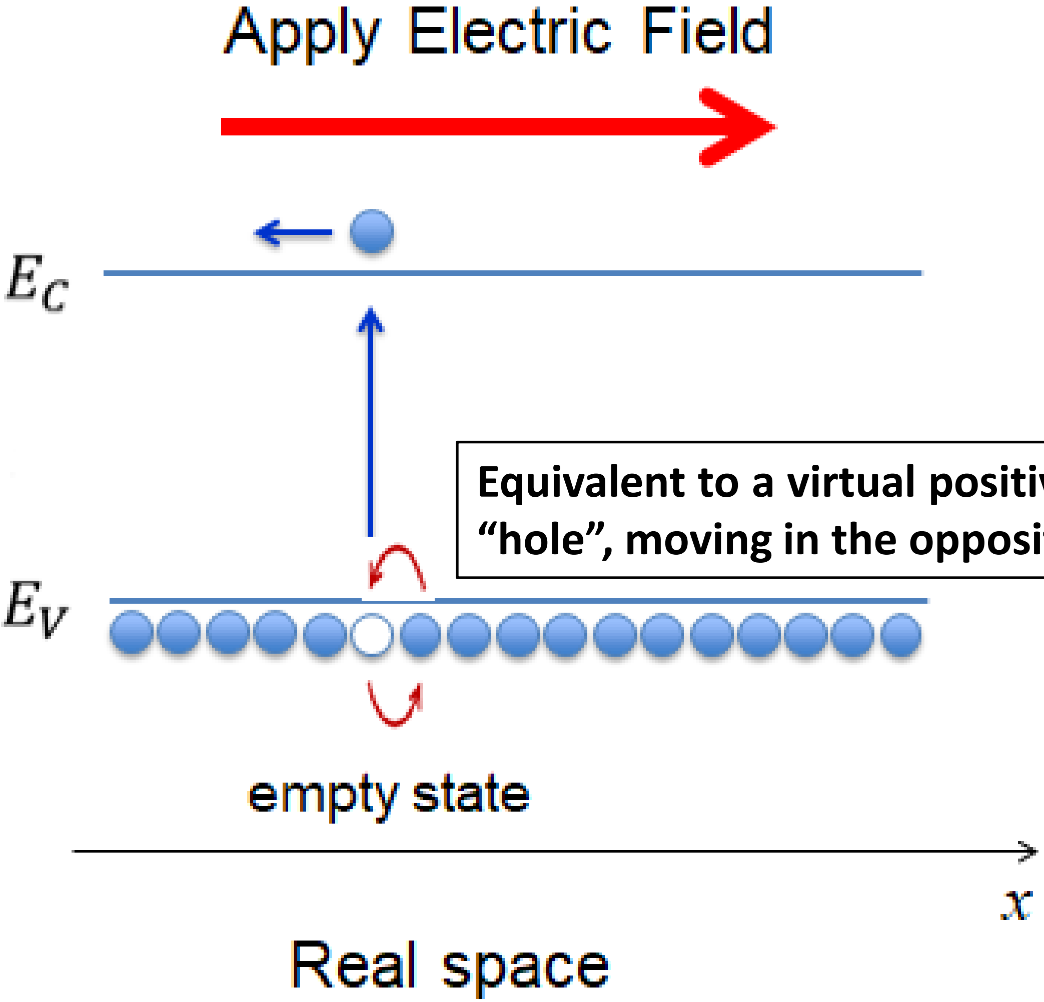
An electron jumping to the conduction band leaves behind an empty energy/momentum state in the valence band



Bottom of the conduction band

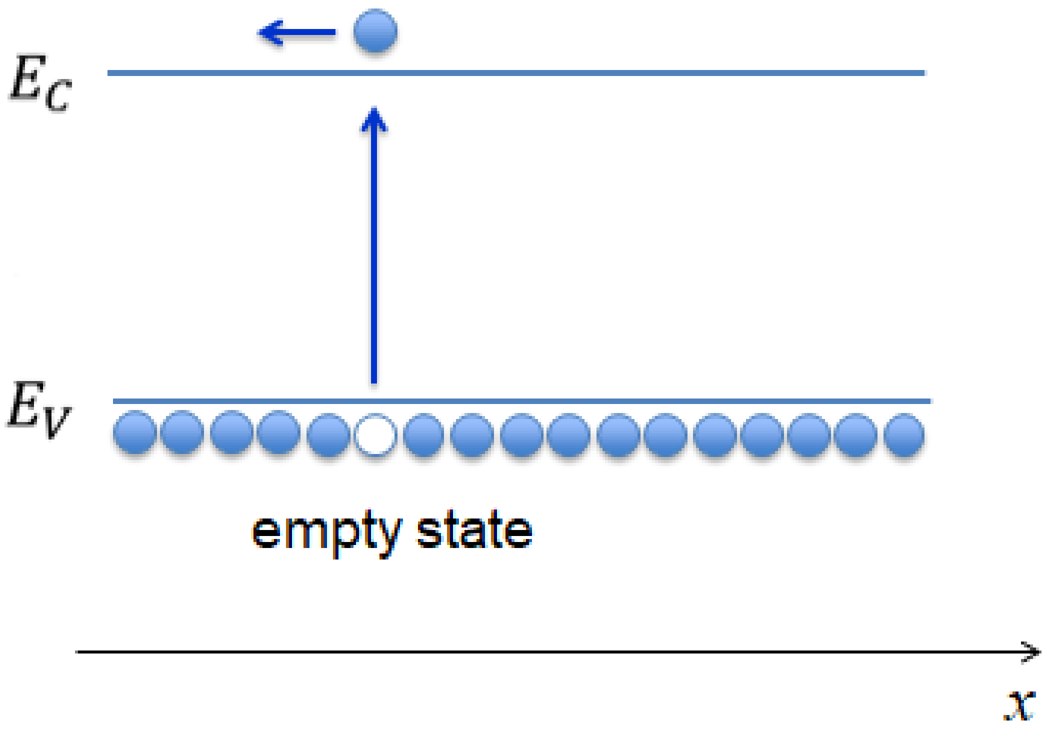
Top of the valence band

Electrons in the conduction band can move if an electric field is applied. An electron in the valence band may occupy the empty space leaving an empty space behind. Both mechanisms contribute to current.



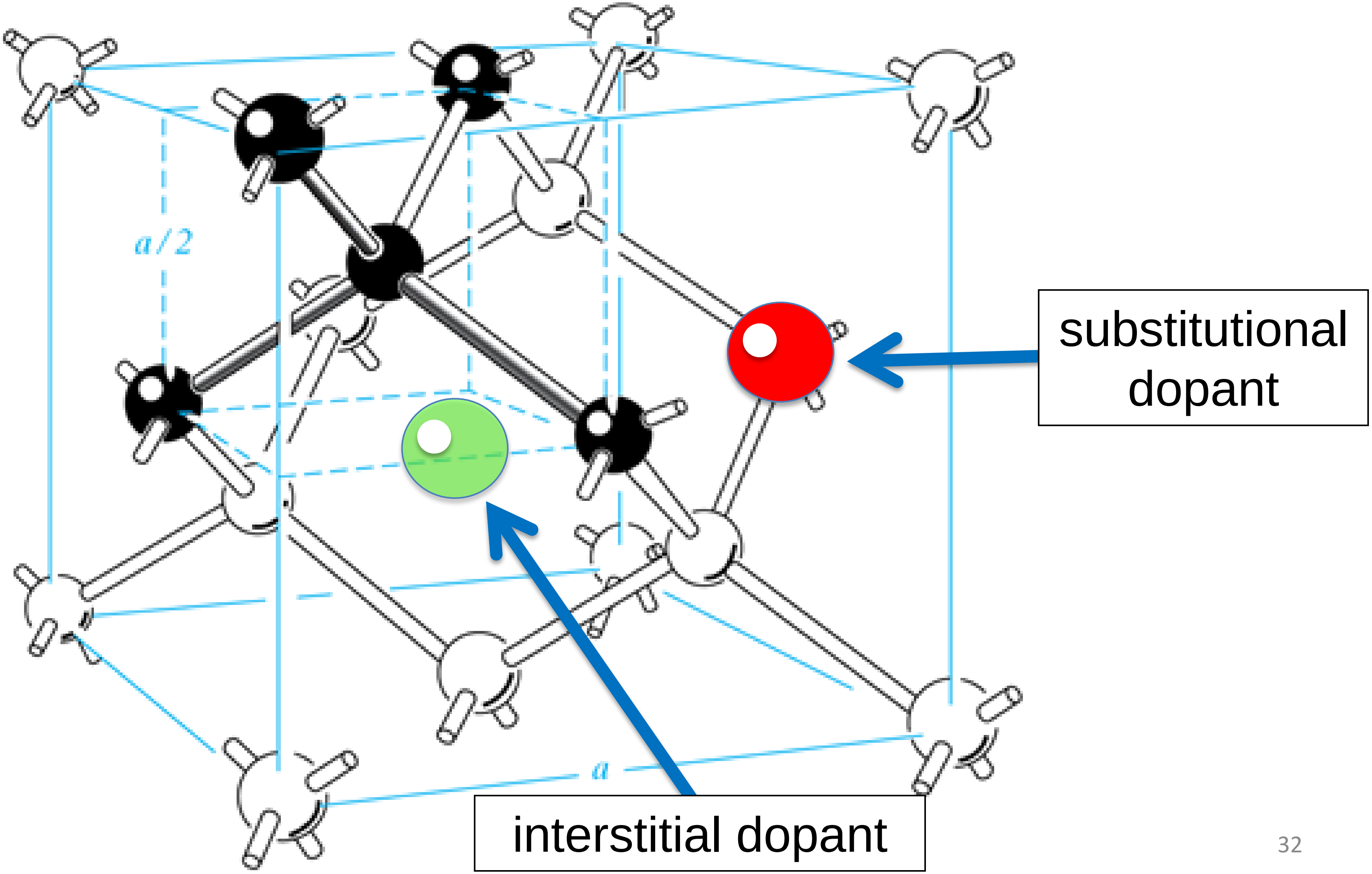
Equivalent to a virtual positive particle, called a "hole", moving in the opposite direction.

A pure semiconductor is called *intrinsic*. There are always as many electrons in the conduction band as holes in the valence band. Concentrations of mobile carriers are low and possible currents are quite small.



Real space

We can modify artificially the concentrations and the relative quantities of electrons and holes by introducing special atomic impurities, called dopants (extrinsic semiconductor)

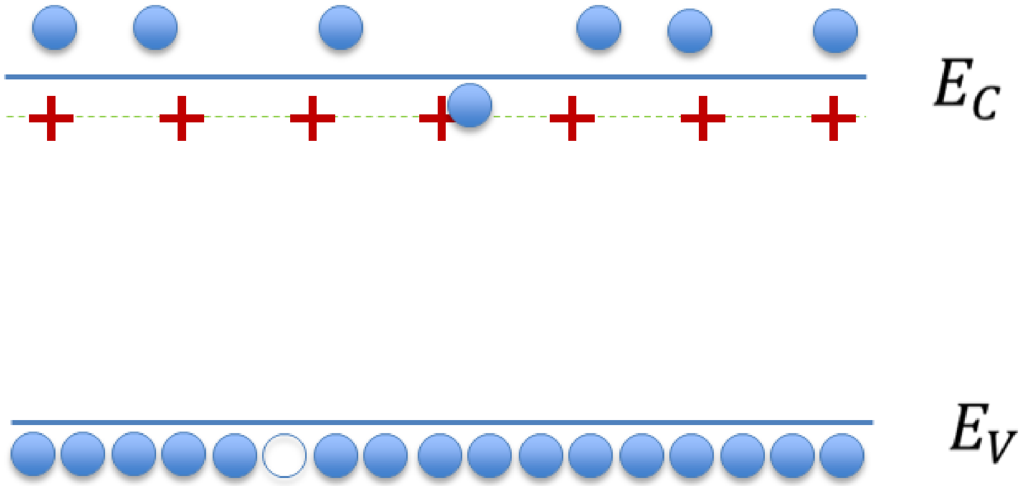


substitutional dopant

interstitial dopant

A donor atomic impurity has an extra electron and it introduces an energy level just below the bottom of the conduction band. A small thermal energy is needed for the extra electron to jump into the conduction band, leaving behind a positive fixed charge.

A semiconductor with predominantly donor impurities is called *n-type* with a large number of free electrons and very few holes.

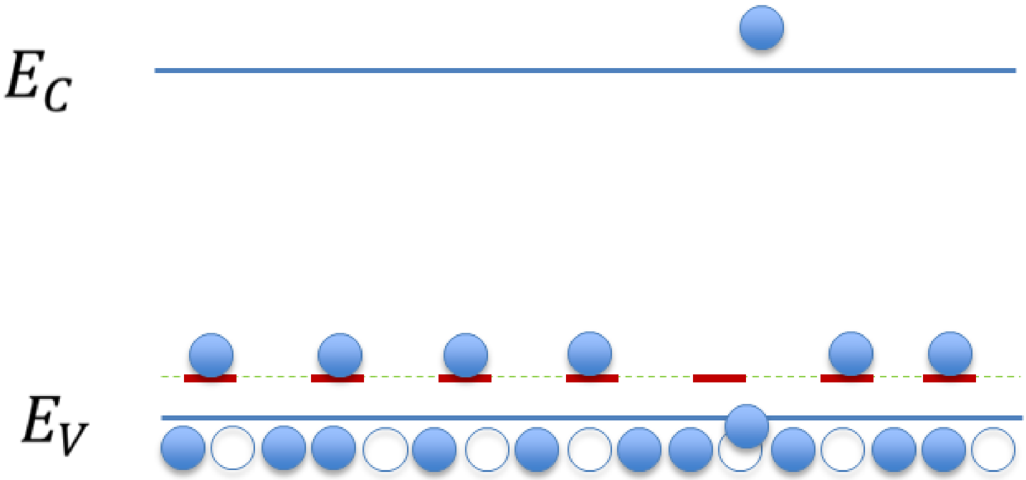


In silicon, a phosphorous (P) atom acts as donor

n-type

An acceptor atomic impurity has one less electron and it introduces an energy level just above the top of the valence band. A small thermal energy is needed for a valence band electron to jump into the acceptor site (which becomes negatively charged) leaving behind a mobile positive hole.

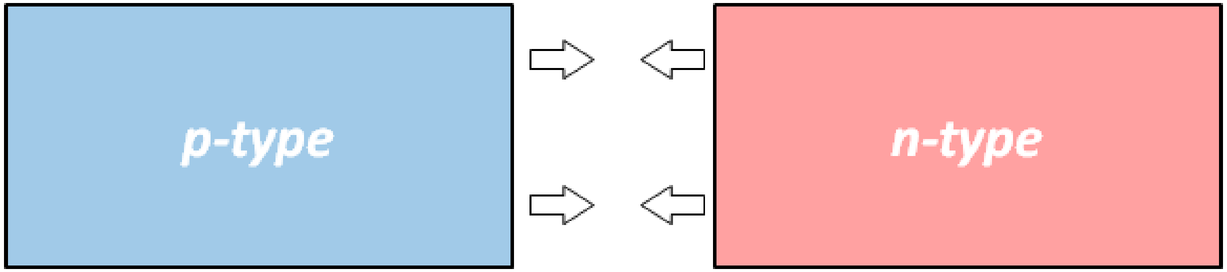
A semiconductor with predominantly acceptor impurities is called *p-type* with a large number free holes and very few conduction band electrons.



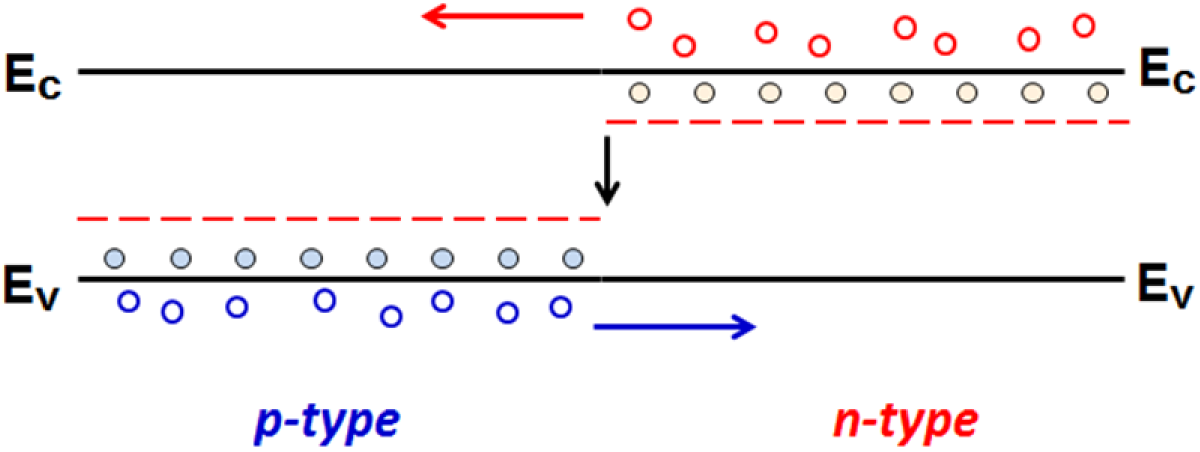
In silicon, a Boron (B) atom acts as acceptor

p-type

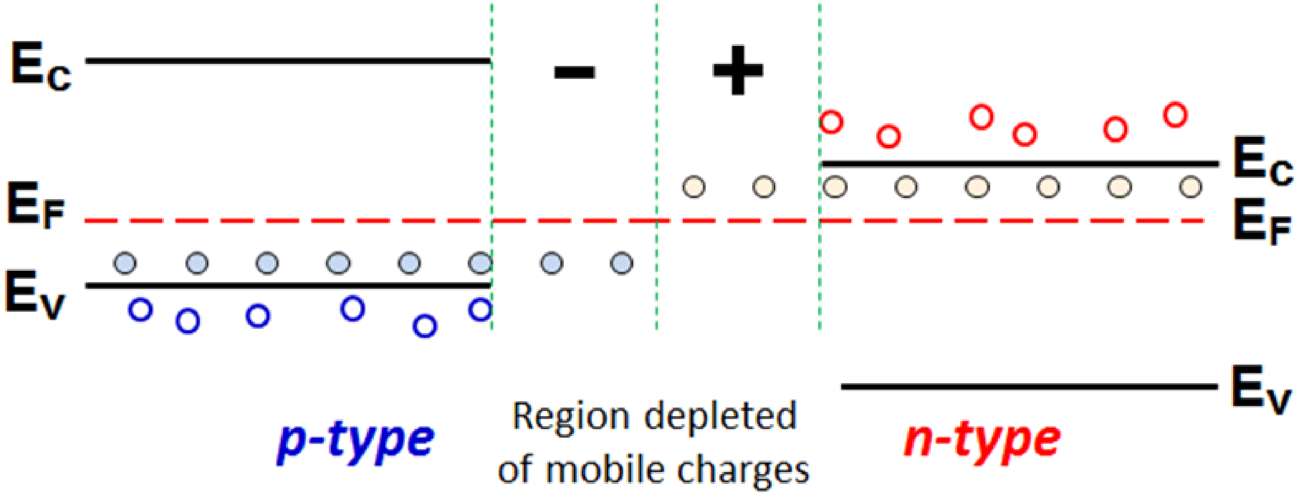
A *p-n* junction is obtained when two regions of semiconductor with different type of dopants are in contact



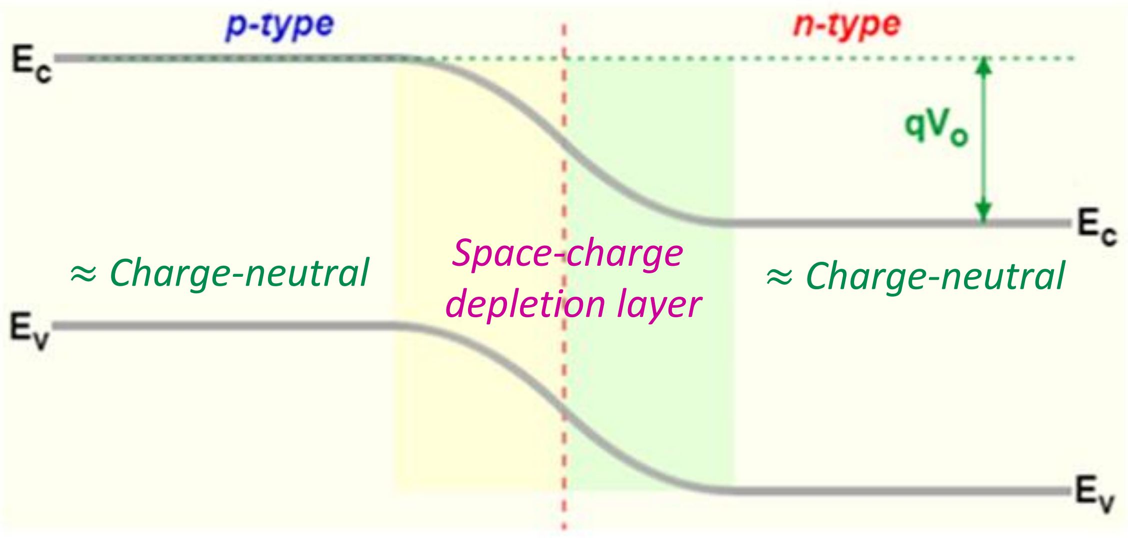
The two sides have different electrochemical potential due to the different doping and equilibrium is reached when a certain region about the junction is depleted of holes on the p-side and of electrons on the n-side.



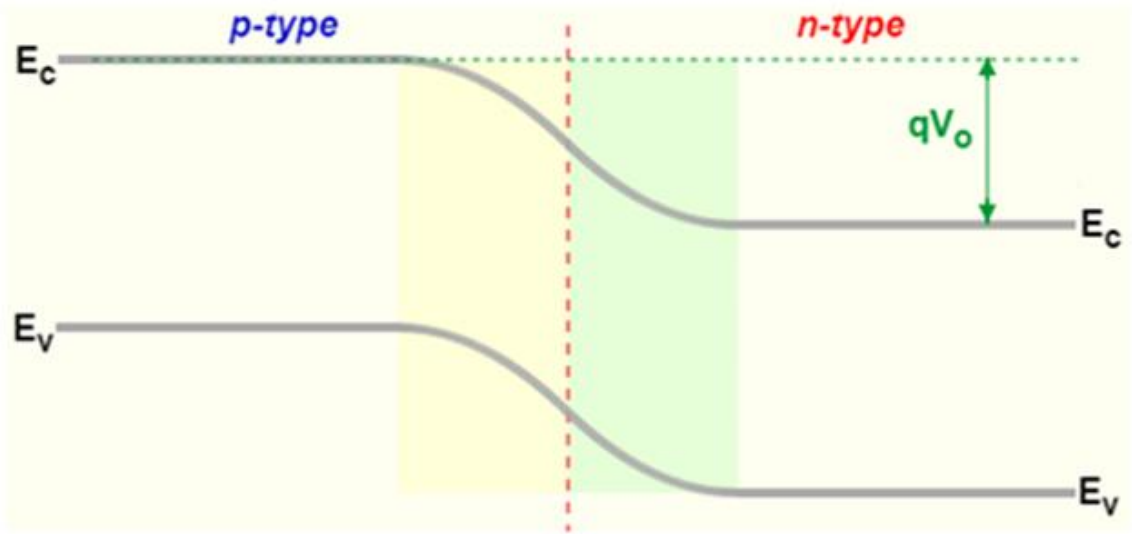
The fixed charge dipole creates a potential barrier preventing further movement of electrons and holes across the junction



Equilibrium potential barrier – No current flows

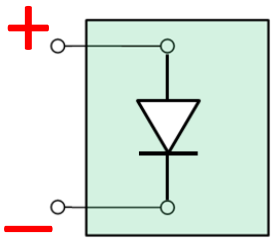


Equilibrium potential barrier – No current flows

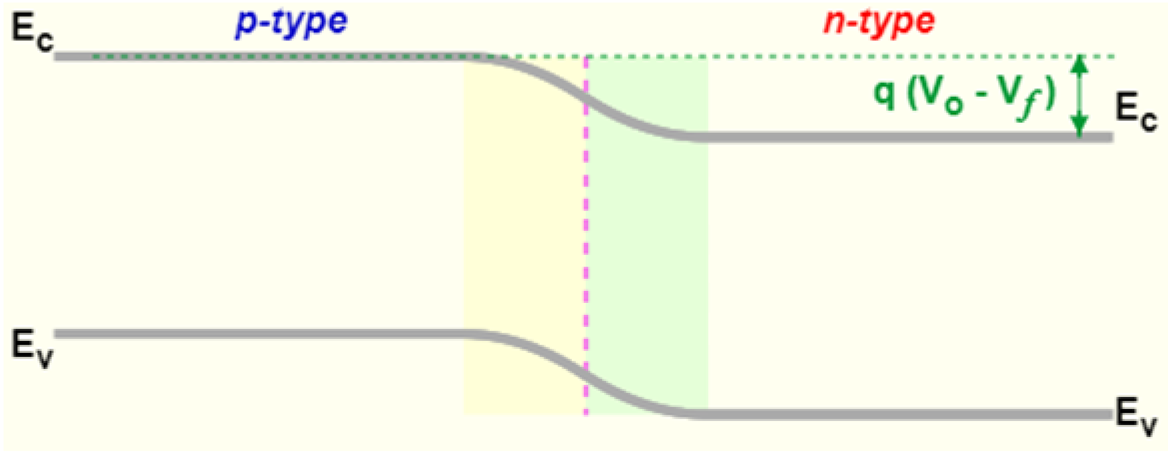


No potential applied

Barrier is lowered – electrons and hole can diffuse across junction



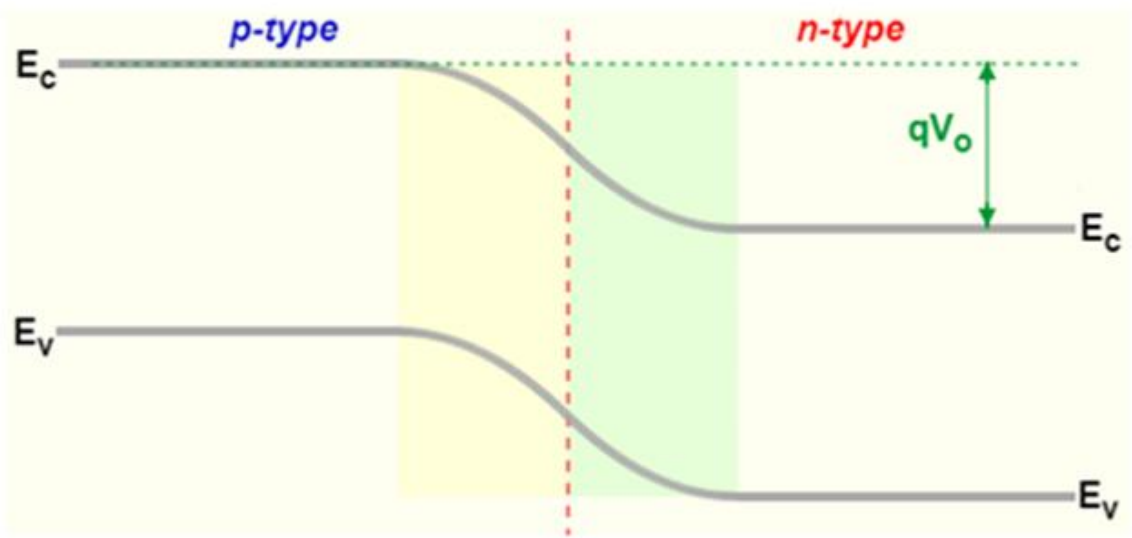
Forward potential applied



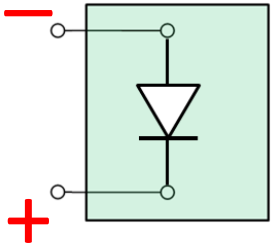
The depletion layer shrinks

Equilibrium potential barrier – No current flow

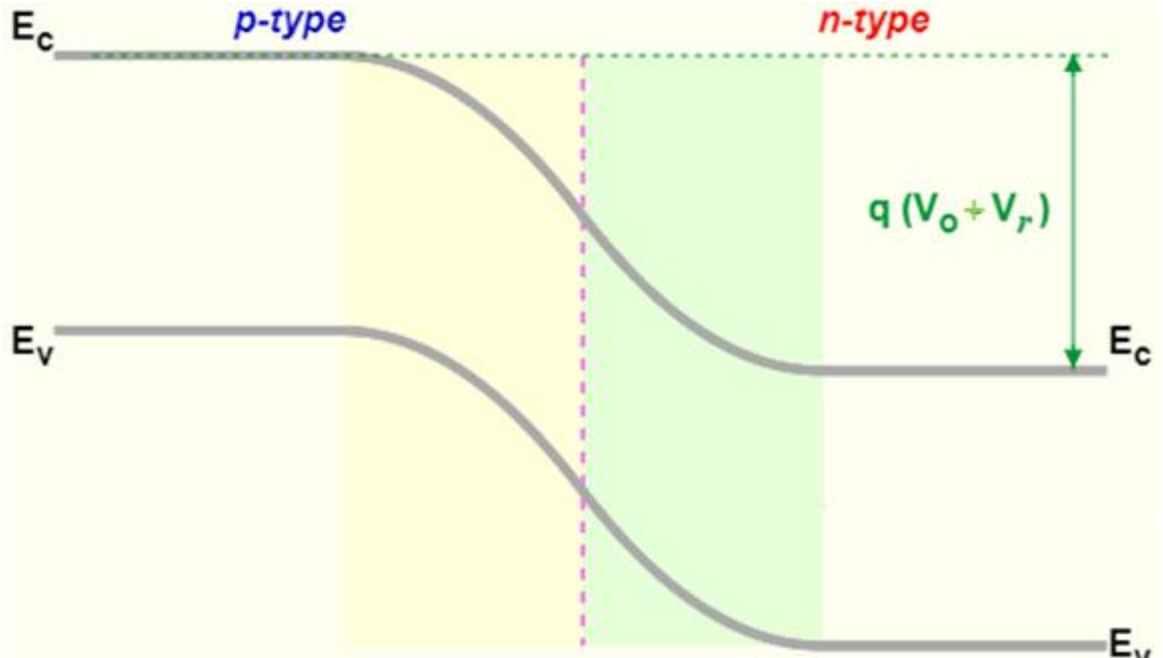
No potential applied



Barrier is higher – Electrons and holes cannot diffuse across junction



Reverse potential applied



The depletion layer is wider

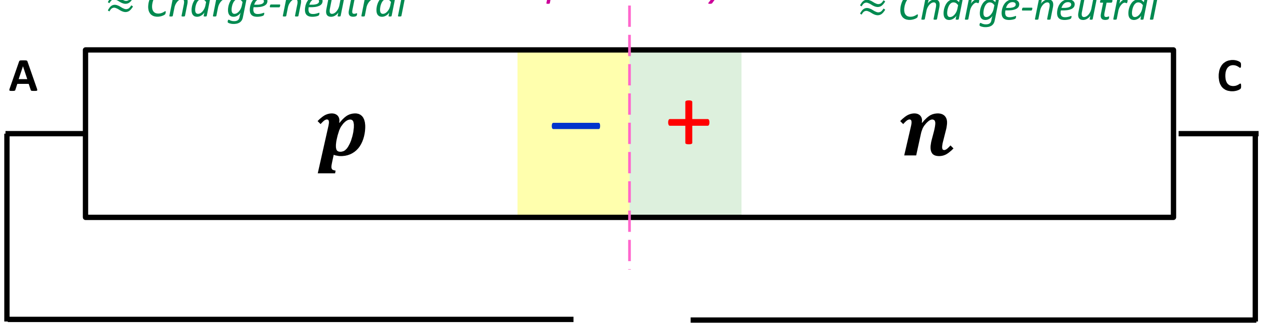
Equilibrium

No potential applied

≈ Charge-neutral

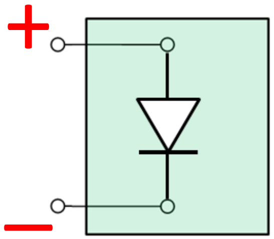
Space-charge depletion layer

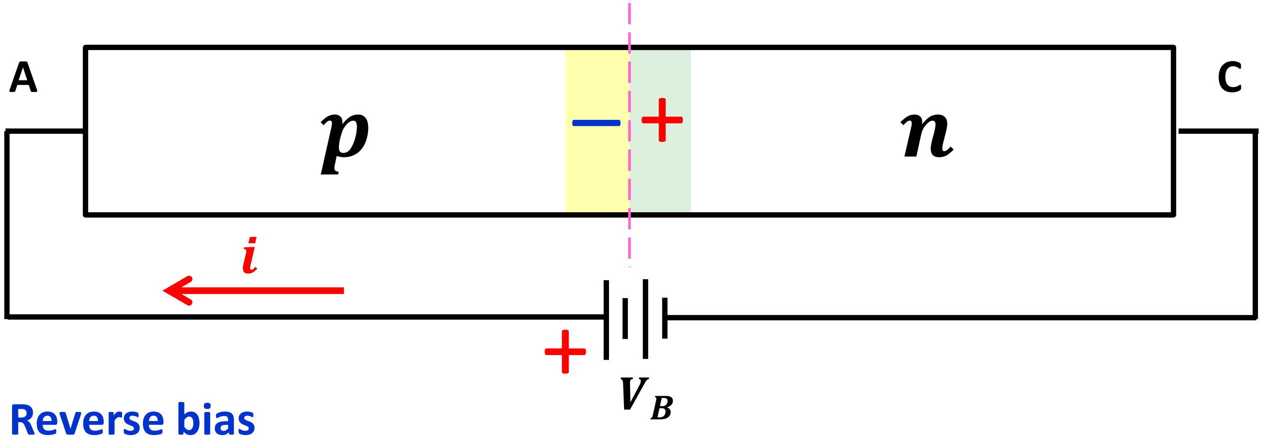
≈ Charge-neutral



Forward bias

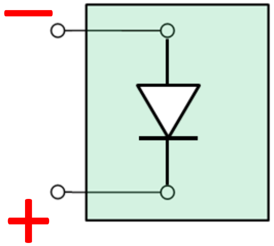
$V_A > V_C$

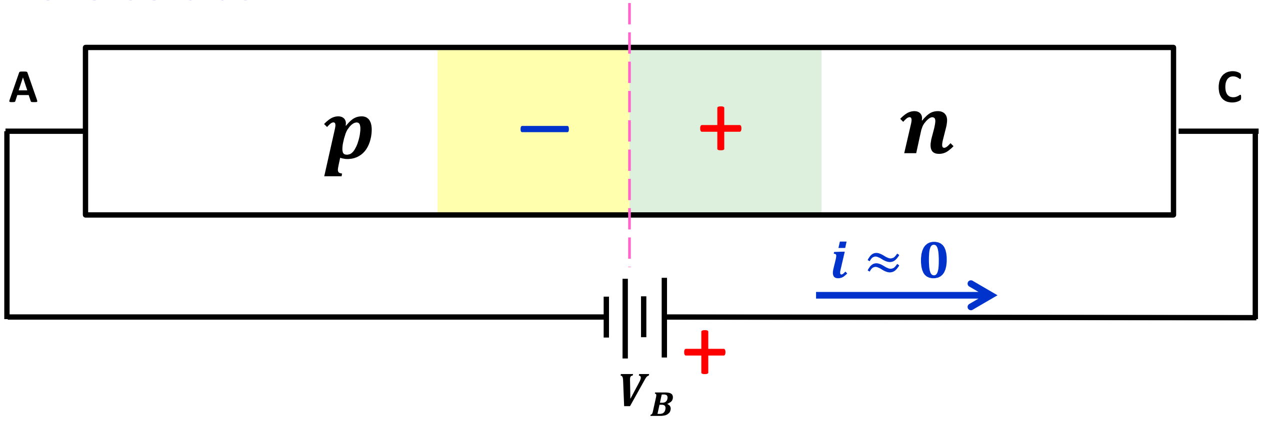




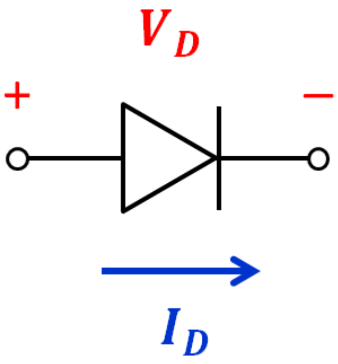
Reverse bias

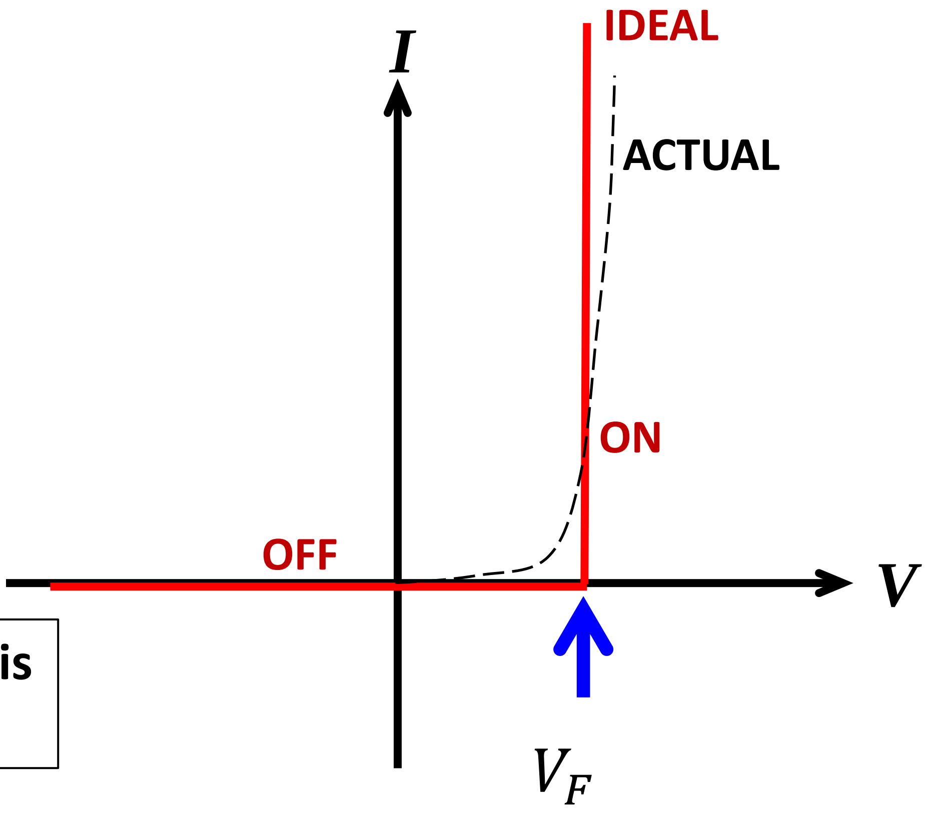
$V_A < V_C$





Ideal diode model for circuit analysis





Typical value for Si diodes is
$V_F = 0.7\ \mathrm{V}$

$V_D < V_F$

Diode is OFF

$V_D = V_F$

Diode is ON

Diode circuit analysis

Diodes are non-linear devices, and we cannot state *a priori* whether a diode is ON or OFF. Therefore, we can start a problem by making an assumption.

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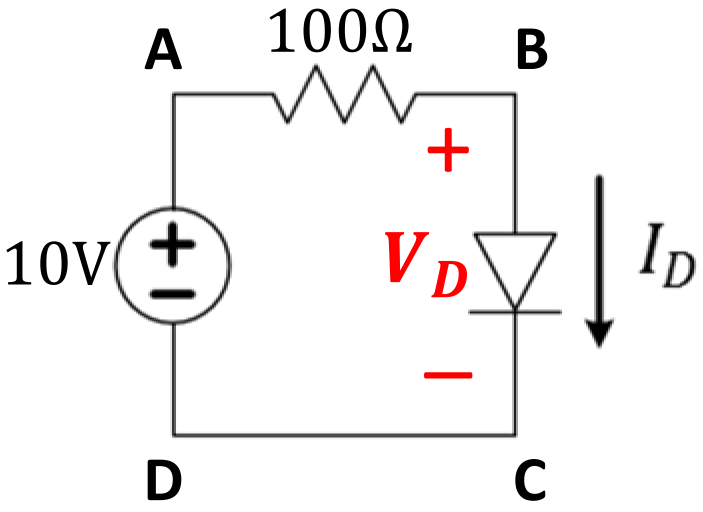
If instead the assumption has generated *unphysical* results, there is a <u>contradiction</u> and we solve the problem again, imposing that the diode is equivalent to an open circuit (OFF).

Example 1A – Solve for I_D



Assume $V_F = 0.7\ \mathrm{V}$

Assume that the diode is conducting (there must be 0.7V across the diode)

KVL $V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$

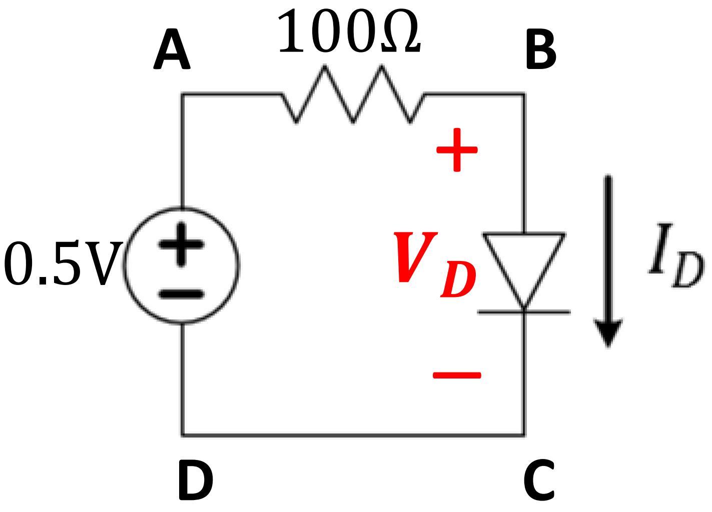
$$100 I_D + 0.7 - 10 = 0$$

$$\boxed{I_D = 9.3\mathrm{V}/100\,\Omega = 93\mathrm{mA}}$$

CHECK: $I_D > 0$ and it flows from Anode to Cathode

Results follow expected physics and there is no contradiction. OK

Example 1B – Solve for I_D



Assume $V_F = 0.7$ V

Assume that the diode is conducting (there must be 0.7V across the diode)

KVL $$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

$$100I_D + 0.7 - 0.5 = 0$$

$$\boxed{I_D = (0.5\text{V} - 0.7\text{V})/100\,\Omega = -2\text{mA}}$$

CHECK: $I_D < 0$ and it flows from Cathode to Anode

Physics is incorrect. Also, DIODE cannot provide power.
There is contradiction.
Conclusion: Diode is OFF and $I_D = 0V$