ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 23

MWF – 12:00pm

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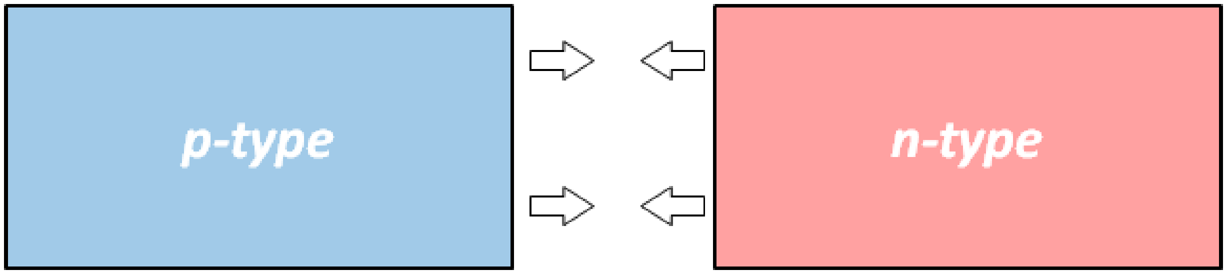
2062 ECE Building

Lecture 23 – Summary

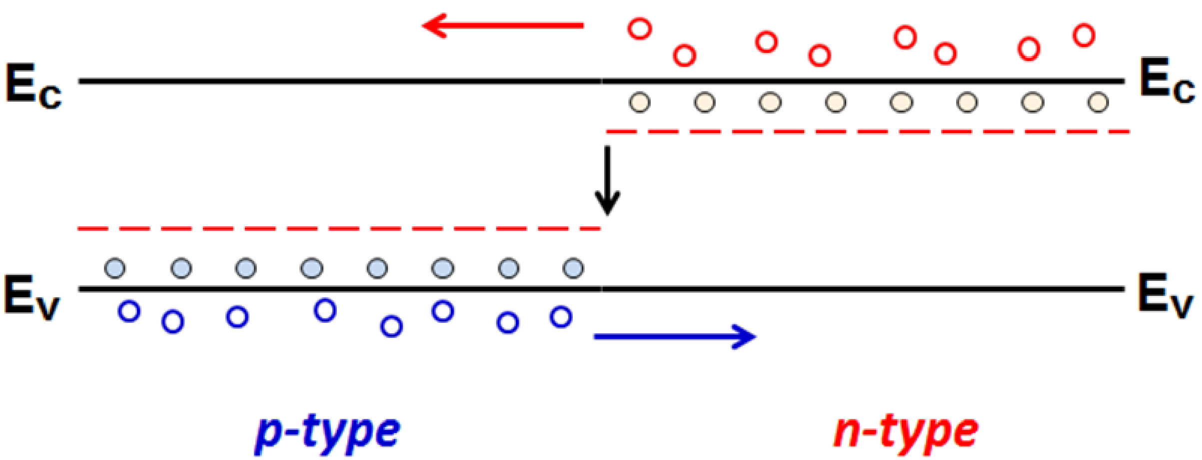
Learning Objectives

1. **Solution of circuits with *p*-*n* junction diodes**

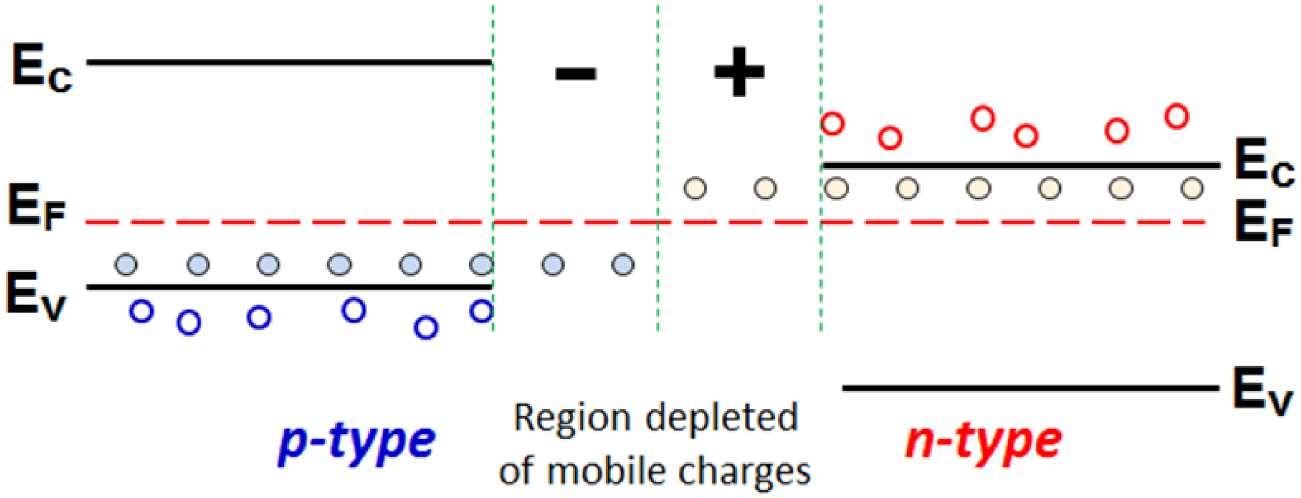
A *p-n* junction is obtained when two regions of semiconductor with different type of dopants are in contact



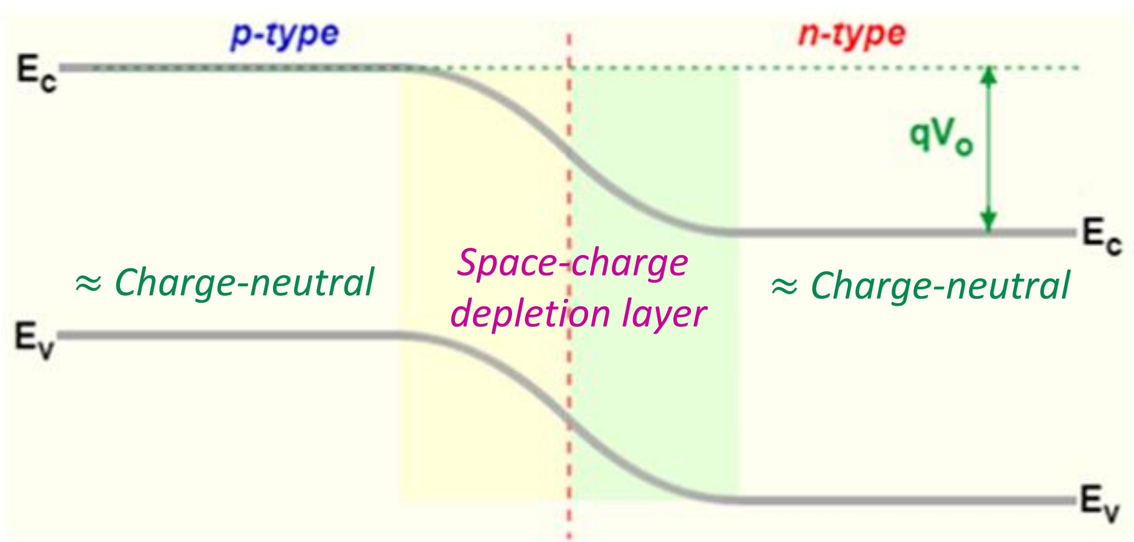
The two sides have different electrochemical potential due to the different doping and equilibrium is reached when a certain region about the junction is depleted of holes on the p-side and of electrons on the n-side.



The fixed charge dipole creates a potential barrier preventing further movement of electrons and holes across the junction



Equilibrium potential barrier – No current flows



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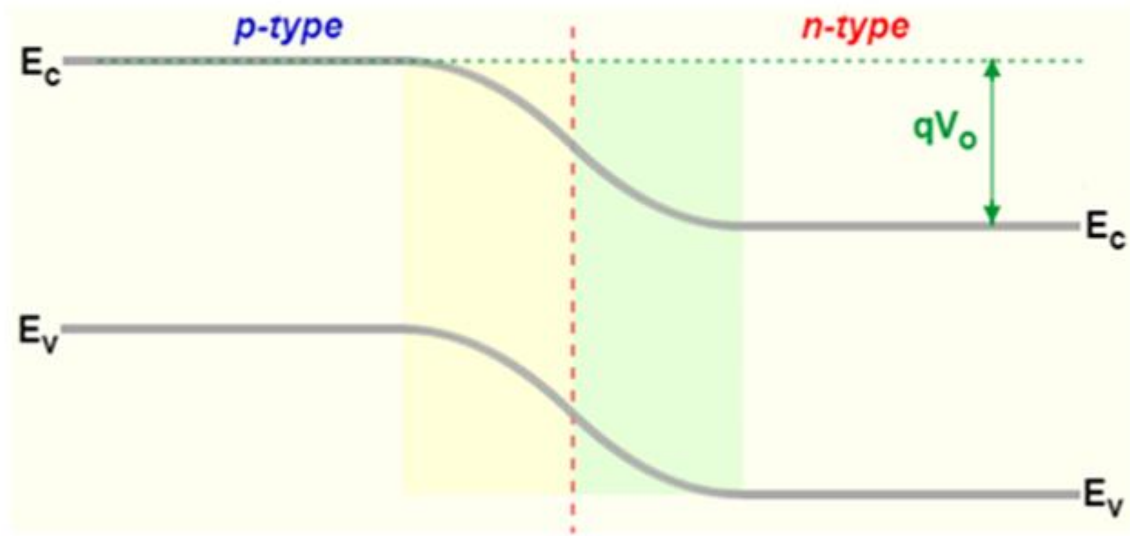
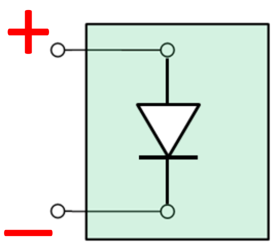
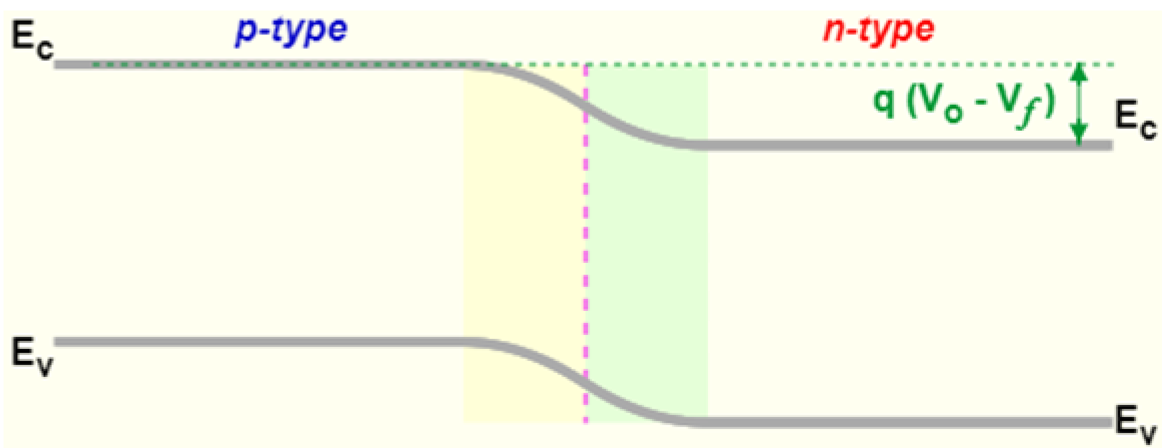


No potential applied

Barrier is lowered – electrons and hole can diffuse across junction

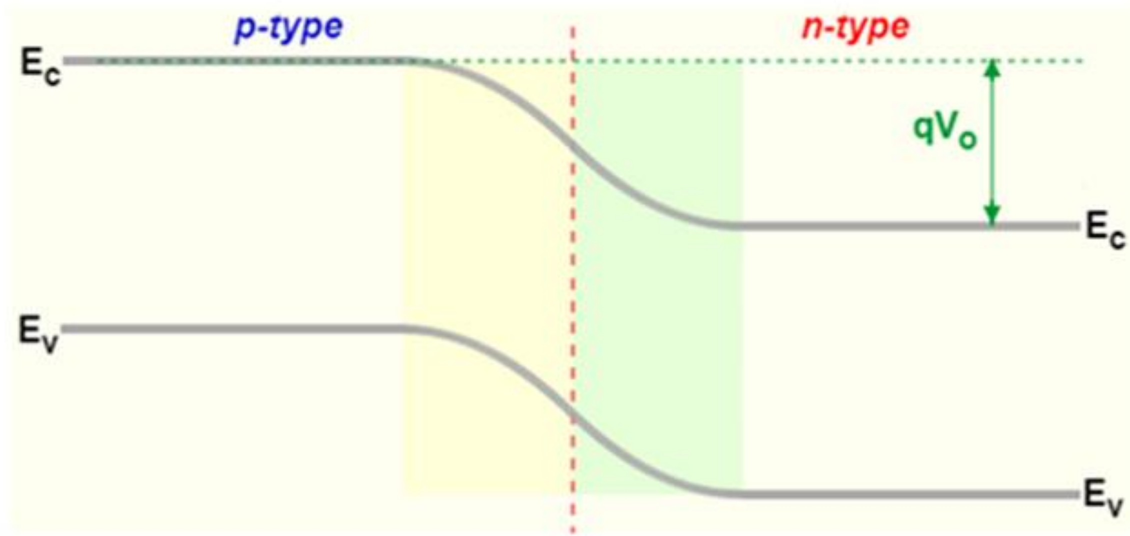




Forward potential applied

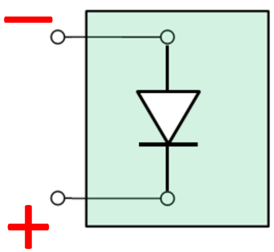
The depletion layer shrinks

Equilibrium potential barrier – No current flow

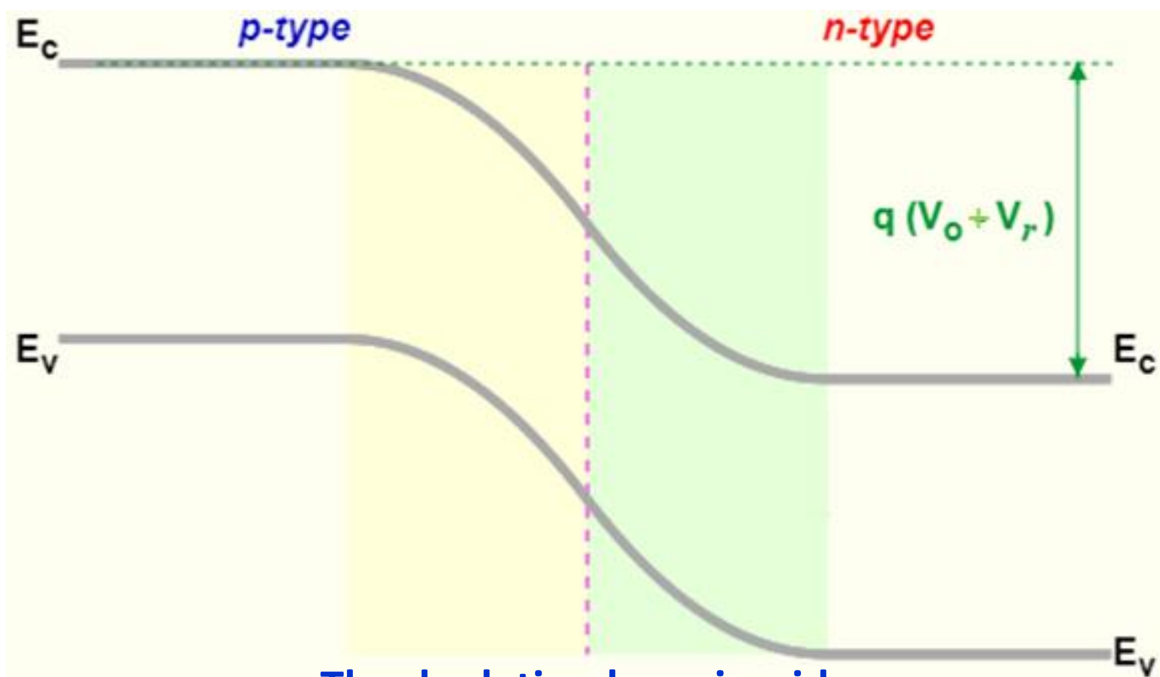


No potential applied

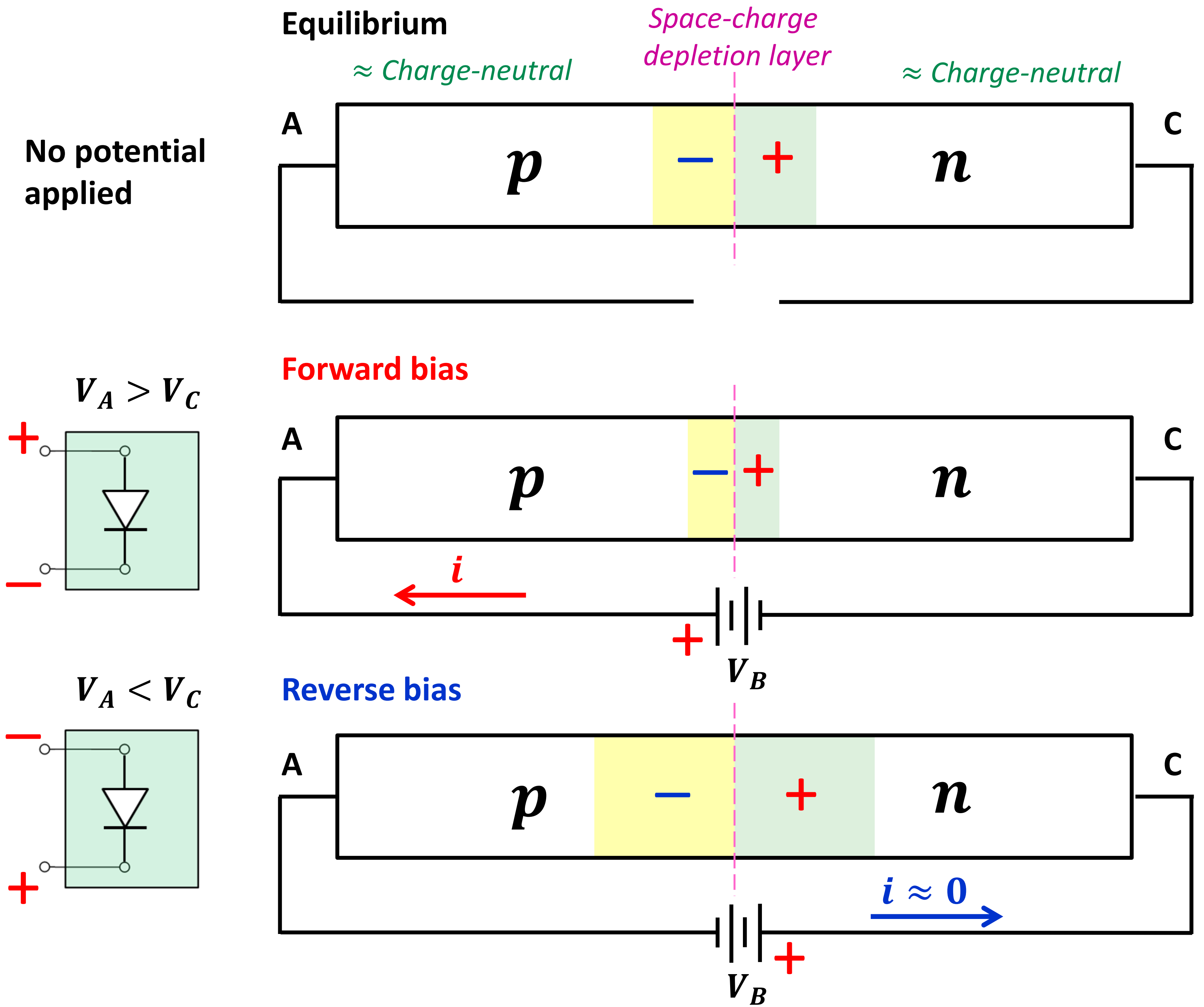
Barrier is higher – Electrons and holes cannot diffuse across junction



Reverse potential applied

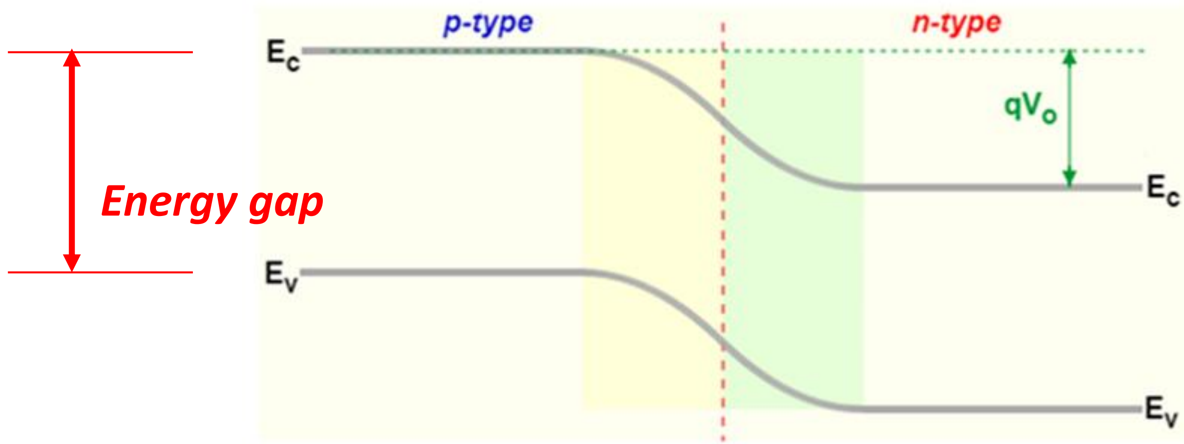


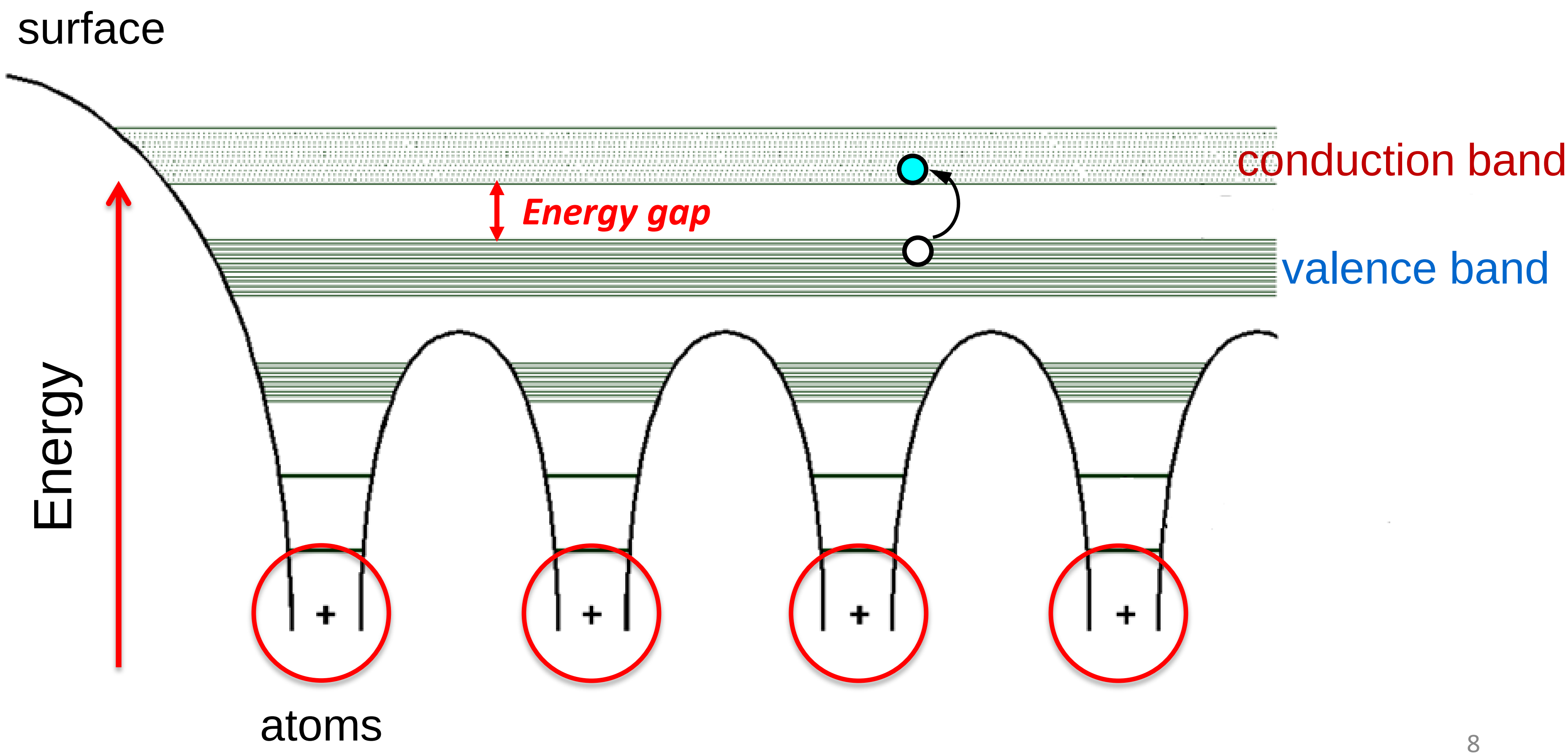
The depletion layer is wider



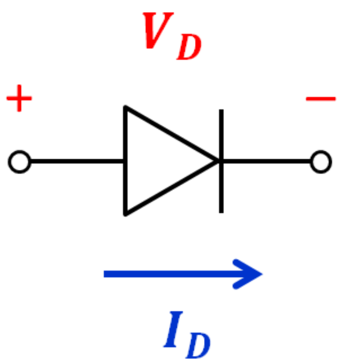
Equilibrium
Space-charge depletion layer
≈ Charge-neutral
≈ Charge-neutral
No potential applied
A
C
p
−
+
n
Forward bias
$V_A > V_C$
+
−
A
C
p
−
+
n
i
+
V_B
Reverse bias
$V_A < V_C$
−
+
A
C
p
−
+
n
$i \approx 0$
+
V_B

The voltage threshold (in Volts) of the diode is a bit smaller than the energy gap (in electron Volts) of the material used. Silicon has energy gap $E_g \approx 1.2\mathrm{eV}$ and $V_F \approx 0.6$ to $0.8\mathrm{V}$.

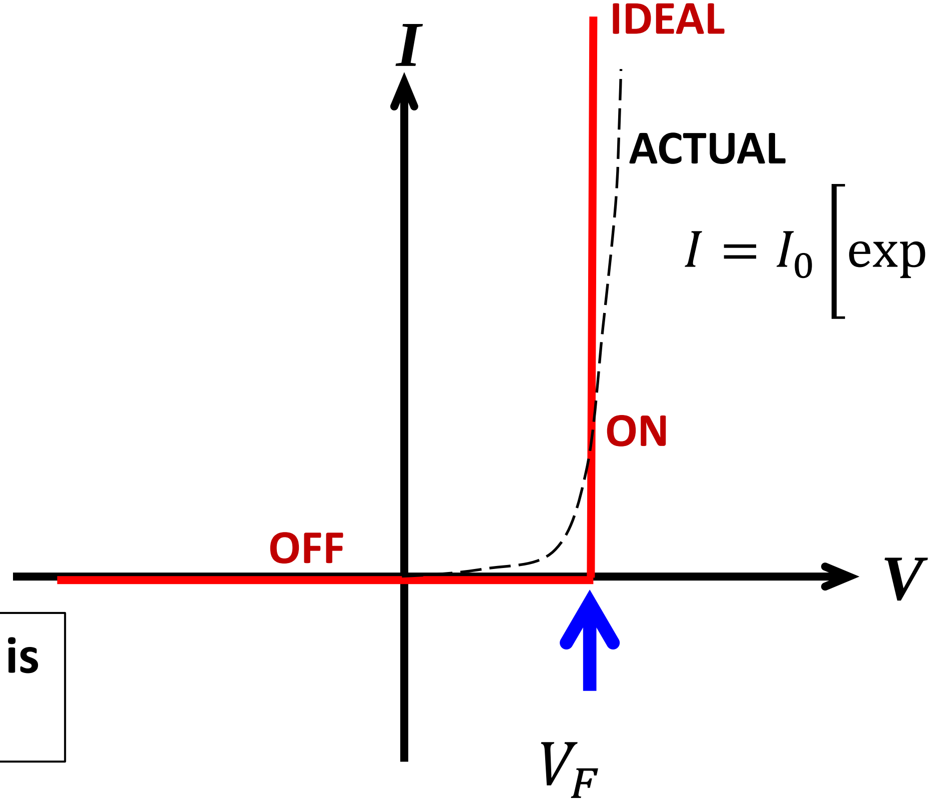




Ideal diode model for circuit analysis



Typical value for Si diodes is
$\boldsymbol{V_F = 0.7\ \mathrm{V}}$



$$I = I_0\left[\exp\left(\frac{qv}{nk_BT}\right) - 1\right]$$

$V_D < V_F$	$V_D = V_F$
Diode is OFF	Diode is ON

Diode circuit analysis

Diodes are non-linear devices, and we cannot state *a priori* whether a diode is ON or OFF. Therefore, we can start a problem by making an assumption.

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If we assume that a diode is conducting (ON), the voltage from anode to cathode is "pinned" to the threshold voltage V_F and we solve the circuit with KVL and KCL linear equations, by imposing that voltage. If the result is *physical*, we accept it.

Diode circuit analysis

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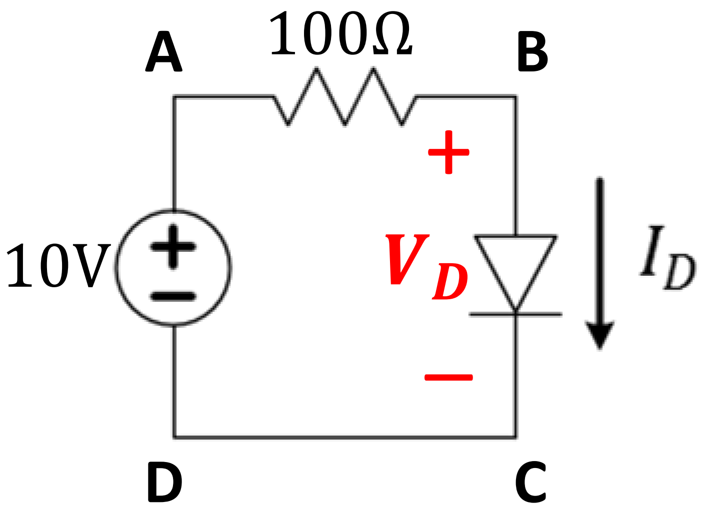
If we assume that a diode is conducting (ON), the voltage from anode to cathode is "pinned" to the threshold voltage V_F and we solve the circuit with KVL and KCL linear equations, by imposing that voltage. If the result is *physical*, we accept it.

If instead the assumption has generated *unphysical* results, there is a contradiction and we solve the problem again, imposing that the diode is equivalent to an open circuit (OFF).

Example 1A – Solve for I_D



Assume $V_F = 0.7$ V

Assume that the diode is conducting (there must be 0.7V across the diode)

KVL $$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

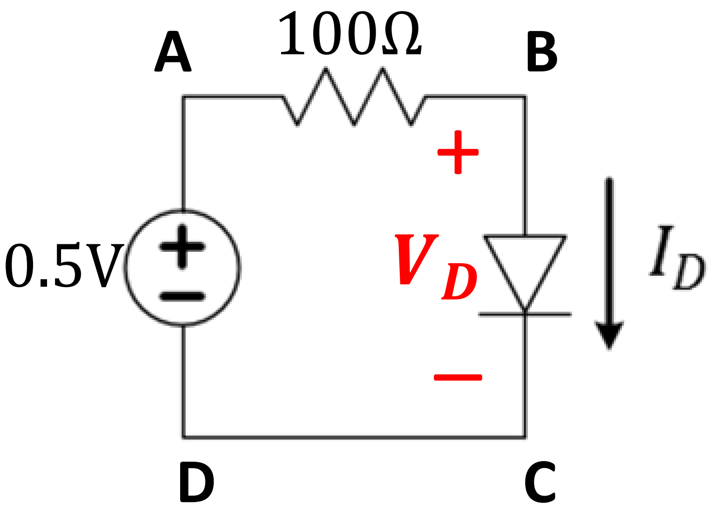
$$100 I_D + 0.7 - 10 = 0$$

$$\boxed{I_D = 9.3\text{V}/100\,\Omega = 93\text{mA}}$$

CHECK: $I_D > 0$ and it flows from Anode to Cathode

Results follow expected physics and there is no contradiction. OK

Example 1B – Solve for I_D



Assume $V_F = 0.7$ V

Assume that the diode is conducting (there must be 0.7V across the diode)

KVL $$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

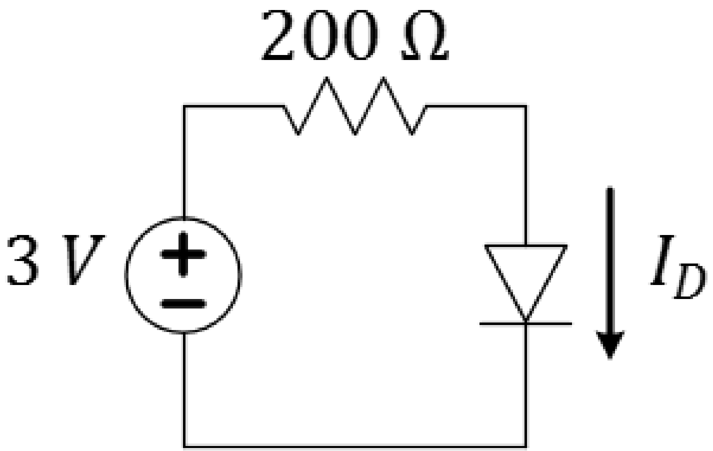
$$100I_D + 0.7 - 0.5 = 0$$

$$\boldsymbol{I_D = (0.5\text{V} - 0.7\text{V})/100\,\Omega = -2\text{mA}}$$

CHECK: $I_D < 0$ and it flows from Cathode to Anode

Physics is incorrect. Also, DIODE cannot provide power.
There is contradiction.
Conclusion: Diode is OFF and $I_D = 0V$

Example 2: Solve for I_D



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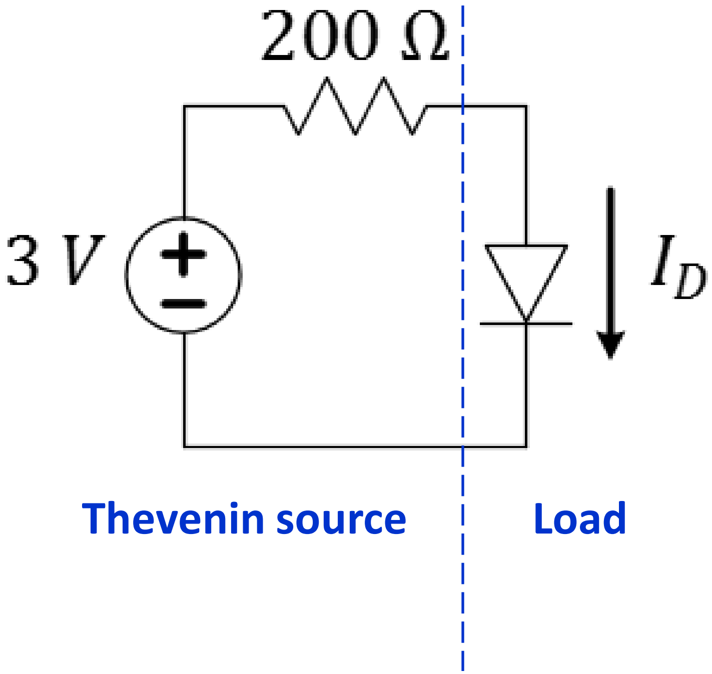
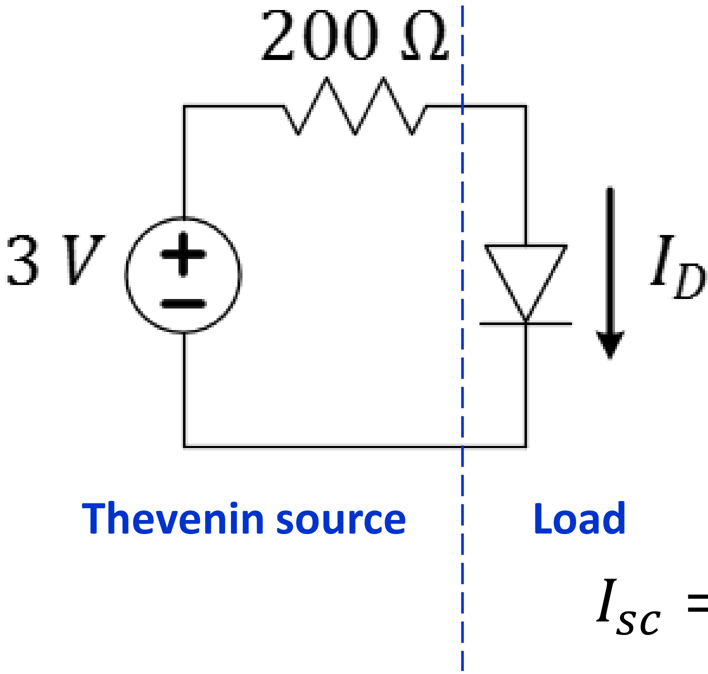
Example 2: Solve for I_D



Assume $V_F = 0.7$ V

Let's solve with the I-V curve.
First, characterize the source.

Example 2: Solve for I_D



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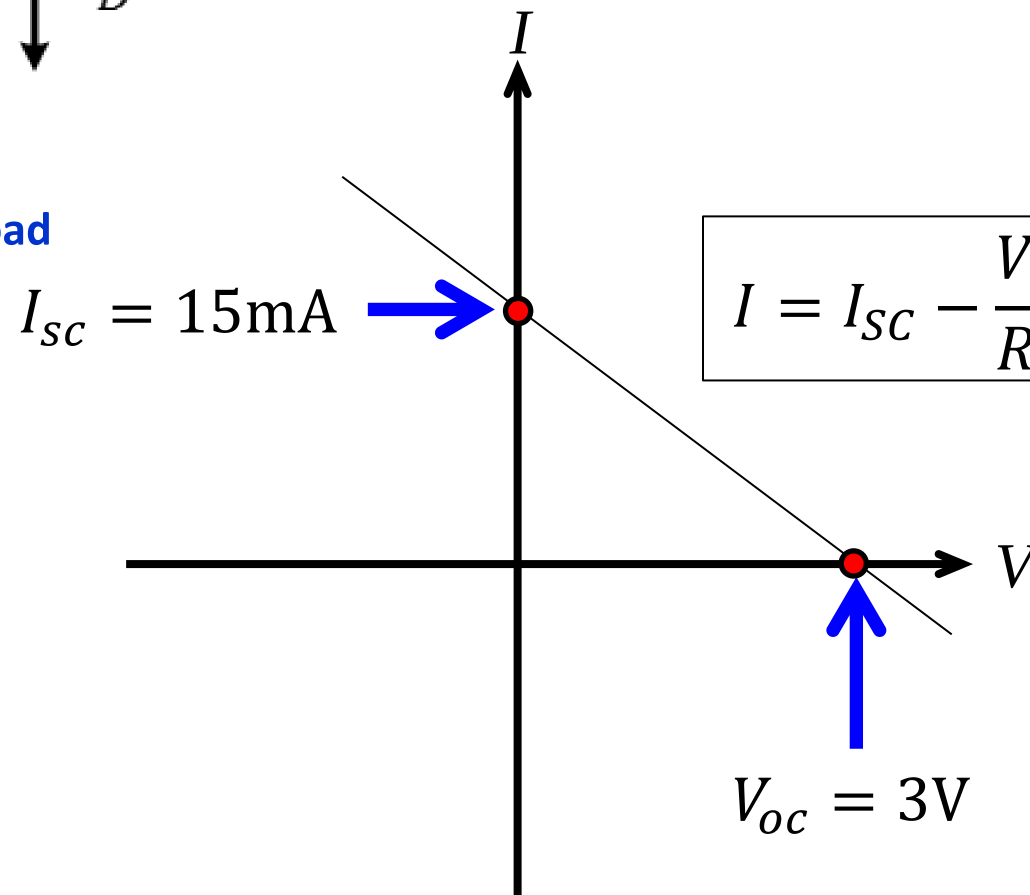
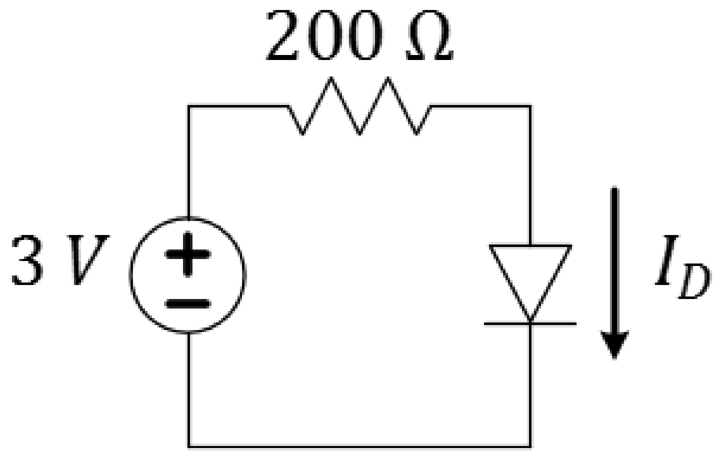


$$I = I_{SC} - \frac{V}{R}$$

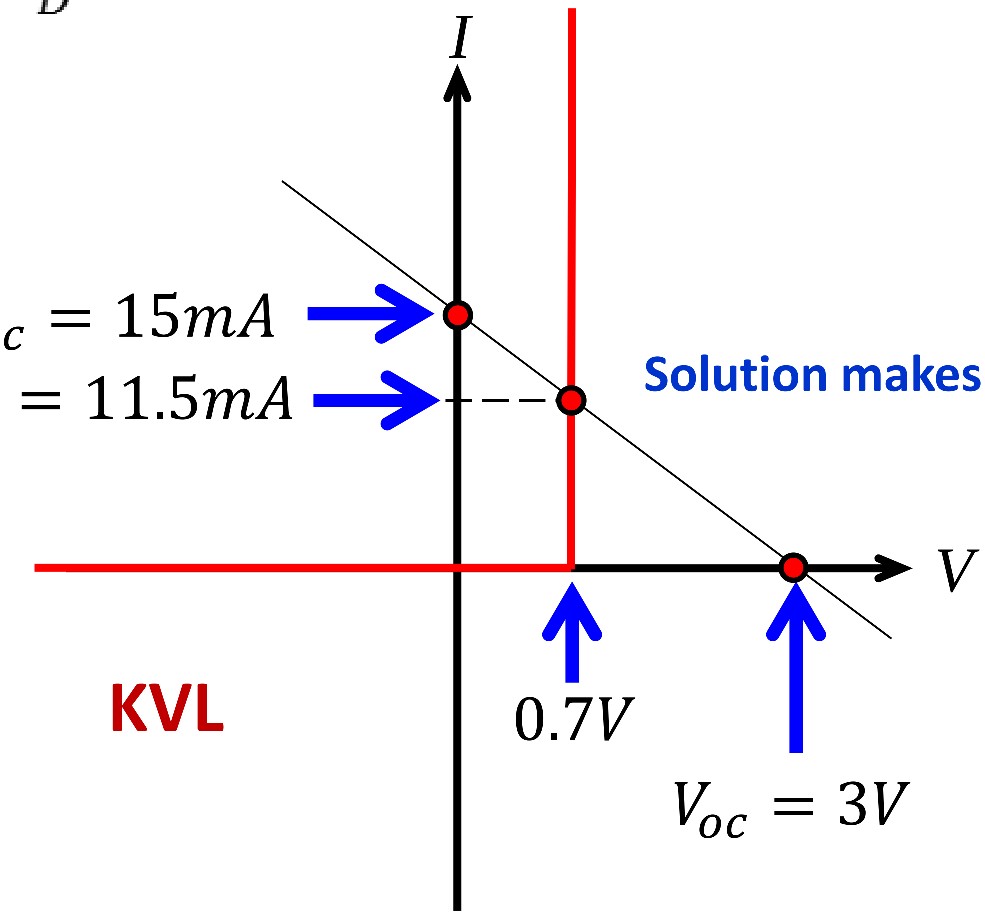
$$I_{sc} = \frac{3}{200} = 15\text{mA}$$

Assume $V_F = 0.7$ V



Assume that the diode is conducting

The diode is included in the KVL as a virtual voltage source of 0.7 V.

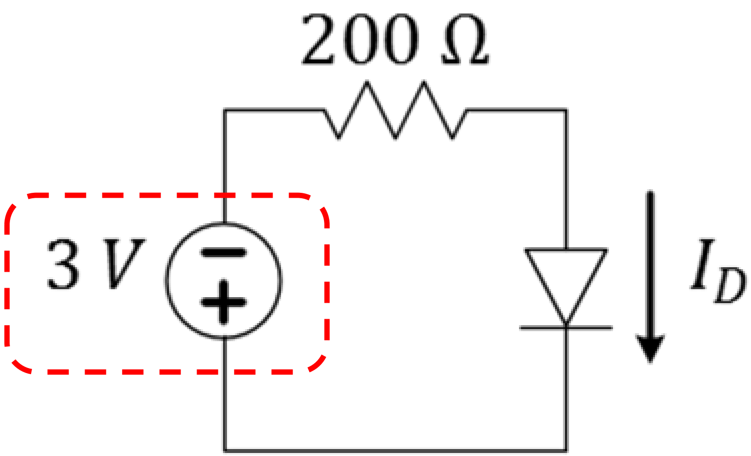


$$I_{sc} = \frac{3}{200} = 15\text{mA}$$

$$-3 + 200I_D + 0.7 = 0 \qquad \text{KVL}$$

$$\boxed{I_D = \frac{2.3}{200} = 11.5\ \text{mA}}$$

Now reverse the bias



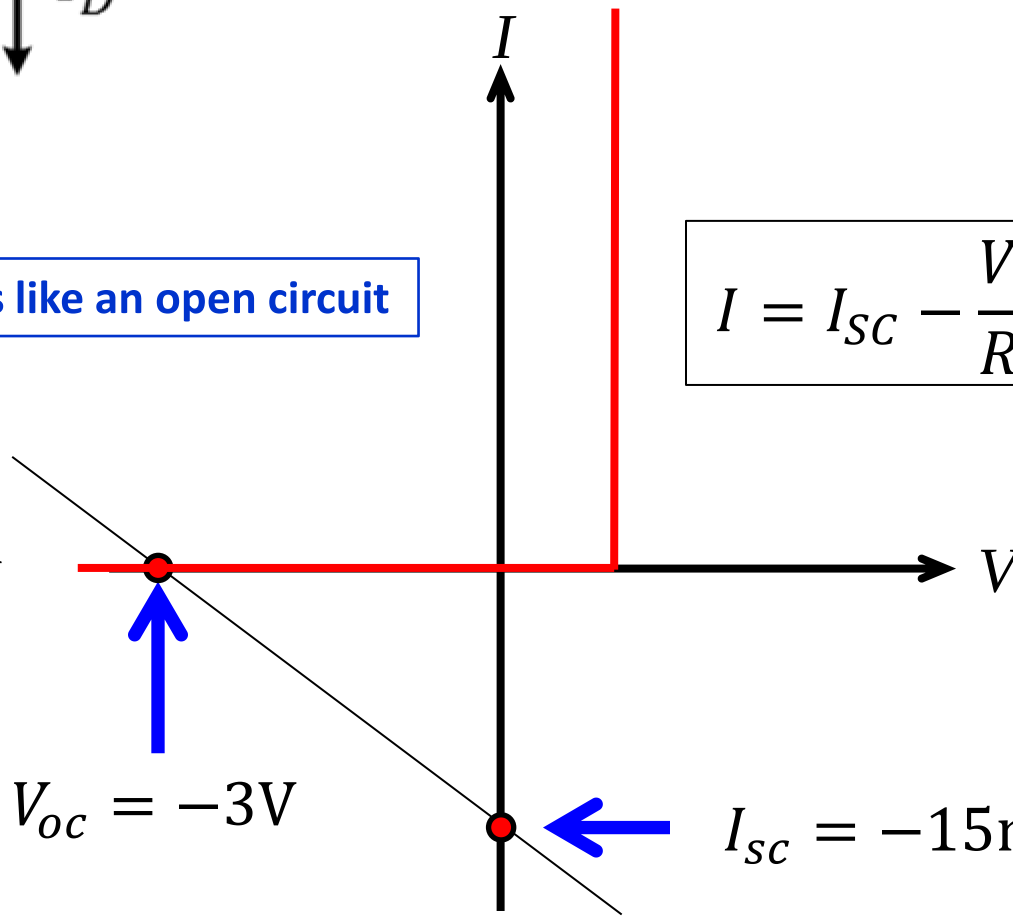
Assume $V_F = 0.7$ V

The current is $I_D = -I_0 \approx 0$ (reverse bias current)

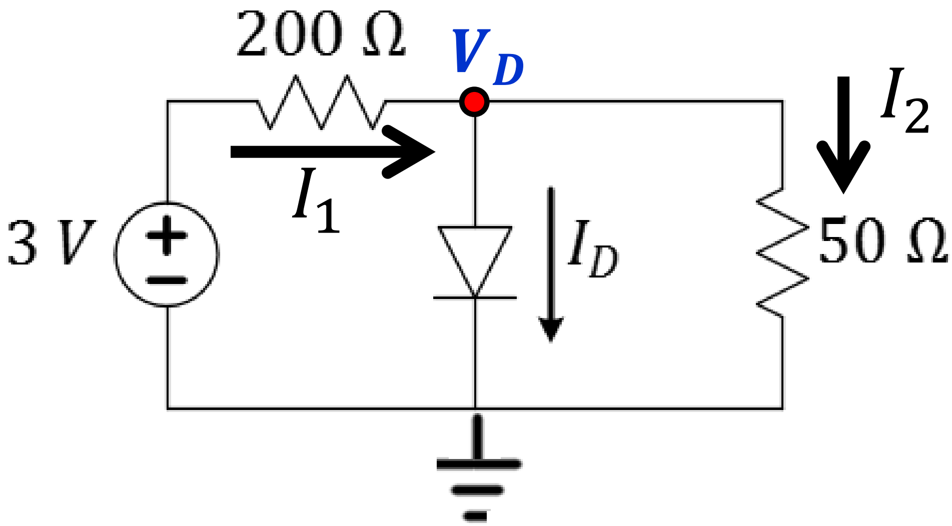
In practice, the diode behaves like an open circuit

$$I = I_{SC} - \frac{V}{R}$$

$$I_{sc} = -\frac{3}{200} = -15\text{mA}$$



Example 3: Solve for I_D



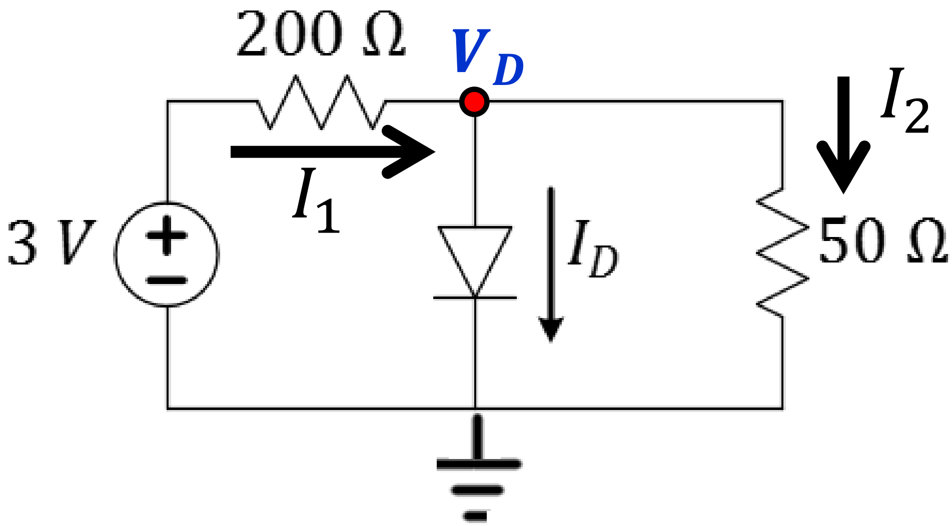
Assume $V_F = 0.7$ V

Assume that the diode is conducting (there must be 0.7V across the diode)

$$I_1 = \frac{3 - 0.7}{200} = \frac{2.3}{200} = 11.5\text{mA}$$

$$I_2 = \frac{0.7}{50} = 14\text{mA}$$

Example 3: Solve for I_D



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From KCL:

$I_D = I_1 - I_2 = -2.5\text{mA}$ $\rightarrow$ NOT PHYSICAL: diode not conducting

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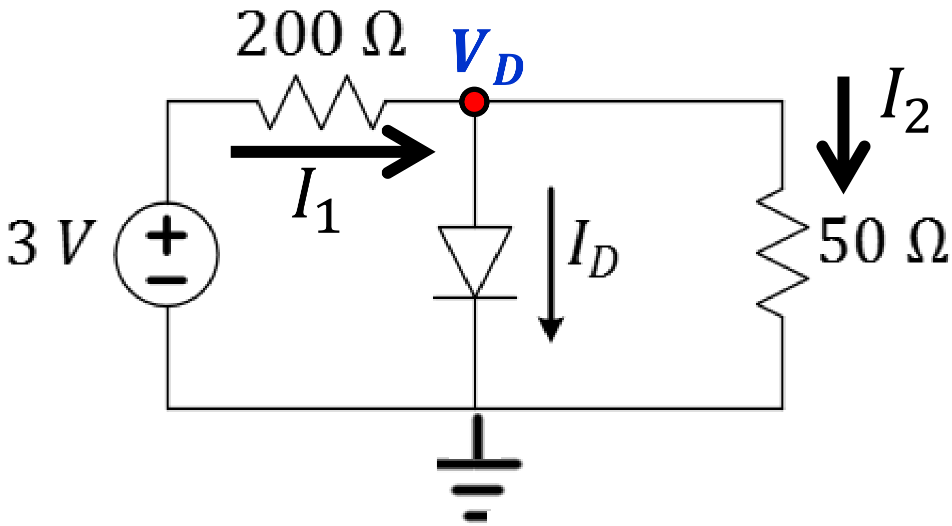

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From KCL:

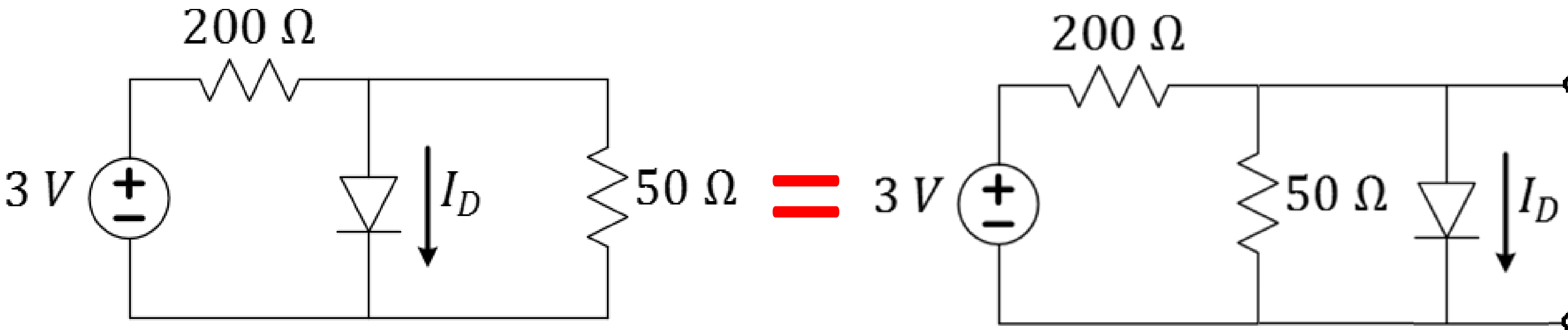
$I_D = I_1 - I_2 = -2.5\text{mA} \quad \rightarrow$ NOT PHYSICAL: diode not conducting

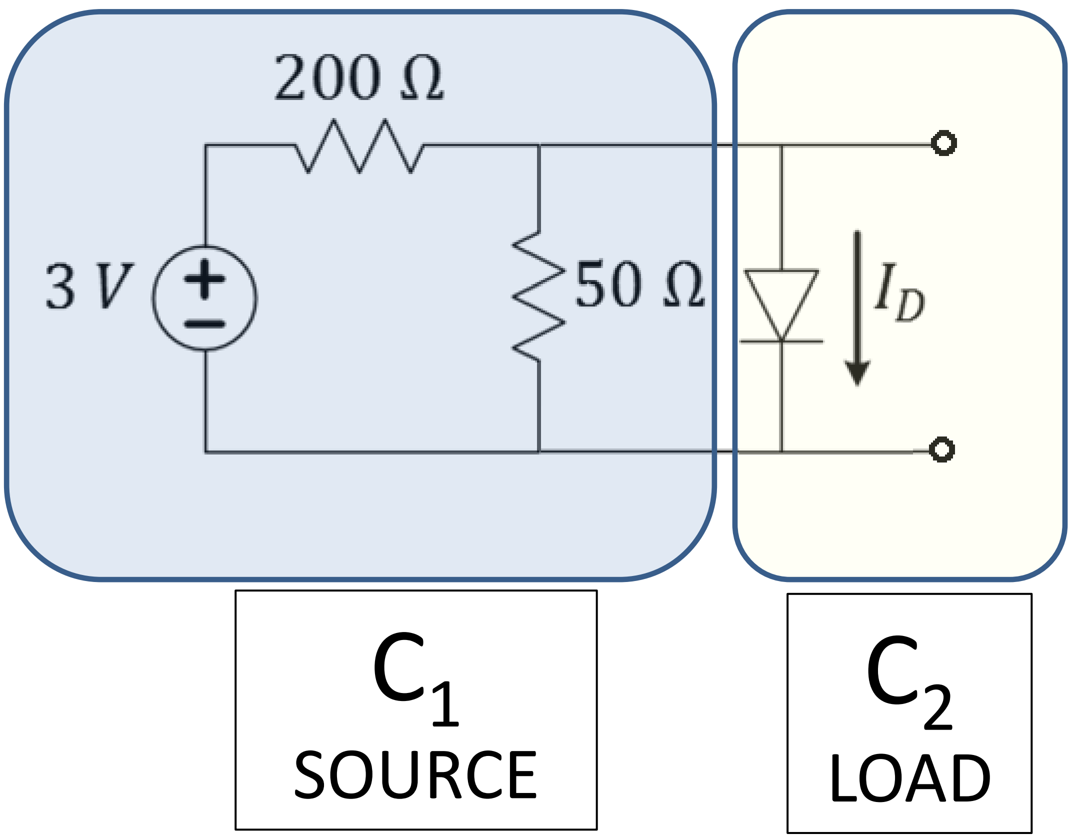
Therefore: $\boxed{I_D = 0}$ $\boxed{I_1 = I_2 = \frac{3}{250} = 12\text{mA}}$

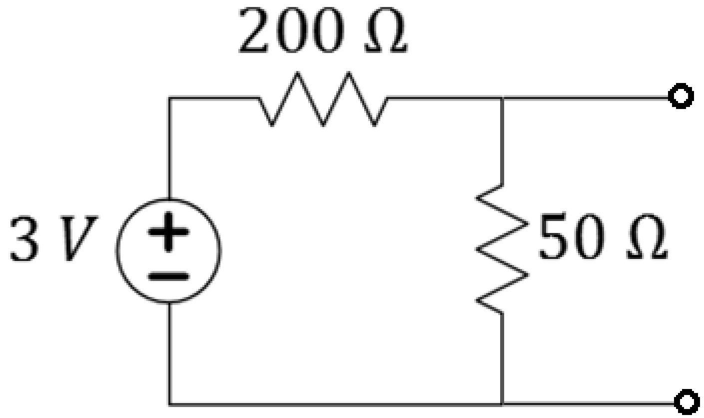
$\boxed{V_D = I_2 \times 50 = 0.6\text{V}}$ **or:** $\boxed{V_D = 3 \times \frac{50}{250} = 0.6\text{V}}$ **from voltage divider**

below threshold

Analysis with IV curve

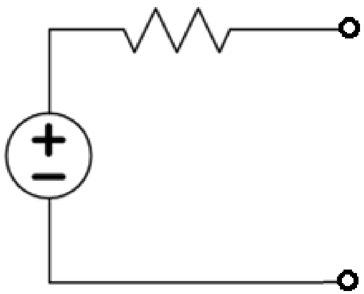




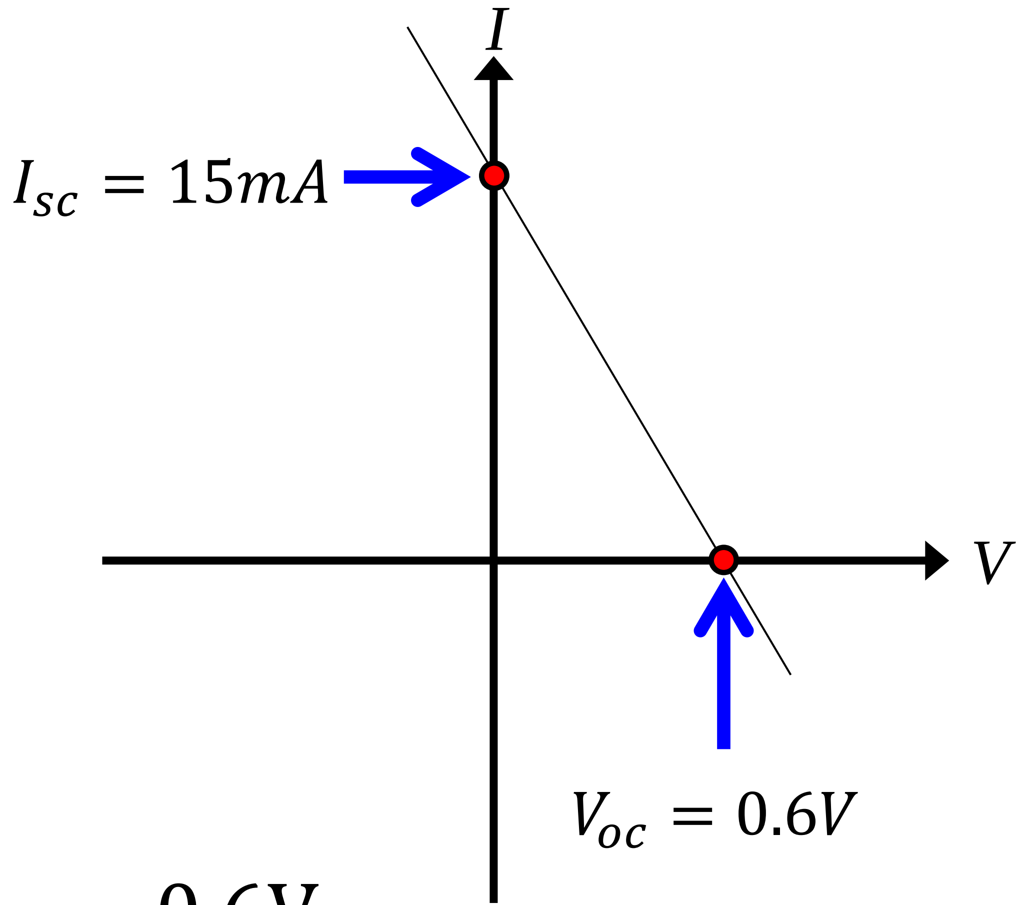


Thevenin source

$R_T = 40\Omega = 200\Omega // 50\Omega$

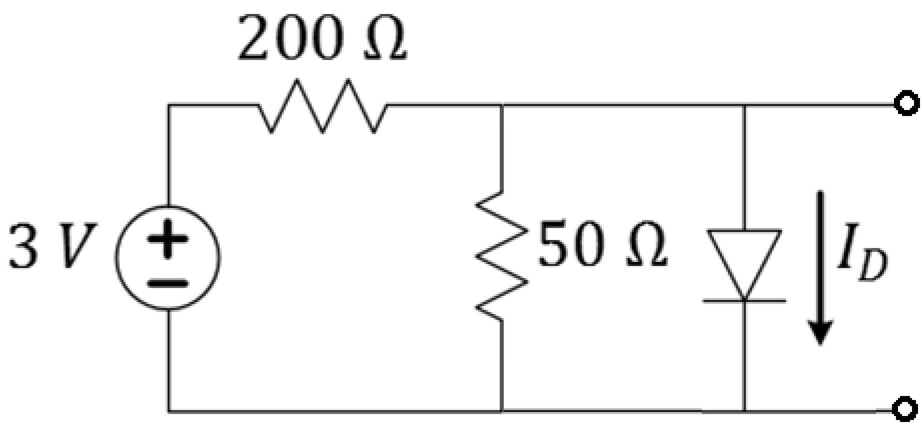


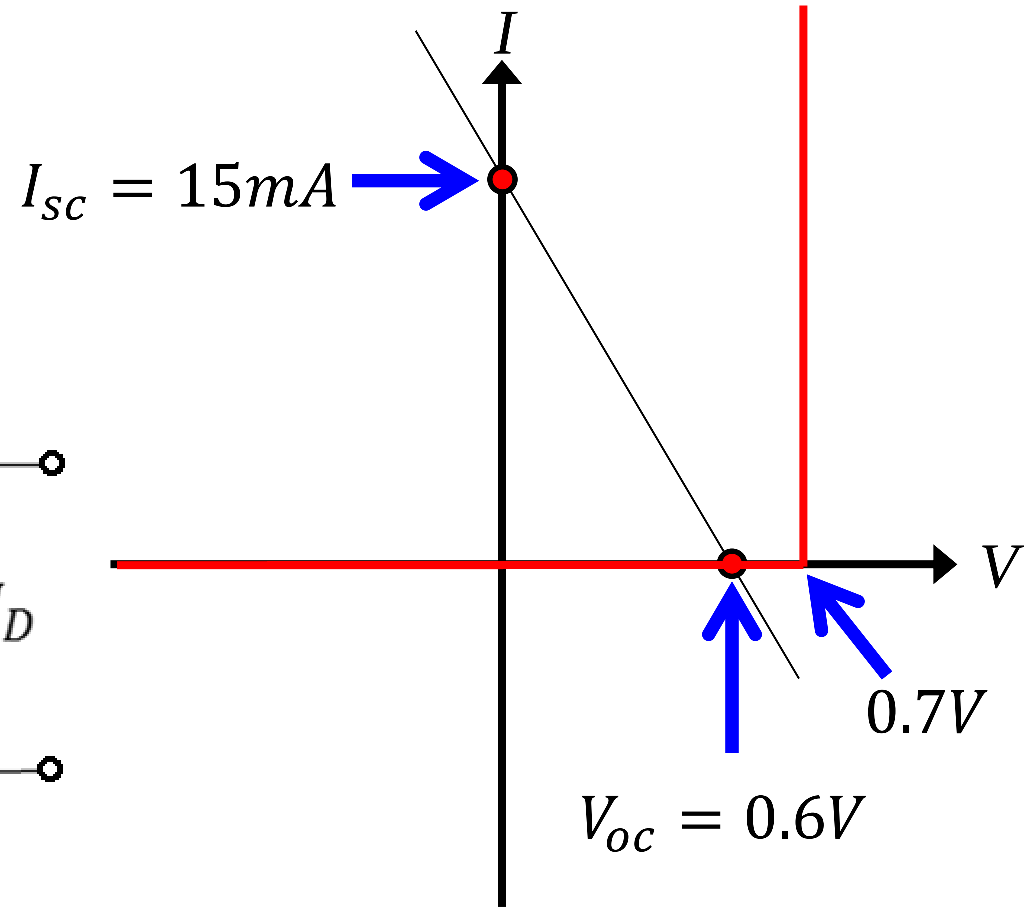
$V_T = 0.6\text{V}$



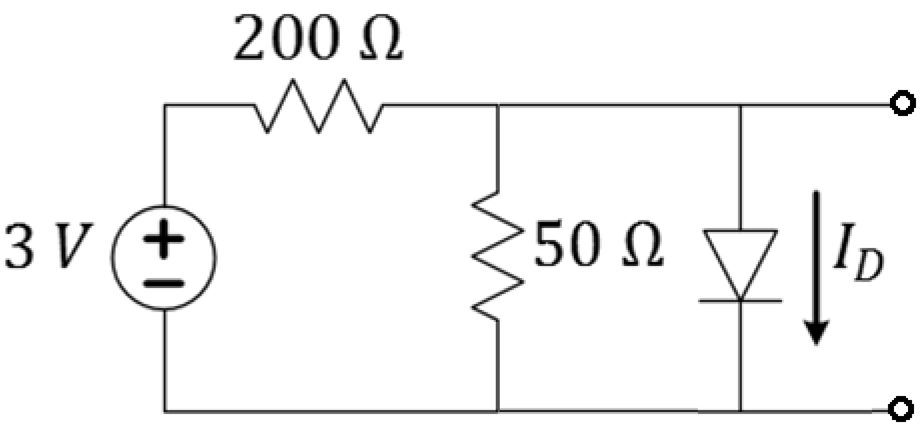
$$V_{OC} = V_T = 3\,\frac{50}{200+50} = 0.6\text{V}$$

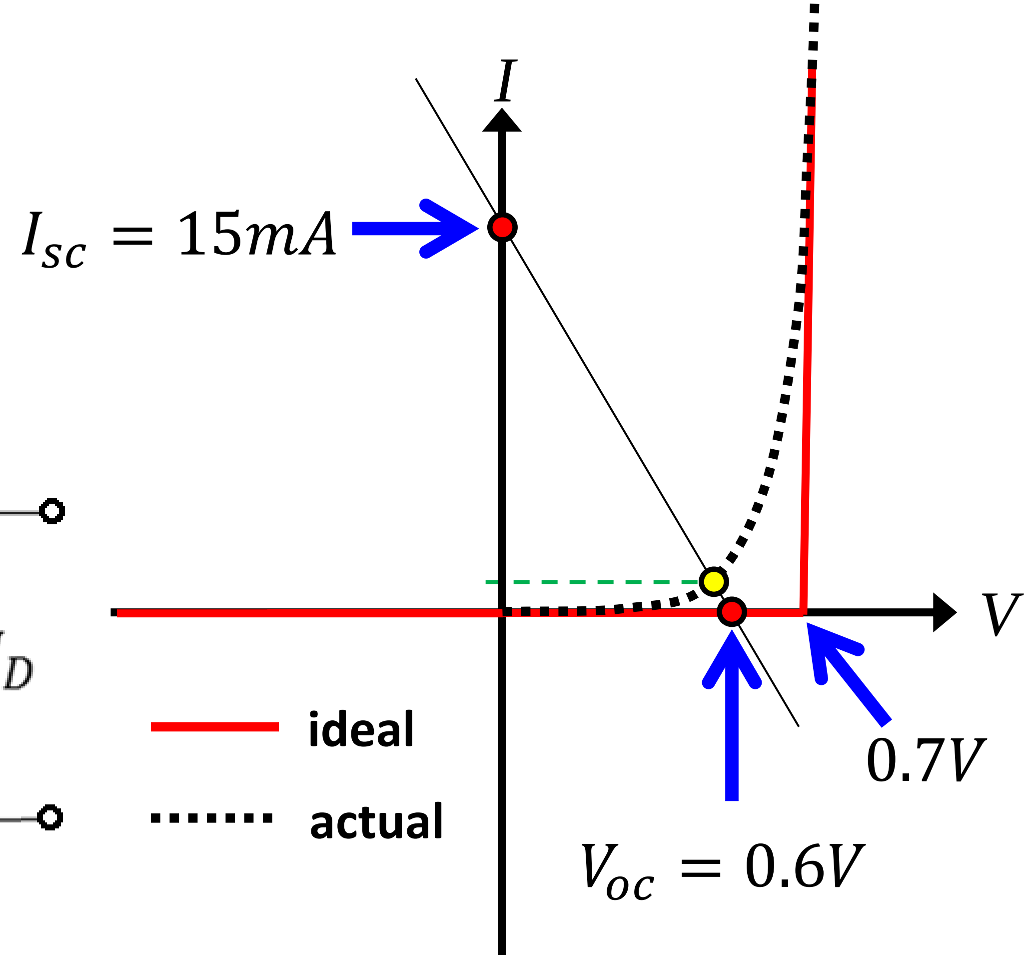
$$I_{SC} = \frac{3}{200} = \frac{V_T}{R_T} = \frac{0.6}{40} = 0.015\text{A} = 15\text{mA}$$





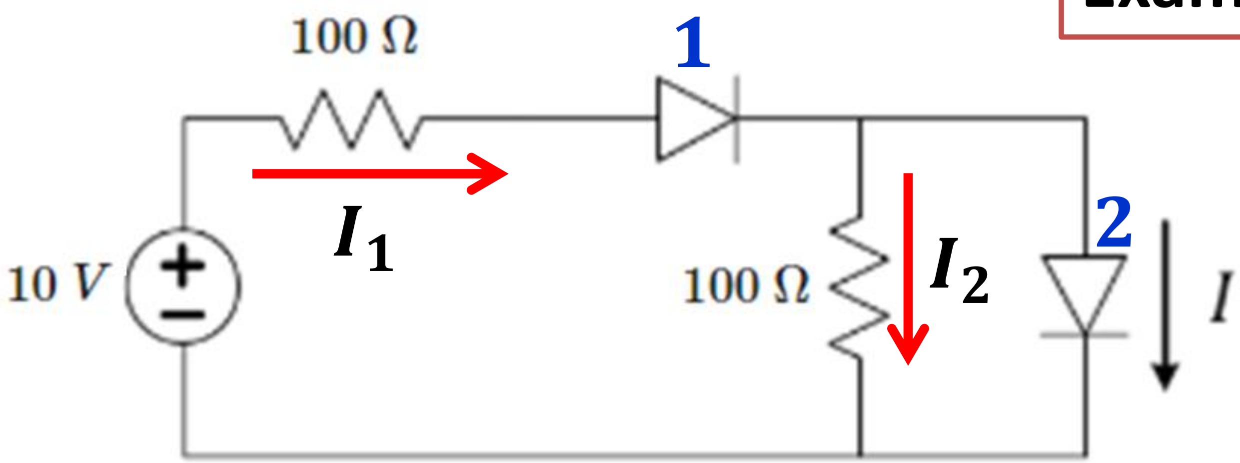
Diode is OFF





Considering an actual I-V characteristic curve, there is a small current flowing, but it is practically negligible.

Example 4A: Solve for I

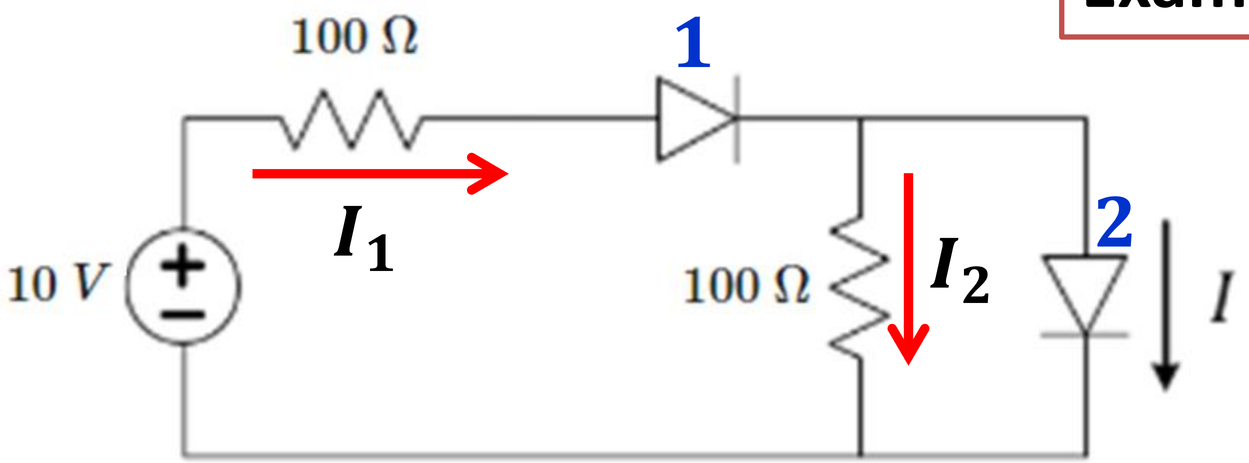


$$\boldsymbol{V_F = 4\text{V}}$$

Assume both diodes conduct

$$I_1 = I_2 + I$$

Example 4A: Solve for I



$\boldsymbol{V_F = 4\mathrm{V}}$

Assume both diodes conduct

$$I_1 = I_2 + I$$

KVL $$-10 + 100I_1 + 4 + 4 = 0$$

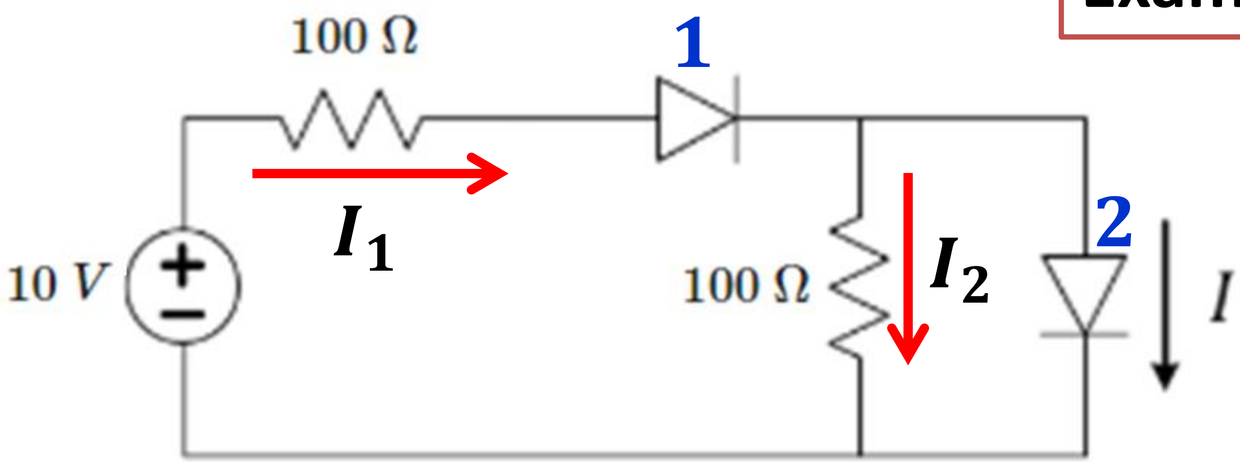
$$I_1 = \frac{2}{100} = 20\text{mA}$$

$$I_2 = \frac{4}{100} = 40mA$$

But $I_2 > I_1 \;\; \rightarrow$ diode **2** cannot conduct

$\boldsymbol{I} = 0$

Example 4A: Solve for I



$$V_F = 4\text{V}$$

Assume both diodes conduct

$$I_1 = I_2 + I$$

KVL

$$-10 + 100I_1 + 4 + 4 = 0$$

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$$I_2 = \frac{4}{100} = 40mA$$

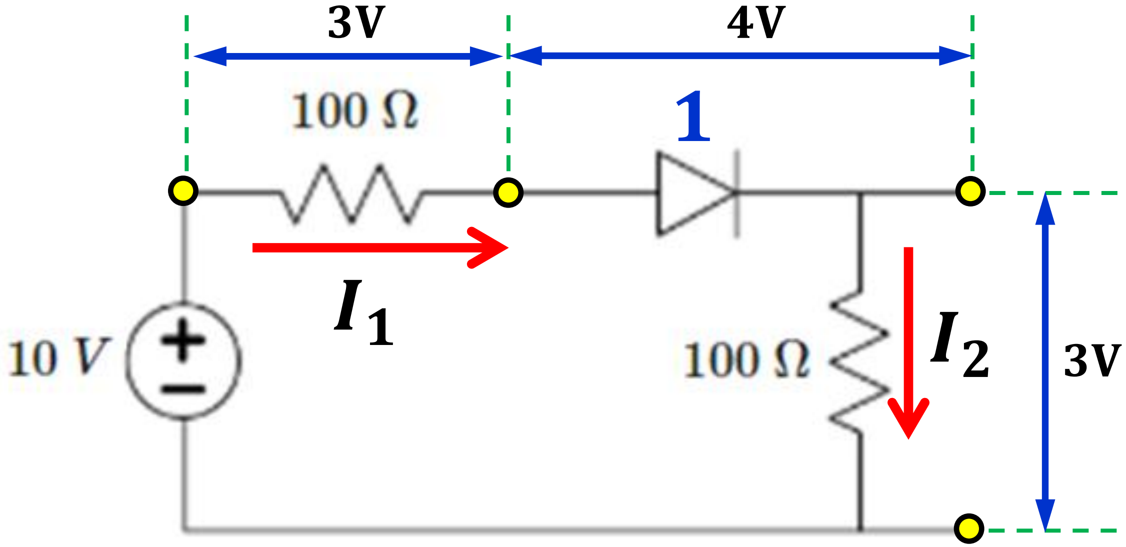
But $I_2 > I_1 \;\; \longrightarrow$ diode **2** cannot conduct

$$I = 0$$

new KVL with diode 2 open circuit

$$-10 + 100I_1 + 4 + 100I_1 = 0$$

$$I_1 = \frac{6}{200} = 30\text{mA}$$

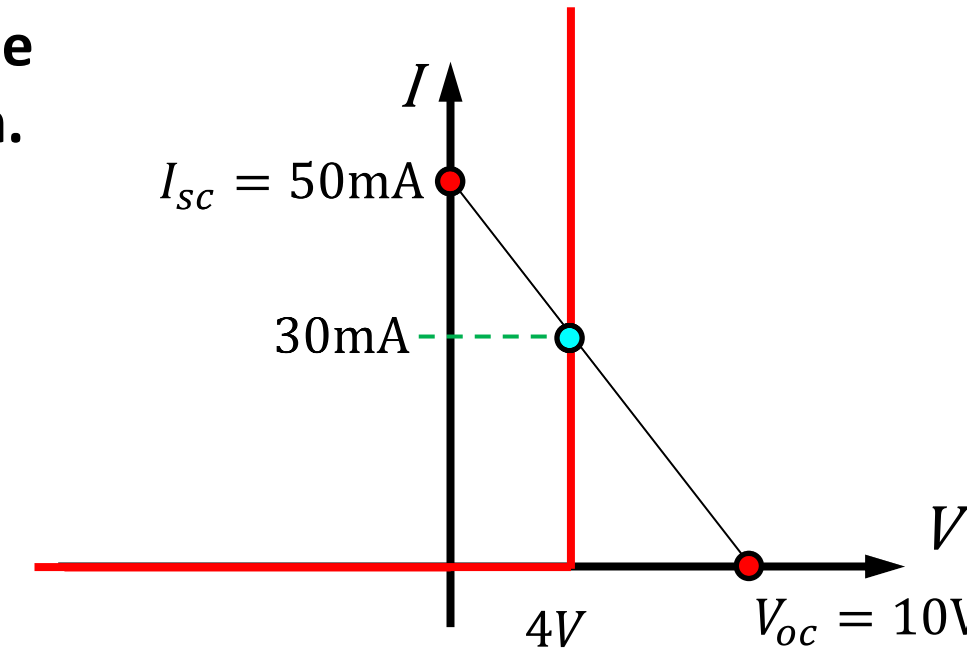


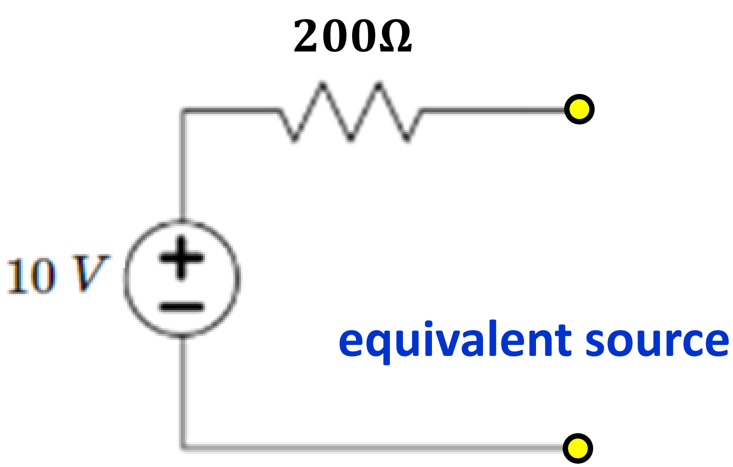
$$\boxed{I = 0}$$

$$\boxed{V_F = 4\text{V}}$$

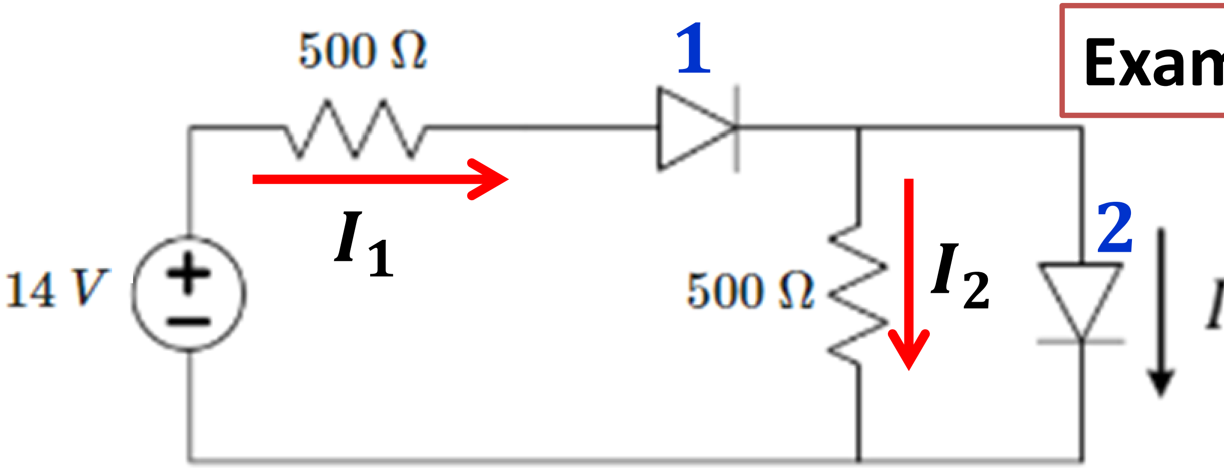
$$\boxed{I_1 = I_2 = 30\text{mA}}$$

Only diode 1 conducts. The two resistors drop 3V each.



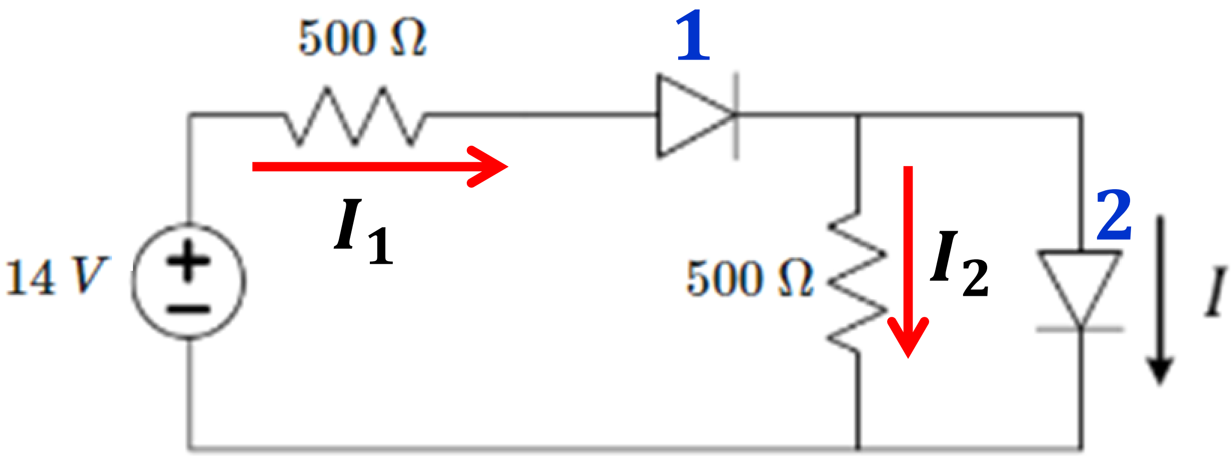


Example 4B: Solve for I



$V_F = 4\mathrm{V}$

The source voltage has been increased



$\boldsymbol{V_F = 4\mathrm{V}}$

Assume both diodes conduct

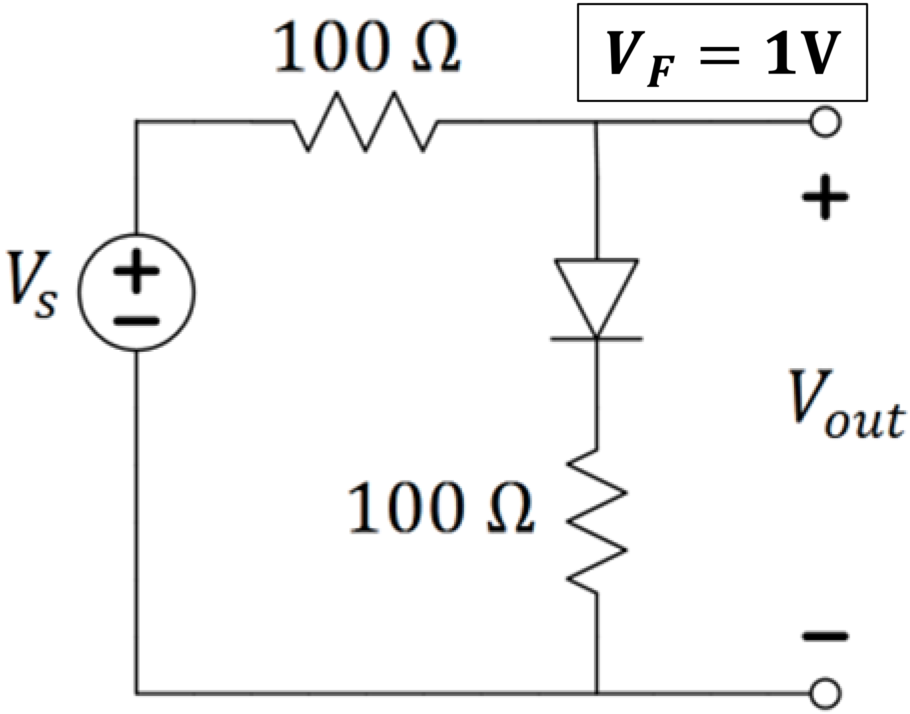
$$\boxed{I_1 = I_2 + I}$$

KVL

$$-14 + 500I_1 + 4 + 4 = 0$$

$$\boxed{I_1 = \frac{6}{500} = 12\mathrm{mA}} \qquad \boxed{I_2 = \frac{4}{500} = 8\mathrm{mA} < I_1}$$

$$\boxed{I = I_1 - I_2 = 4\ \mathrm{mA}}$$

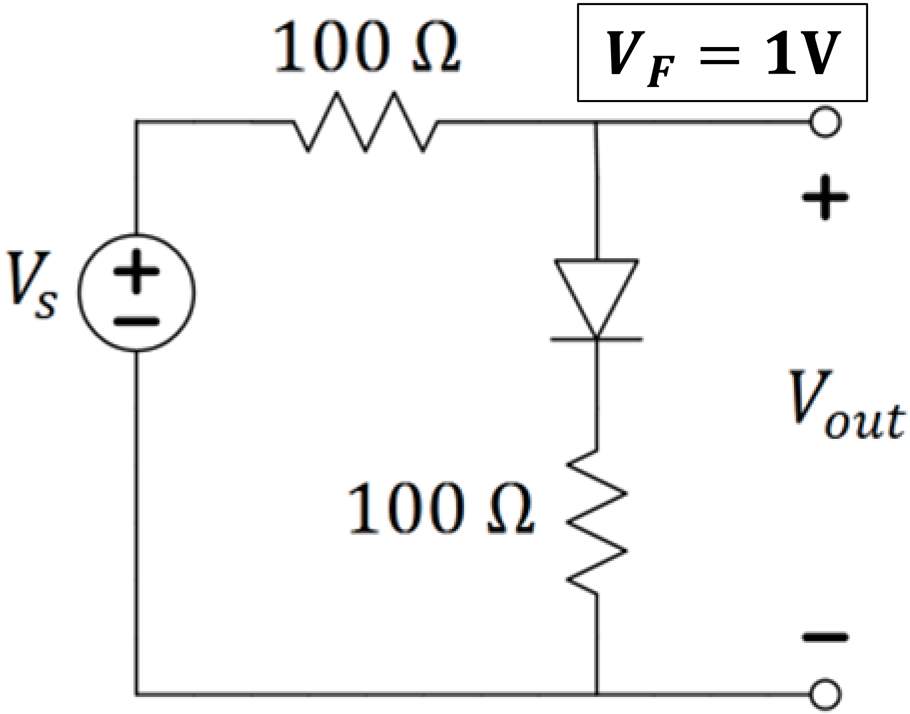


Example 5: Solve for V_{out}

Find V_{out} when

a) $V_S = 5\text{V}$

b) $V_S = -12\text{V}$



Example 5: Solve for V_{out}

Find V_{out} when

a) $V_S = 5\text{V}$

b) $V_S = -12\text{V}$

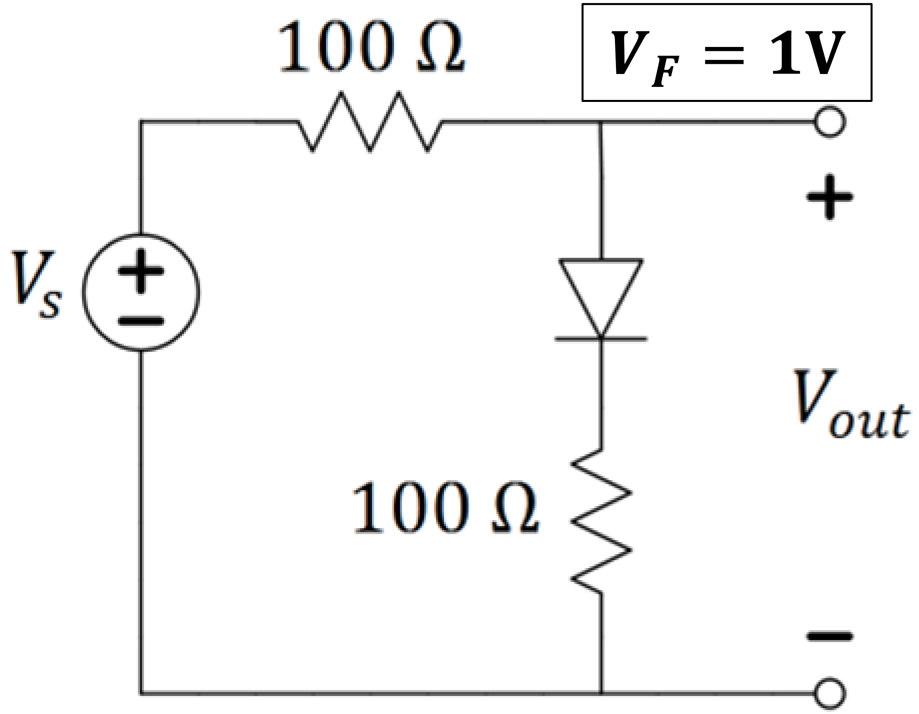
a)

$$-5 + 100I_1 + 1 + 100\,I = 0$$

$$200I_1 = 4 \quad \rightarrow \quad I = 20\text{mA}$$

$$V_{out} = 100 \times 20\text{m} + 1 = 3\text{V}$$

Example 5: Solve for V_{out}



Find V_{out} when

a) $V_S = 5V$

b) $V_S = -12V$

a)

$$-5 + 100I_1 + 1 + 100\,I = 0$$

$$200I_1 = 4 \quad \rightarrow \quad I = 20\text{mA}$$

$$V_{out} = 100 \times 20\text{m} + 1 = 3\text{V}$$

b)

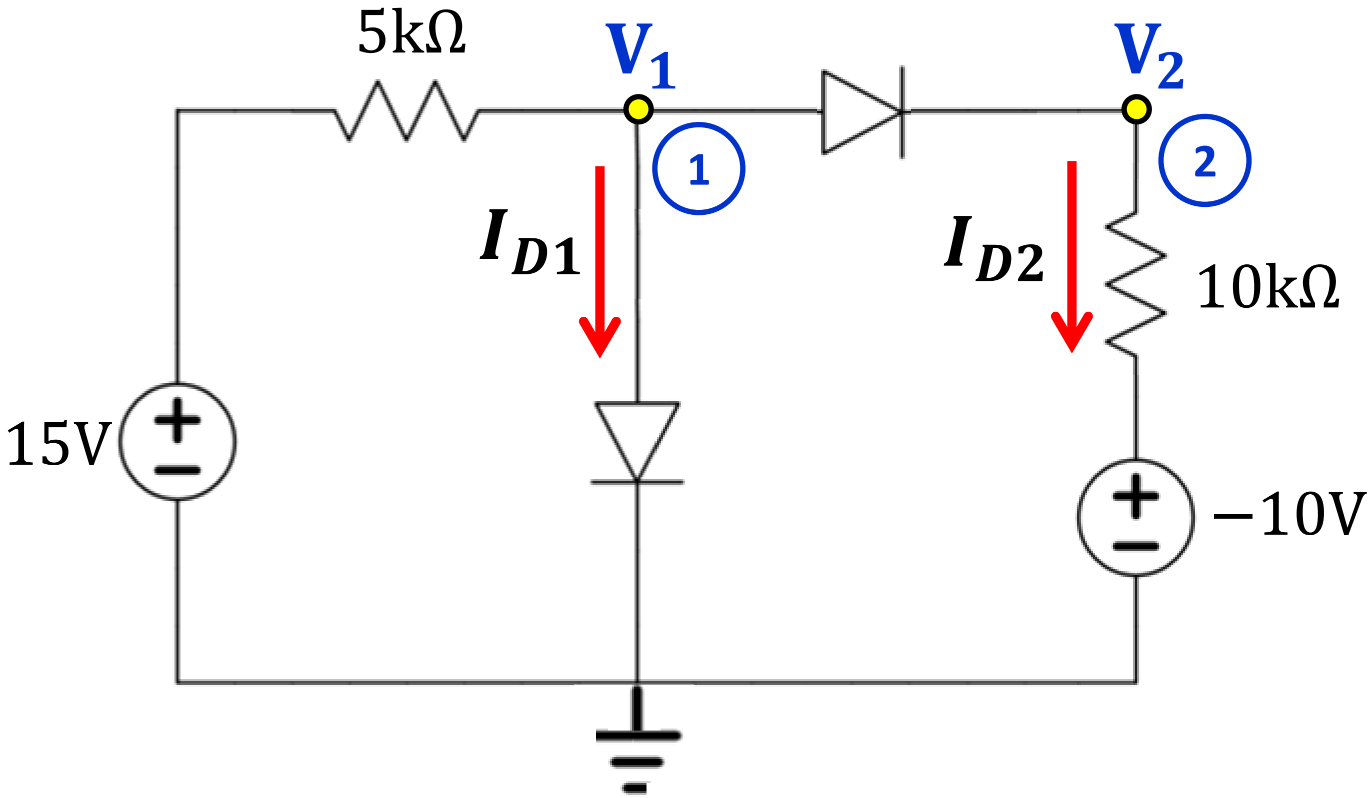
$$I = 0\text{ A}$$

Diode does not conduct (reverse bias)

$$V_{out} = -12\text{V}$$

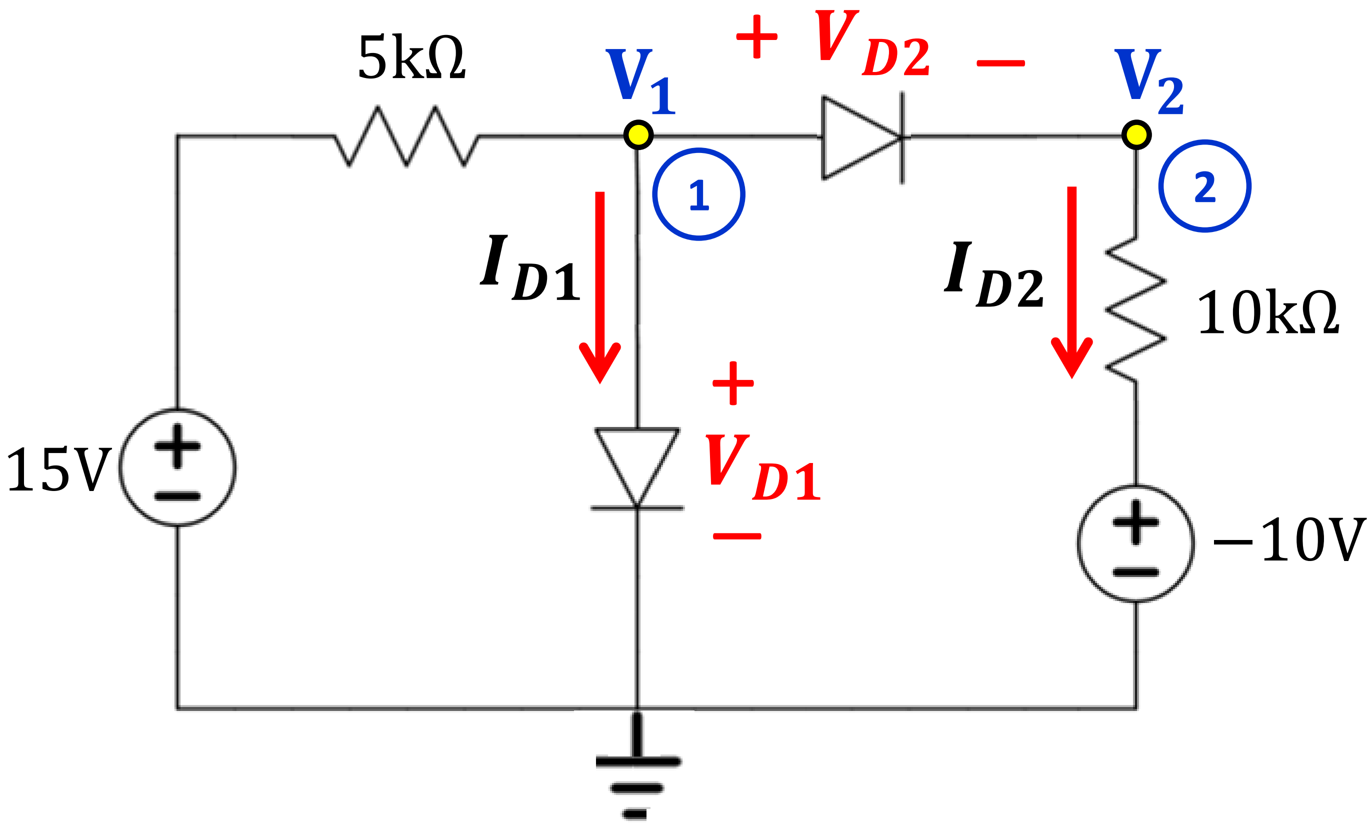
Example 6: Solve for I_{D1} and I_{D2}

$V_{D1} = V_{D2} = 0.7\text{V}$



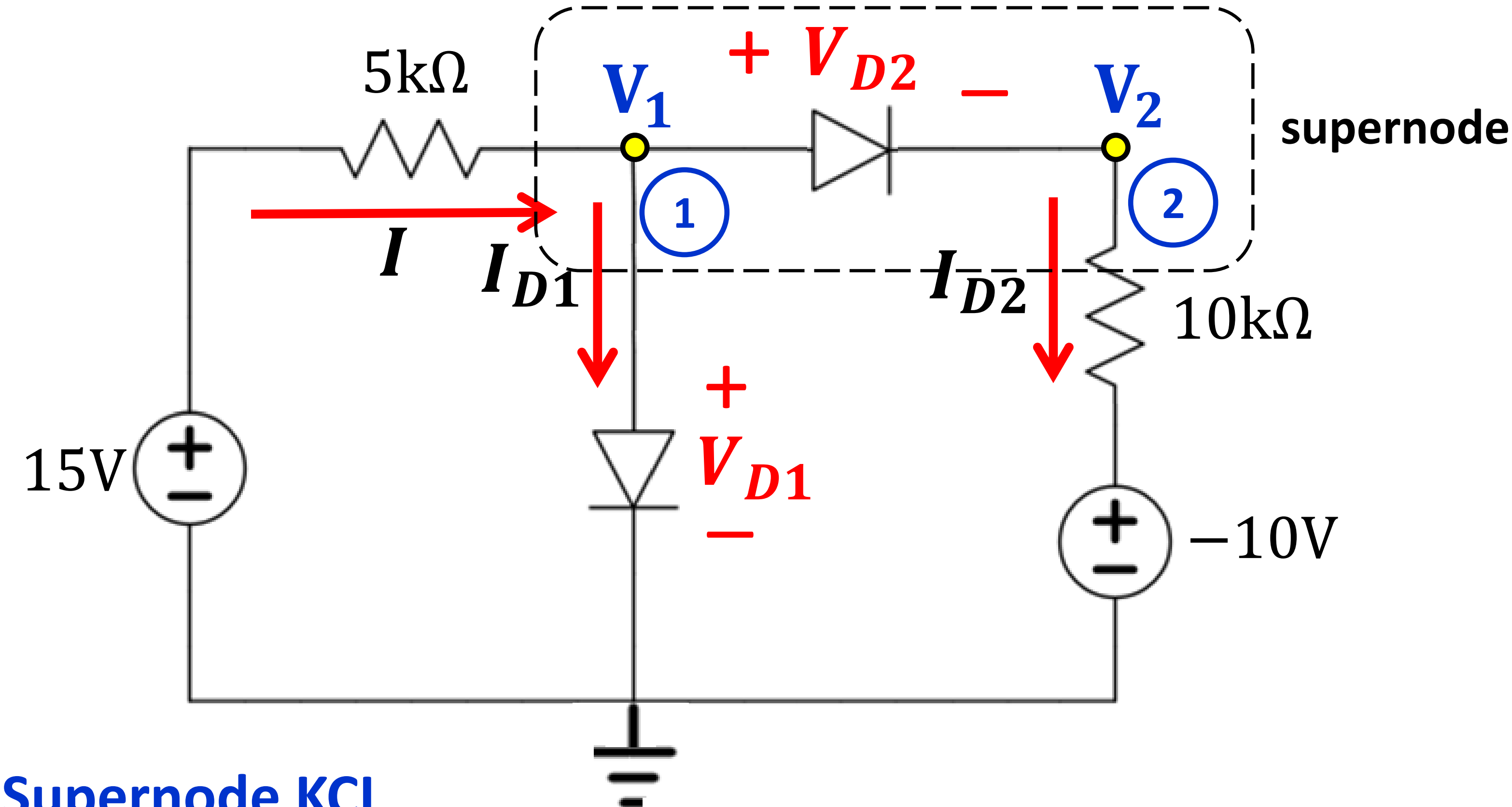
Example 6: Solve for I_{D1} and I_{D2}

$\boxed{V_{D1} = V_{D2} = 0.7\text{V}}$



- **Assume both diodes are ON**
- **By inspection:** $\boxed{\mathbf{V}_1 = V_{D1} = 0.7\text{V}}$ $\boxed{\mathbf{V}_1 - \mathbf{V}_2 = 0.7\text{V}}$

$$\boxed{\mathbf{V}_2 = \mathbf{V}_1 - V_{D2} = 0.7 - 0.7 = 0\text{V}}$$

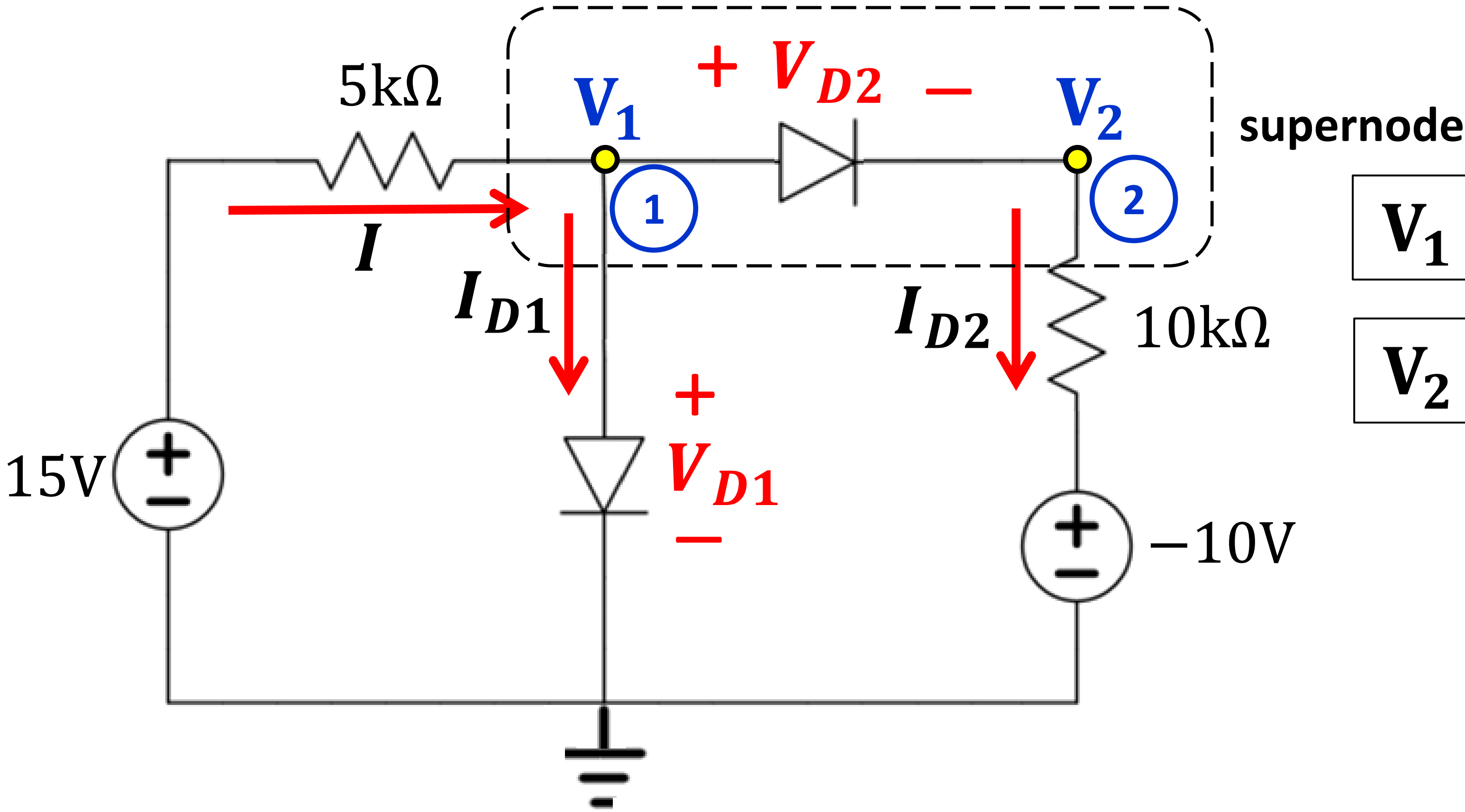


Supernode KCL

$$\frac{V_1 - 15}{5k} + I_{D1} + \frac{V_2 - (-10)}{10k} = 0$$

$$\frac{0.7 - 15}{5k} + I_{D1} + \frac{0 + 10}{10k} = 0$$

$$\boxed{I_{D1} = \frac{9.3}{5k} = 1.86\text{mA}}$$



$$\boxed{V_1 = 0.7V}$$

$$\boxed{V_2 = 0V}$$

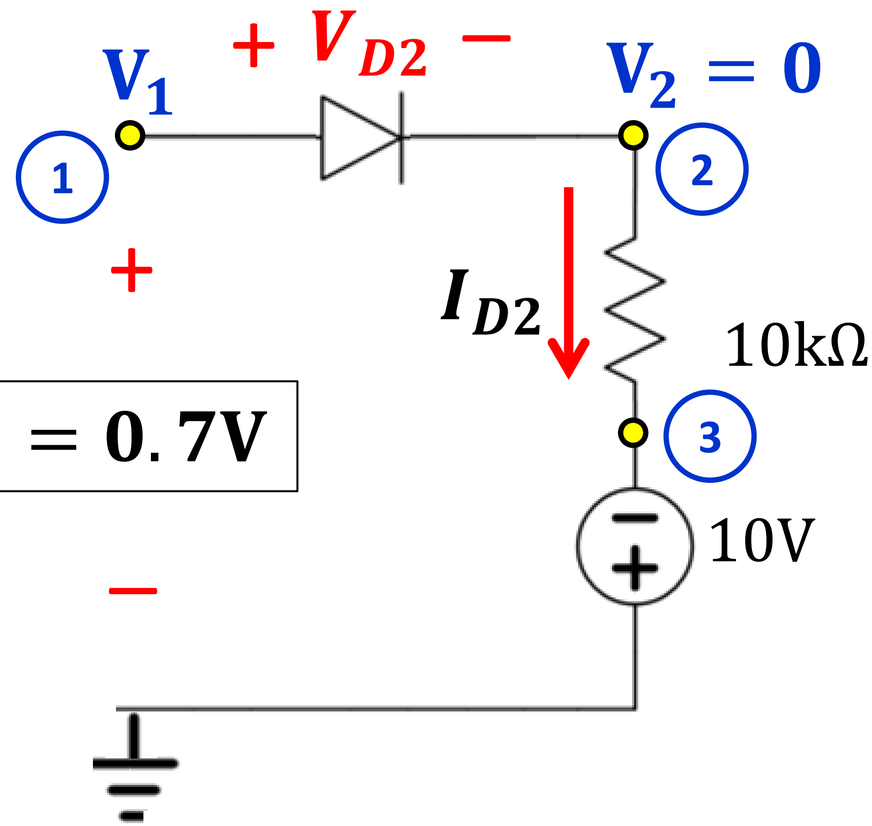
$$\boxed{I_{D1} = \frac{9.3}{5k} = 1.86mA}$$

$$\boxed{I_{D2} = \frac{V_2 + 10}{10k} = 1mA}$$

$$\boxed{I = \frac{15 - 0.7}{5k} = 2.86mA = I_{D1} + I_{D2}}$$

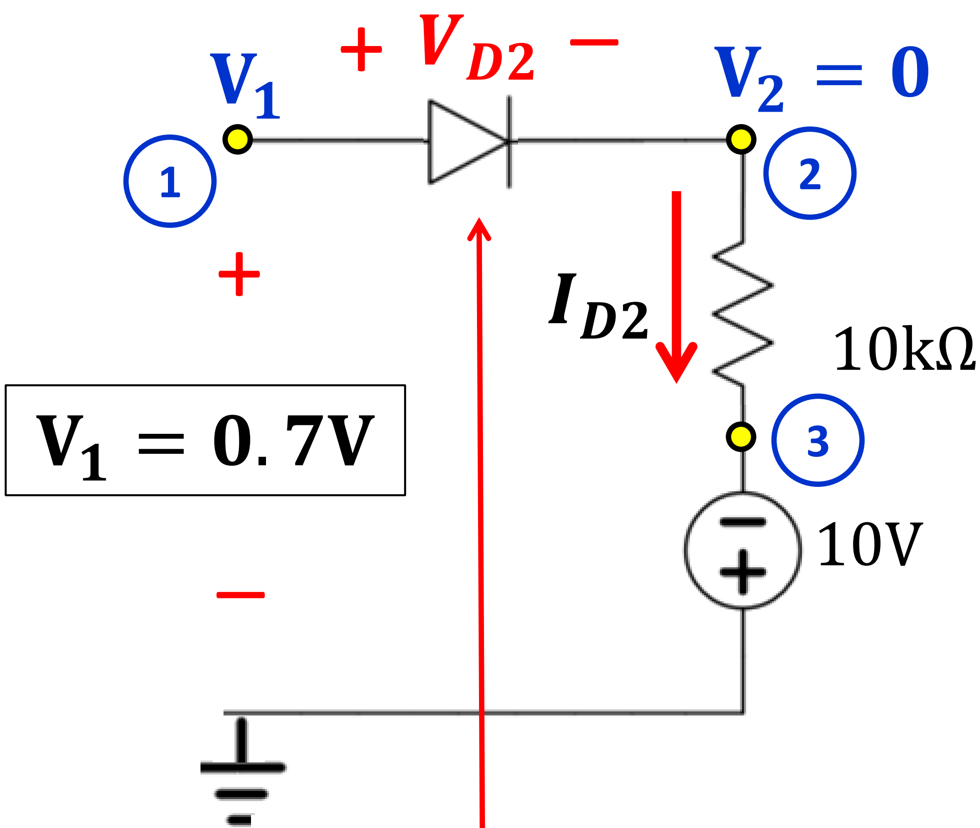
Results present no contradiction, both diodes are ON

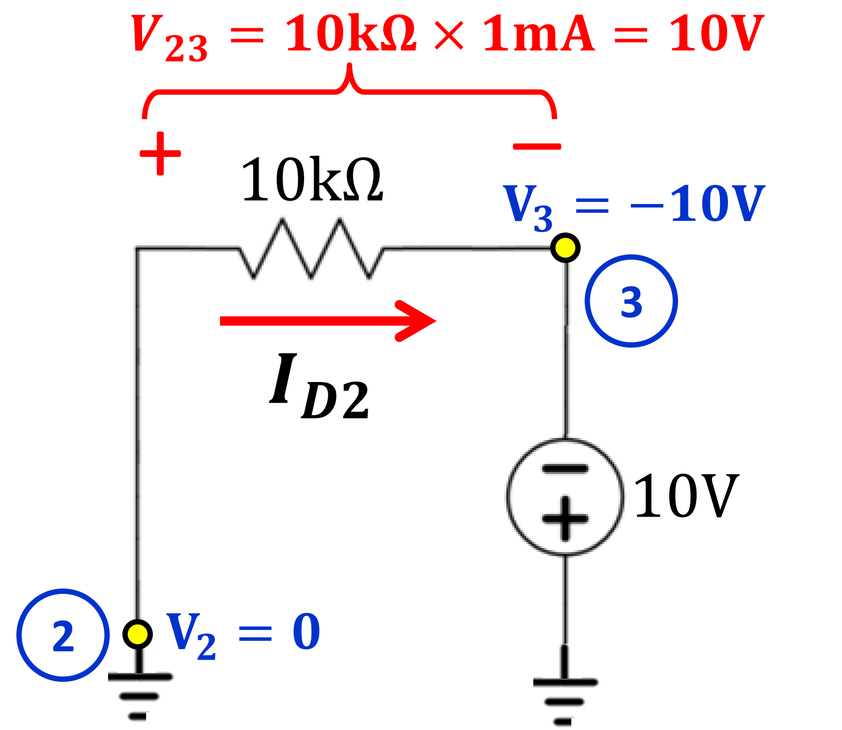
Right side of the circuit



$$\boxed{V_1 = 0.7V}$$

Right side of the circuit





This is the effective circuit seen by the 10V source

$$-\underbrace{\cancel{V_2}}_{=0} + 10\mathrm{k}\, I_{D2} - 10 = 0$$

Because of continuity (KCL) at node 2, this is the current which flows also through the diode

$$\boxed{I_{D2} = \frac{10}{10\mathrm{k}} = 1\mathrm{mA}}$$