ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 33

MWF – 12:00pm

Prof. Umberto Ravaioli

2062 ECE Building

Lecture 33 – Summary

Learning Objectives

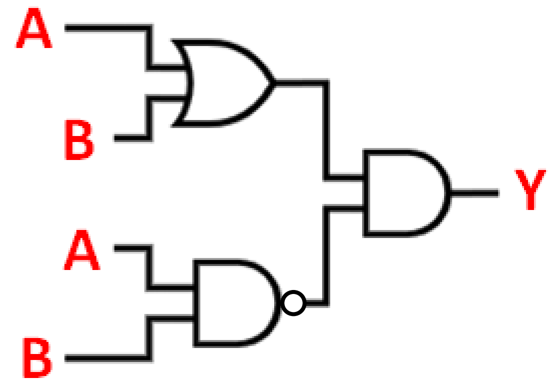
1. Frequency Response of Circuits
2. Low-Pass Passive Filters

Quiz 4 Reminder

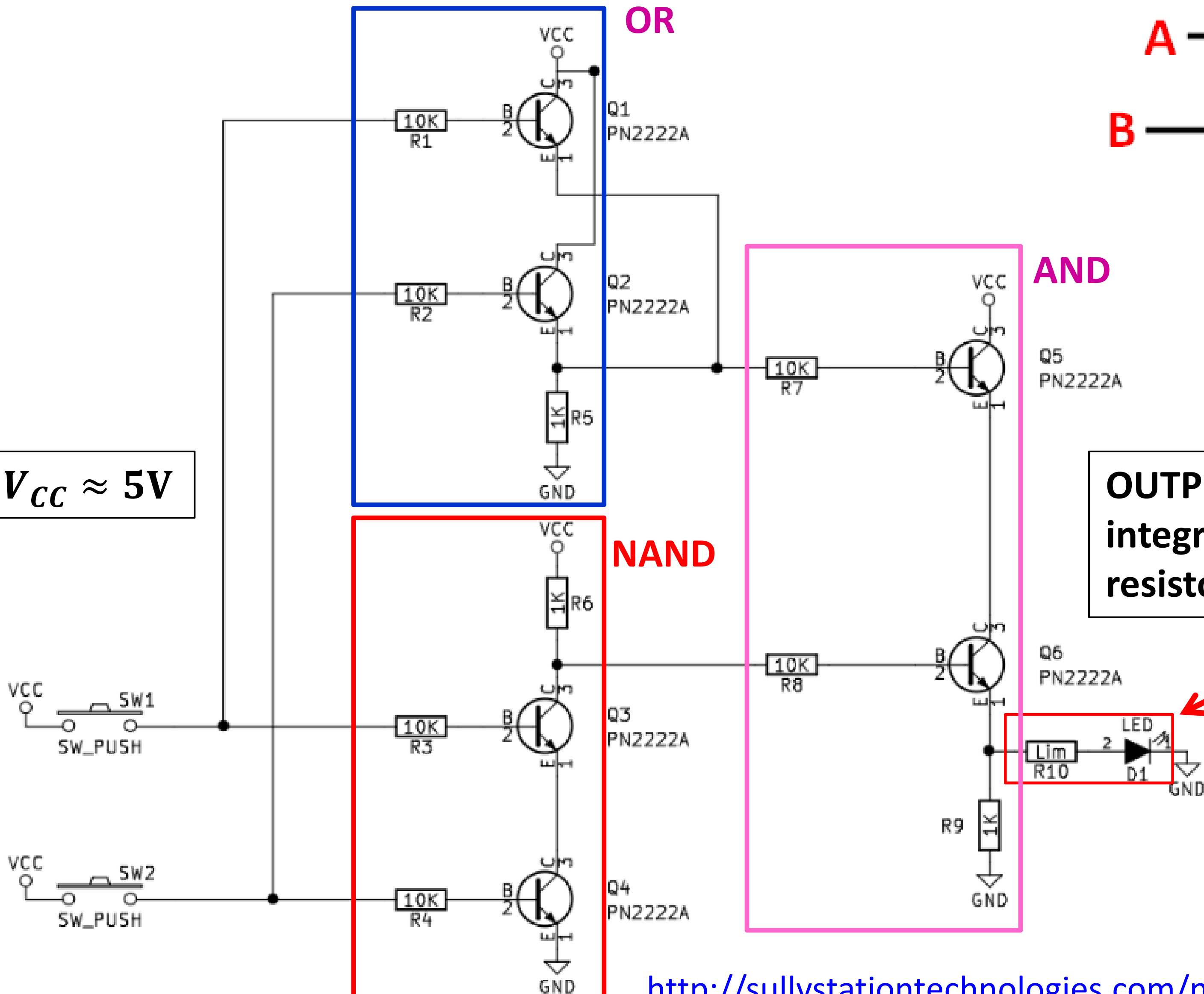
- April 22-24 (next week)
- **5 Questions:**
  - BJT Analysis
    - 2 Questions on *n-p-n*
    - 1 Question on *p-n-p*
  - 1 Question on boolean expression reduction
  - 1 Question on BJT logic circuits
- **Covering from Lectures 24 to Lecture 32**

From Lecture 30

$$(A+B)\overline{(AB)}$$



XOR circuit realization with BJT

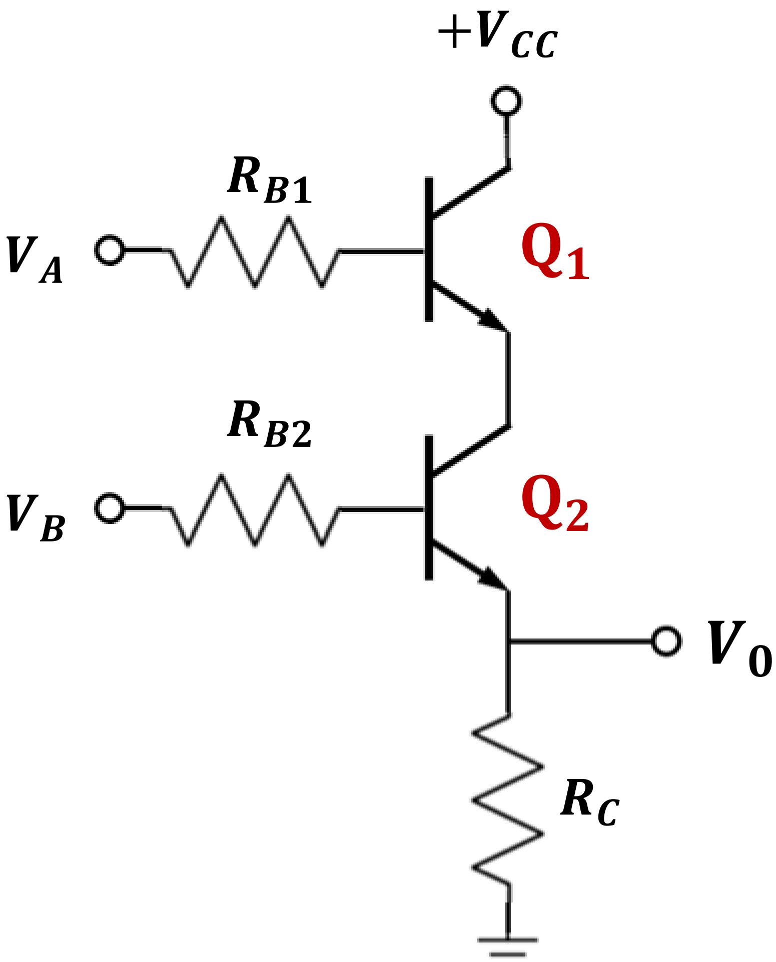


$V_{CC} \approx 5V$

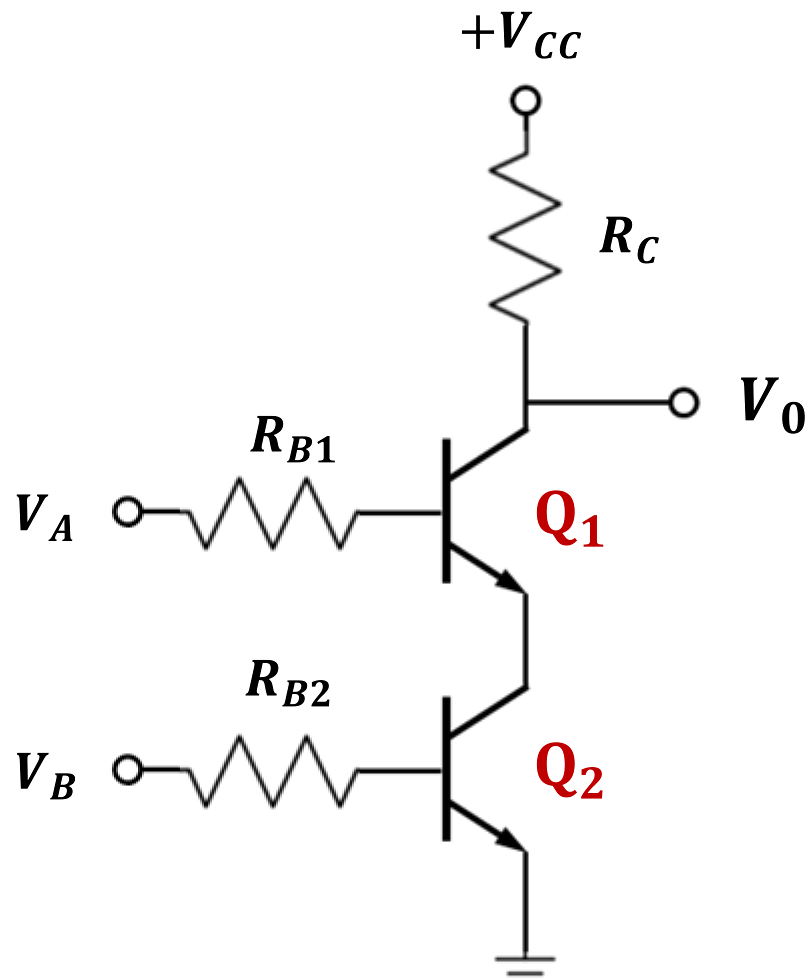
OUTPUT: LED LIGHT with integrated current limiting resistor

http://sullystationtechnologies.com/npnxorgate.html

Minimal implementation



AND



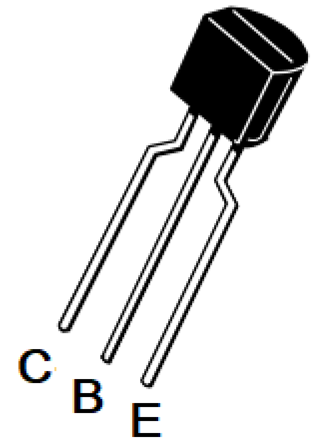
NAND

$V_{BE}(\mathrm{ON}) \approx 0.6\ \mathrm{V}$

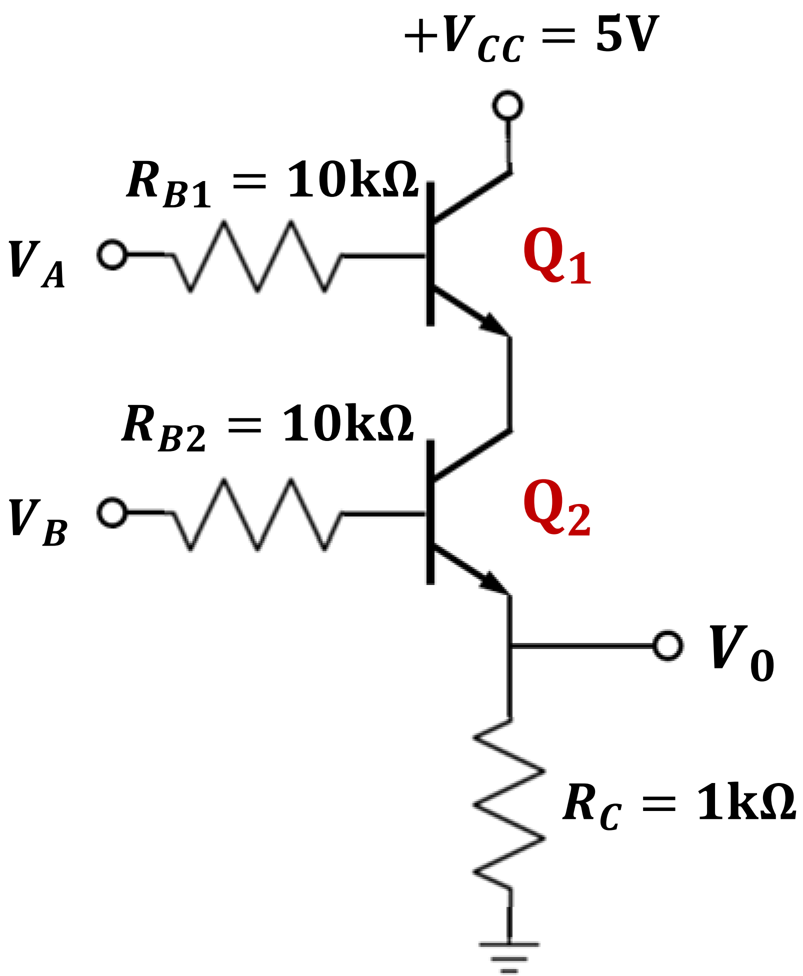
$V_{CE}(\mathrm{sat}) \approx 0.3\ \mathrm{V}$

General Purpose PN2222A BJT

$\beta \approx 100\ \text{to}\ 300$



Pick: $\beta = 100$



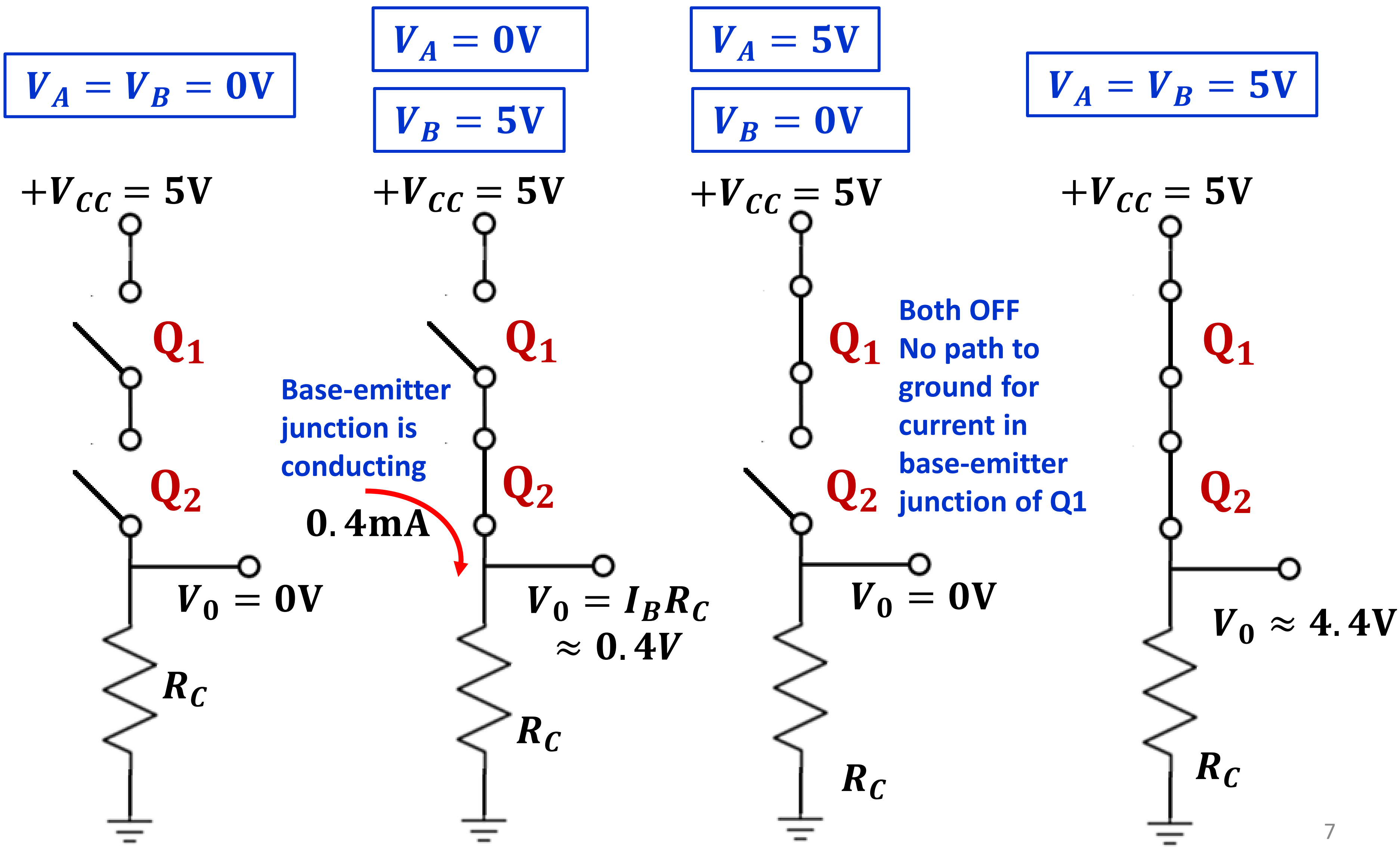
AND

Two transistors in series

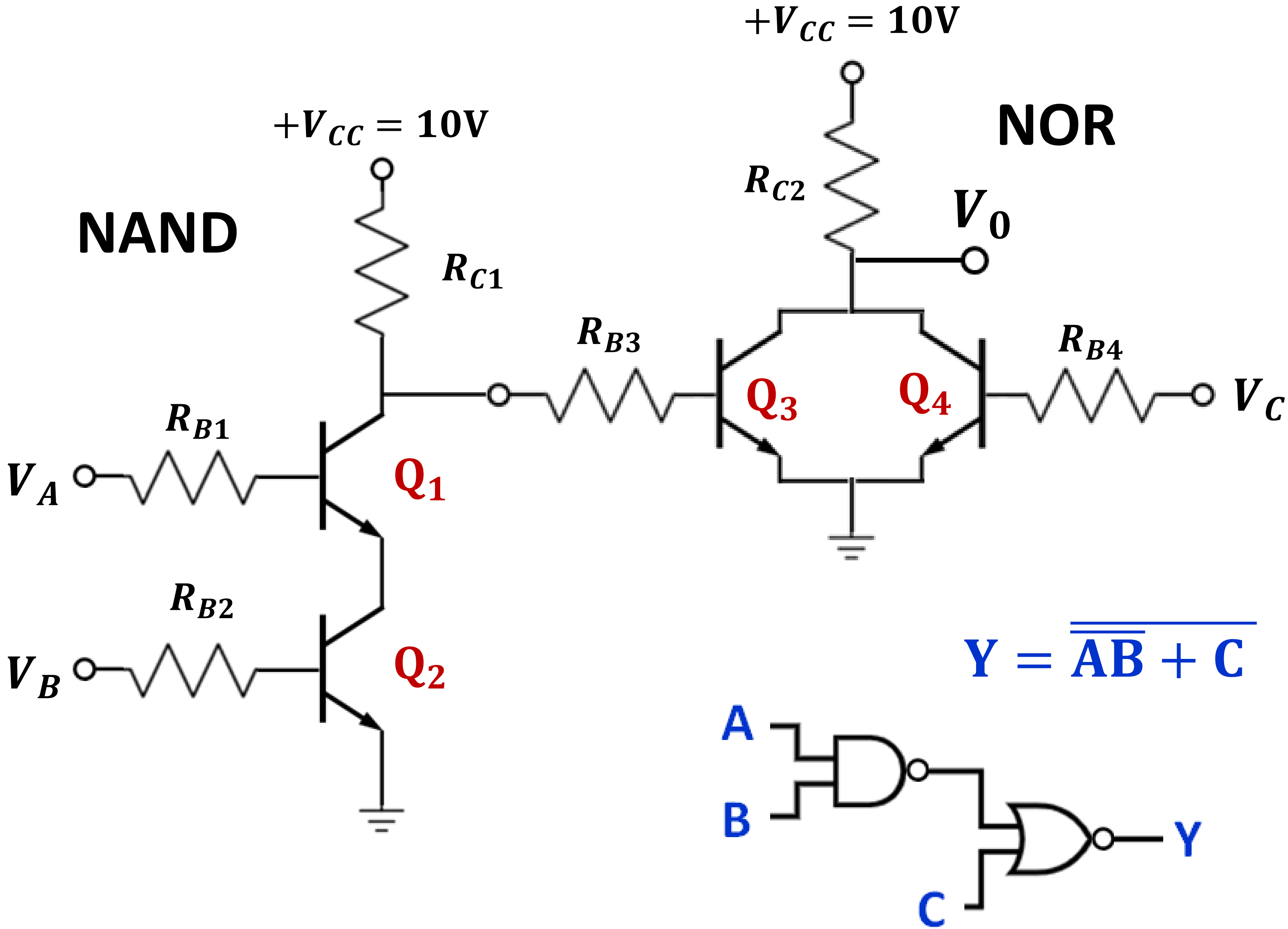
$V_{BE}(\mathrm{ON}) \approx 0.6\ \mathrm{V}$

$V_{CE}(\mathrm{sat}) \approx 0.3\ \mathrm{V}$

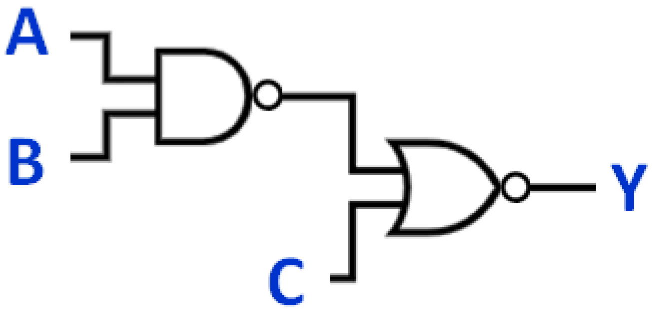
$\beta = 100$



Another Example



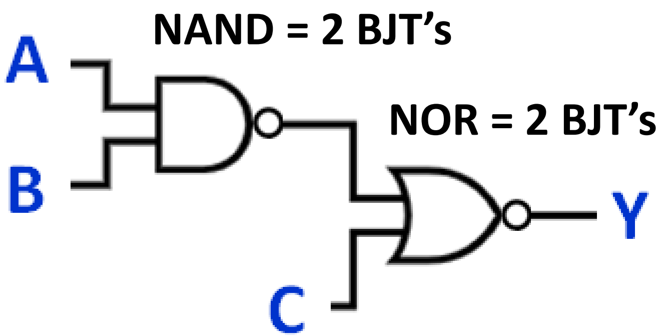
$$Y = \overline{\overline{AB} + C}$$



$$Y = \overline{\overline{AB} + C}$$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	0

What if we transform the circuit?

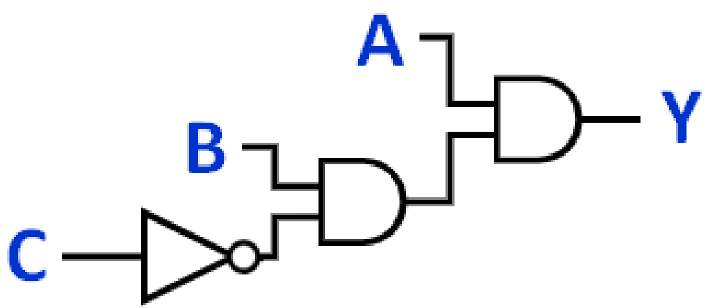


TOTAL = 4 BJT's

$$Y = \overline{\overline{AB} + C}$$

Using De Morgan's theorem

$$Y = AB\overline{C}$$



NOT = 1 BJT
Two 2-inputs NAND's = 4 BJT's
Two NOT's to obtain AND's = 2 BJT

TOTAL = 7 BJT's

NOT = 1 BJT
One 3-inputs NAND = 3 BJT's
NOT to obtain AND = 1 BJT

TOTAL = 5 BJT's

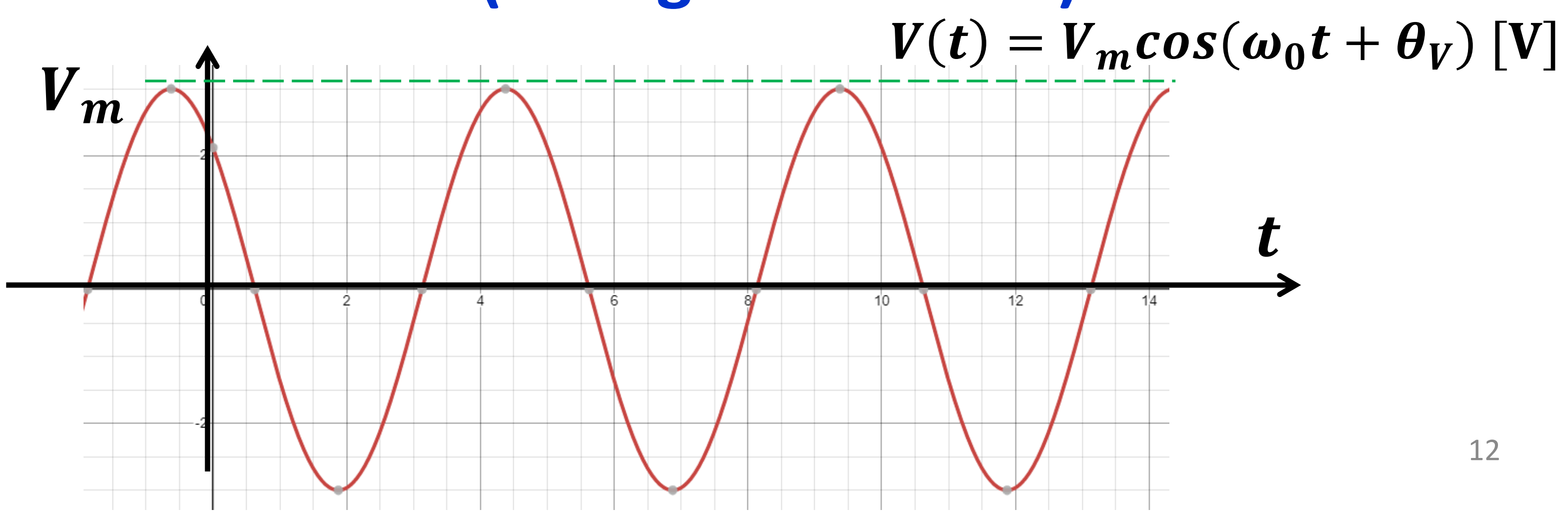
Frequency response of circuits

Frequency response

Until now we have considered the following forms of circuit excitation:

- **Constant source (voltage or current)**

- **Sinusoidal source (voltage or current)**



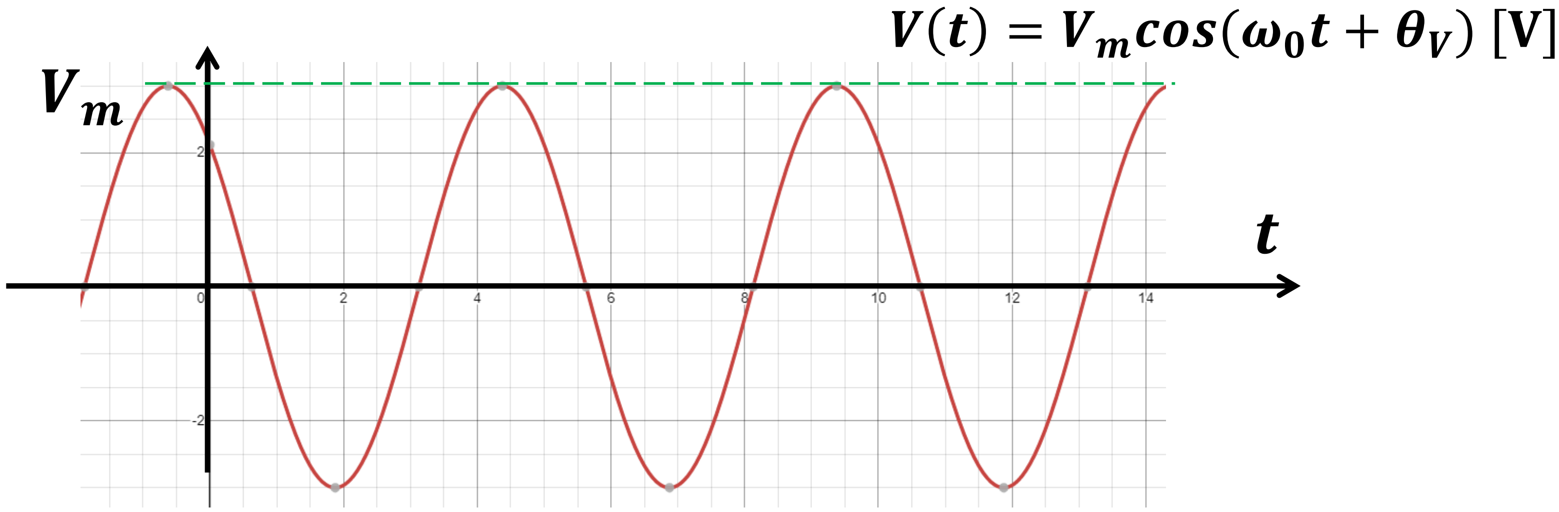
- **Constant source (voltage or current)**

$$V(t) = K\ [\mathrm{V}] = K\cos(0t)$$

$$\omega = 0\ \ \mathrm{rad/s}$$

It can be interpreted as a signal with zero frequency

- **Sinusoidal source (voltage or current)**



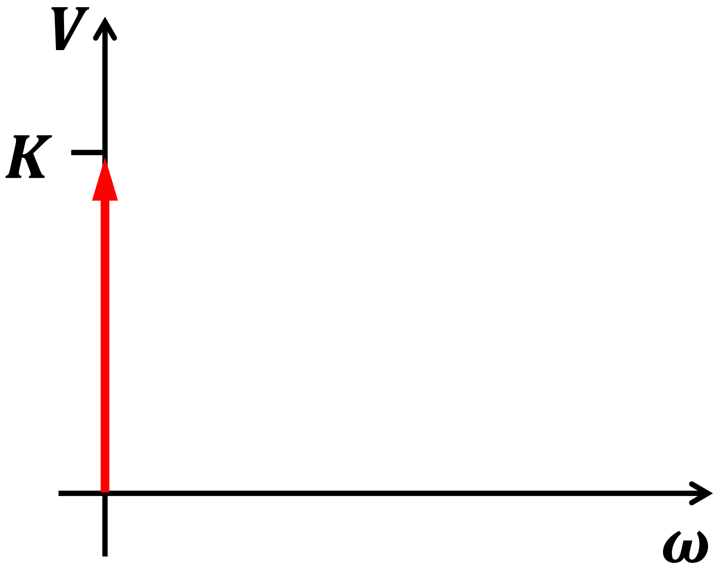
$$V(t) = V_m \cos(\omega_0 t + \theta_V)$$

$$\omega = 2\pi f_0 \ \text{rad/s}$$

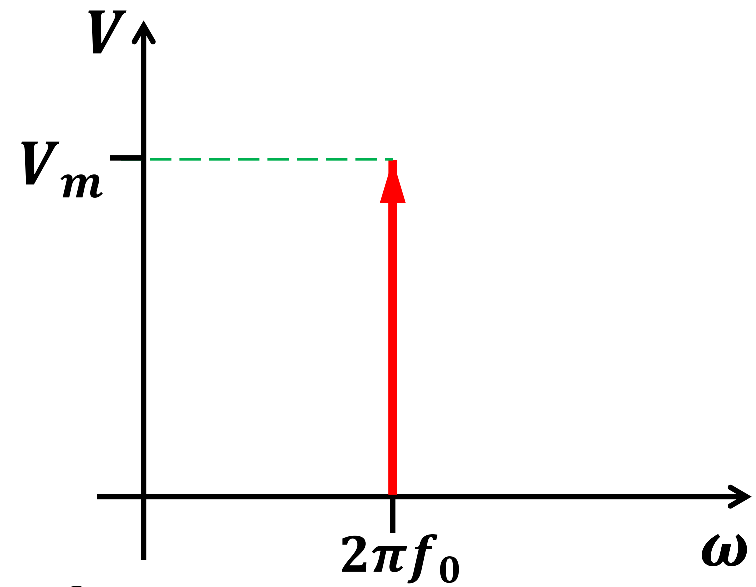
$$\theta_V = \frac{\pi}{4} \ \text{rad}$$

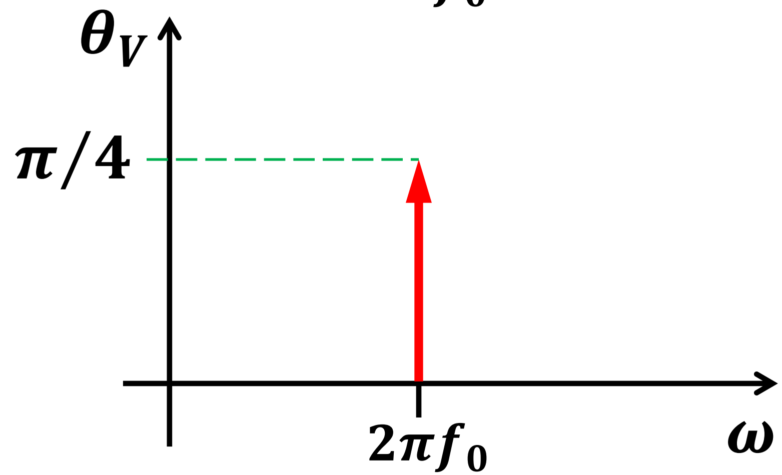
This is a signal with a single frequency component f_0

Representation in the frequency domain



$$V(t) = K$$

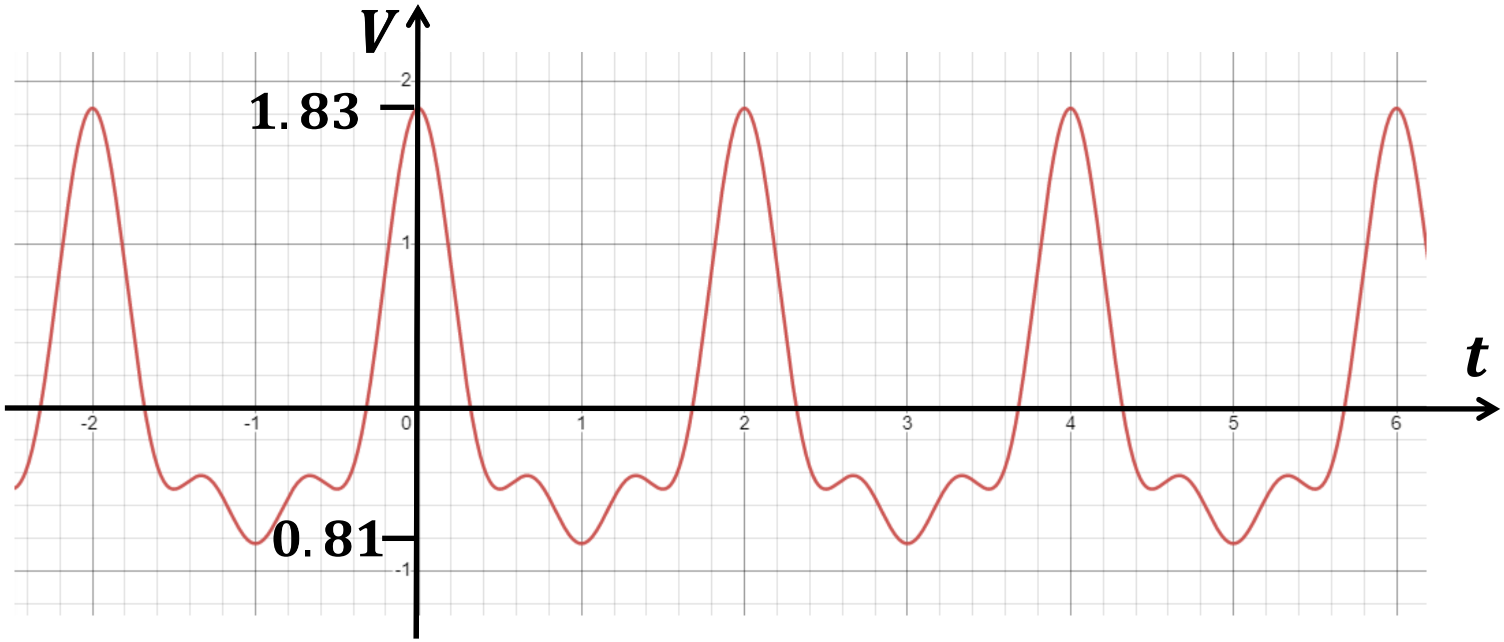


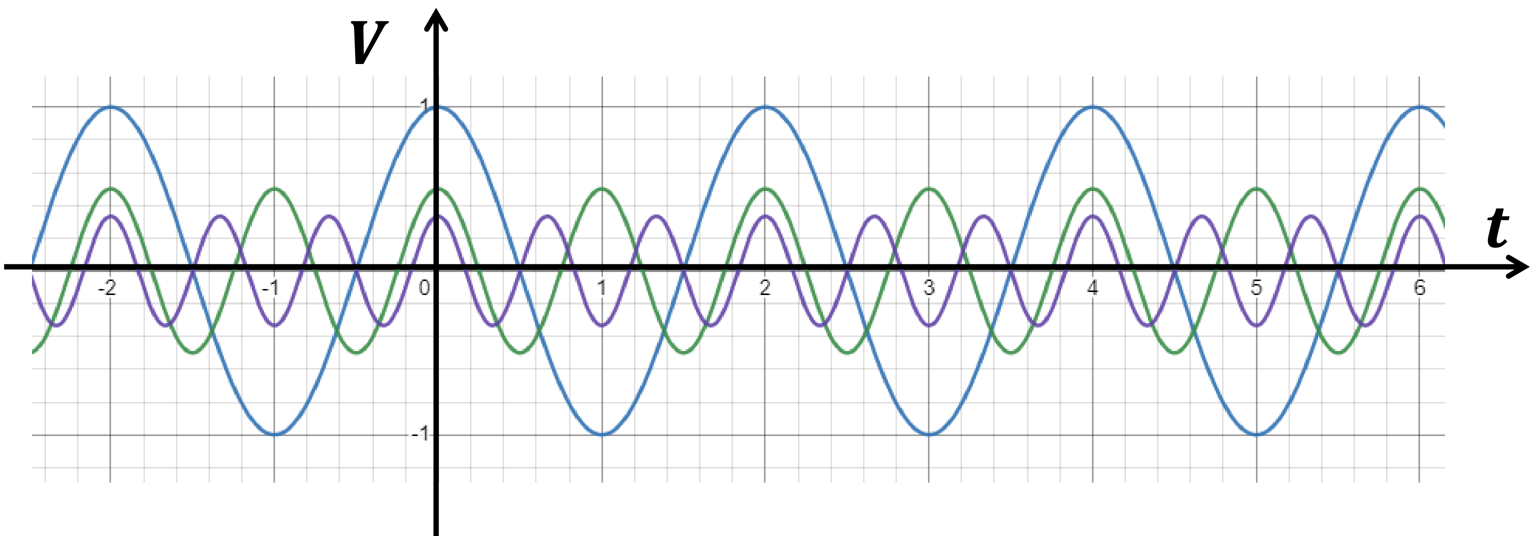


$$V(t) = V_m \cos(\omega_0 t + \pi/4)$$

Signal with several frequencies

$$V(t) = \cos(\omega_0 t) + \frac{1}{2}\cos(2\omega_0 t) + \frac{1}{3}\cos(3\omega_0 t)$$

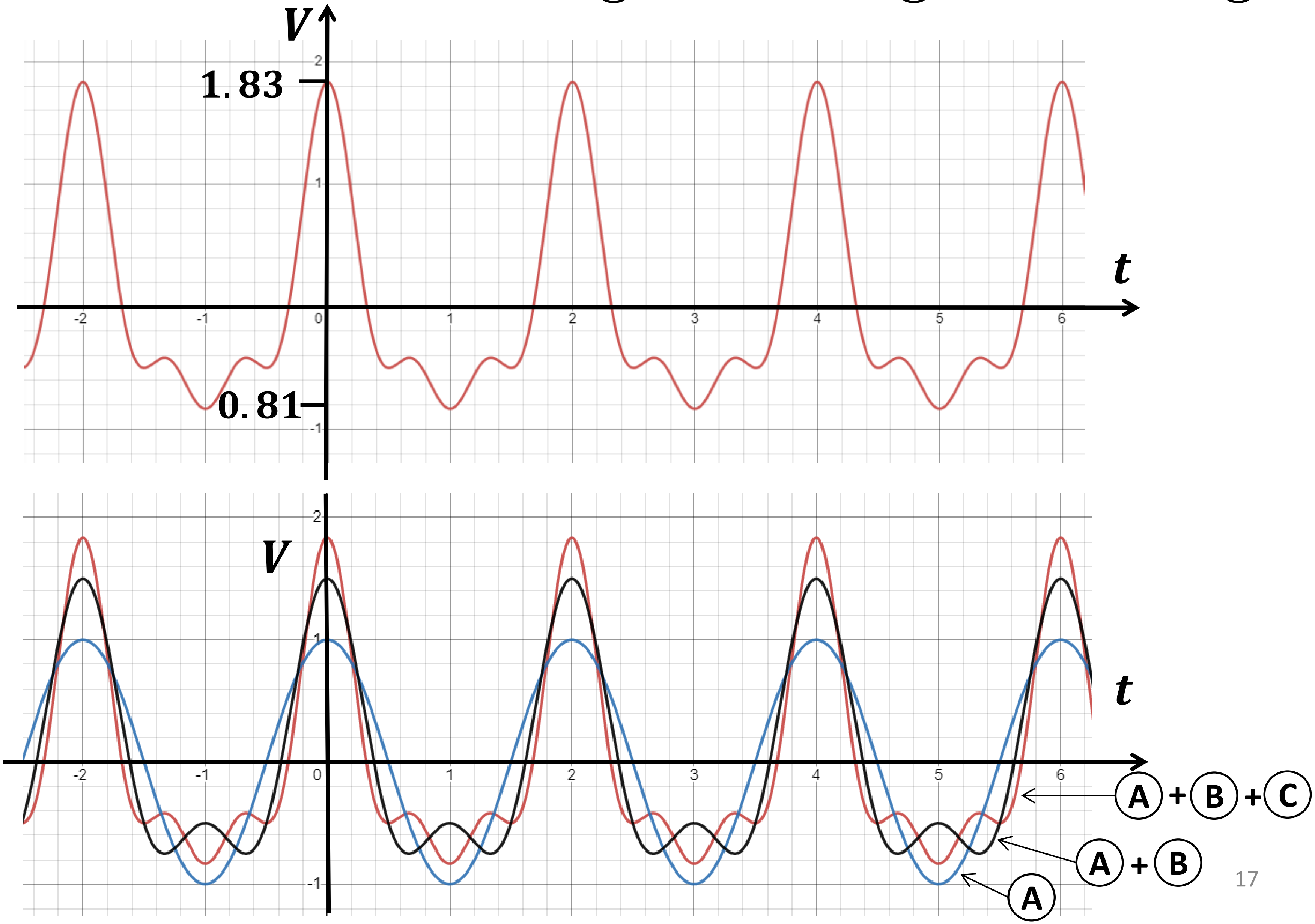




Signal with several frequencies

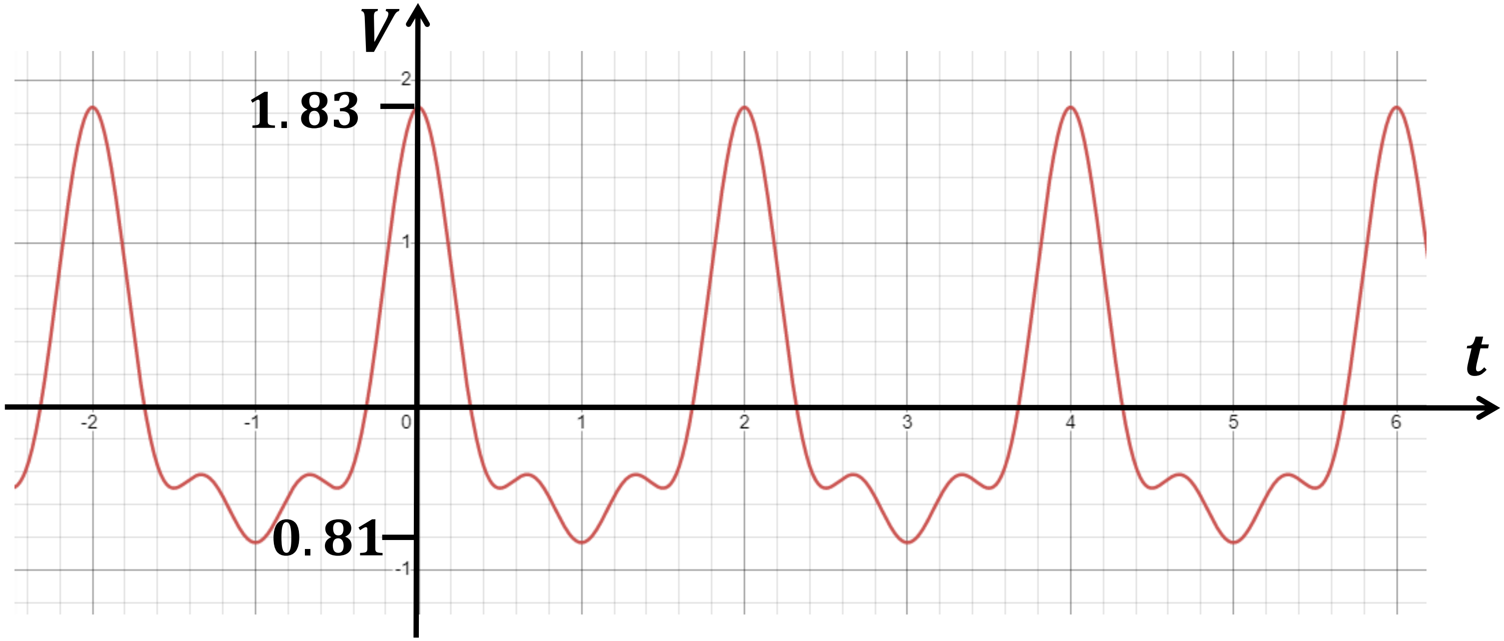
$$V(t) = \cos(\omega_0 t) + \frac{1}{2}\cos(2\omega_0 t) + \frac{1}{3}\cos(3\omega_0 t)$$

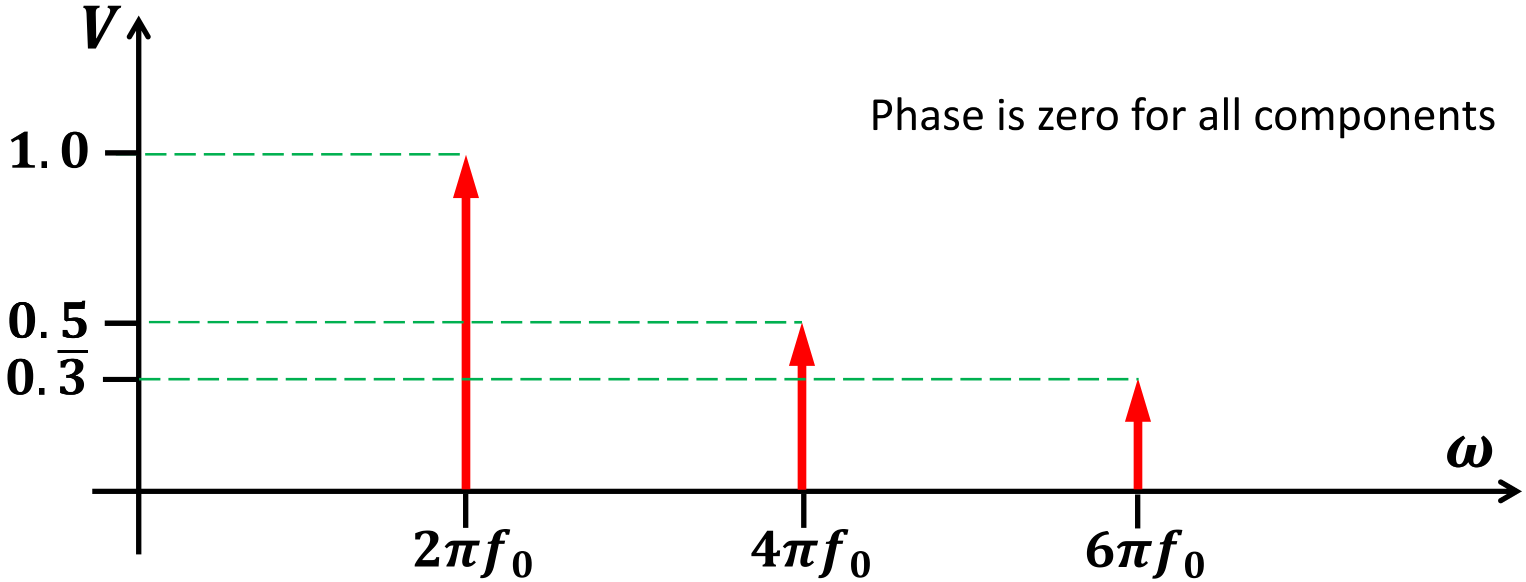
Ⓐ Ⓑ Ⓒ



Signal with several frequencies

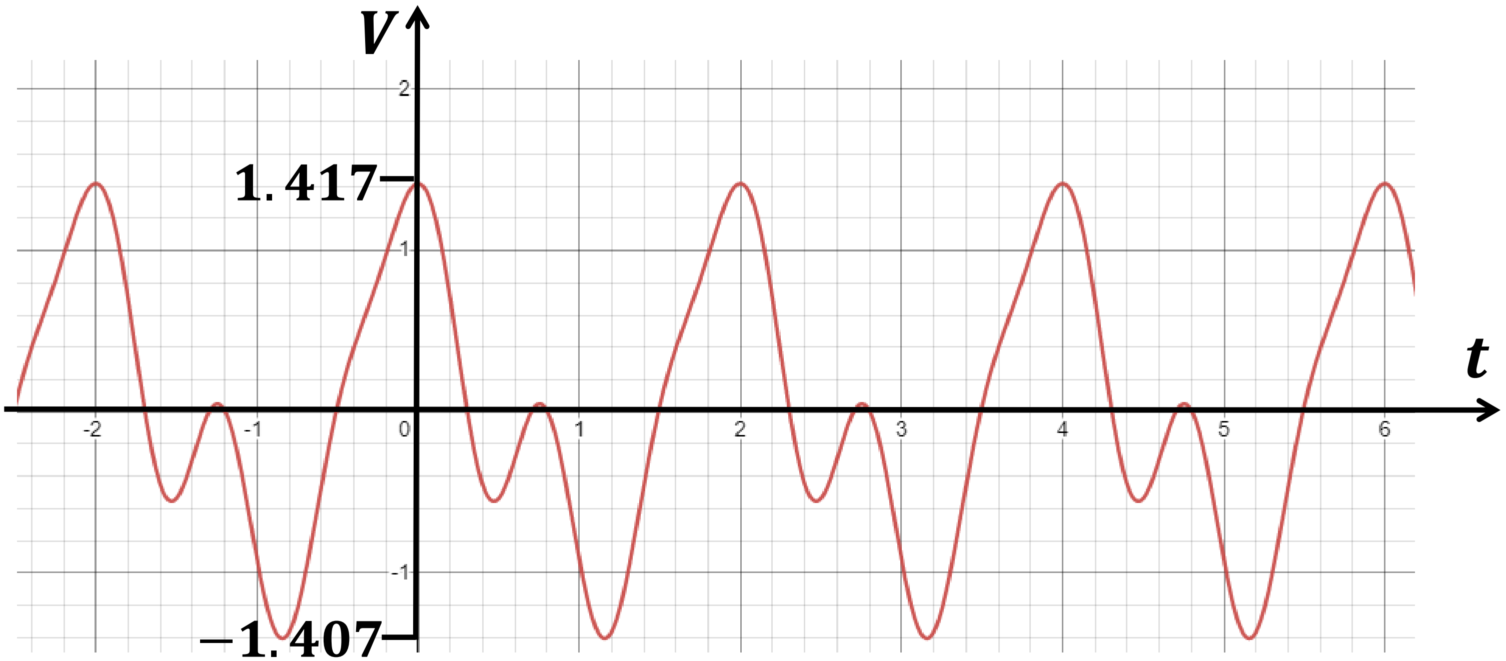
$$V(t) = \cos(\omega_0 t) + \frac{1}{2}\cos(2\omega_0 t) + \frac{1}{3}\cos(3\omega_0 t)$$

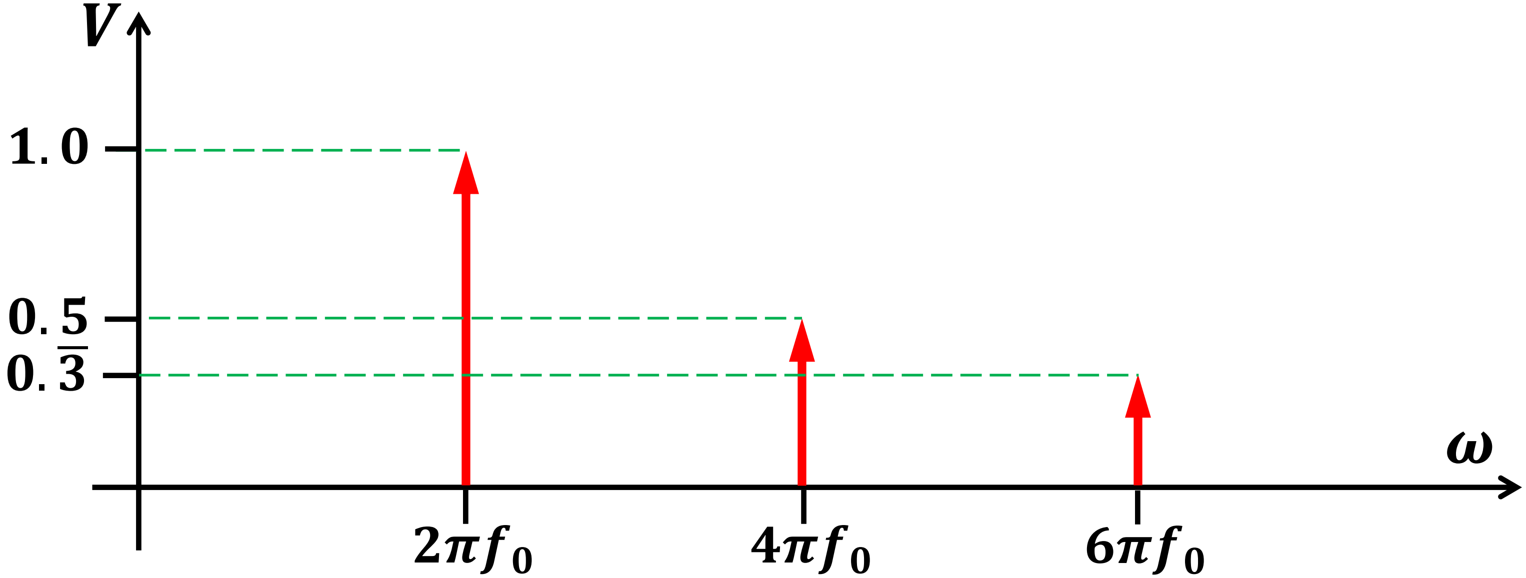




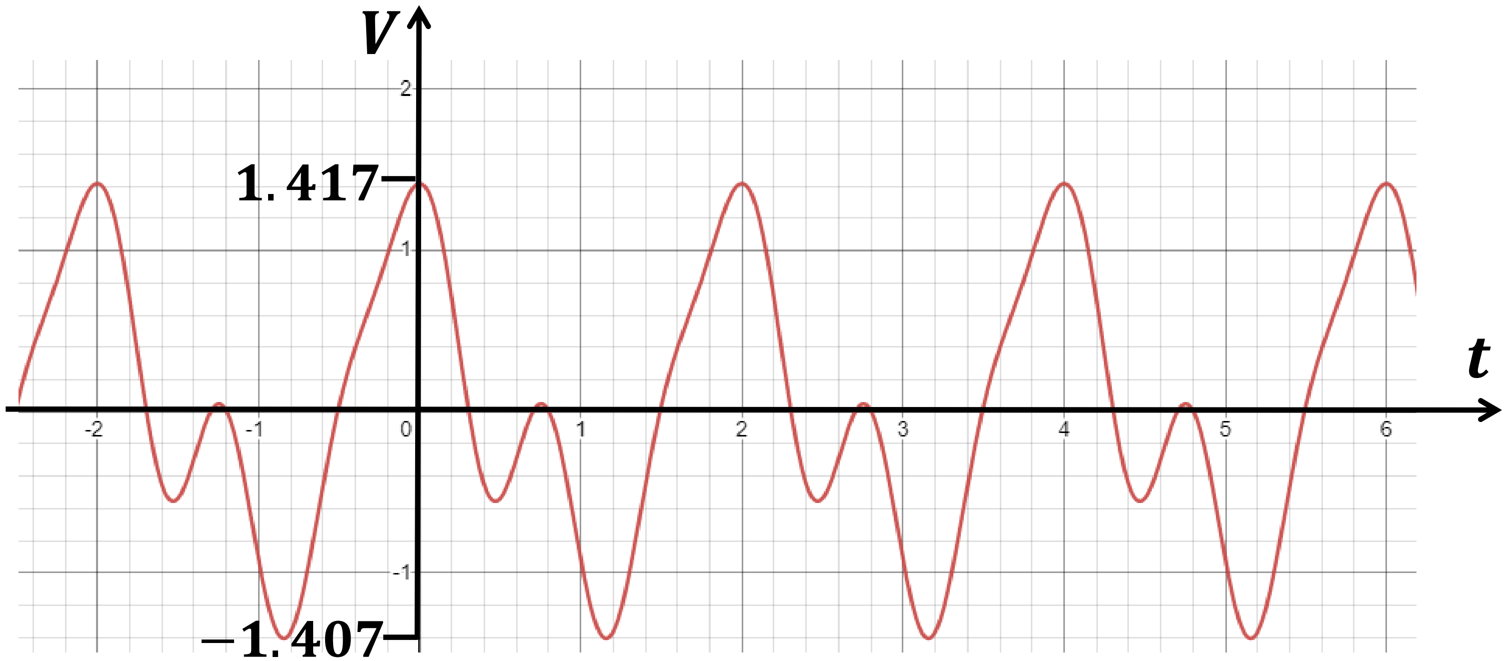
Phase is zero for all components

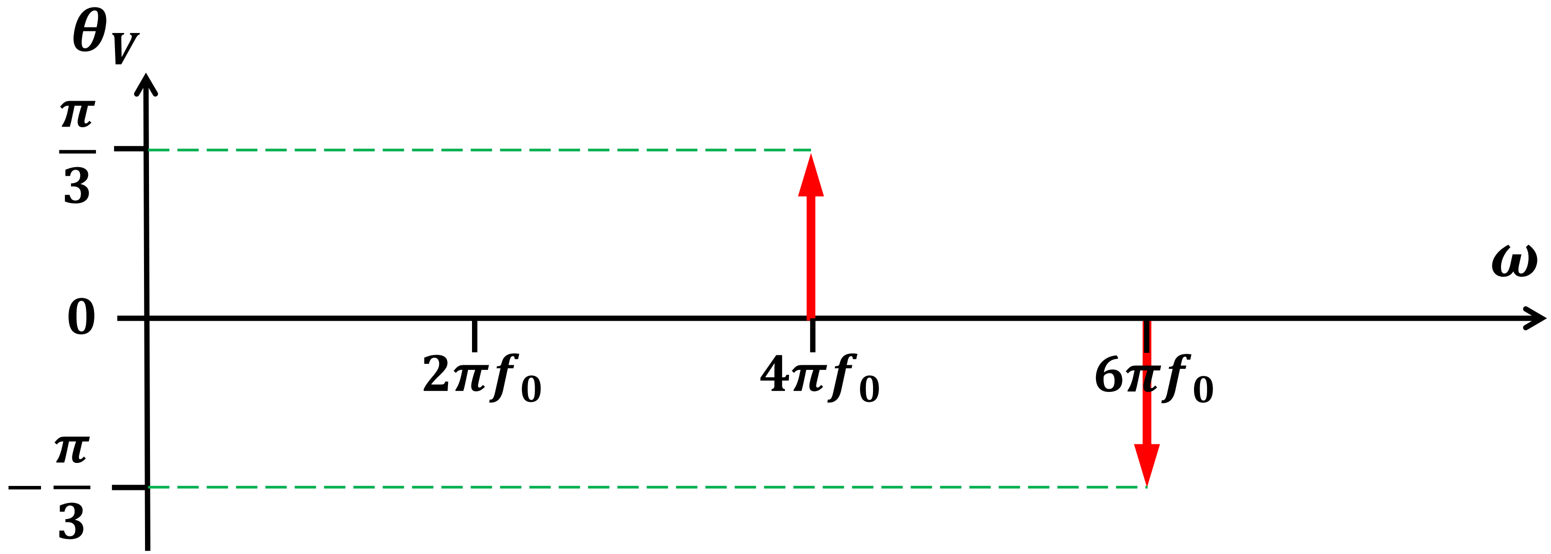
$$V(t) = \cos(\omega_0 t) + \frac{1}{2}\cos\left(2\omega_0 t + \frac{\pi}{3}\right) + \frac{1}{3}\cos\left(3\omega_0 t - \frac{\pi}{3}\right)$$



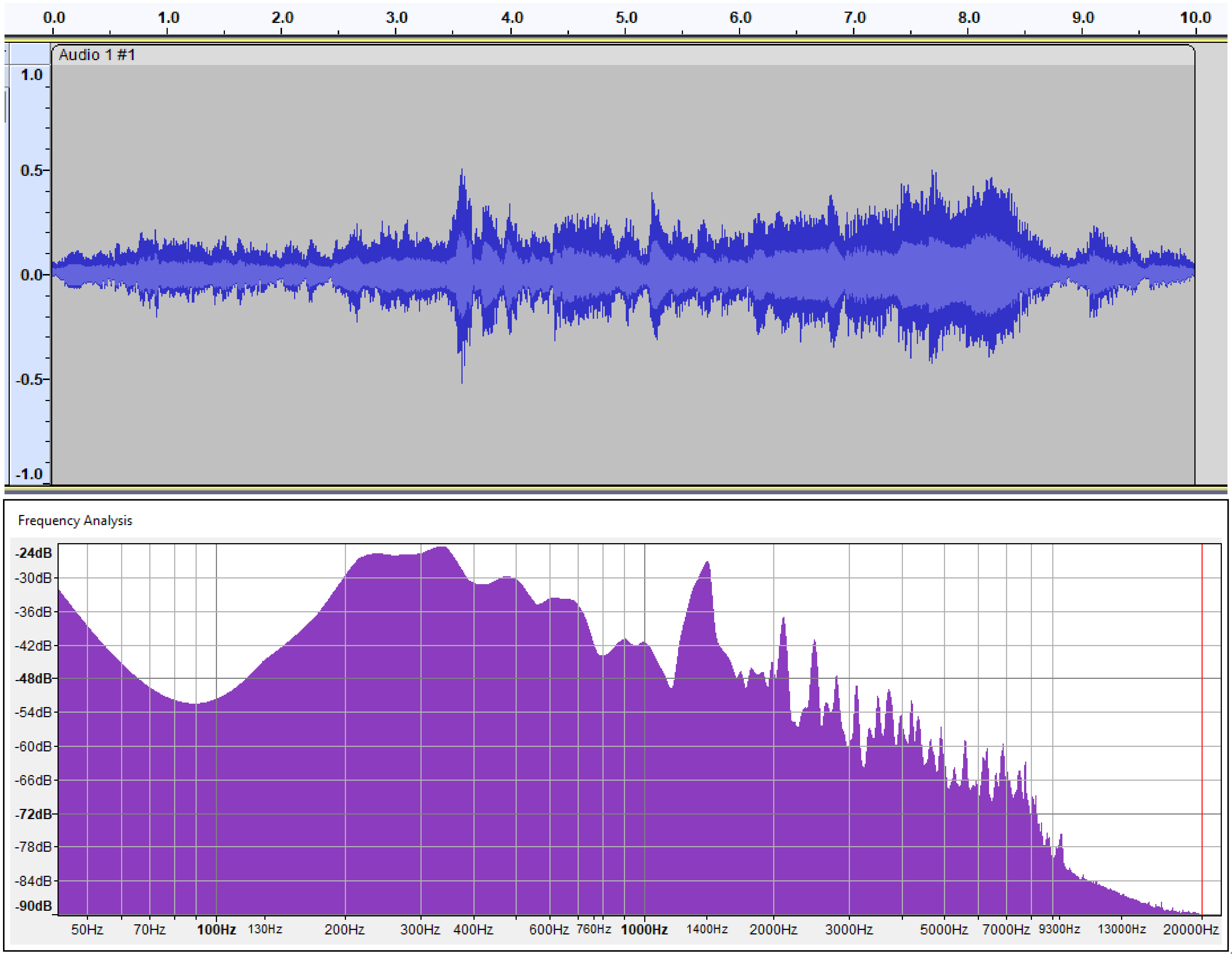


$$V(t) = \cos(\omega_0 t) + \frac{1}{2}\cos\left(2\omega_0 t + \frac{\pi}{3}\right) + \frac{1}{3}\cos\left(3\omega_0 t - \frac{\pi}{3}\right)$$





Ten seconds of music and its frequency "spectrum"



Excerpt from: Dmitri Shostakovich, Concerto No.1 for piano, trumpet and strings in C minor, Op. 35 (ORFEO - C220011)

Fourier transform

Converts a time domain signal $V(t)$ to a frequency domain signal $\tilde{\mathrm{V}}(\omega)$

$$\tilde{\mathbf{V}}(\boldsymbol{\omega}) = \int_{-\infty}^{\infty} \boldsymbol{V}(\boldsymbol{t})\, \boldsymbol{e}^{-\mathbf{j}\boldsymbol{\omega t}}\, \boldsymbol{dt}$$

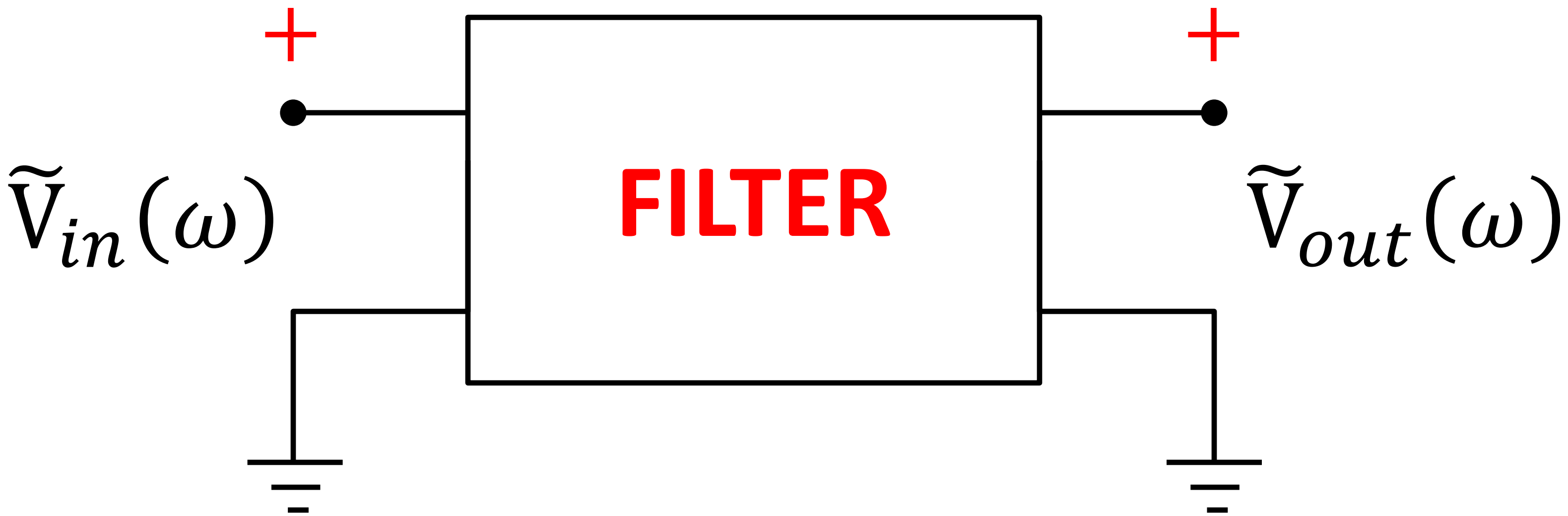
In general, the Fourier Transform is a complex function

Anti-Transform

$$\boldsymbol{V}(\boldsymbol{t}) = \frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\pi}} \int_{-\infty}^{\infty} \tilde{\boldsymbol{V}}(\boldsymbol{\omega})\, \boldsymbol{e}^{\mathbf{j}\boldsymbol{\omega t}}\, \boldsymbol{d\omega}$$

Filter

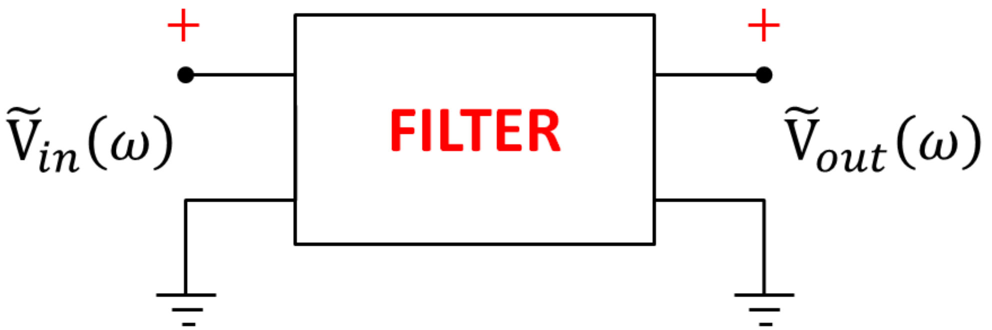
A circuit which manipulates a signal, typically by changing the relative amplitudes of the frequency components.



We will consider filters (systems) which are "single-input" and "single-output," consisting of "linear" and "time-invariant" circuits.

Transfer Function

The relationship linking the frequency-dependent input and output



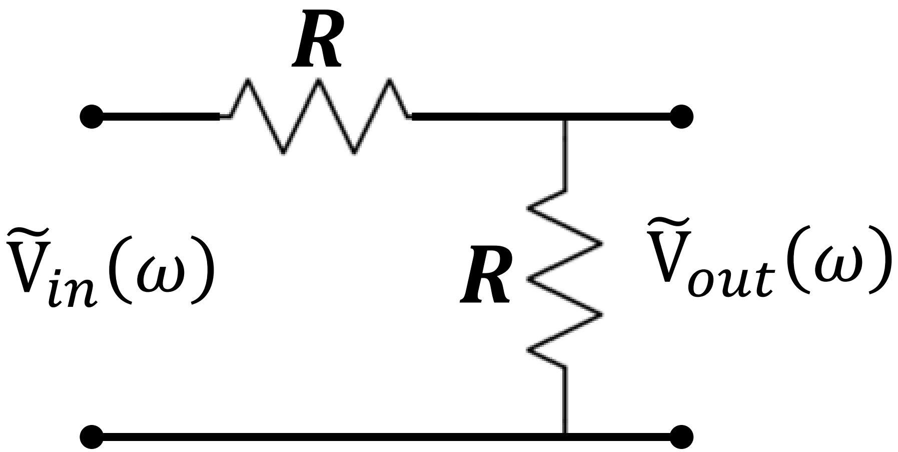
$$\tilde{\mathbf{V}}_{out}(\omega) = \mathbf{H}(\omega)\,\tilde{V}_{in}(\omega)$$

$$\mathbf{H}(\omega) = \frac{\tilde{V}_{out}(\omega)}{\tilde{V}_{in}(\omega)} = |\mathbf{H}(\omega)|\,\angle\theta(\omega)$$

Transfer Function | **Magnitude Response** | **Phase Response**

1 – Voltage Divider

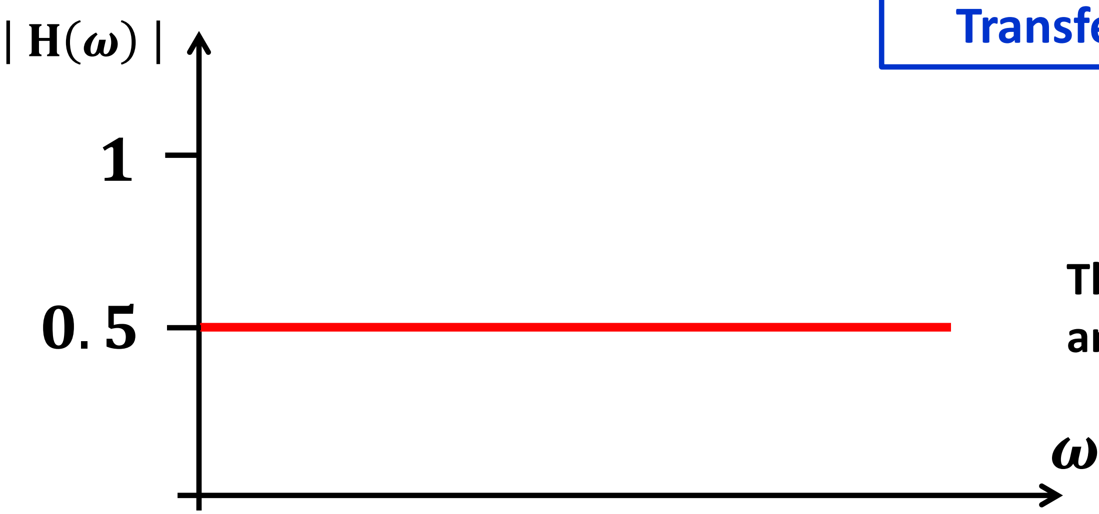
The voltage divider is a very simple filter



$$\tilde{V}_{out}(\omega) = \frac{1}{2}\tilde{V}_{in}(\omega)$$

$$H(\omega) = |H(\omega)| = \frac{1}{2}$$

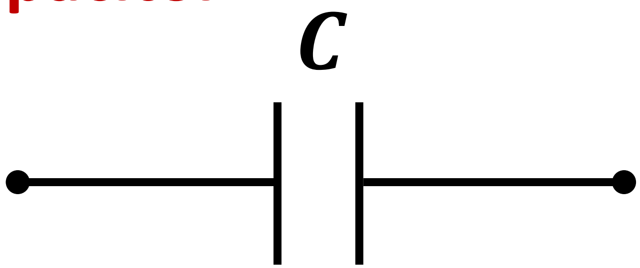
Transfer Function



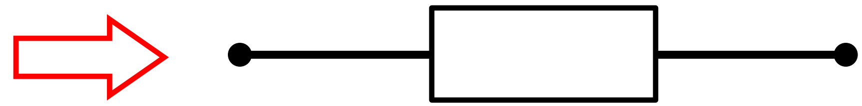
This circuit works as an "attenuator"

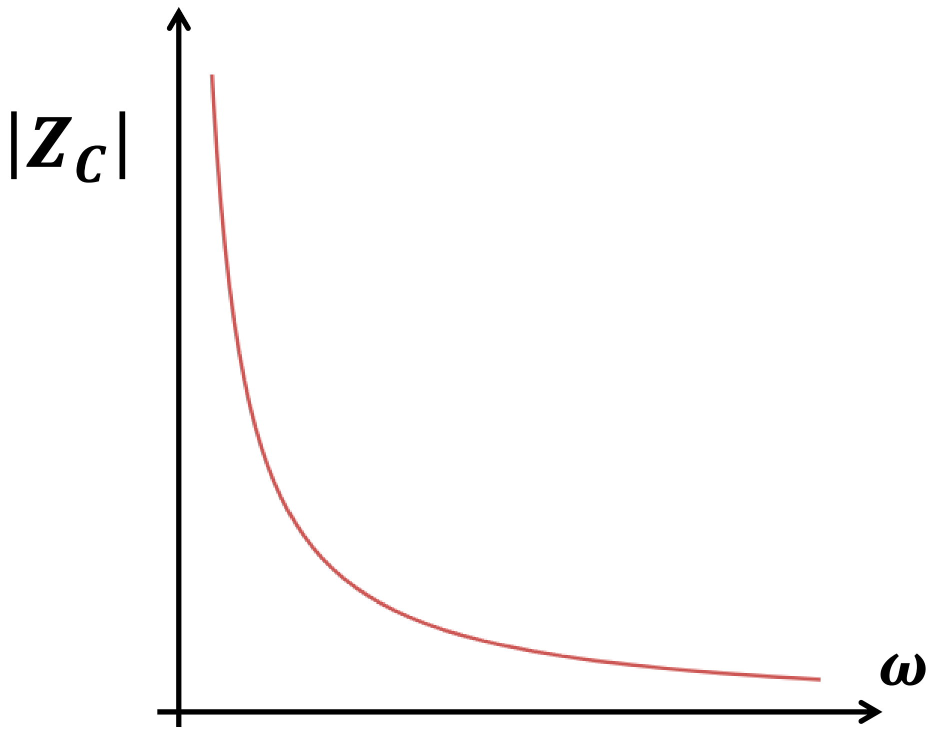
Behavior of Reactive Circuit Elements

Capacitor



$$Z_C(\omega) = \frac{1}{j\omega C}$$

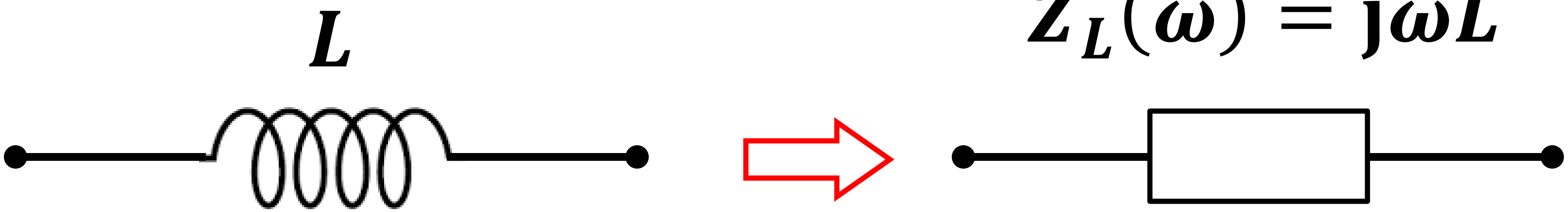


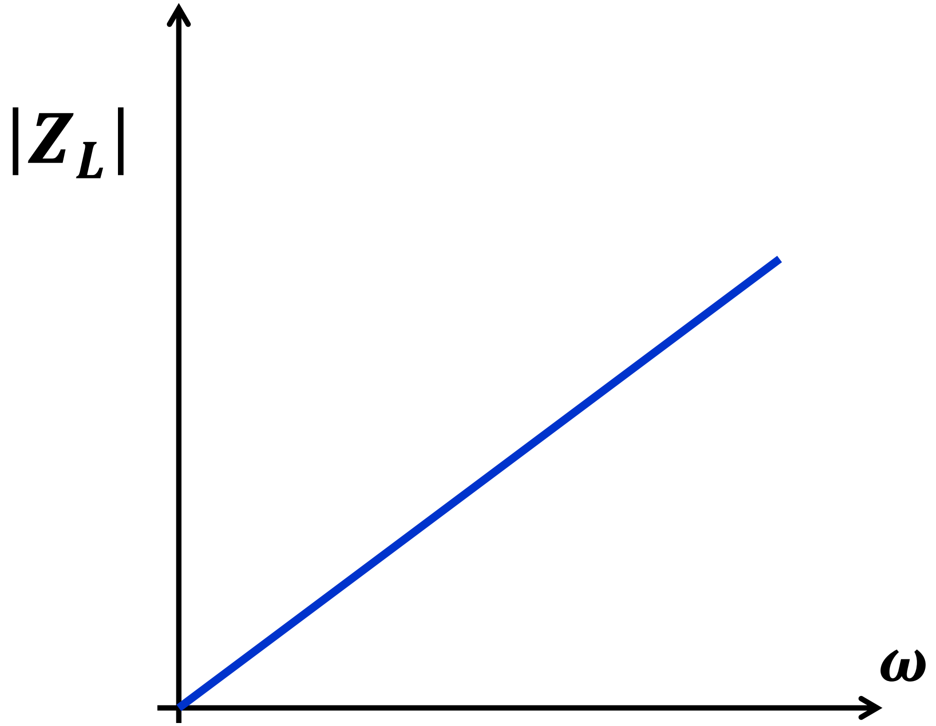


Behavior of Reactive Circuit Elements

Inductor

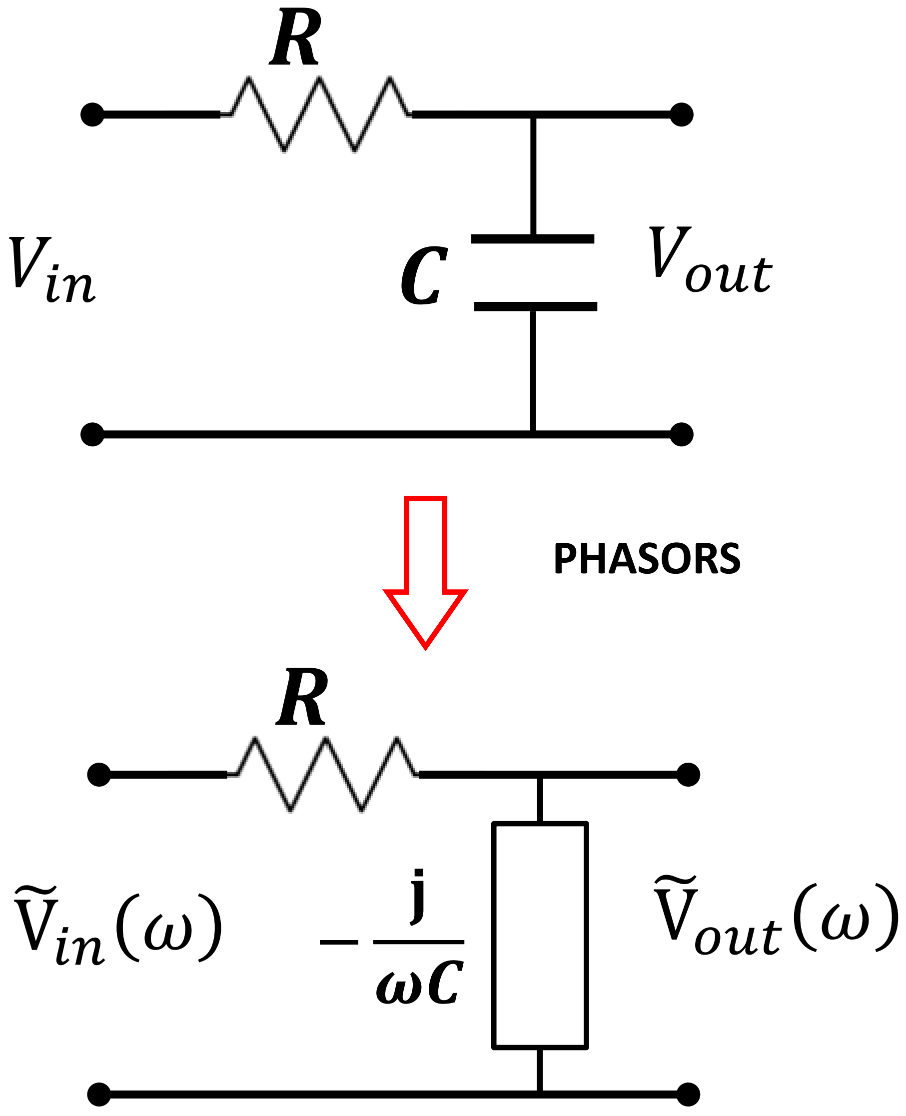
$$Z_L(\omega) = \mathrm{j}\omega L$$

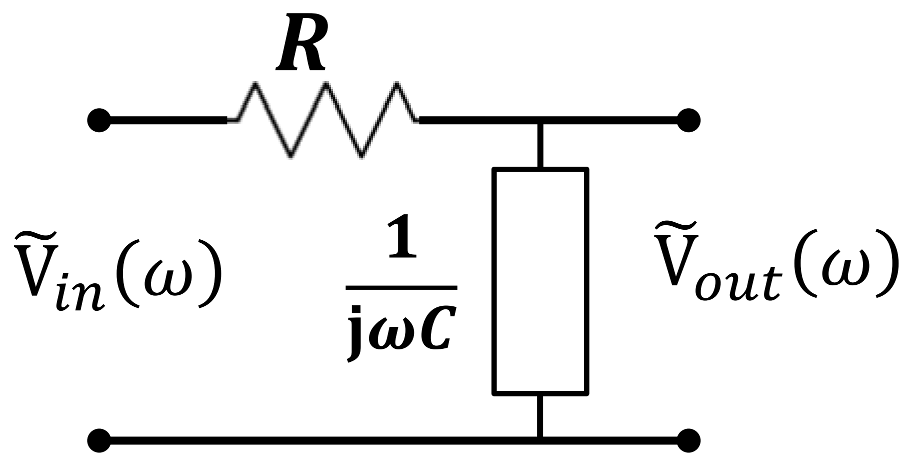




2 – Low Pass RC filter

RC filter
(1st order)





Let the input be a phasor of the form

$$\widetilde{\mathbf{V}}_{in}(\omega) = \mathbf{V}_I \angle 0^\circ$$

$$\widetilde{\mathbf{V}}_{out}(\omega) = \mathbf{V}_I \angle 0^\circ \frac{1/j\omega C}{R + 1/j\omega C} = \underbrace{\mathbf{V}_I \angle 0^\circ}_{\widetilde{\mathbf{V}}_{in}(\omega)} \frac{1}{1 + j\omega RC}$$

$$\frac{\widetilde{\mathbf{V}}_{out}(\omega)}{\widetilde{\mathbf{V}}_{in}(\omega)} = \mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

Transfer Function

$$\mathbf{H}(\omega) = \frac{1}{1 + j\omega RC}$$

$$\mathbf{H}(\omega) = \frac{1 - j\omega RC}{(1 + j\omega RC)(1 - j\omega RC)} = \underbrace{\frac{1 - j\omega RC}{1 + (\omega RC)^2}}_{\text{Cartesian Form}}$$

Magnitude

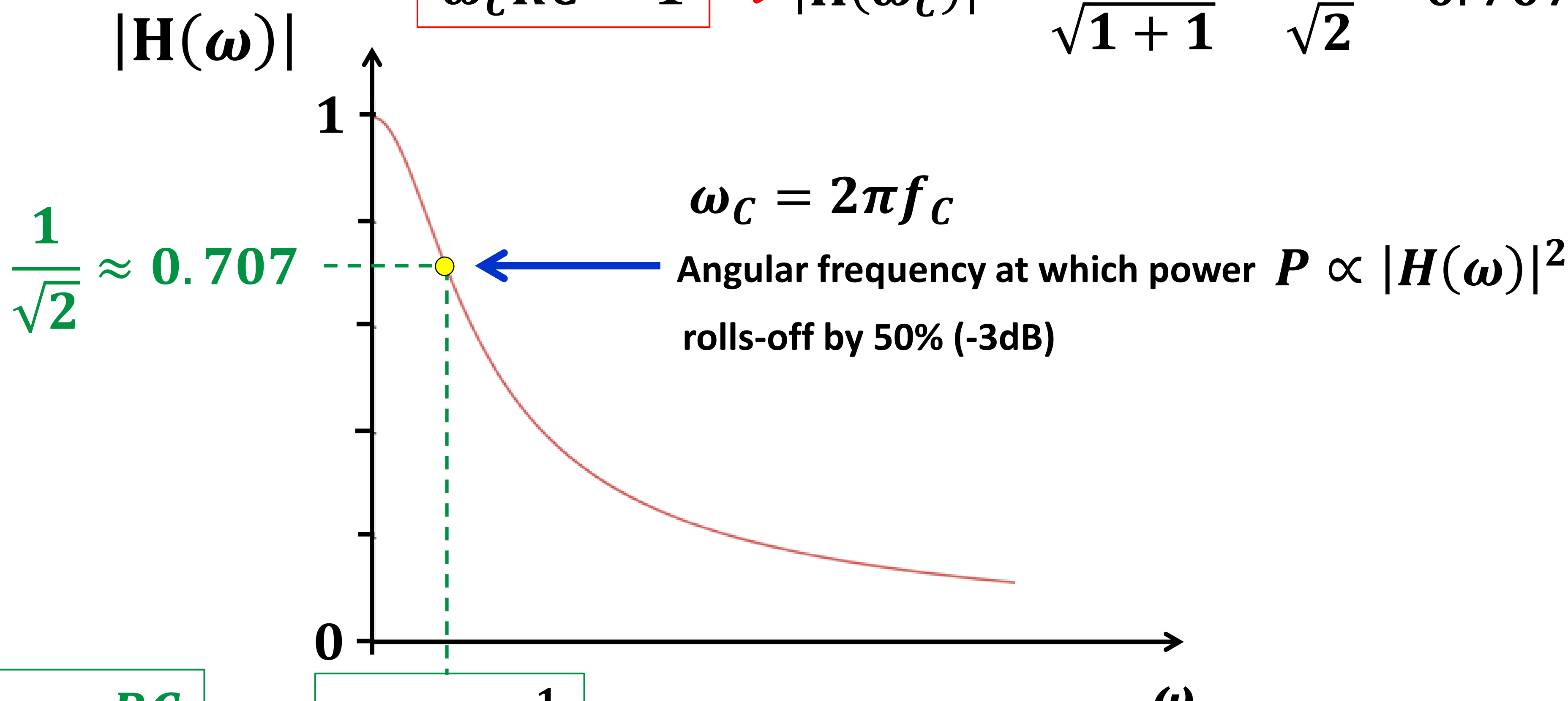
$$|\mathbf{H}(\omega)| = \frac{1}{|1 + j\omega RC|}$$

$$|\mathbf{H}(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

Magnitude of $H(\omega)$ for RC low-pass filter

$$|\mathrm{H}(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$\omega_C RC = 1 \rightarrow |\mathrm{H}(\omega_C)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$$

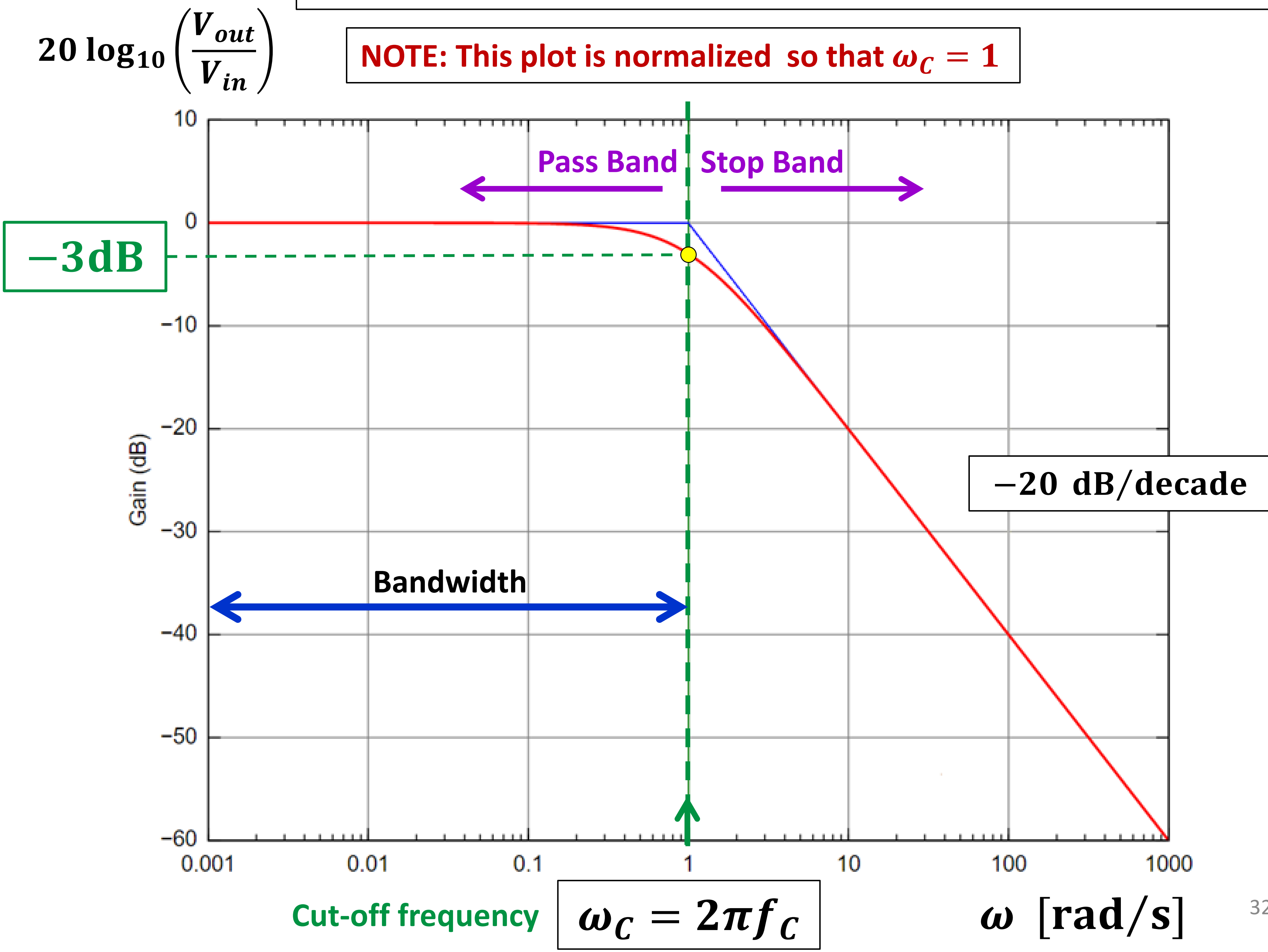


$$\omega_C = 2\pi f_C$$

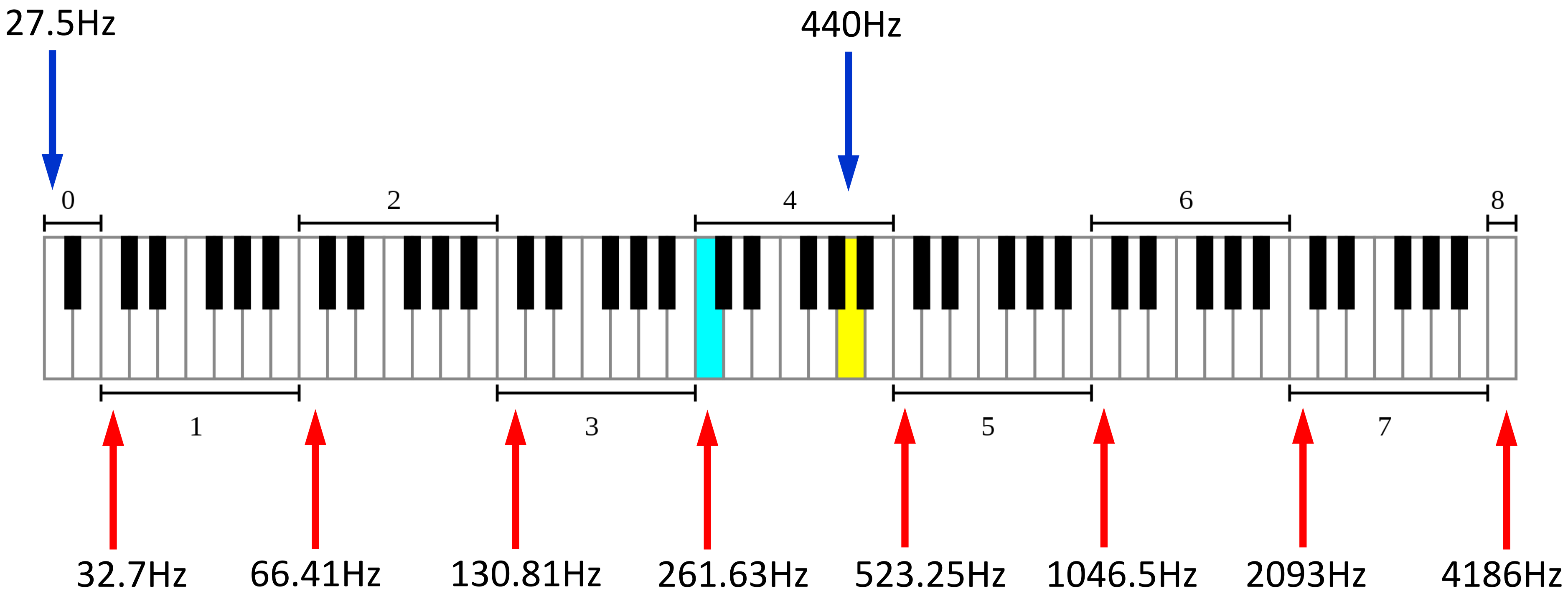
$$\tau = RC$$

$$\omega_C = \tau^{-1}$$

log-decibel representation – Bode Plot for magnitude



The piano keyboard uses octaves (instead of decades)



Phase of $H(\omega)$ for RC low-pass filter

$$\mathrm{H}(\omega)=\frac{1-\mathrm{j}\omega RC}{(1+\mathrm{j}\omega RC)(1-\mathrm{j}\omega RC)}=\underbrace{\frac{1-\mathrm{j}\omega RC}{1+(\omega RC)^2}}_{\text{Cartesian Form}}$$

$$\angle \mathrm{H}(\omega)=\tan^{-1}\frac{\Im m\{\mathrm{H}(\omega)\}}{\Re e\{\mathrm{H}(\omega)\}}=\tan^{-1}\frac{-\omega RC/\left(1+(\omega RC)^2\right)}{1/(1+(\omega RC)^2)}$$

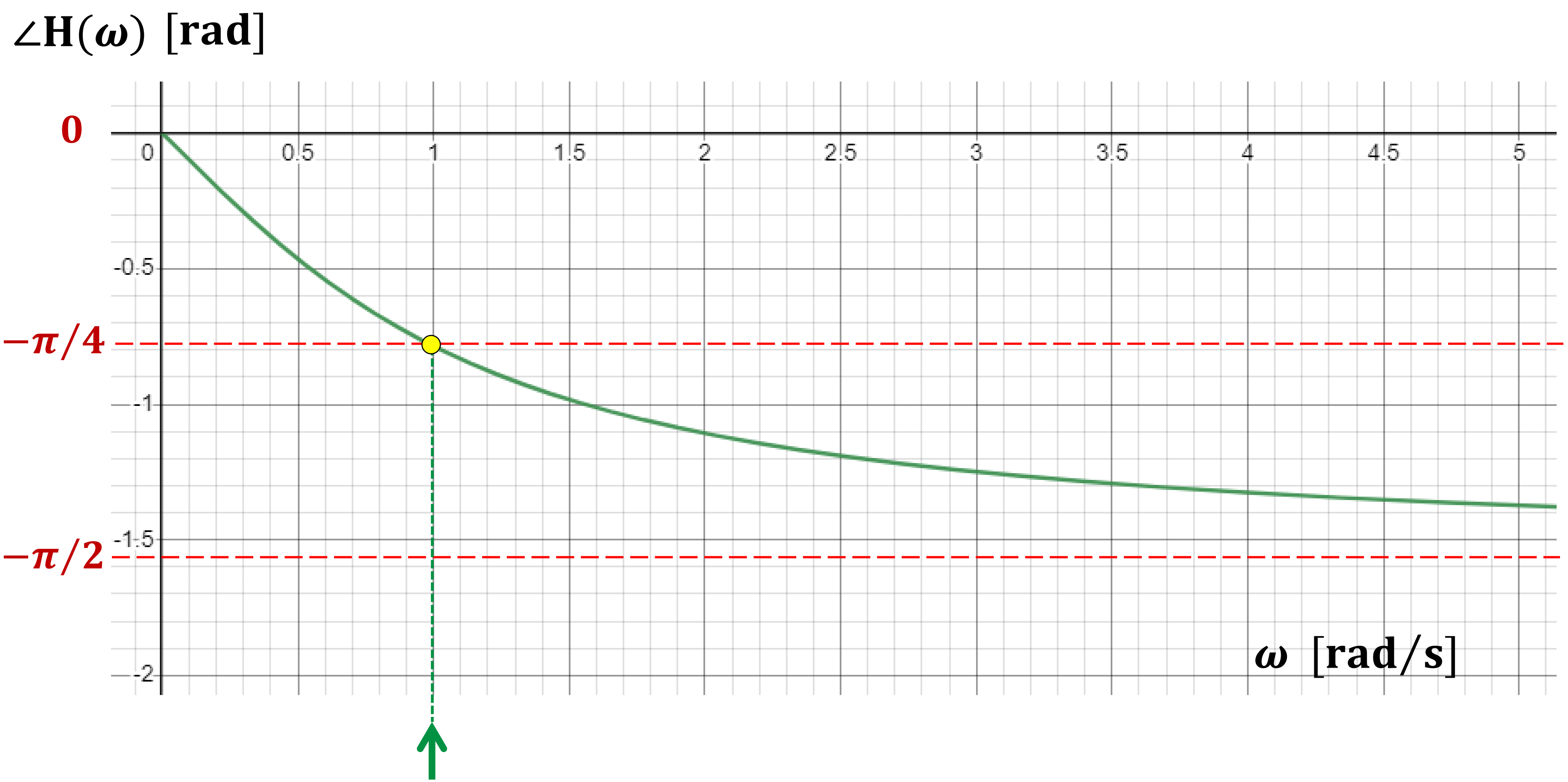
$$\angle \mathrm{H}(\omega)=\tan^{-1}(-\omega RC)=-\tan^{-1}(\omega RC)$$

When $\omega=\omega_C$ we have $\omega_C RC=1$

$$\angle \mathrm{H}(\omega)=\tan^{-1}(-1)=-\frac{\pi}{4}=-45^{\circ}$$

Phase for RC low-pass filter

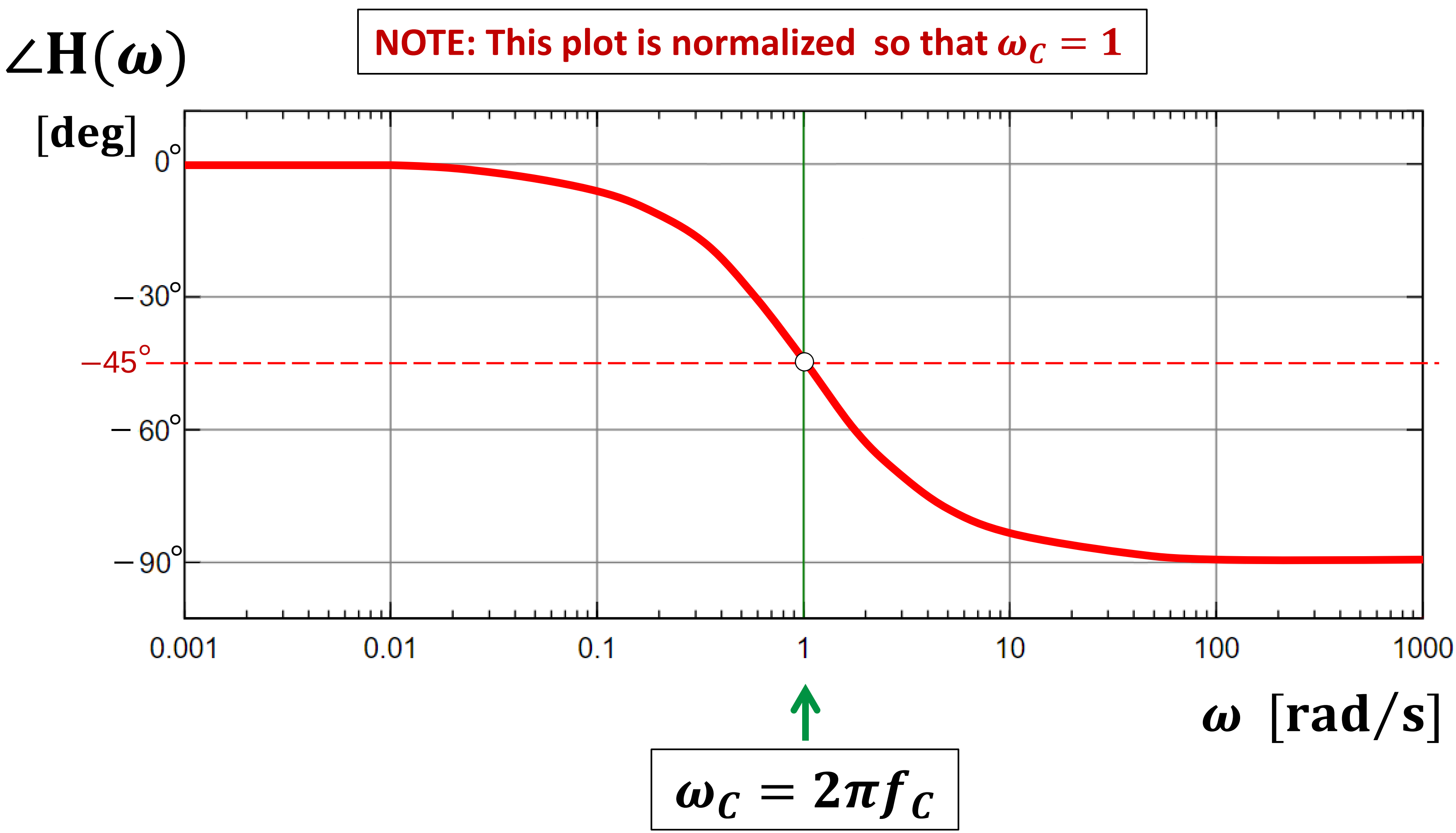
Linear scale representation



$$\omega_C = 2\pi f_C$$

Phase for RC low-pass filter

semi-log scale representation – Bode Plot for phase



Example - RC low-pass filter ω_1

Consider $R = 2.5\ \mathrm{k\Omega}$ **and** $C = 400\ \mathrm{nF}$

$$\omega_0 = 1/(RC) = \left(2.5\mathrm{k} \times 40 \times 10^{-9}\right)^{-1} = 10^3\ \ \mathrm{rad/s}$$

$f = 60\mathrm{Hz}$

$$\omega_1 \approx 377\ \mathrm{rad/s}$$

$$\omega_1 RC = 2\pi \times 60 \times 2.5\mathrm{k} \times 400 \times 10^{-9} \approx 377.0$$

$$|\mathrm{H}(\omega_1)| = \frac{1}{\sqrt{1 + (\omega_1 RC)^2}} = 0.9357$$

$$|\mathrm{H}(\omega_1)|_{\mathrm{dB}} = 20\log_{10}(0.9357) = -0.5772\ \mathrm{dB}$$

Example - RC low-pass filter ω_2

Consider $R = 2.5\ \mathrm{k\Omega}$ and $C = 400\ \mathrm{nF}$

$$\omega_0 = 1/(RC) = \left(2.5\mathrm{k} \times 40 \times 10^{-9}\right)^{-1} = 10^3\ \mathrm{rad/s}$$

$$\boxed{f = 160\mathrm{Hz}} \qquad \omega_2 = 1005.3\ \mathrm{rad/s}$$

$$\omega_2 RC = 2\pi \times 160 \times 2.5\mathrm{k} \times 400 \times 10^{-9} \approx 1.0$$

$$|\mathrm{H}(\omega_2)| = \frac{1}{\sqrt{1 + (\omega_2 RC)^2}} = 0.7052$$

$$|\mathrm{H}(\omega_2)|_{\mathrm{dB}} = 20\log_{10}(0.7052) = -3.03\overline{3}\ \mathrm{dB}$$

Example - RC low-pass filter ω_3

Consider $R = 2.5\ \text{k}\Omega$ and $C = 400\ \text{nF}$

$$\omega_0 = 1/(RC) = \left(2.5\text{k} \times 40 \times 10^{-9}\right)^{-1} = 10^3\ \text{rad/s}$$

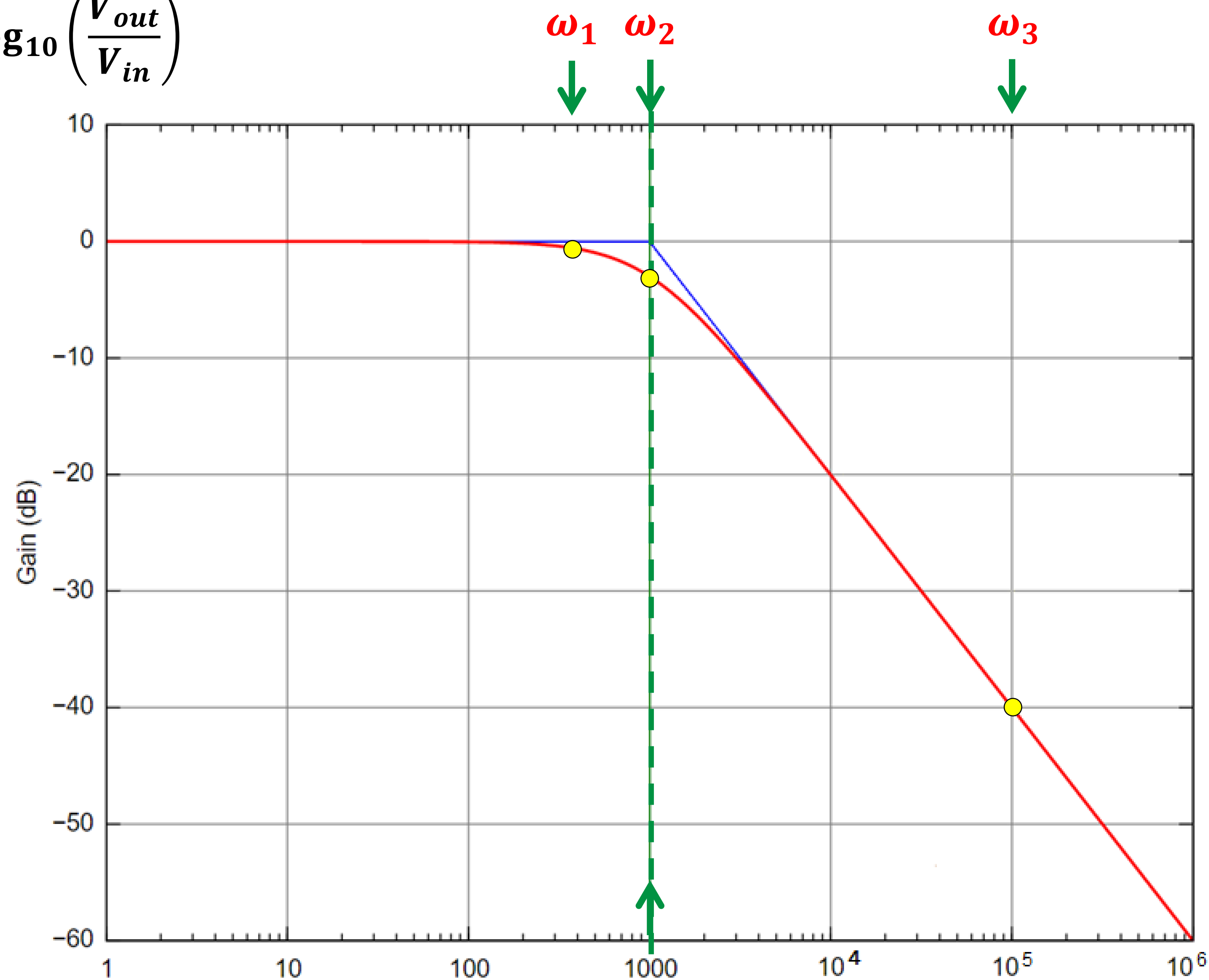
$$f = 16\ \text{kHz} \qquad \omega_3 = 100{,}530\ \text{rad/s}$$

$$\omega_3 RC = 2\pi \times 16\text{k} \times 2.5\text{k} \times 400 \times 10^{-9} \approx 1.0$$

$$|\text{H}(\omega_3)| = \frac{1}{\sqrt{1 + (\omega_3 RC)^2}} = 0.0099$$

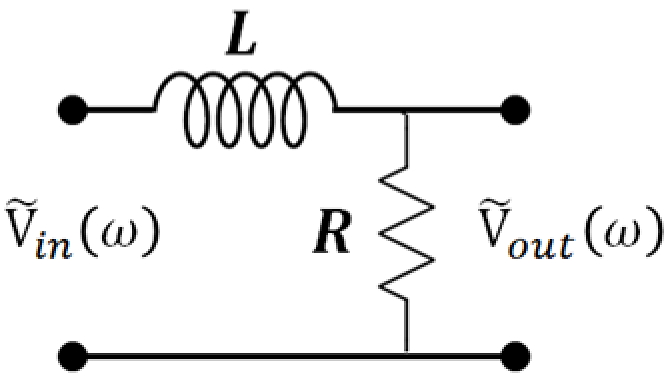
$$|\text{H}(\omega_3)|_{\text{dB}} = 20\log_{10}(0.7052) = -40.046\ \text{dB}$$

$$20 \log_{10}\left(\frac{V_{out}}{V_{in}}\right)$$

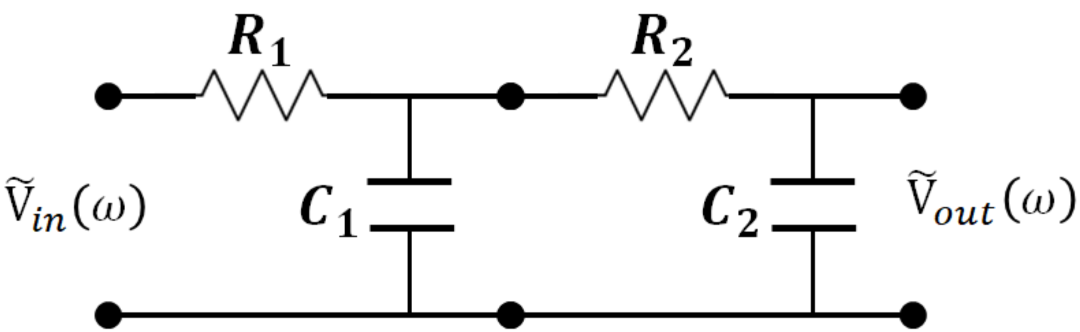


$$\omega_C = 2\pi f_C$$

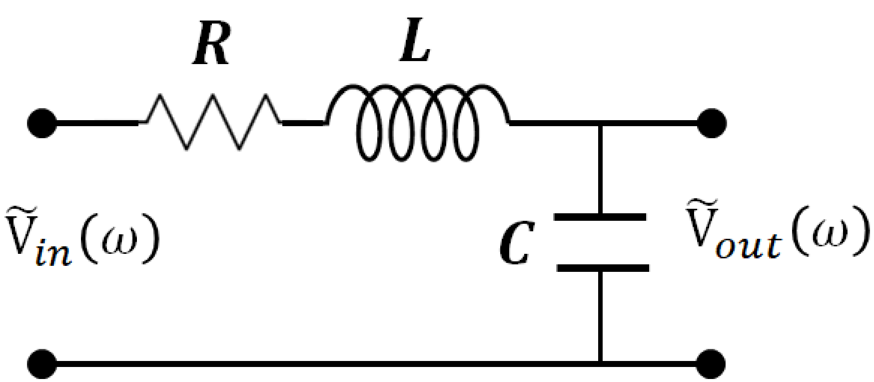
Other Low-Pass Passive filter configurations



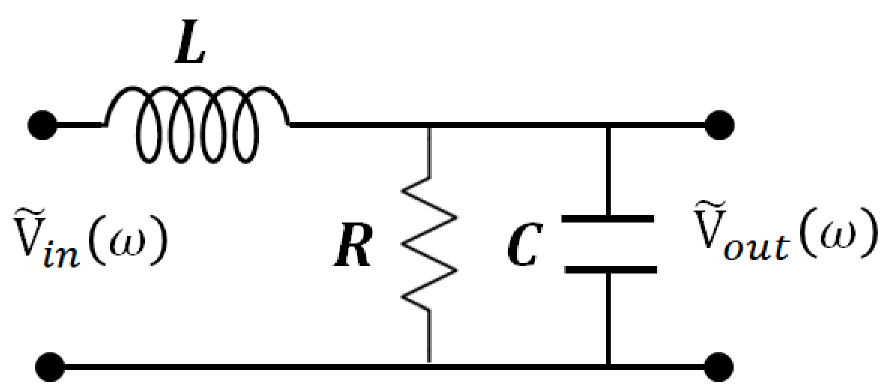
RL filter (1st order)



RC filter (2nd order)



RLC filter (2nd order)



RLC filter (2nd order)