ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 35

MWF – 12:00pm

Prof. Umberto Ravaioli

2062 ECE Building

Lecture 35 – Summary

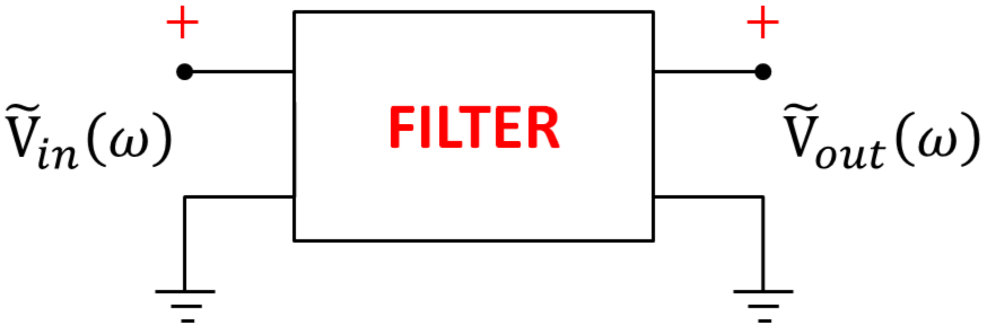
Learning Objectives

1. Low-Pass and High-Pass Filters
2. Behavior of Operational Amplifiers

Sinusoidal input signal

$$V_{in}(t) = V_{\mathrm{m}} \cos(\omega t + \theta_V)$$

$$\mathrm{H}(\omega) = |\mathrm{H}(\omega)| \angle \mathrm{H}(\omega)$$



$$\tilde{V}_{out}(\omega) = \mathrm{H}(\omega)\, \tilde{V}_{in}(\omega)$$

$$V_{out}(t) = |\mathrm{H}(\omega)| V_{\mathrm{m}} \cos(\omega t + \theta_V + \angle \mathrm{H}(\omega))$$

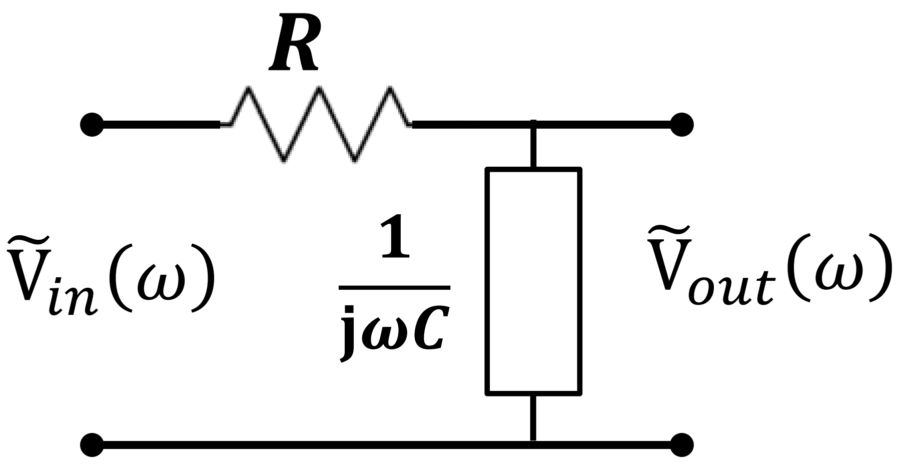
Example – Low Pass RC filter

$V_{in}(t) = 2\cos(3000t)$

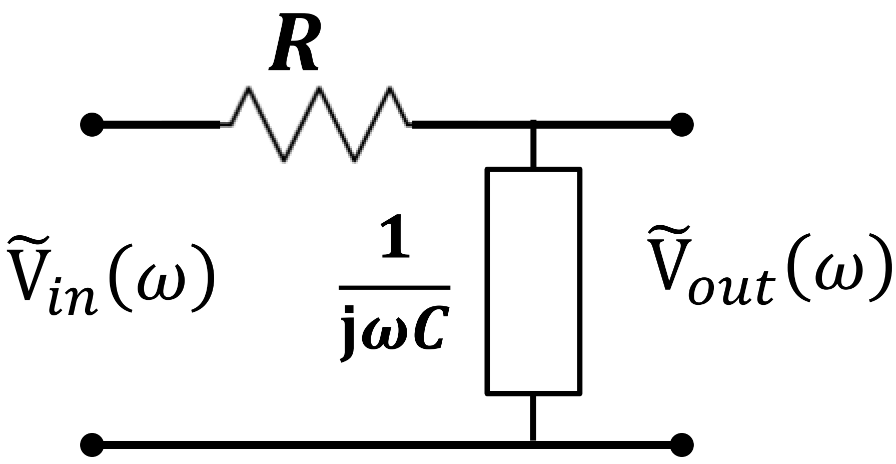
Find $V_{out}(t)$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Let $R = 1\text{k}\Omega$ and $C = 1\mu\text{F}$.



Example – Low Pass RC filter



$V_{in}(t) = 2\cos(3000t)$

Find $V_{out}(t)$

$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Let $R = 1\text{k}\Omega$ and $C = 1\mu\text{F}$.

$$\omega RC = 3000 \times 1k \times 1\mu = 3$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$$|H(\omega)| = \frac{1}{\sqrt{1 + (3)^2}} = \frac{1}{\sqrt{10}}\ \text{V}$$

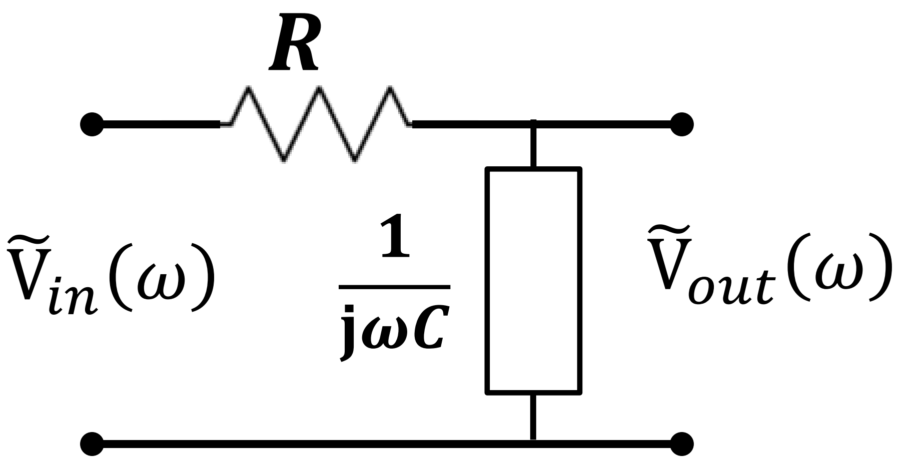
$$\angle H(\omega) = -\tan^{-1}(\omega RC)$$

$$\angle H(\omega) = -\tan^{-1}(3) = -1.249\ \text{rad}$$

Example – Low Pass RC filter

$V_{in}(t) = 2\cos(3000t)$

Find $V_{out}(t)$



$$H(\omega) = \frac{1}{1 + j\omega RC}$$

Let $R = 1\text{k}\Omega$ and $C = 1\mu\text{F}$.

$$|H(\omega)| = \frac{1}{\sqrt{1 + (3)^2}} = \frac{1}{\sqrt{10}}\ \text{V}$$

$$\angle H(\omega) = -\tan^{-1}(3) = -1.249\ \text{rad}$$

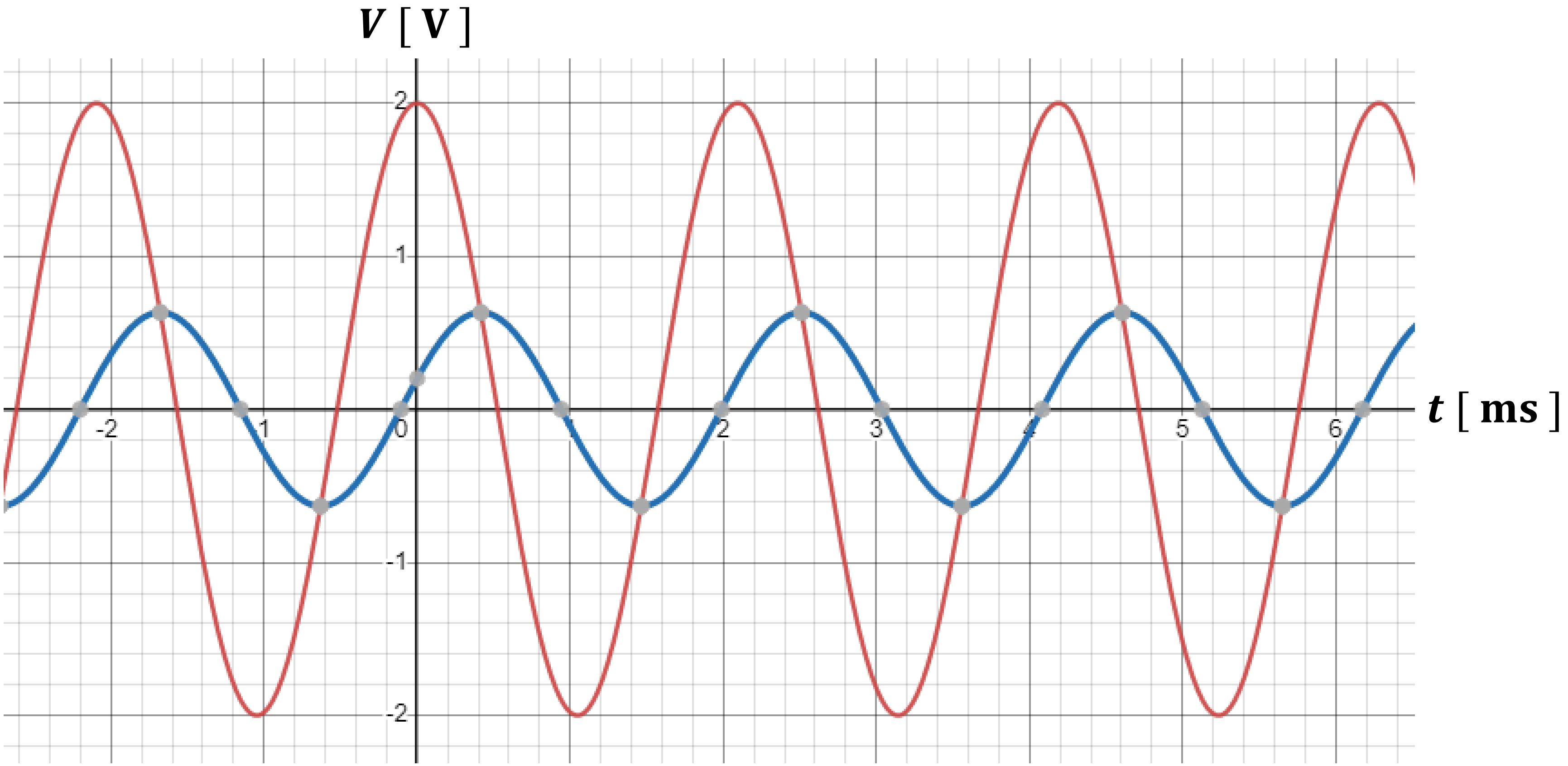
$$V_{out}(t) = |H(\omega)| \times 2\cos(3000t + \angle H(\omega))$$

$$V_{out}(t) = \frac{2}{\sqrt{10}}\cos(3000t - 1.249)$$

Valid for sinusoidal signals

$V_{in}(t) = 2\cos(3000t)$

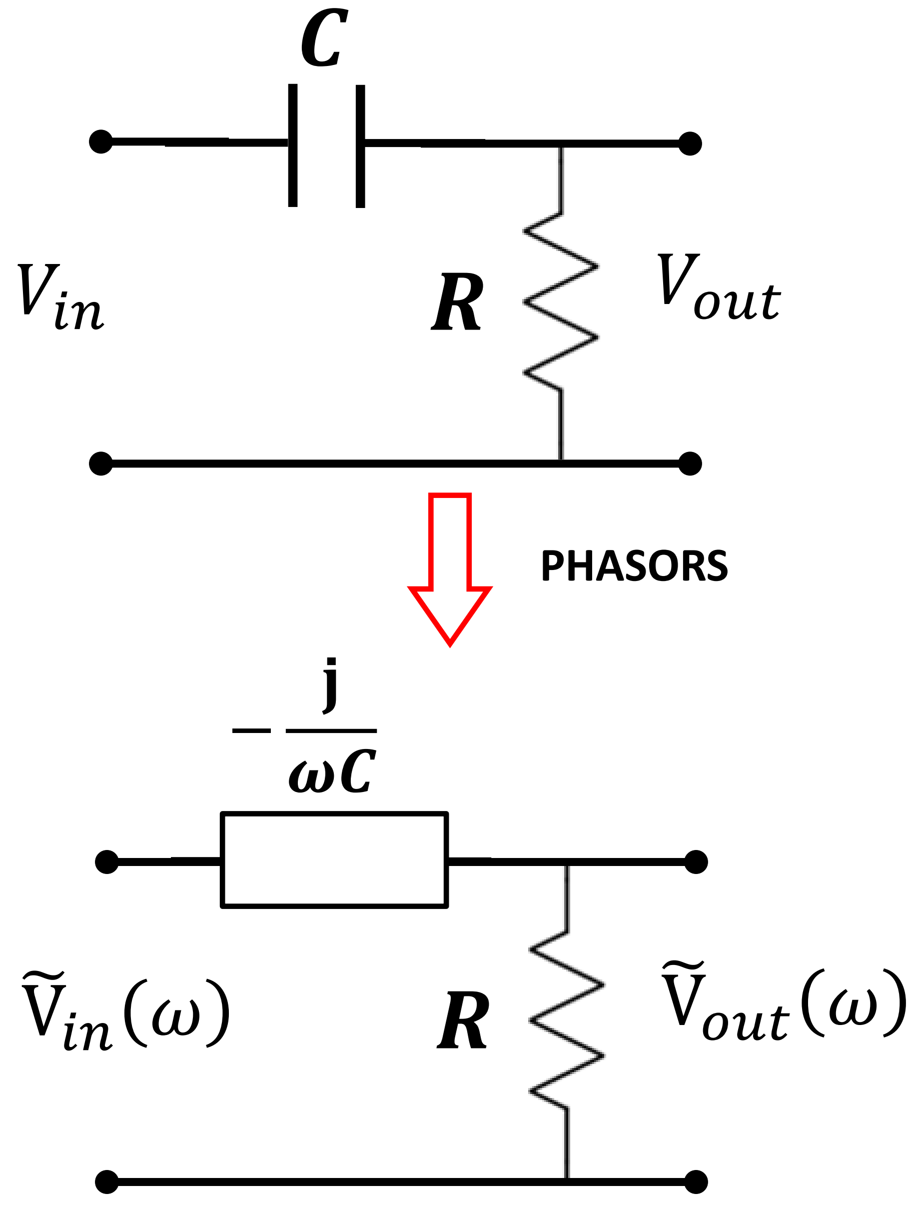
$V_{out}(t) = \frac{2}{\sqrt{10}}\cos(3000t - 1.249)$

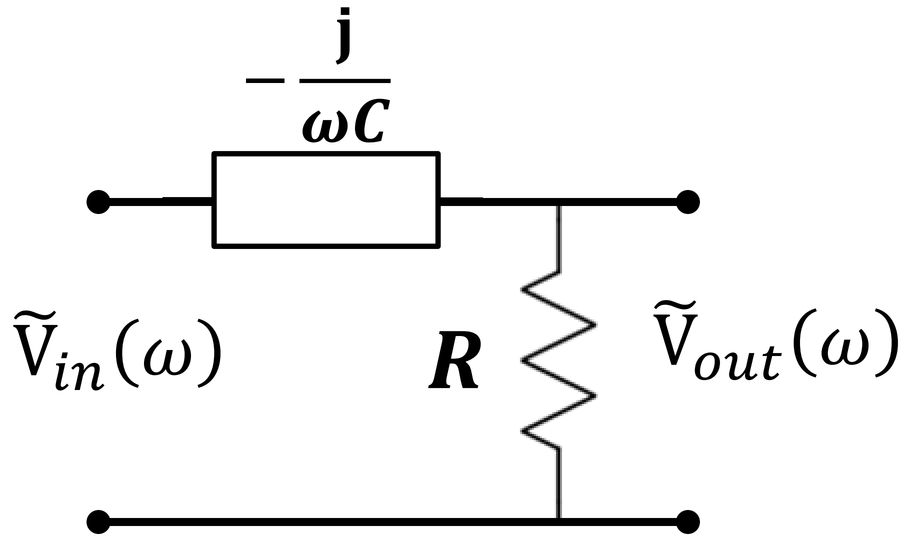


The filter introduces a time delay for the output signal

High Pass RC filter

RC filter
(1st order)





Let the input be a phasor of the form

$$\tilde{\mathbf{V}}_{in}(\omega) = \mathbf{V}_I \angle 0°$$

$$\tilde{\mathbf{V}}_{out}(\omega) = \mathbf{V}_I \angle 0° \frac{R}{R + 1/\mathrm{j}\omega C} = \underbrace{\mathbf{V}_I \angle 0°}_{\tilde{\mathbf{V}}_{in}(\omega)} \frac{\mathrm{j}\omega RC}{1 + \mathrm{j}\omega RC}$$

$$\frac{\tilde{\mathbf{V}}_{out}(\omega)}{\tilde{\mathbf{V}}_{in}(\omega)} = \mathbf{H}(\omega) = \frac{\mathrm{j}\omega RC}{1 + \mathrm{j}\omega RC}$$

Transfer Function

$$\mathrm{H}(\omega) = \frac{\mathrm{j}\omega RC}{1 + \mathrm{j}\omega RC}$$

$$\mathrm{H}(\omega) = \frac{\mathrm{j}\omega RC(1 - \mathrm{j}\omega RC)}{(1 + \mathrm{j}\omega RC)(1 - \mathrm{j}\omega RC)} = \underbrace{\frac{(\omega RC)^2 + \mathrm{j}\omega RC}{1 + (\omega RC)^2}}_{\text{Cartesian Form}}$$

Magnitude

$$|\mathrm{H}(\omega)| = \frac{|\mathrm{j}\omega RC|}{|1 + \mathrm{j}\omega RC|}$$

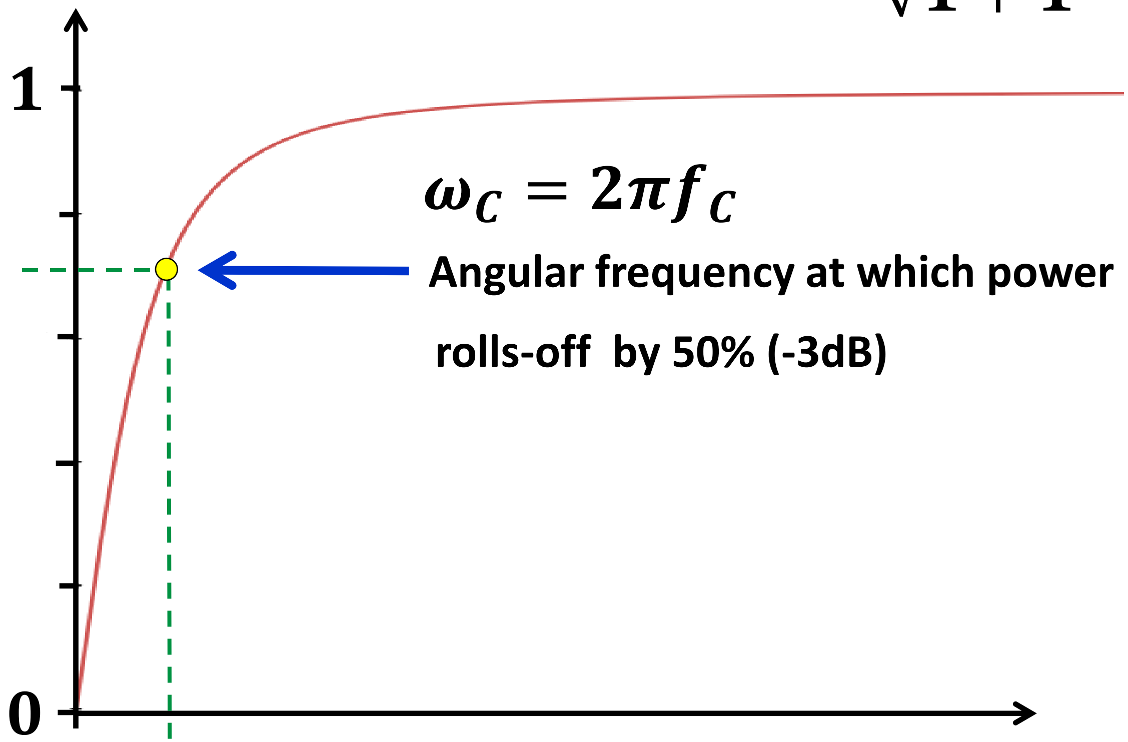
$$|\mathrm{H}(\omega)| = \frac{\omega RC}{\sqrt{1 + (\omega RC)^2}}$$

Magnitude of $H(\omega)$ for RC high-pass filter

$$|\mathrm{H}(\omega)| = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}}$$

$\omega_C RC = 1$ → $|\mathrm{H}(\omega_C)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$

$|\mathrm{H}(\omega)|$



$\frac{1}{\sqrt{2}} \approx 0.707$

$\omega_C = 2\pi f_C$

Angular frequency at which power rolls-off by 50% (-3dB) $P \propto |H(\omega)|^2$

$\tau = RC$

$\omega_C = \tau^{-1}$

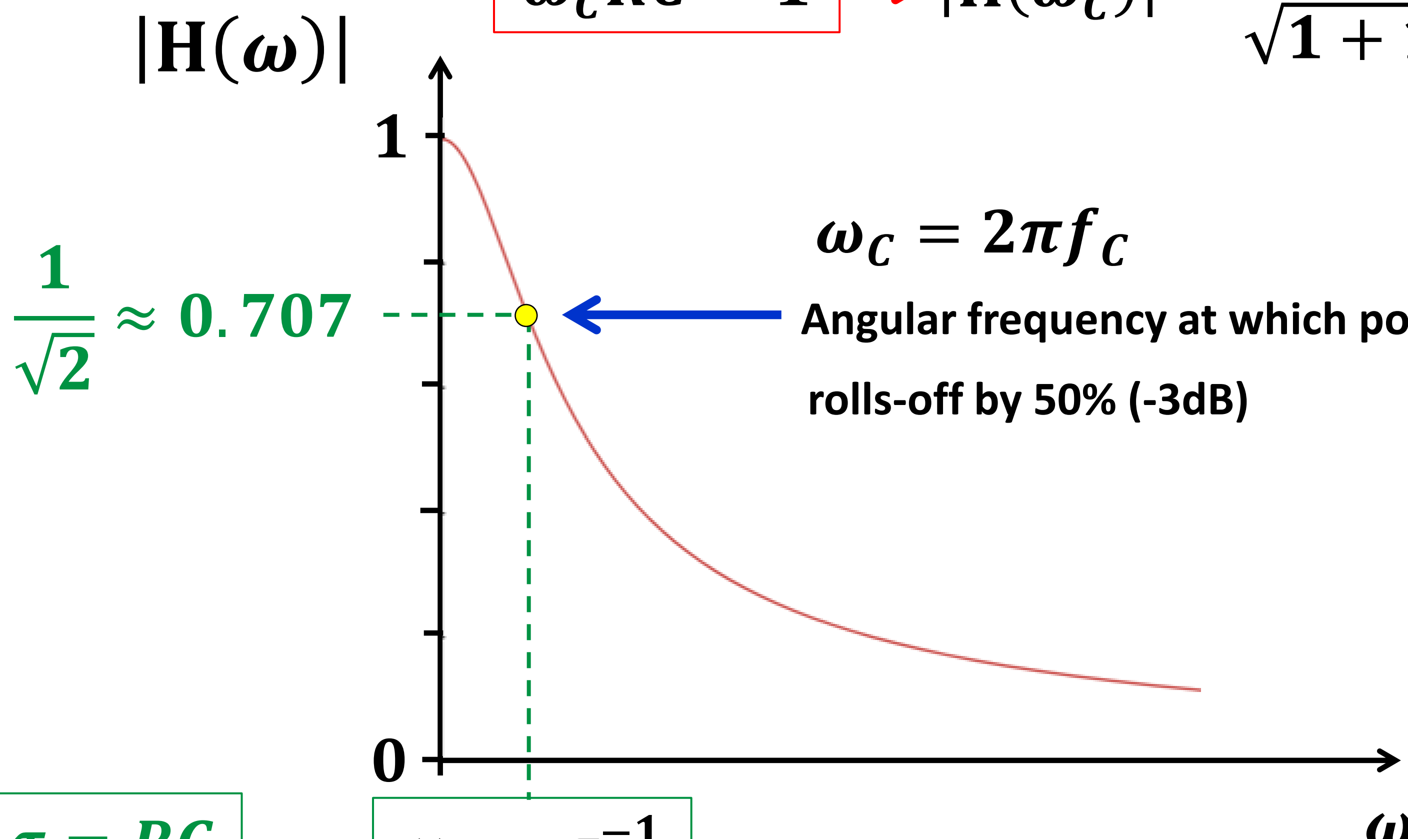
ω

REMEMBER – for the RC low-pass filter:

At $\omega = \omega_C$: $R = 1/\omega C$

$$|\mathrm{H}(\omega)| = \frac{1}{\sqrt{1 + (\omega RC)^2}}$$

$\omega_C RC = 1$ → $|\mathrm{H}(\omega_C)| = \frac{1}{\sqrt{1+1}} = \frac{1}{\sqrt{2}} = 0.707$



$\omega_C = 2\pi f_C$

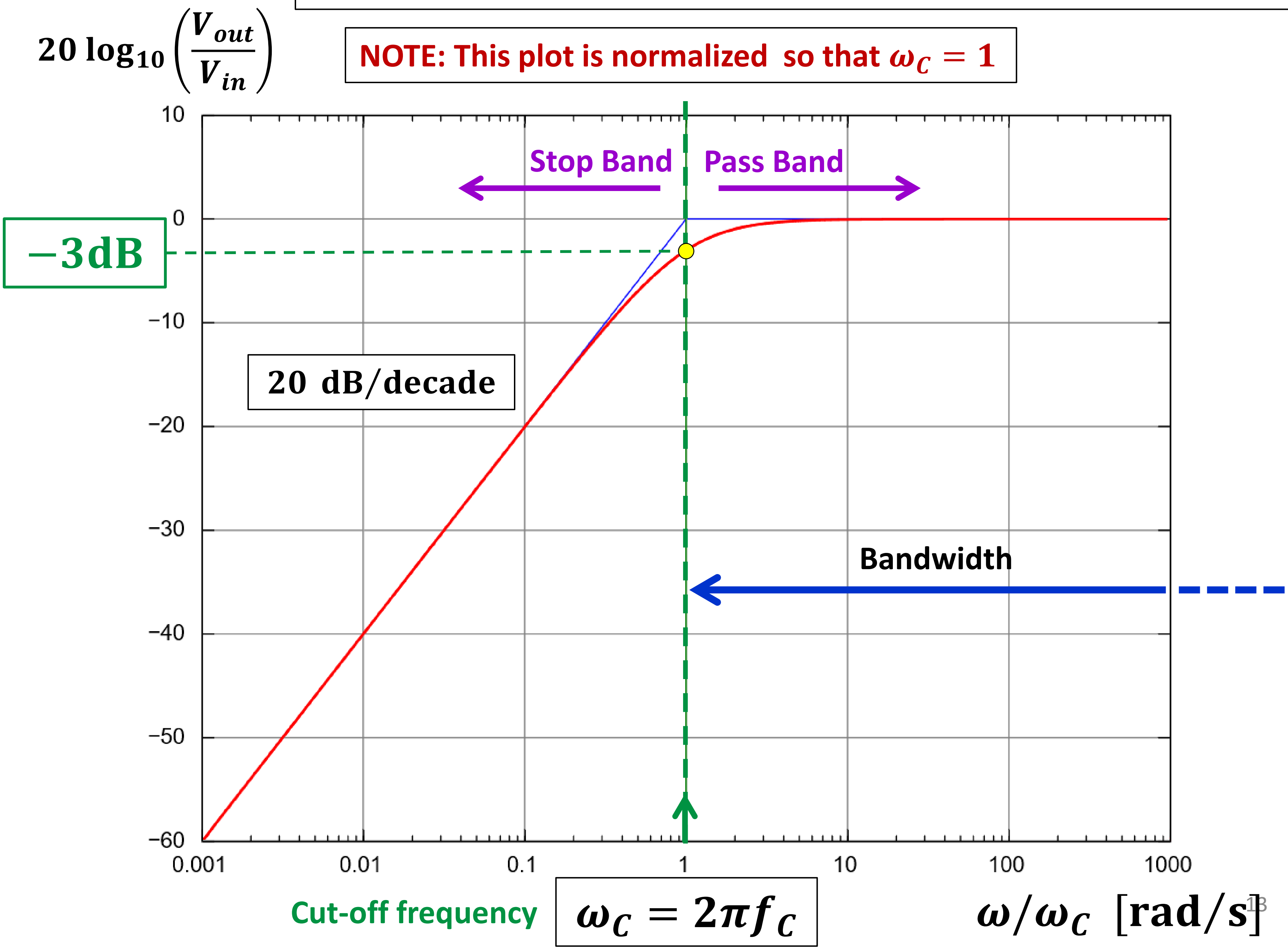
Angular frequency at which power $P \propto |H(\omega)|^2$ rolls-off by 50% (-3dB)

$\tau = RC$

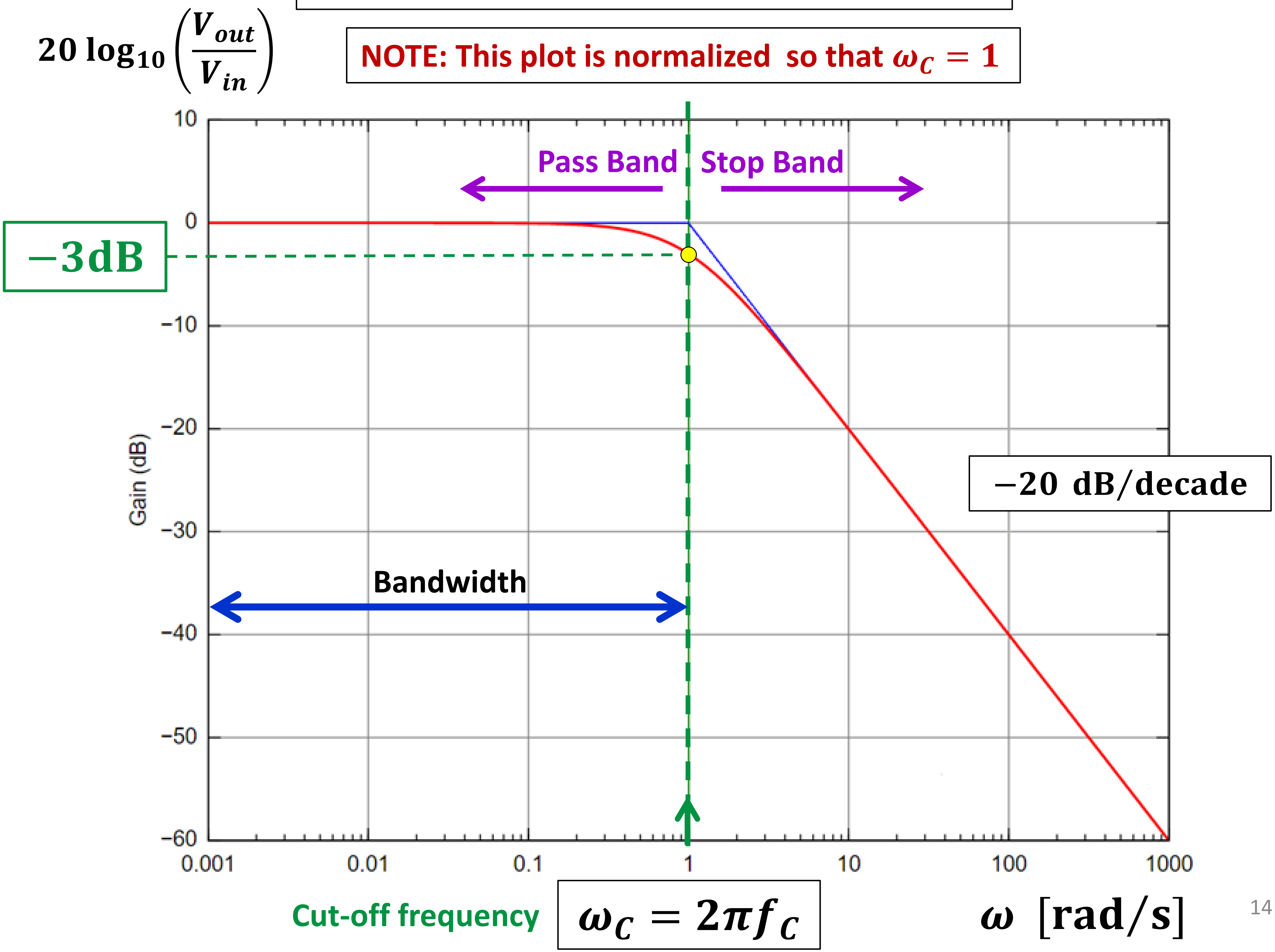
$\omega_C = \tau^{-1}$

log-decibel representation – Bode Plot for magnitude

NOTE: This plot is normalized so that $\omega_C = 1$



REMEMBER – for the low-pass filter:



Phase of $H(\omega)$ for RC high-pass filter

$$\mathrm{H}(\omega) = \frac{\mathrm{j}\omega RC(1 - \mathrm{j}\omega RC)}{(1 + \mathrm{j}\omega RC)(1 - \mathrm{j}\omega RC)} = \underbrace{\frac{(\omega RC)^2 + \mathrm{j}\omega RC}{1 + (\omega RC)^2}}_{\text{Cartesian Form}}$$

$$\angle \mathrm{H}(\omega) = \tan^{-1}\frac{\Im m\{\mathrm{H}(\omega)\}}{\Re e\{\mathrm{H}(\omega)\}} = \tan^{-1}\frac{\omega RC/\left(1 + (\omega RC)^2\right)}{(\omega RC)^2/(1 + (\omega RC)^2)}$$

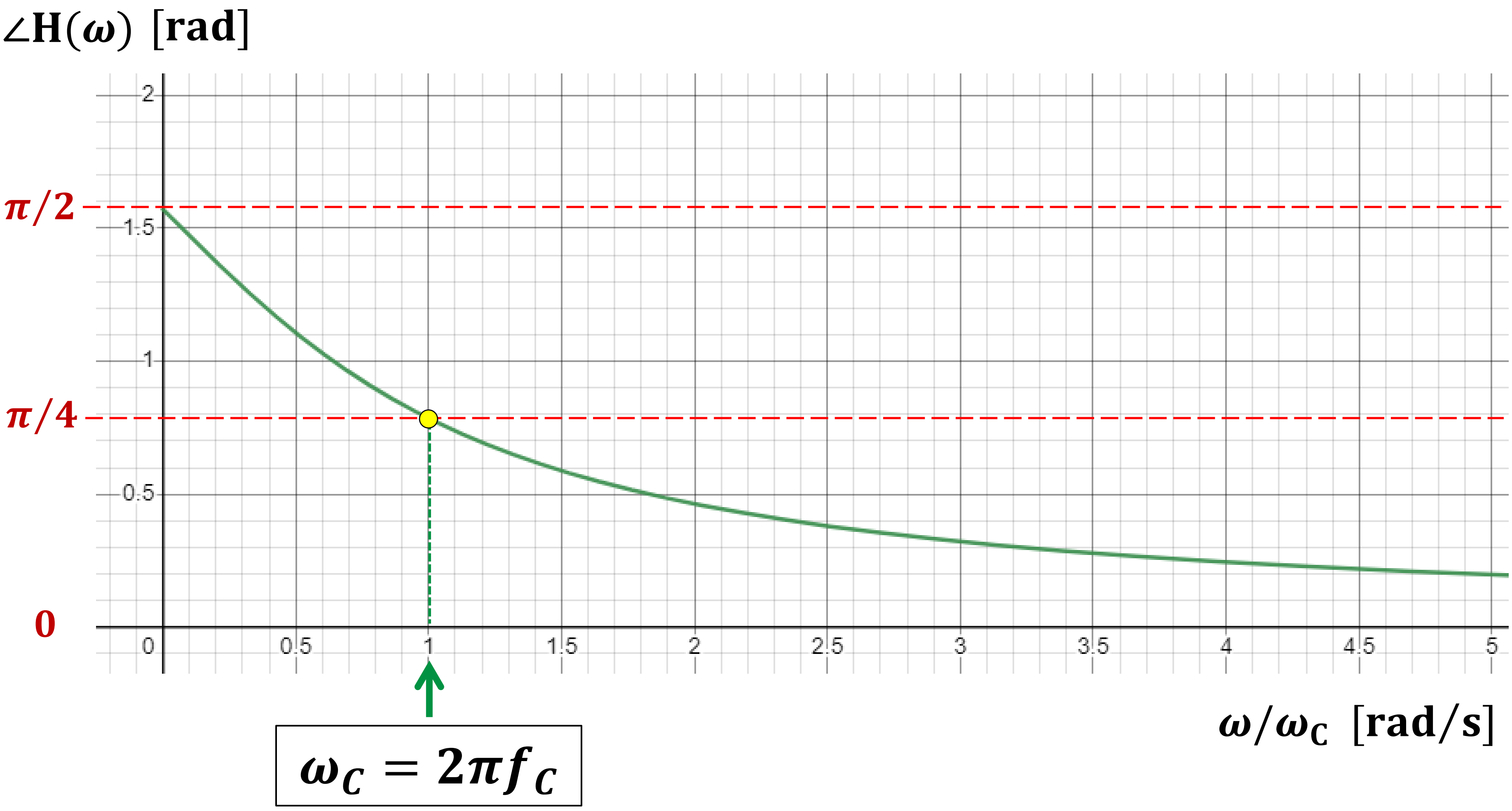
$$\angle \mathrm{H}(\omega) = \tan^{-1}\left(\frac{1}{\omega RC}\right)$$

When $\omega = \omega_C$ we have $\omega_C RC = 1$

$$\angle \mathrm{H}(\omega) = \tan^{-1}(1) = \frac{\pi}{4} = 45^\circ$$

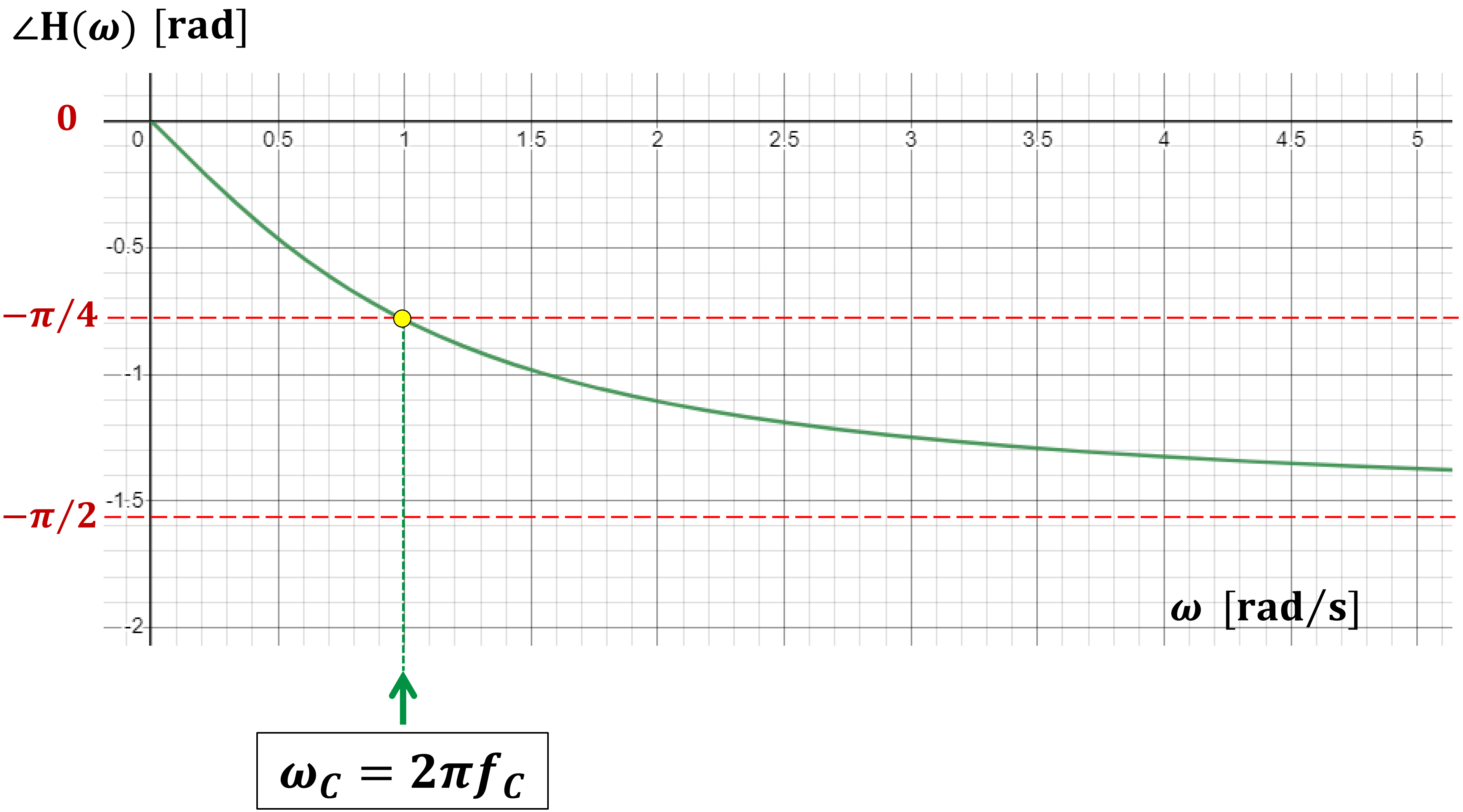
Phase for RC high-pass filter

Linear scale representation



REMEMBER – for the RC low-pass filter:

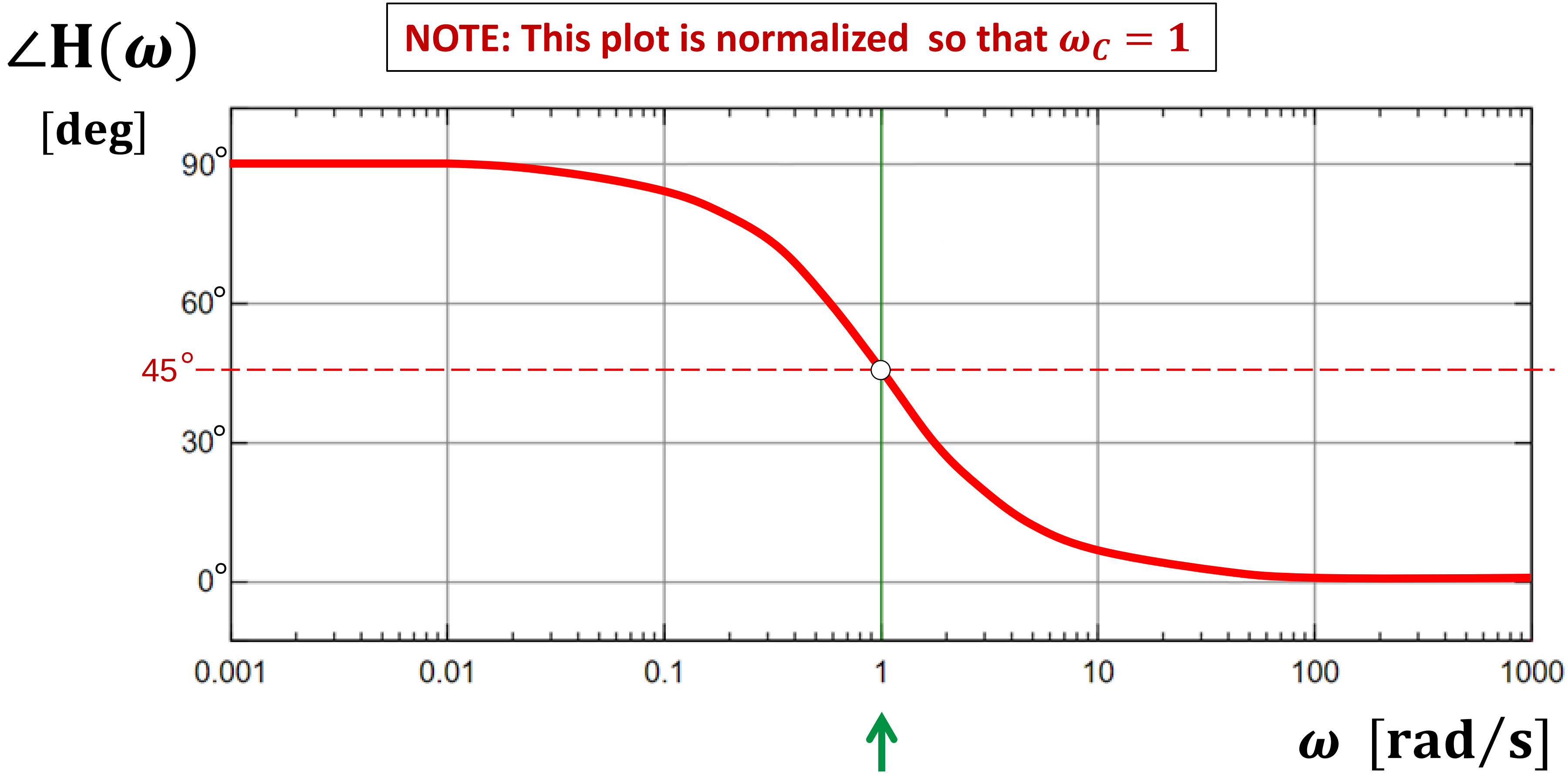
Linear scale representation



Phase for RC high-pass filter

semi-log scale representation – Bode Plot for phase

NOTE: This plot is normalized so that $\omega_C = 1$

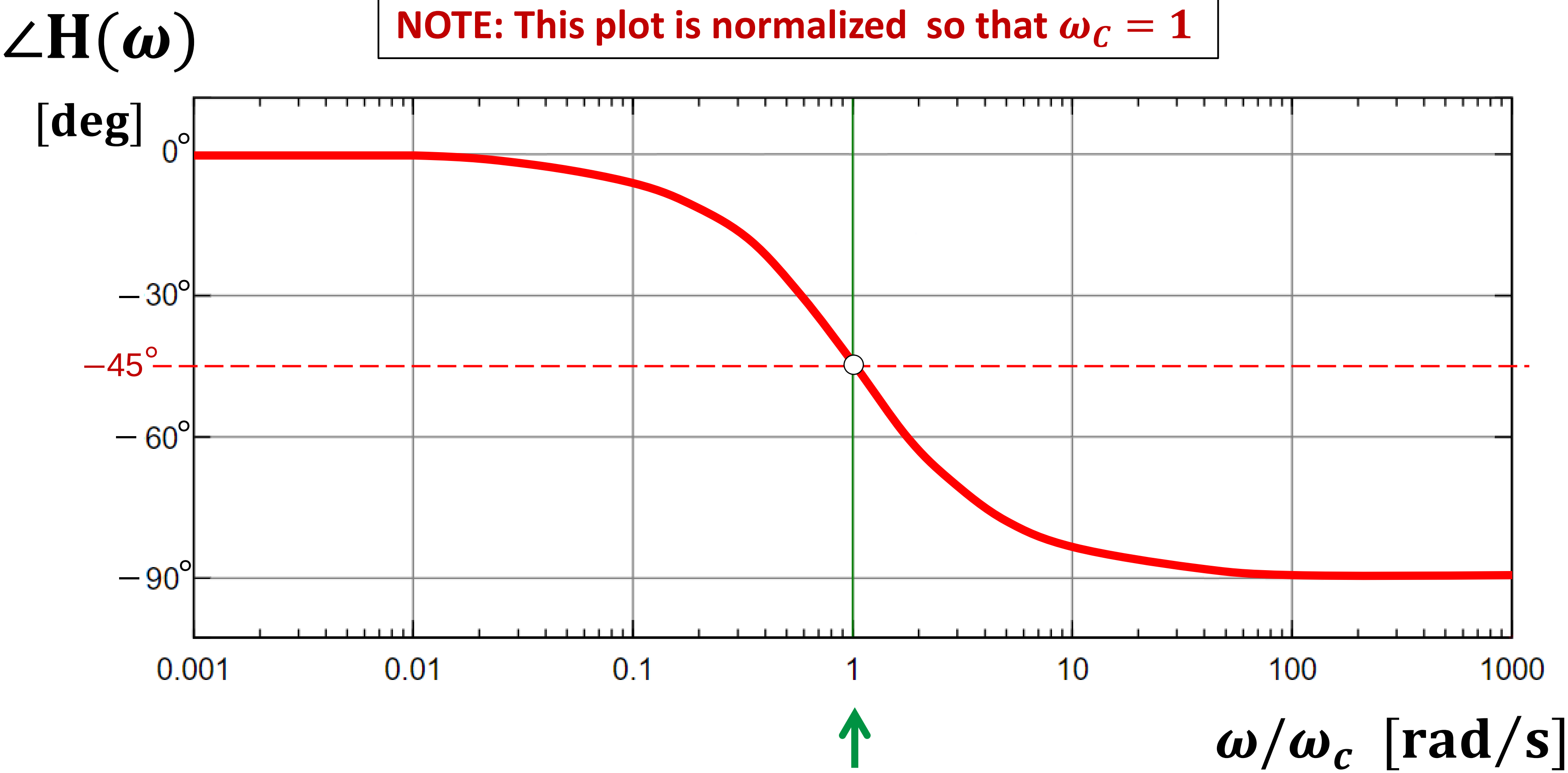


$$\omega_C = 2\pi f_C$$

REMEMBER – for the RC low-pass filter:

semi-log scale representation – Bode Plot for phase

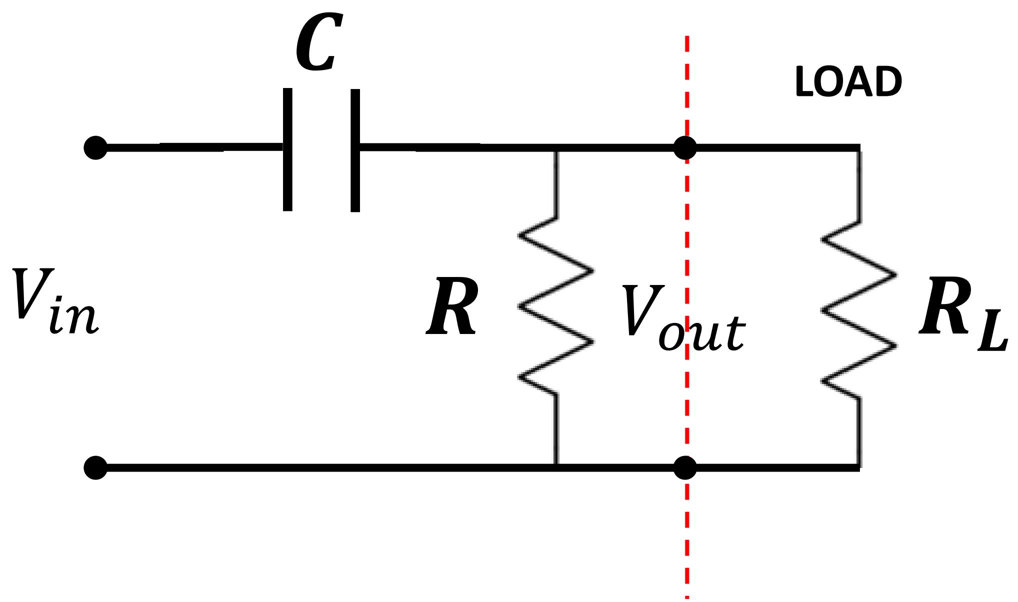
NOTE: This plot is normalized so that $\omega_C = 1$



$$\omega_C = 2\pi f_C$$

Limitations of simple passive filters

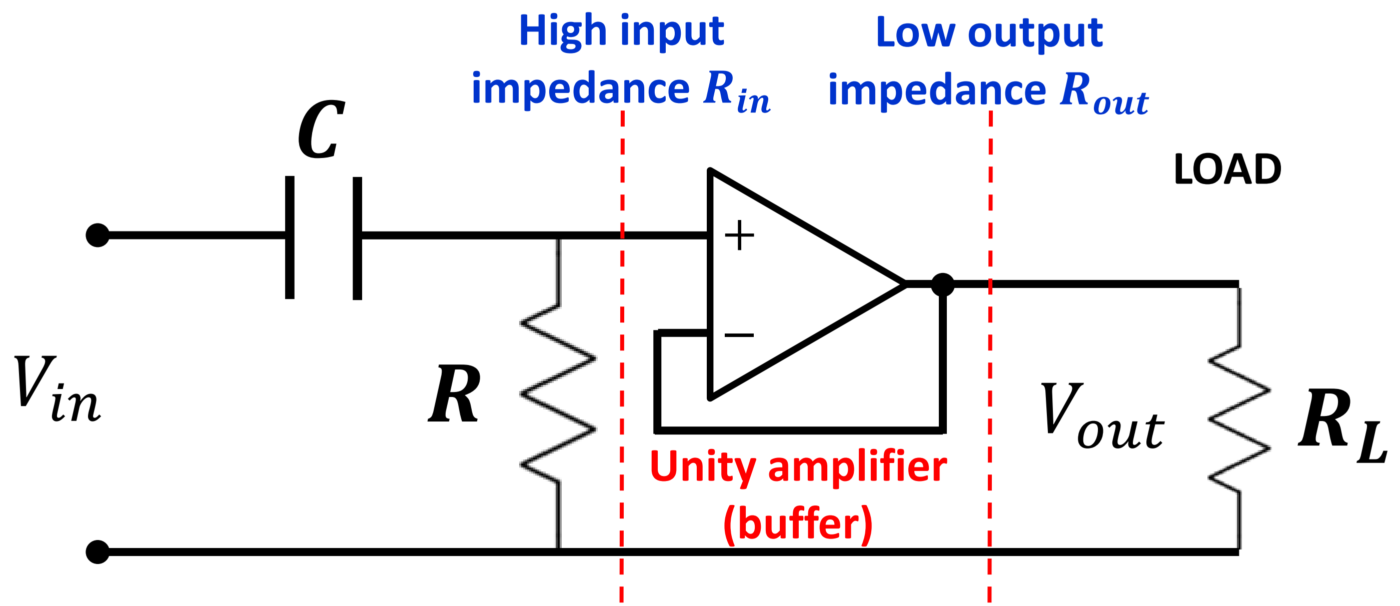
The response of a passive filter is affected by the load connected directly to it. For example, consider a high-pass RC filter:



The parallel $\mathrm{R_{eff}} = R//R_L$ yields an equivalent resistance lower than either R or R_L. In particular, if connected to a small resistor R_L, the resulting cutoff frequency $\omega_C' = (R_{\mathrm{eff}}C)^{-1}$ may change considerably with respect to the original $\omega_C = (RC)^{-1}$.

Overcome limitations by using active filters

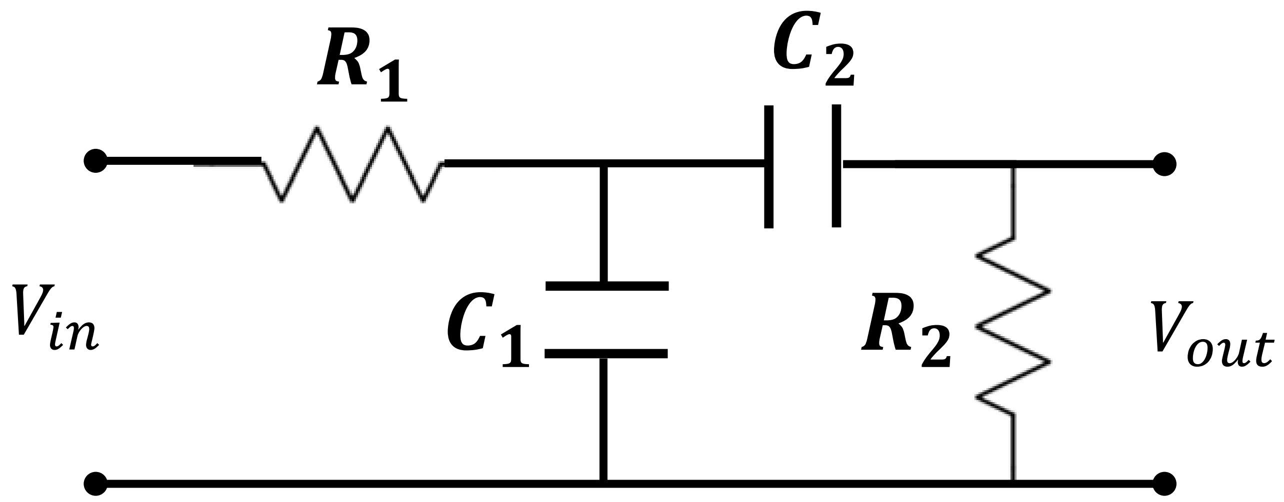
In the final part of the course, we will learn how a high input impedance operational amplifier can be used as an intermediate stage to improve the interconnection between filter and load.



The filter sees $R//R_{in} \approx R$ and is not affected. In output the voltage V_{out} drives a total resistance $R_{in} + R_L$. If R_{out} is much smaller, power is delivered mainly to the load R_L.

Band-Pass Filter

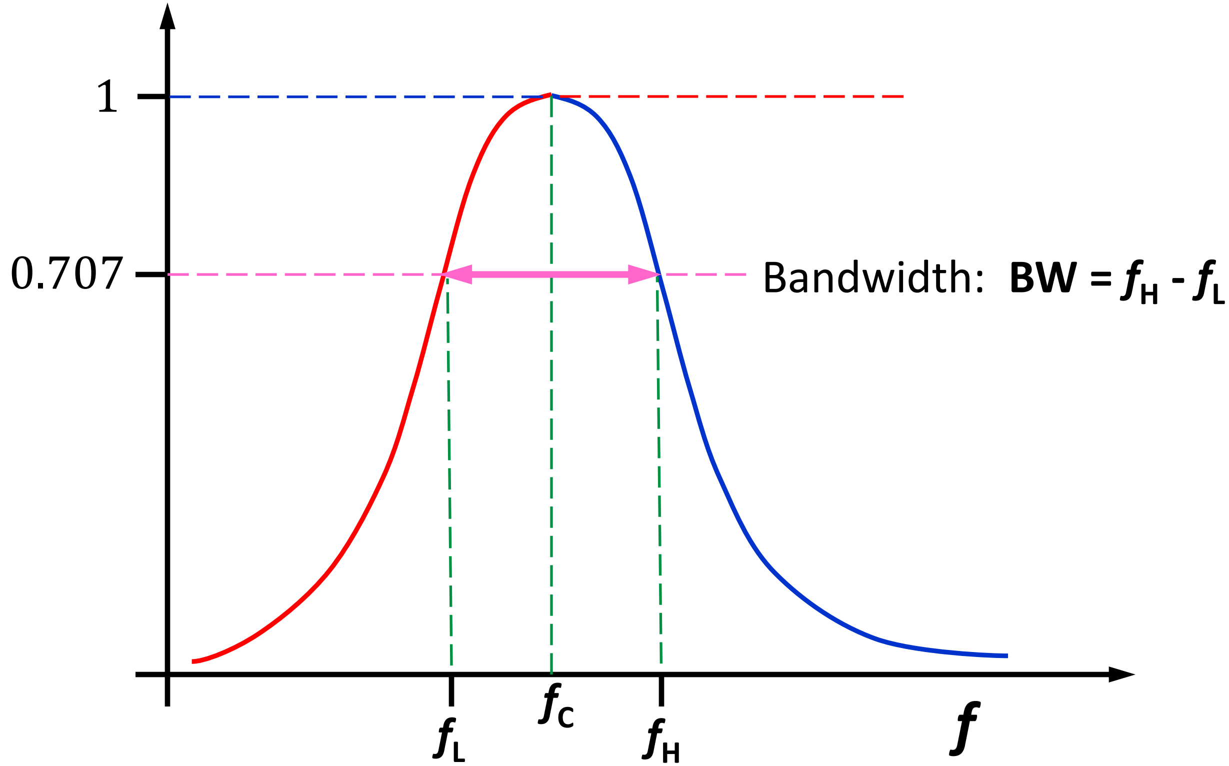
Cascade of a low pass and a high-pass filter can be designed so that $\omega_{CLP} > \omega_{CHP}$. The two filter characteristics combine, letting only an intermediate frequency band pass through.



Looking at frequency extremes, one can see that:

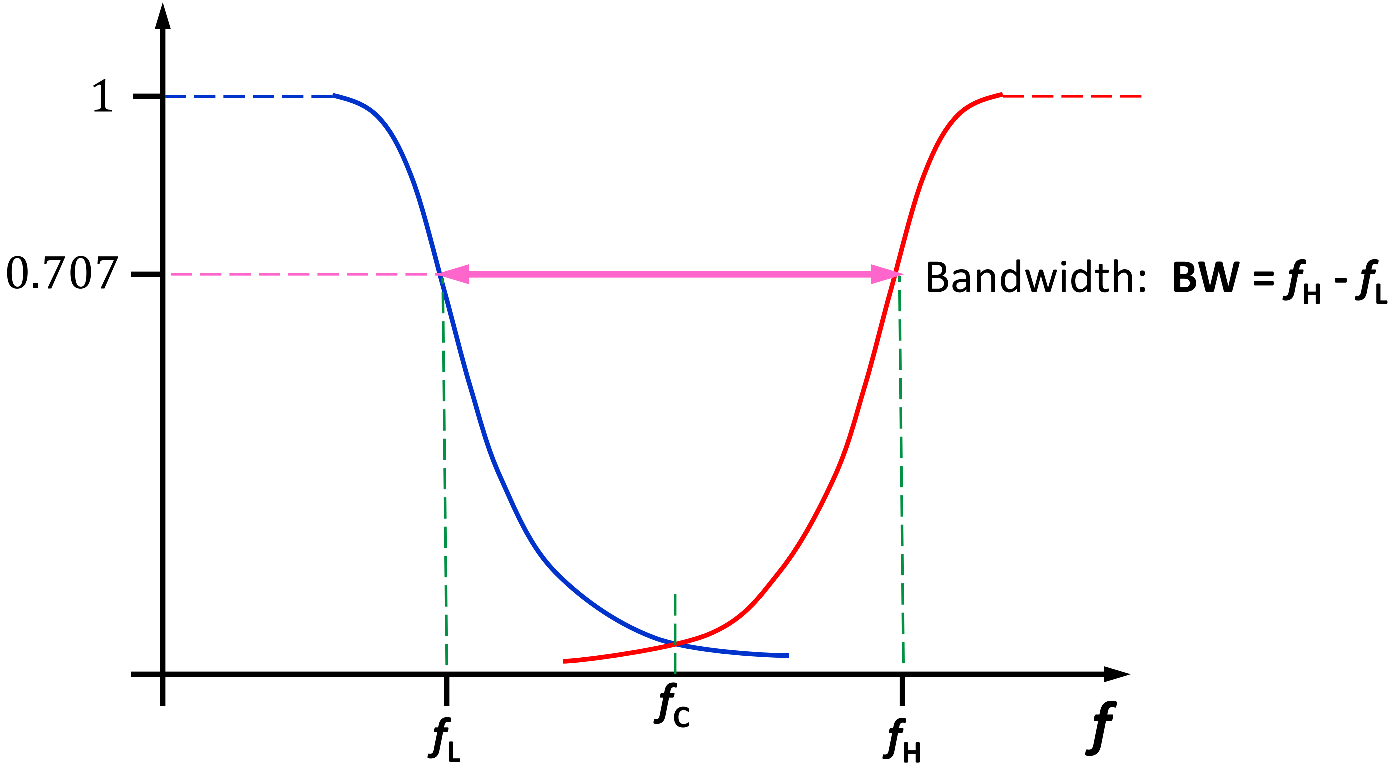
- $\omega = 0$ **capacitors behave like open circuit** $\rightarrow V_{out} = 0$
- $\omega \rightarrow \infty$ **capacitors behave like short circuit** $\rightarrow V_{out} = 0$

Band-Pass Filter



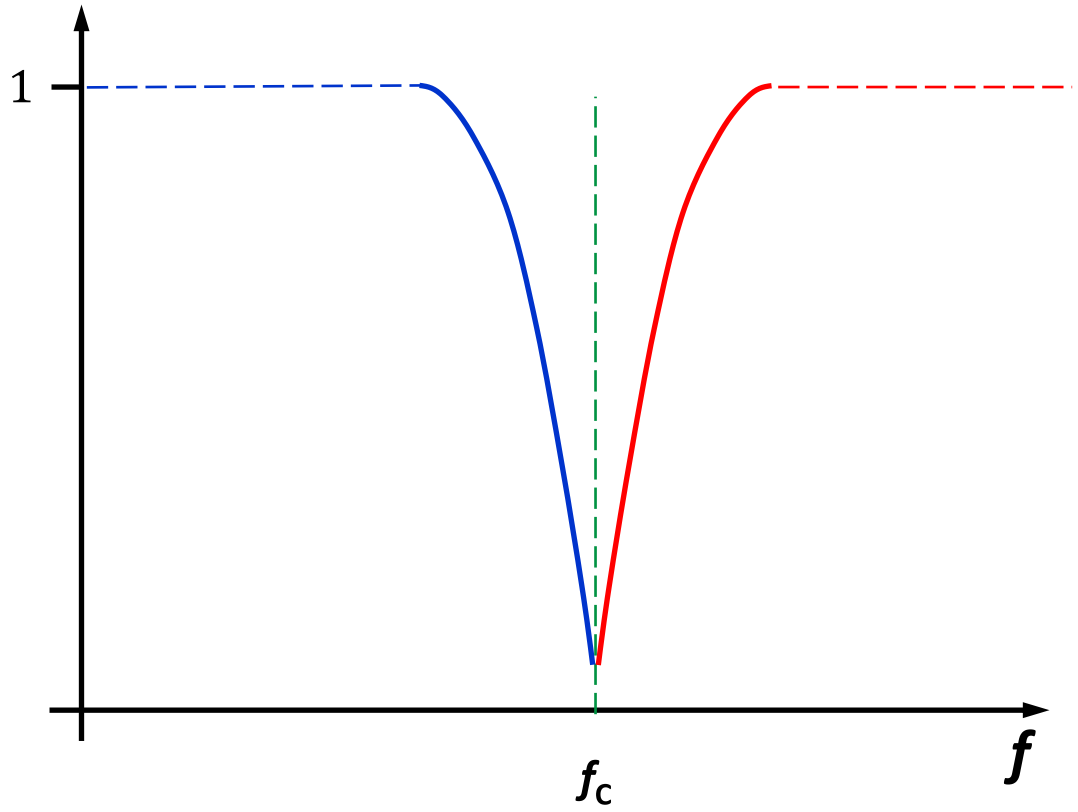
Frequencies $f < f_L$ and $f > f_H$ are strongly attenuated

Band-Stop Filter



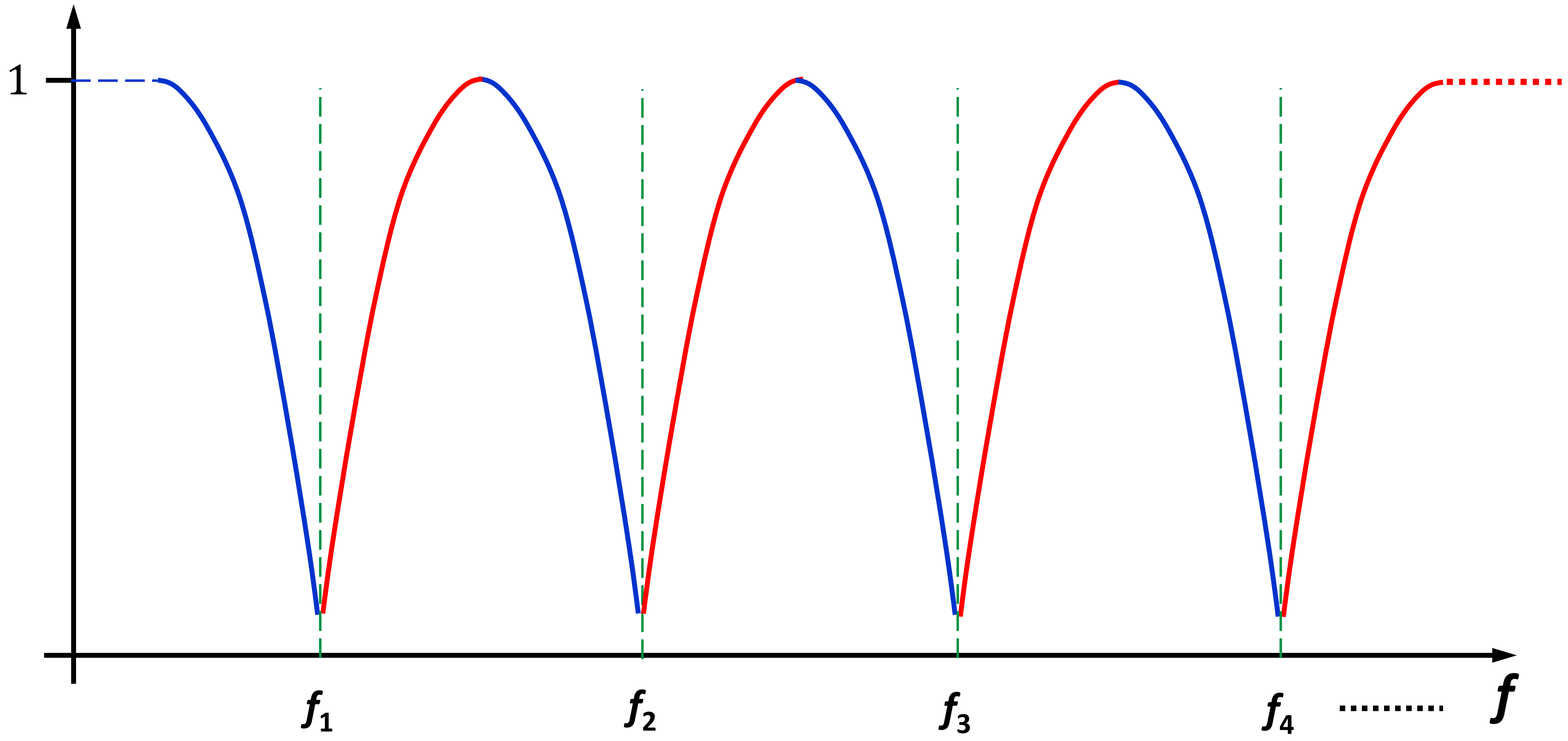
Frequencies and $f_L < f < f_H$ are strongly attenuated

Notch Filter



A very narrow band-stop filter, designed to reject a specific frequency, is called a notch filter.

Comb Filter



The comb filter consists of a series of regularly spaced notches and peaks (also called *teeth*).

Operational Amplifiers

Passive Circuit Response

The frequency response of passive circuits has notable limitations.

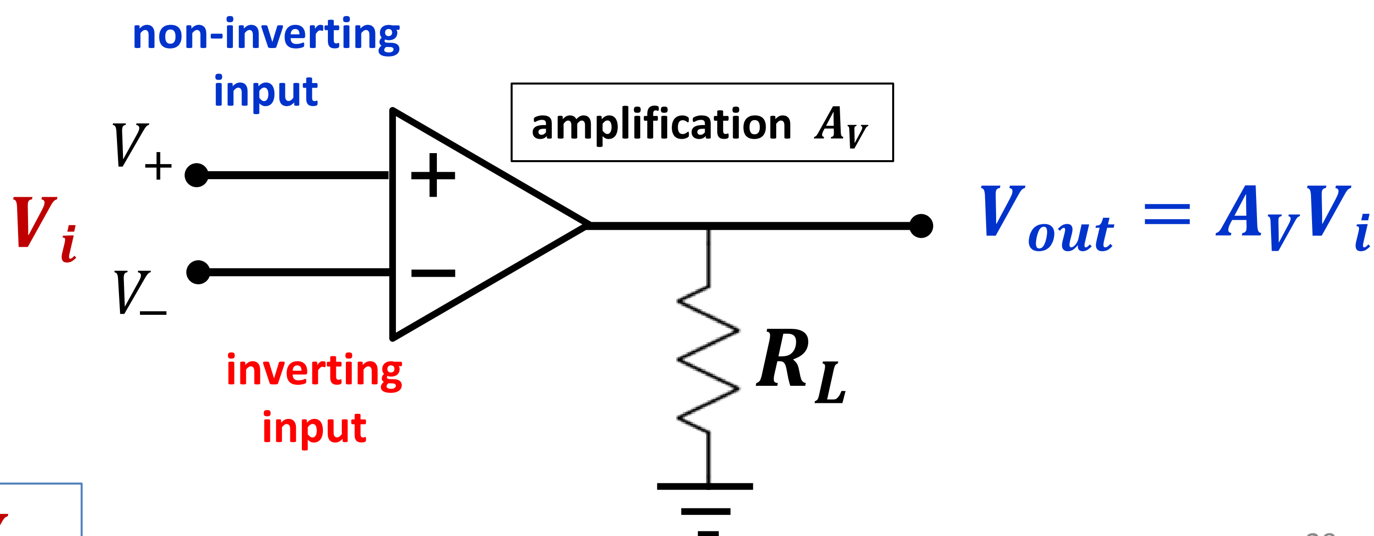
- **The magnitude of the transfer function is always $|\mathbf{H}(\boldsymbol{\omega})| \leq \mathbf{1}$**
- **When a load is applied, the frequency response changes**

Operational amplifiers can be used to address these limitations.

Operational Amplifier

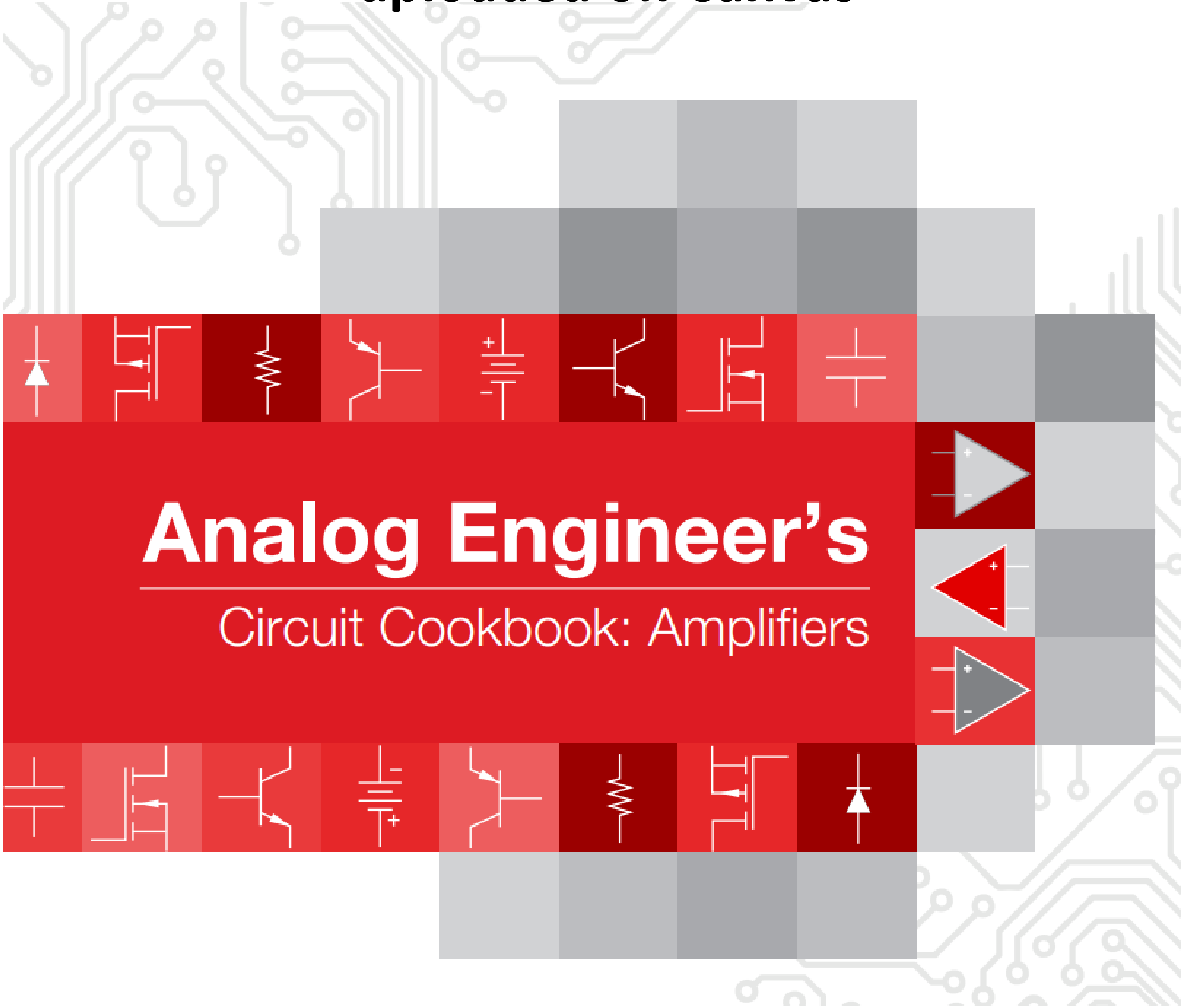
The Operational Amplifier (or OP AMP) is a versatile high gain circuit whose response characteristics can be controlled with the application of feedback.

It is commonly available in a compact integrated circuit package and it can be used as the basic building block for designing many analog circuit applications.

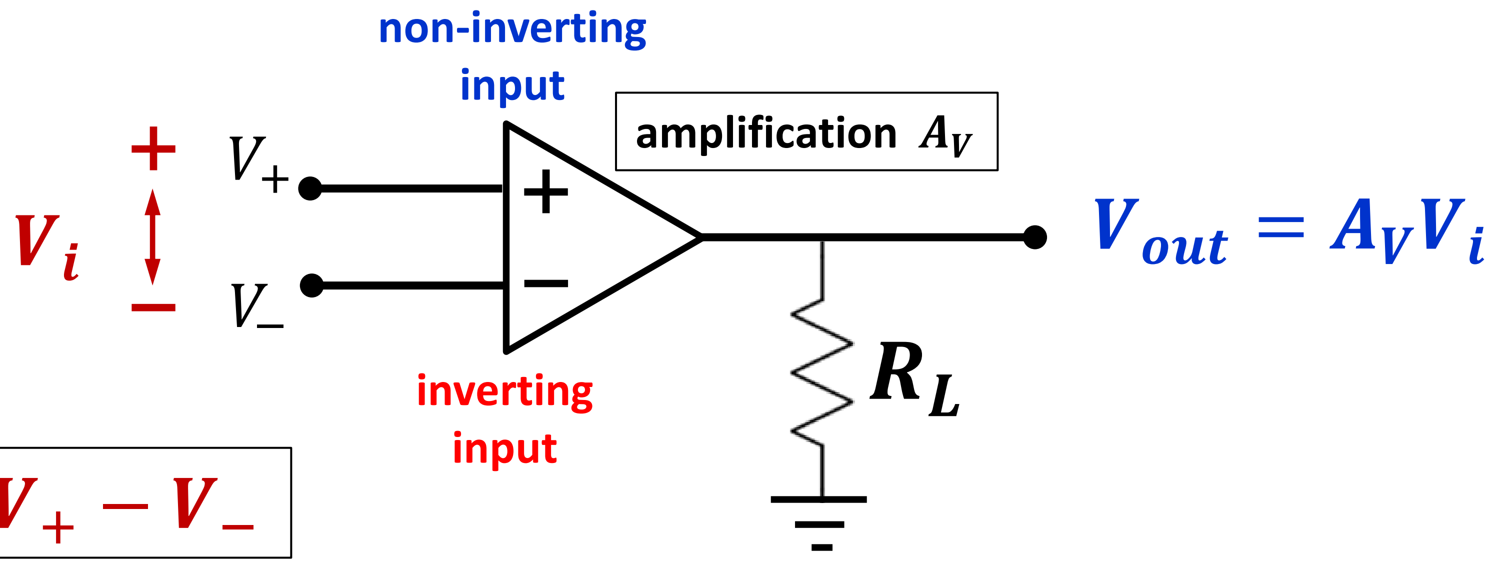


$$V_i = V_+ - V_-$$

Extensive technical reference from Texas Instruments uploaded on Canvas



The OP AMP has usually two inputs and one output



$$V_i = V_+ - V_-$$

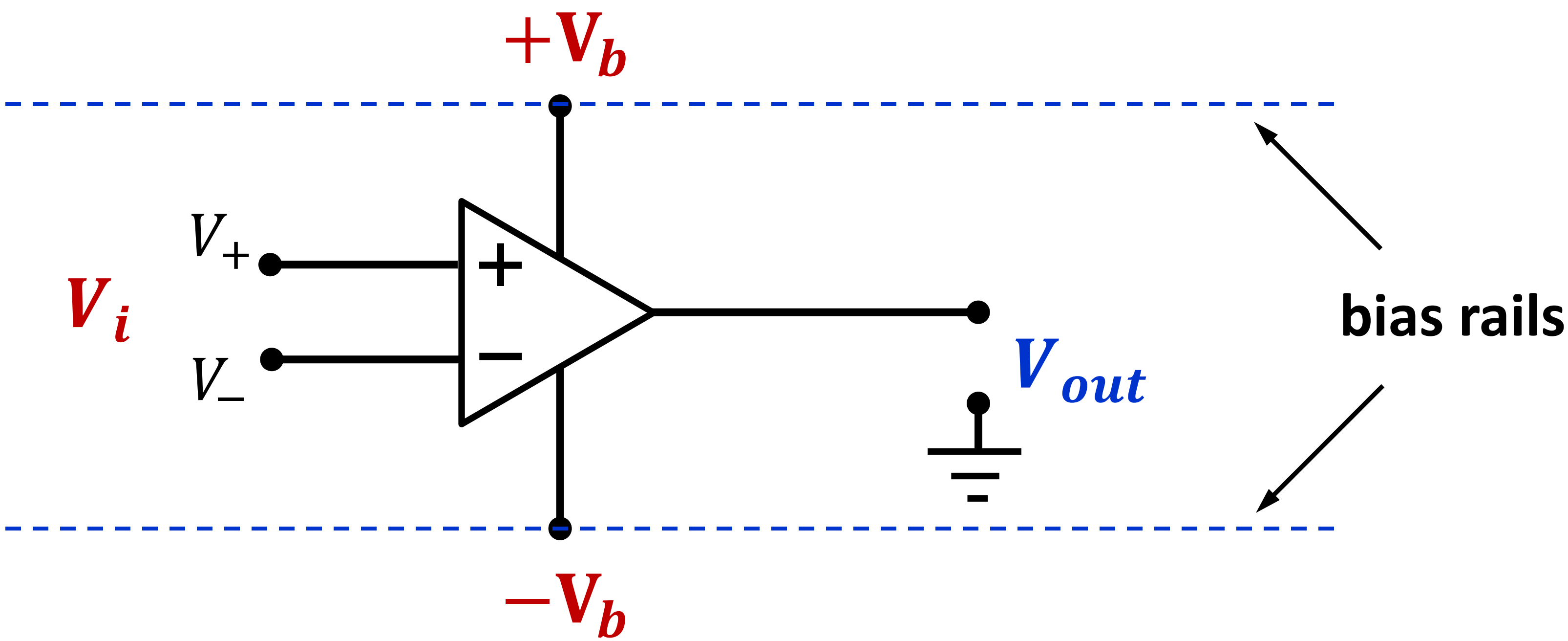
The gain between V_{out} and V_+ is positive (non-inverting)

$$\frac{V_{out}}{V_+} > 0$$

The gain between V_{out} and V_- is negative (inverting)

$$\frac{V_{out}}{V_-} < 0$$

The OP AMP is an active device requiring power from a DC power supply. A common design has two balanced $\pm V_b$ bias voltages.



The output signal V_{out} can swing between $\pm V_b$ but it will saturate if trying to exceed $|V_b|$.

NOTE: An OMP AMP is a "direct coupled" amplifier and it works also in DC conditions, not only AC.

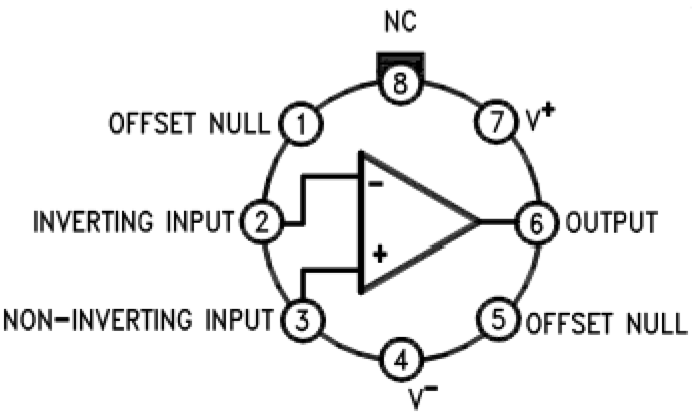
A very popular general purpose Op Amp: the “741”

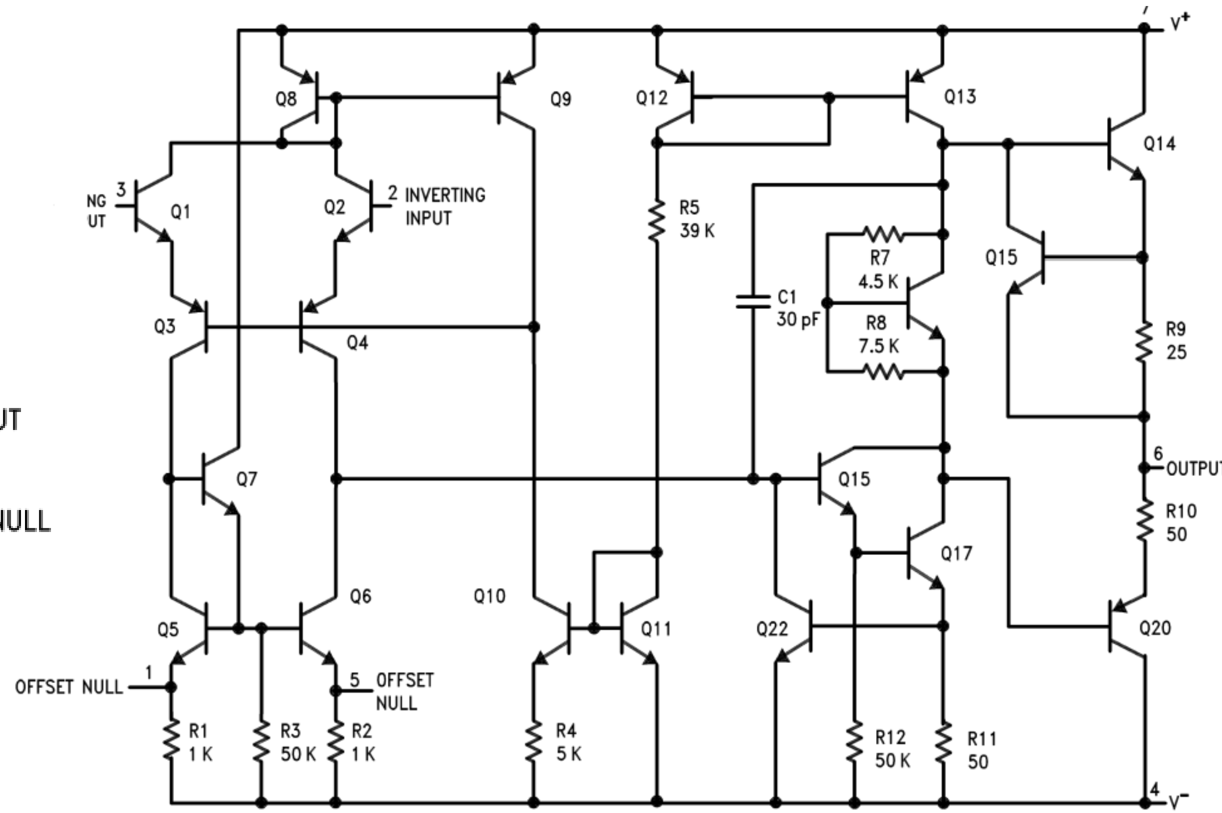
Texas Instruments

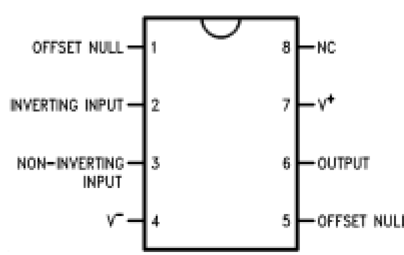
~ 20 transistors

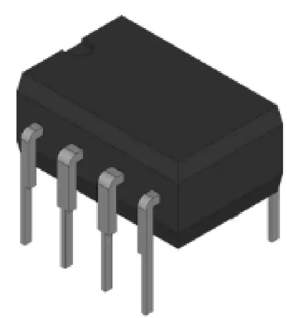
LM741 Operational Amplifier

Metal Can Package



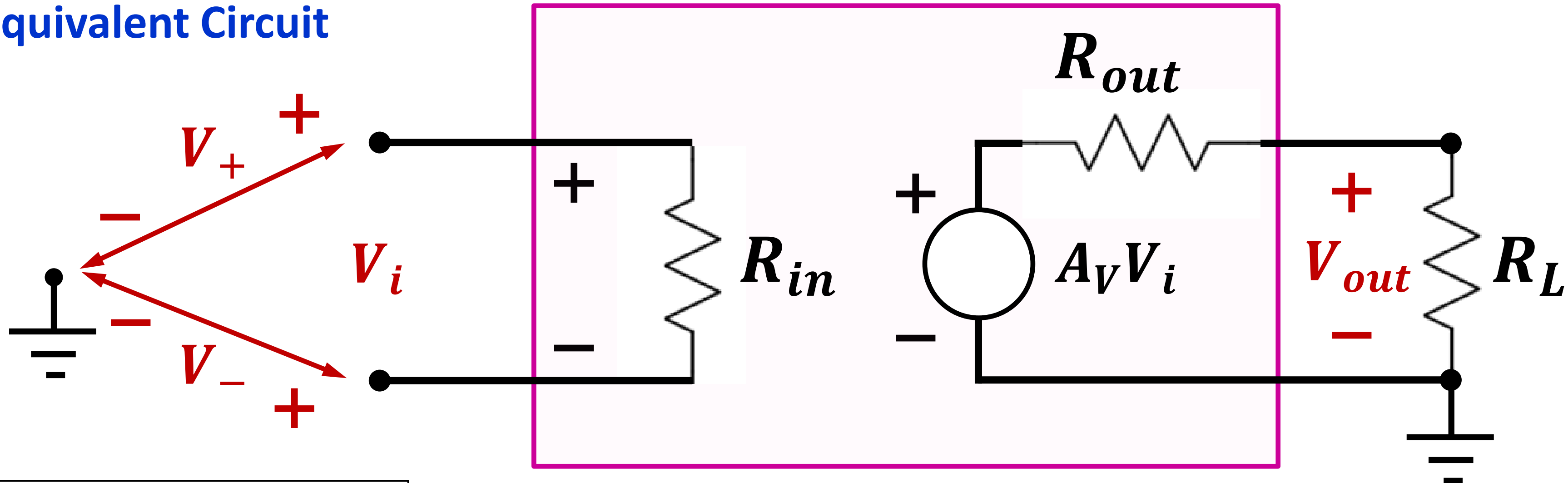






Ideal Operational Amplifier

Equivalent Circuit

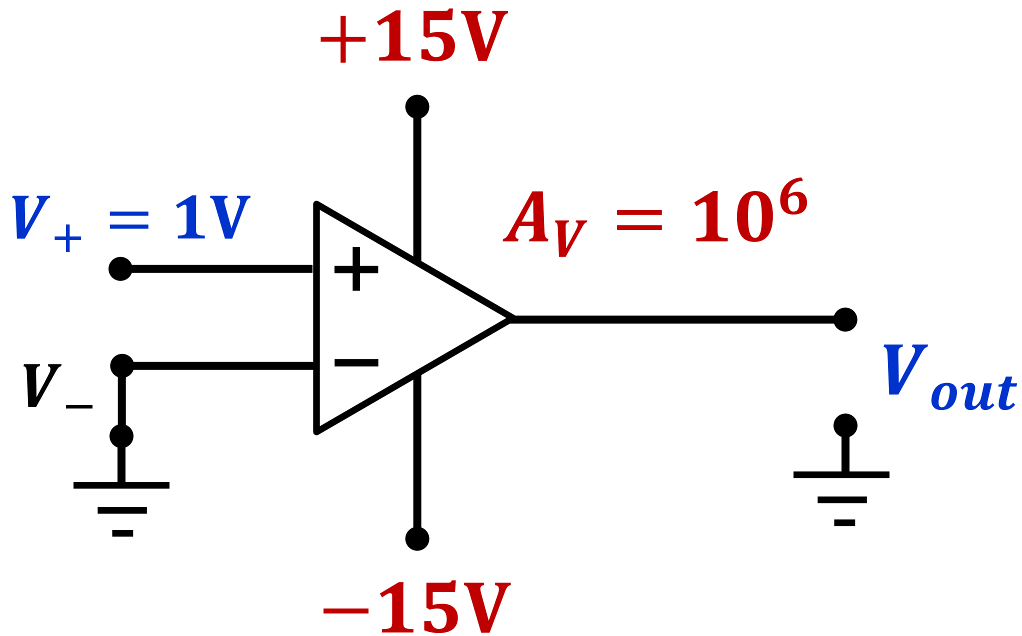


$V_i = V_+ - V_-$

open circuit voltage gain A_V

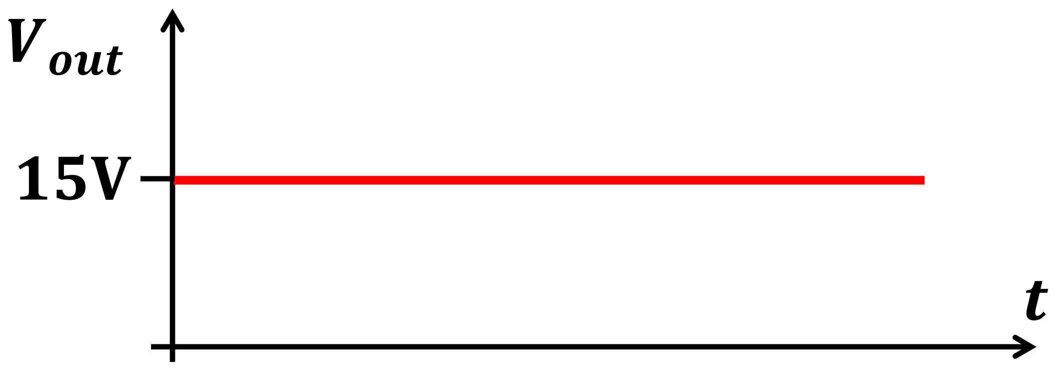
- **Input Resistance** $R_{in} \rightarrow \infty$
- **Output Resistance** $R_o \rightarrow 0$
- **Open-circuit voltage gain** $A_V \rightarrow -\infty$**. In real devices** $A_V \approx 10^6$.
- **Bandwidth** $\rightarrow \infty$ **(same operation up to any frequency)**
- $V_{out} = 0$ **when** $V_+ = V_-$
- **Behavior is independent of temperature**

Example 1

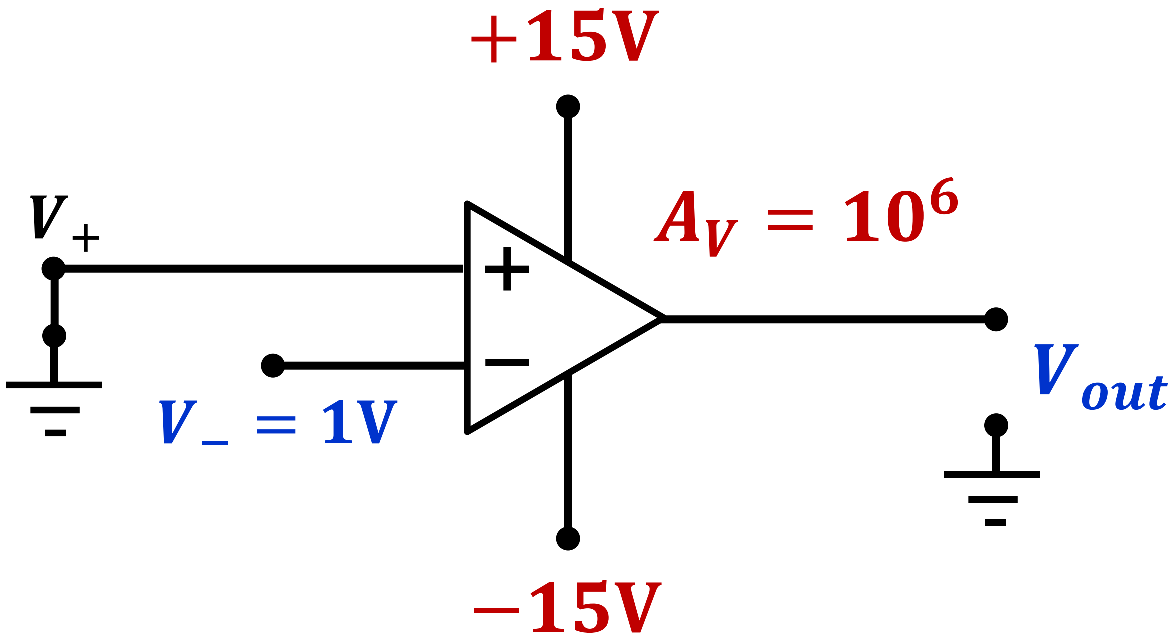


$$V_{out} = A_V \times (V_+ - V_-) = 10^6(1-0) = 10^6\text{V}$$

Saturation occurs at +15 Volts. ⇨ $V_{out} = 15\text{V}$

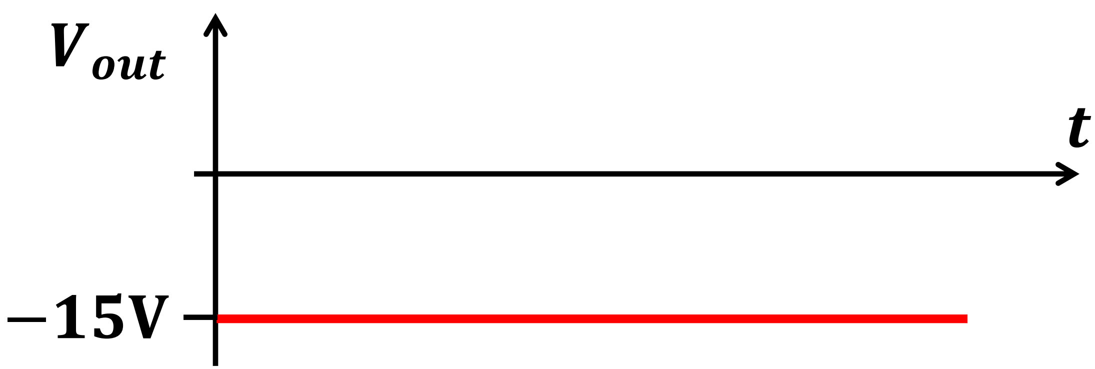


Example 2

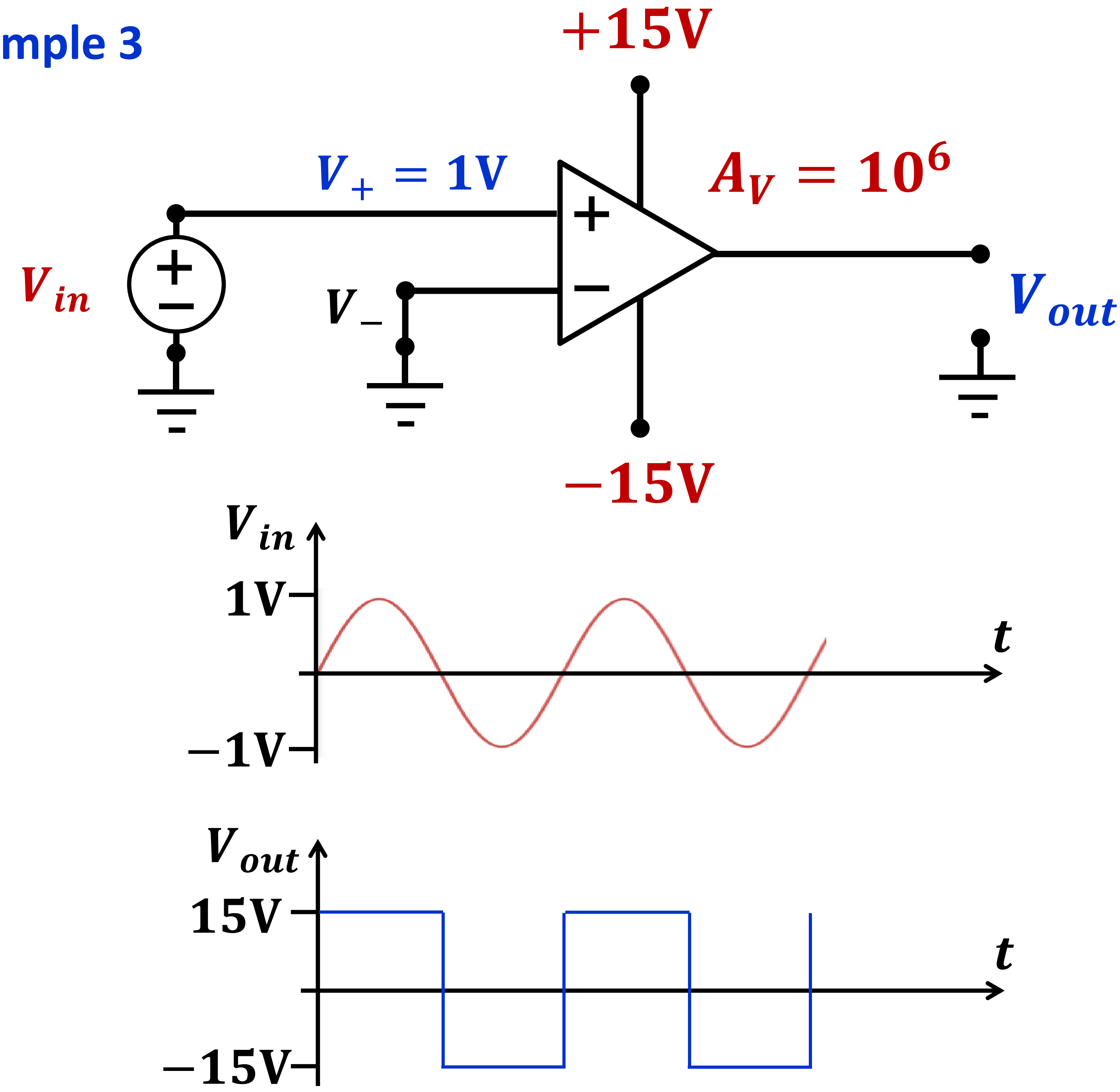


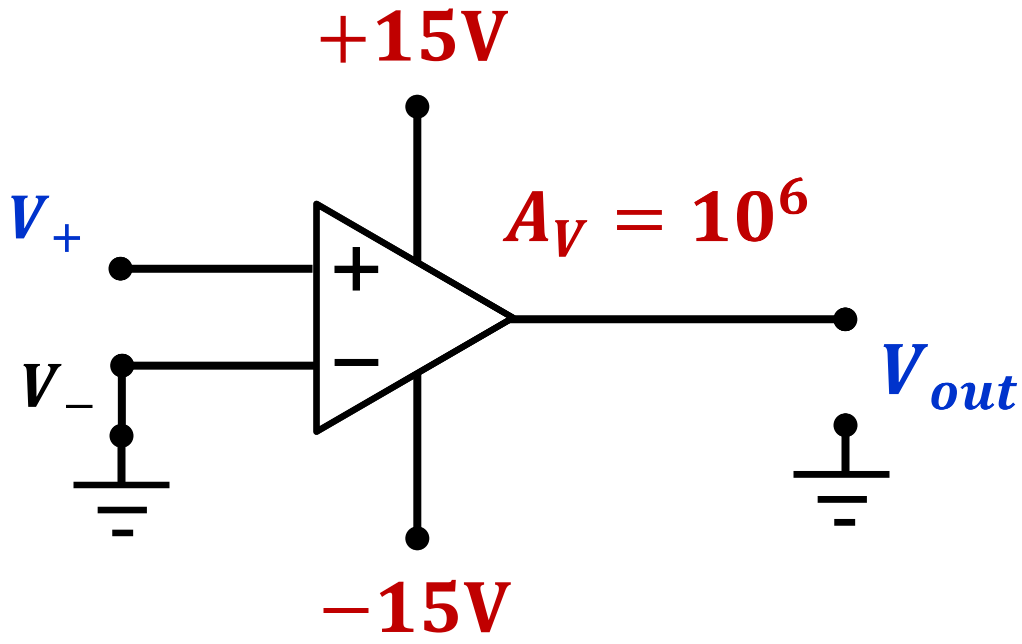
$$V_{out} = A_V \times (V_+ - V_-) = 10^6(0 - 1) = -10^6 V$$

Saturation occurs at -15 Volts. ⇒ $V_{out} = -15V$



Example 3





The circuit in Examples 1, 2, and 3 is called a "COMPARATOR"

$V_{out} = +15\text{V}$	if	$V_+ > V_-$
$V_{out} = -15\text{V}$	if	$V_+ < V_-$
$V_{out} = 0\text{V}$	if	$V_+ = V_-$

The gain A_V of the OP AMP is extremely large and the output saturates easily to $\pm V_b$.

This occurs even for minute values of the voltage between input terminals when

$$|(V_+ - V_-)| > V_b/A_V$$

Feedback

When $|(V_+ - V_-)| < V_b/A_V$, the output V_{out} does not saturate and follows the functional form of the input as

$$V_{out} = A_V(V_+ - V_-)$$

implying

$$V_+ \approx V_-$$

since the gain is so large.

To achieve these conditions, the OP AMPS are operated in *FEEDBACK* configuration.