ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 36

MWF – 12:00pm

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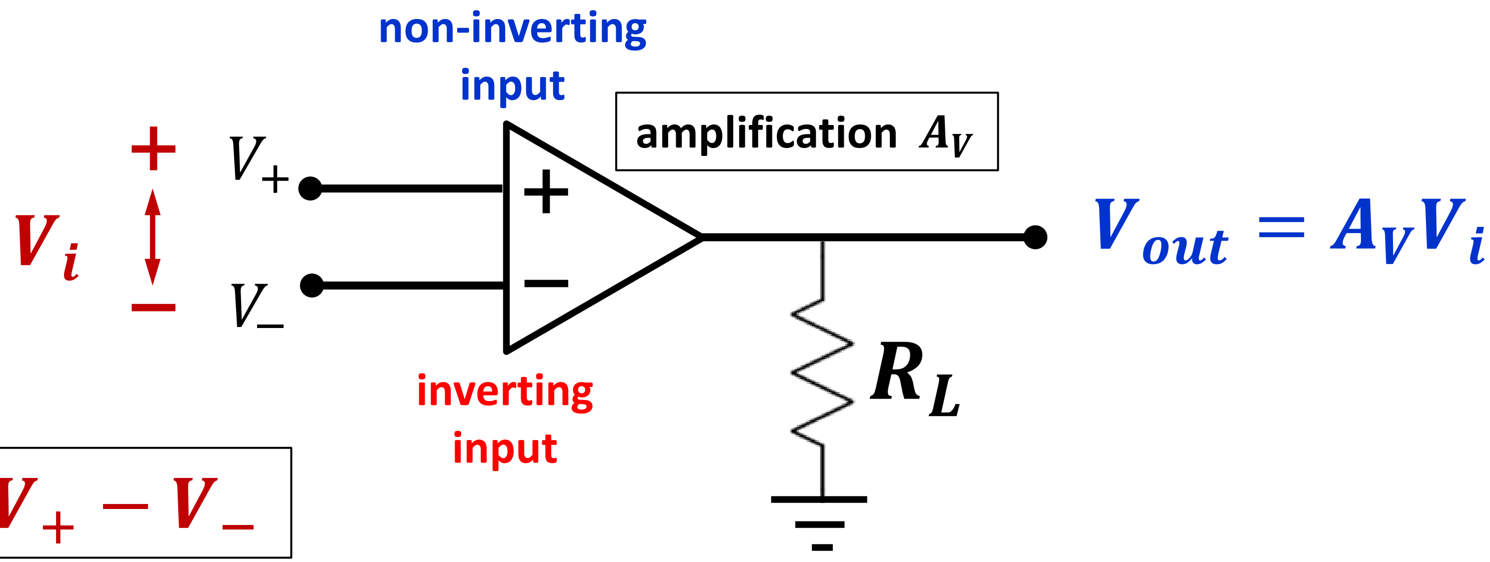
2062 ECE Building

Lecture 36 – Summary

Learning Objectives

1. Behavior of Operational Amplifiers
2. Important applications

The OP AMP has usually two inputs and one output



$$\boldsymbol{V_i = V_+ - V_-}$$

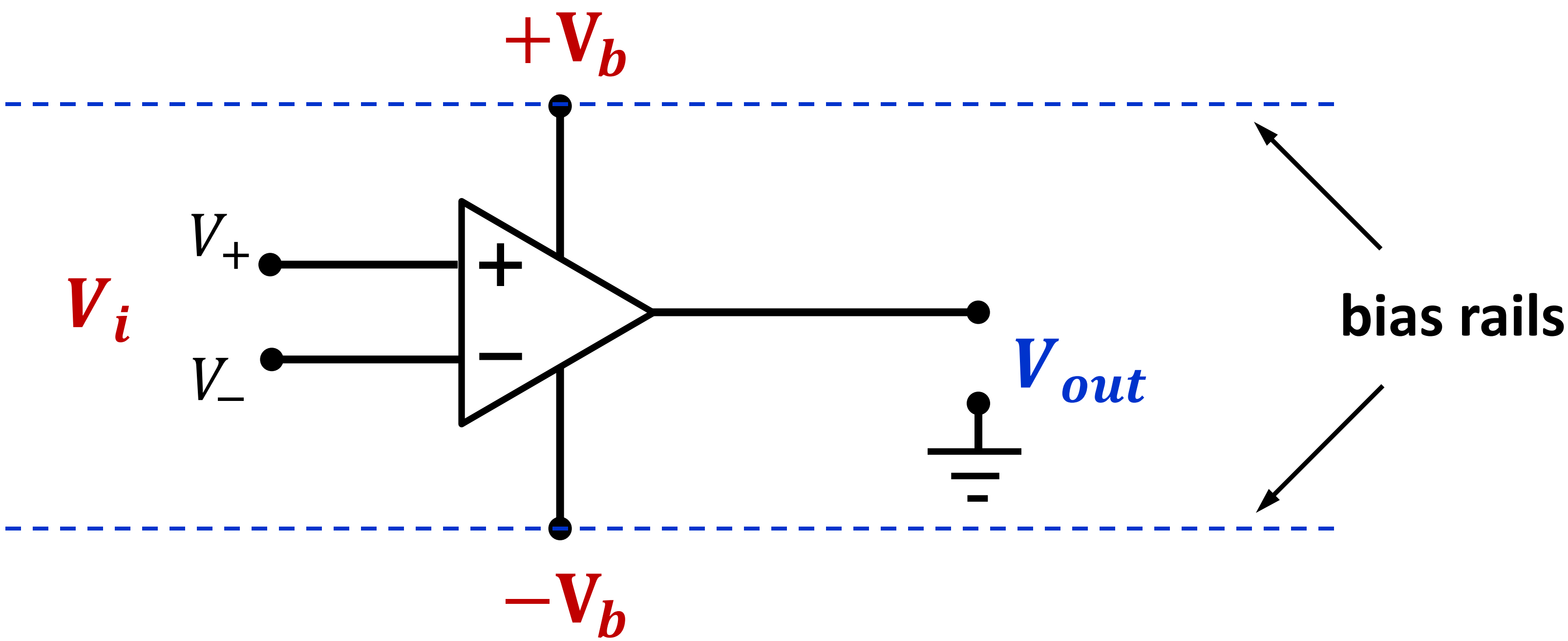
The gain between V_{out} and V_+ is positive (non-inverting)

$$\frac{V_{out}}{V_+} > 0$$

The gain between V_{out} and V_- is negative (inverting)

$$\frac{V_{out}}{V_-} < 0$$

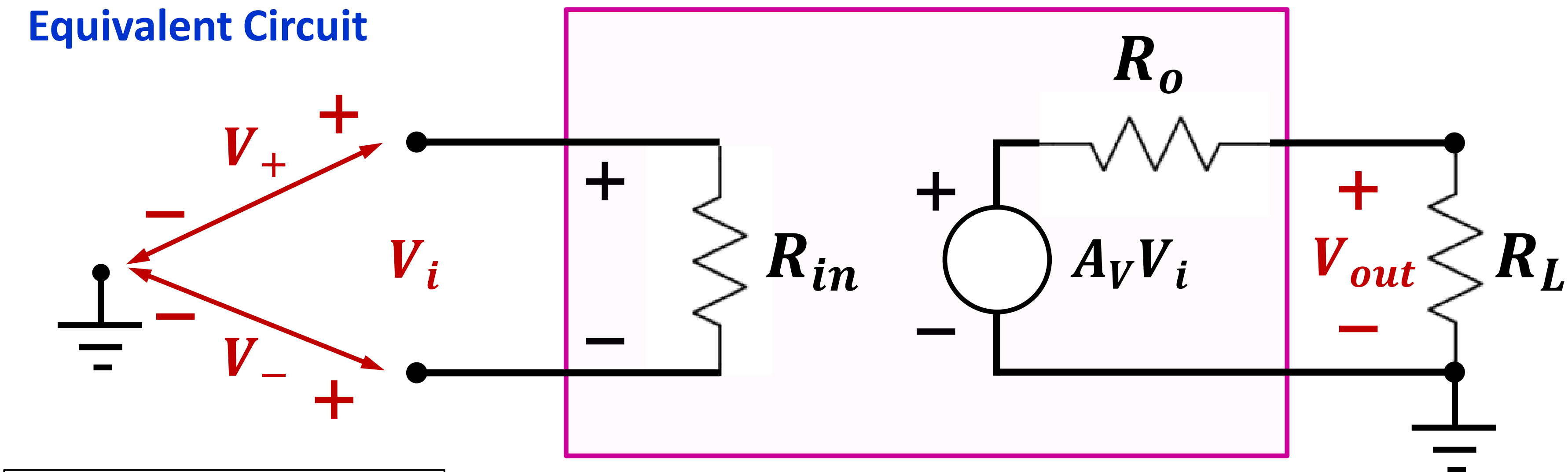
The OP AMP is an active device requiring power from a DC power supply. A common design has two balanced $\pm\mathrm{V}_b$ bias voltages.



The output signal V_{out} can swing between $\pm\mathrm{V}_b$ but it will saturate if trying to exceed $|\mathrm{V}_b|$.

NOTE: An OMP AMP is a "direct coupled" amplifier and it works also in DC conditions, not only AC.

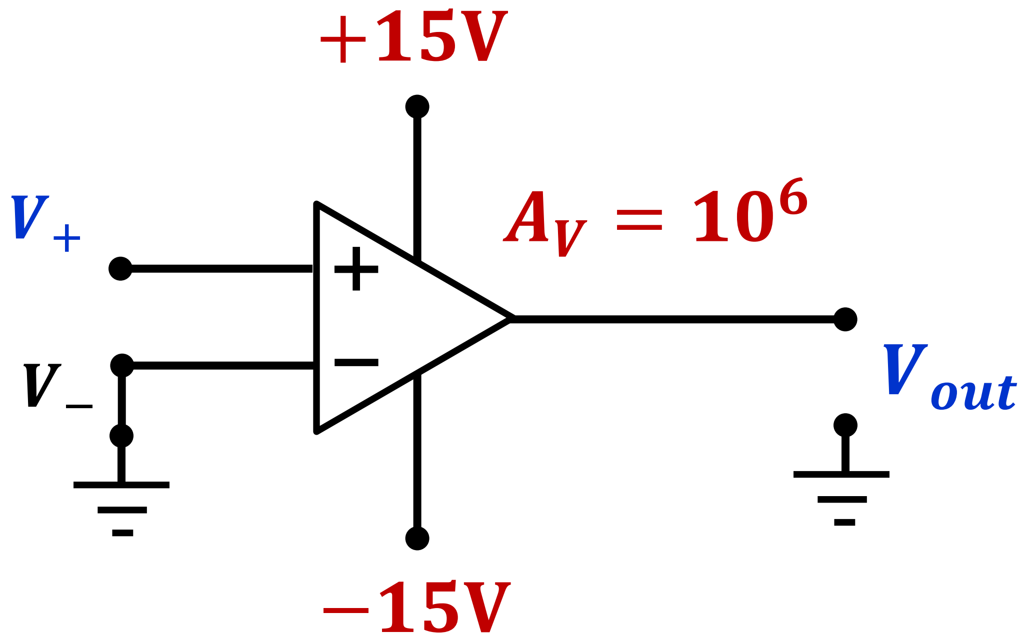
Ideal Operational Amplifier



$$\boxed{V_i = V_+ - V_-}$$

open circuit voltage gain A_V

- Input Resistance $R_{in} \to \infty$
- Output Resistance $R_o \to 0$
- Open-circuit voltage gain $A_V \to \infty$. In real devices $A_V \approx 10^6$.
- Bandwidth $\to \infty$ (same operation up to any frequency)
- $V_{out} = 0$ when $V_+ = V_-$
- Behavior is independent of temperature



"COMPARATOR"

$$\boxed{\begin{array}{lll} V_{out} = +15\text{V} & \text{if} & V_+ > V_- \\ V_{out} = -15\text{V} & \text{if} & V_+ < V_- \\ V_{out} = 0\text{V} & \text{if} & V_+ = V_- \end{array}}$$

In the following, let's call the bias voltage V_{CC}.
The OP AMP gain is very large and the output saturates to $\pm V_{CC}$ when $|(V_+ - V_-)|$ is not negligible.

When $|(V_+ - V_-)| < V_{CC}/A_V$, the output V_{out} does not saturate and follows the functional form of the input as

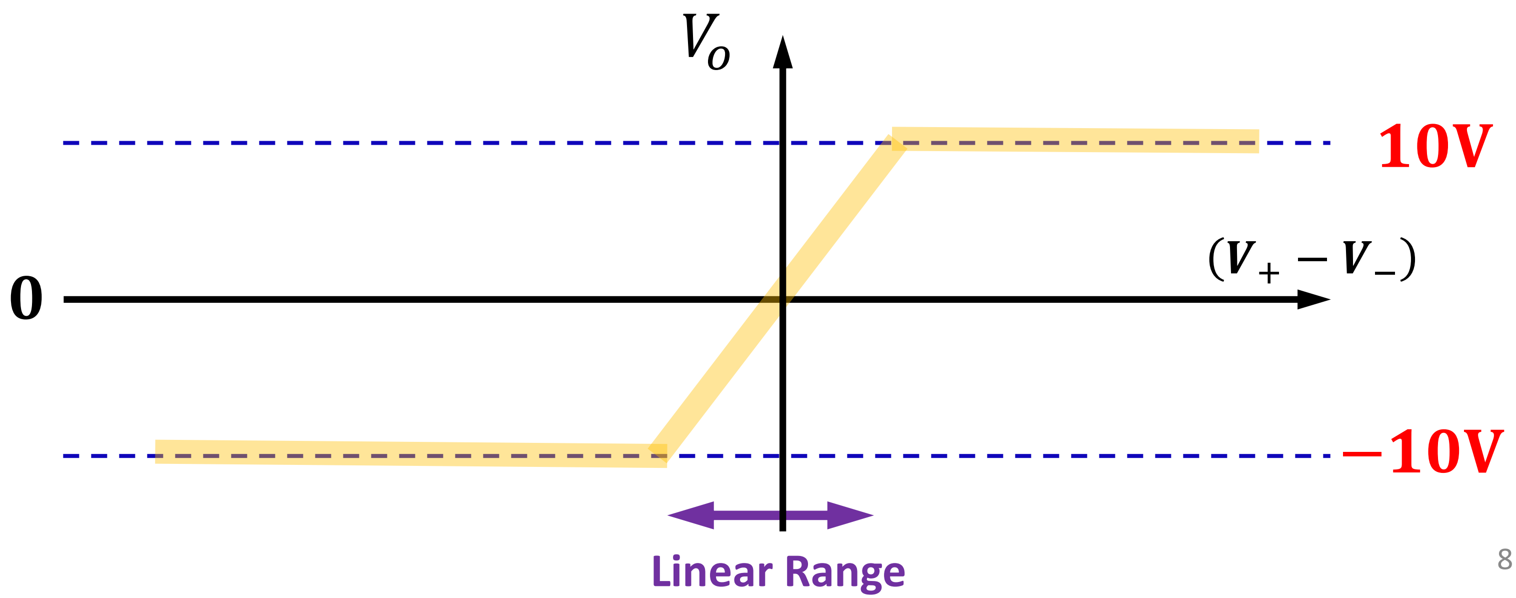
$$V_{out} = A_V(V_+ - V_-)$$

implying $$V_+ \approx V_-$$

when A_V is very large.

To give an example, consider an Op Amp with voltage amplification (gain) $A_V = 10^6$ and $V_{CC} = 10\mathrm{V}$. To keep the variation of the output voltage V_o in the linear range $\pm 10\mathrm{V}$ we need:

$$|(V_+ - V_-)| < \frac{|V_{CC}|}{A_V} = 10\ [\mu\mathrm{V}]$$



Feedback

To achieve these conditions:

$$V_{out} = A_V(V_+ - V_-)$$

$$V_+ \approx V_-$$

in practical circuits, the OP AMPS are operated by applying feedback.

Most frequently, the *NEGATIVE FEEDBACK* configuration is used.

Negative Feedback

We will consider circuits with negative feedback where the output has a connection path to the inverting input terminal of the OP AMP.

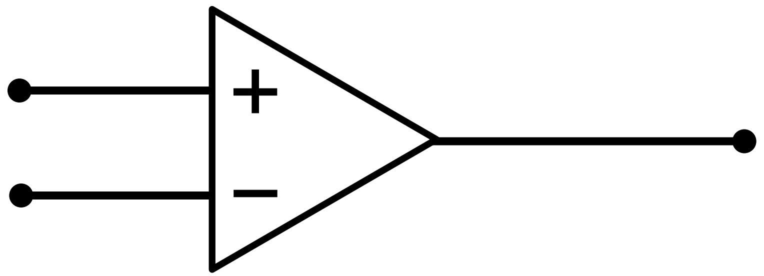
The following conditions (consistent with the assumption of ideal OP AMP) will be considered in all the circuits involving negative feedback:

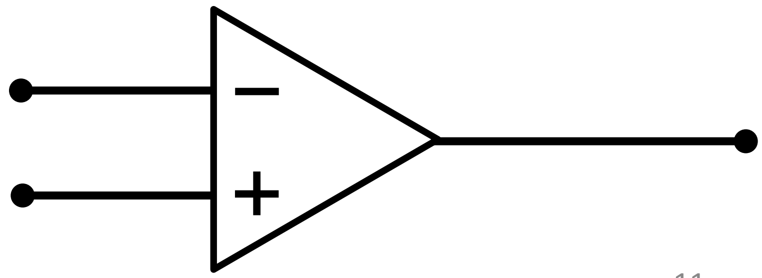
- Voltage inputs: $V_+ = V_-$
- Current inputs $i_+ = i_- = 0$
- Output cannot exceed the bias (rail) voltages

To avoid too much clutter in the schematics, the bias terminals are most often omitted. You should assume, however, that bias is present.

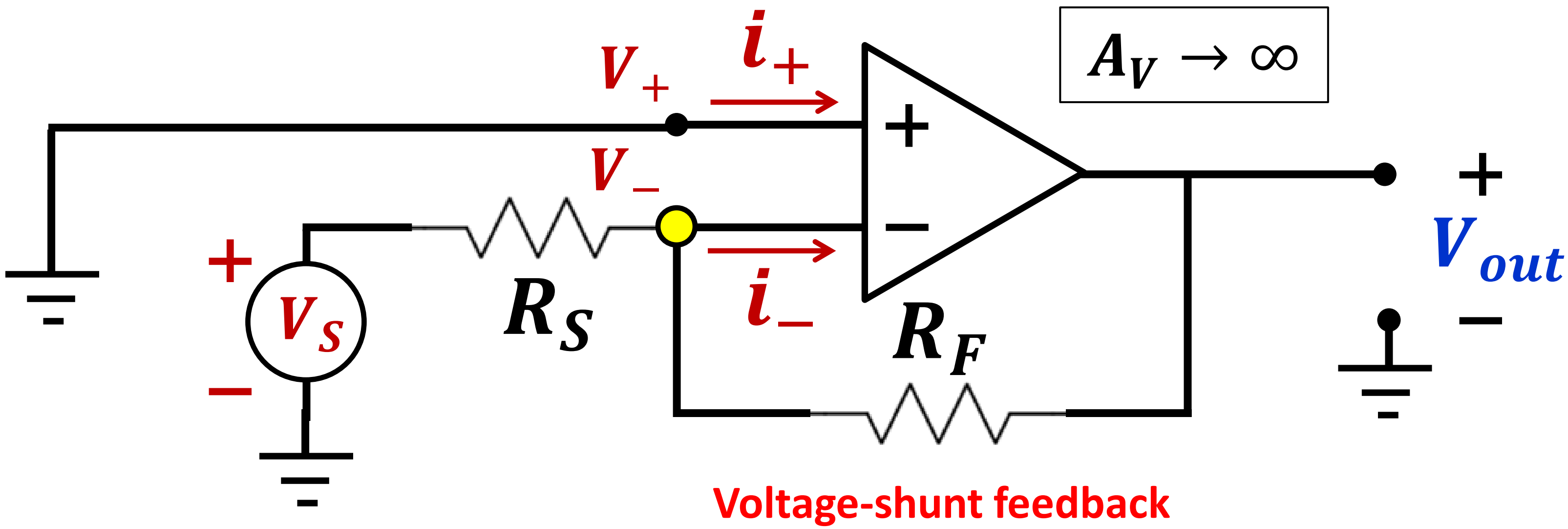
Also, in practical schematics, the OP AMP may be oriented with the inverting terminal UP or DOWN as convenient.

We will represent it DOWN whenever possible, but always check the orientation first!





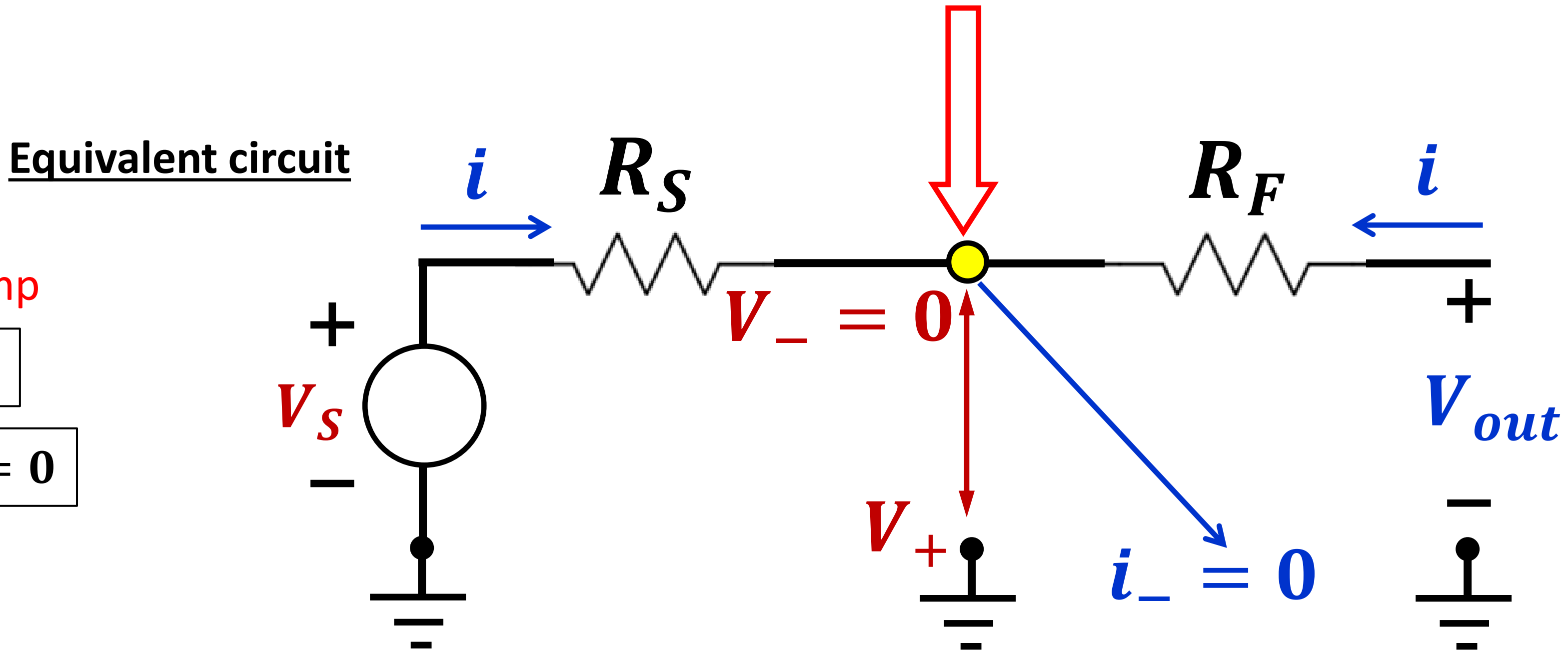
Ideal Inverting Amplifier



The **voltage gain with feedback** is

$$A_{VF} = \frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

"Virtual ground" or "virtual short-circuit"



Ideal Op Amp

$$\boxed{V_+ = V_-}$$

$$\boxed{i_+ = i_- = 0}$$

Node voltage method

$$\frac{V_S - V_-}{R_S} + \frac{V_{out} - V_-}{R_F} = i_- = 0$$

$$\frac{V_S}{R_S} + \frac{V_{out}}{R_F} = 0 \quad \Rightarrow \quad A_{VF} = \frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

The output voltage is constrained by the supply voltages $\pm V_{CC}$

$$|V_{out}| < V_{CC}$$

From

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

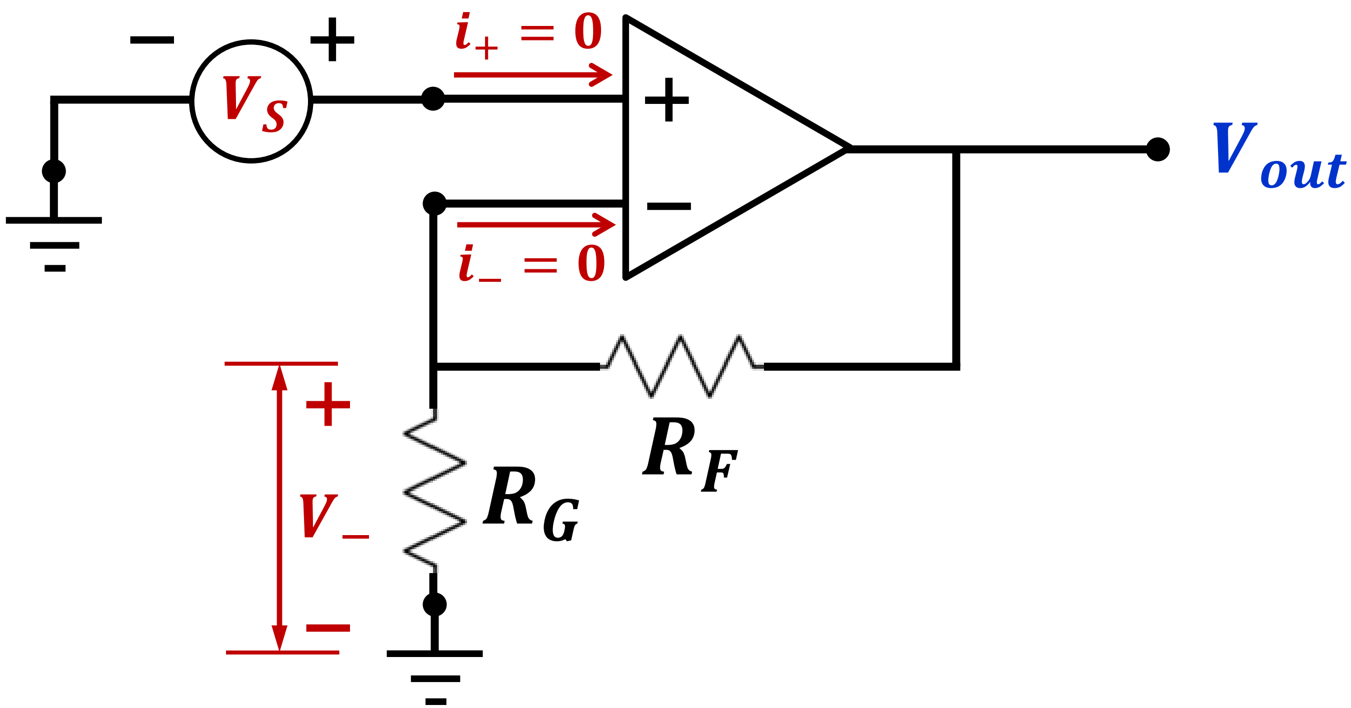
we obtain the maximum resistor ratio for a given input V_S

$$\frac{R_F}{R_S} < \left|\frac{V_{CC}}{V_S}\right|$$

Again, for an ideal OP AMP

- ***The current to each input is zero.***
- ***The voltage between the two input terminals is zero.***

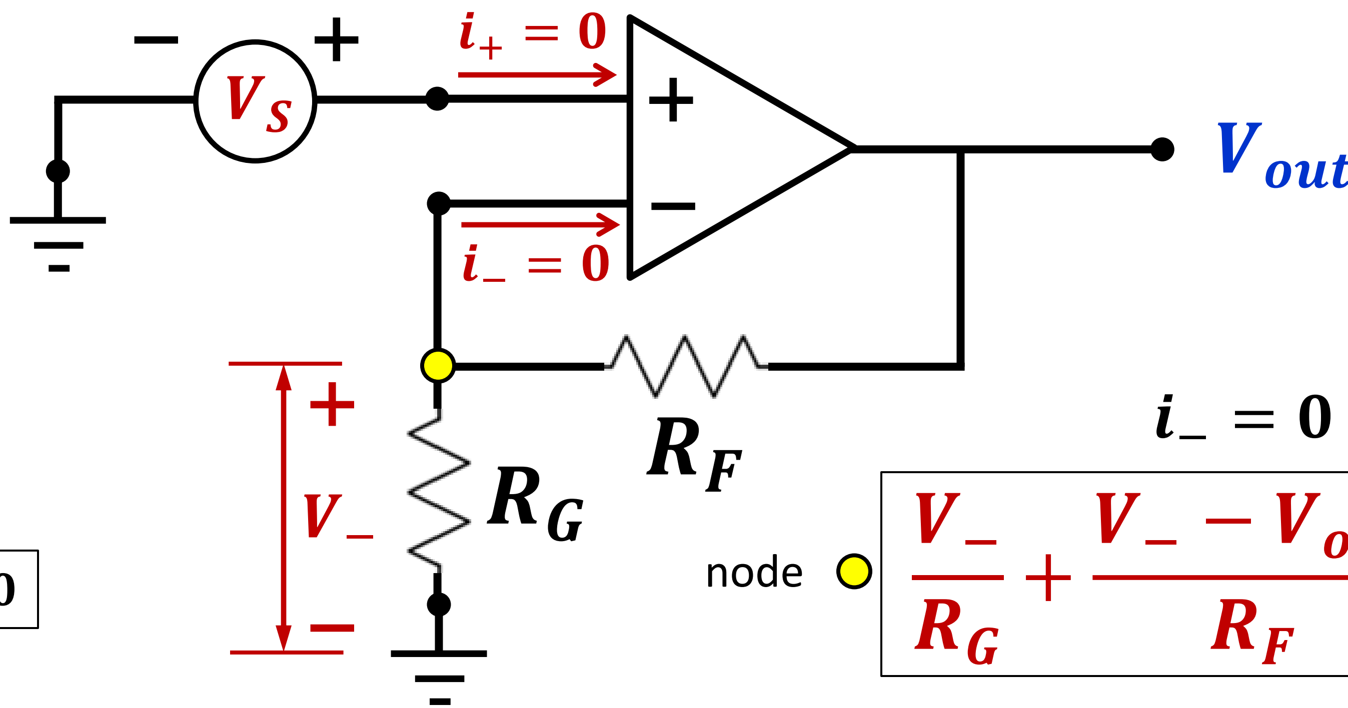
Ideal Non-inverting Amplifier



- Signal goes to the non-inverting input
- Feedback voltage is applied to the inverting input

Source and load have in-phase voltages but are effectively isolated.

Ideal Non-inverting Amplifier



Ideal Op Amp

$$\boxed{V_+ = V_-}$$

$$\boxed{i_+ = i_- = 0}$$

$i_- = 0$

node

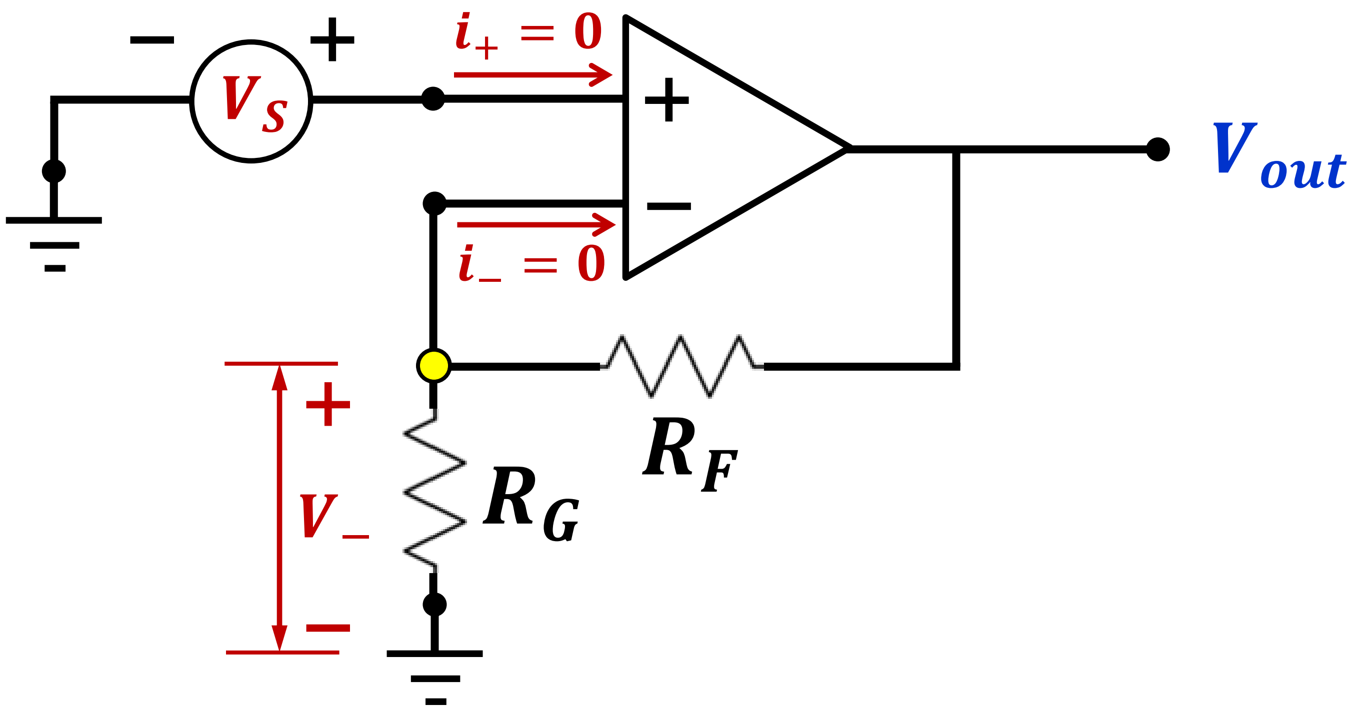
$$\boxed{\frac{V_-}{R_G} + \frac{V_- - V_{out}}{R_F} = 0}$$

$$\Rightarrow \boxed{V_- = V_{out}\frac{R_G}{R_G + R_F}}$$

(same as voltage divider)

$$\boxed{A_{VF} = \frac{V_{out}}{V_S} = \frac{V_{out}}{V_-} = \frac{R_G + R_F}{R_G} = 1 + \frac{R_F}{R_G}}$$

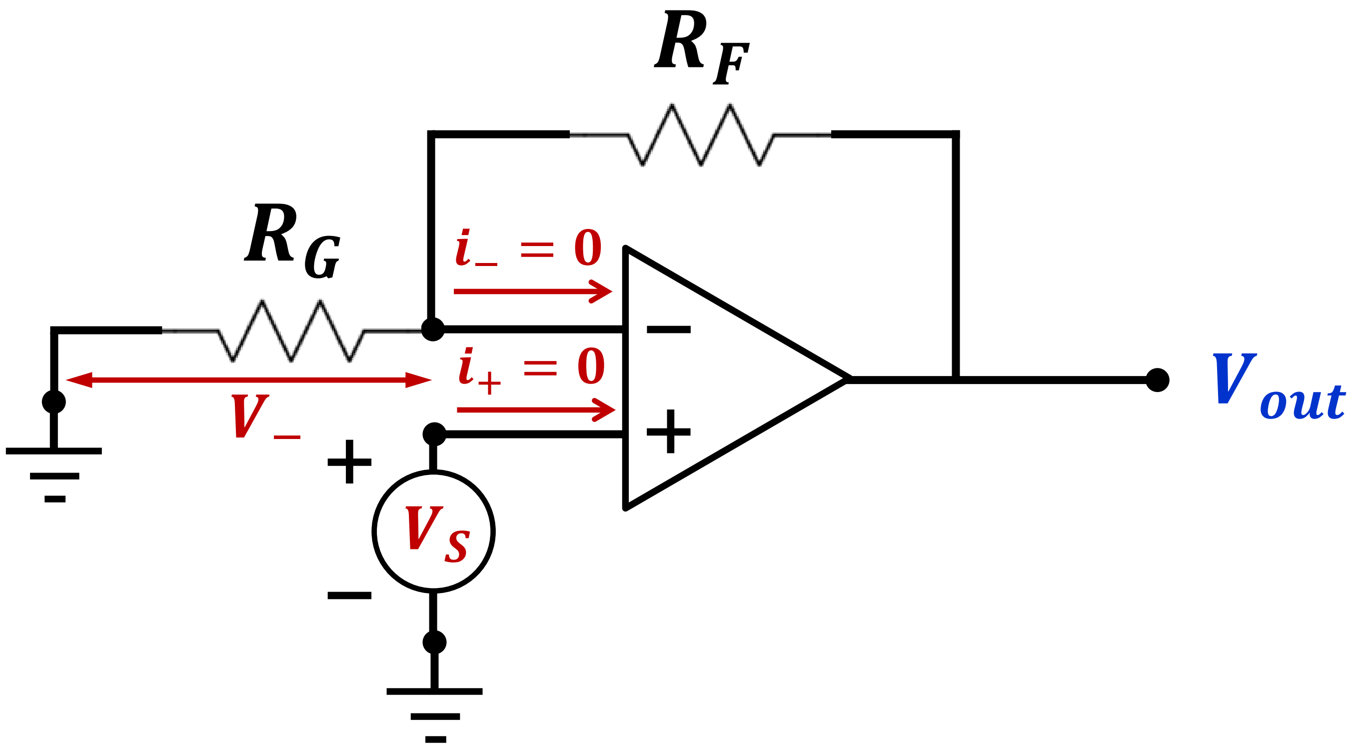
Ideal Non-inverting Amplifier



Gain is positive $\boxed{A_{VF} = 1 + \dfrac{R_F}{R_G}}$ Input and output in phase

Amplifier operates in linear region when $\boxed{1 + \dfrac{R_F}{R_G} < \left|\dfrac{V_{CC}}{V_S}\right|}$

The same circuit is often found drawn as below, with the inverting input up.



Similar relationships are established in the OP AMP when simple resistors are replaced with impedances

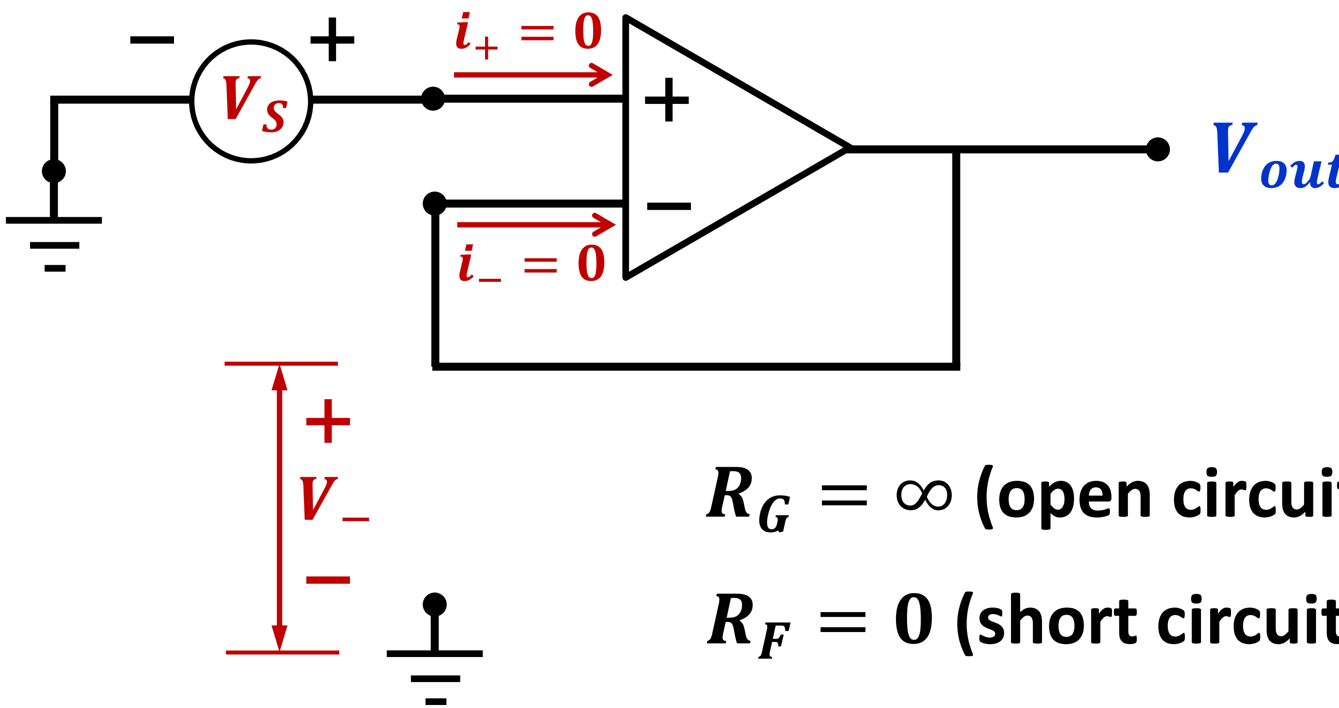
- ***Inverting amplifier***

$$A_{VF} = \frac{V_{out}}{V_S} = -\frac{Z_F}{Z_S}$$

- ***Non-inverting amplifier***

$$A_{VF} = \frac{V_{out}}{V_S} = \frac{Z_G + Z_F}{Z_G} = 1 + \frac{Z_F}{Z_G}$$

Example 4



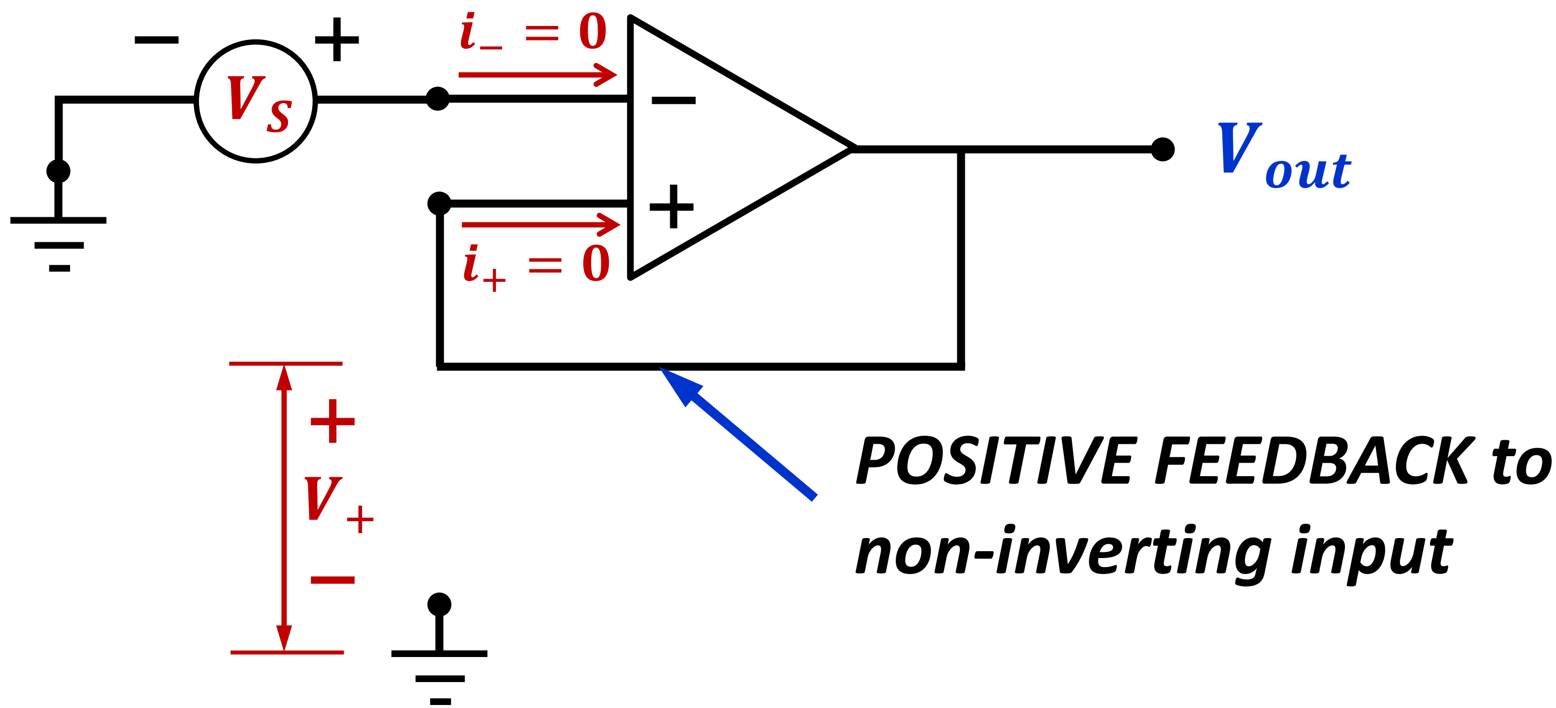
$R_G = \infty$ **(open circuit)**

$R_F = 0$ **(short circuit)**

$$\boxed{A_{VF} = \frac{V_{out}}{V_S} = \frac{V_{out}}{V_-} = \frac{R_G + R_F}{R_G} = 1 + \cancel{\frac{R_F}{R_G}}}$$

$$\Rightarrow \quad V_{out} = V_S$$

Unity amplifier (buffer) also called *"Voltage Follower"*

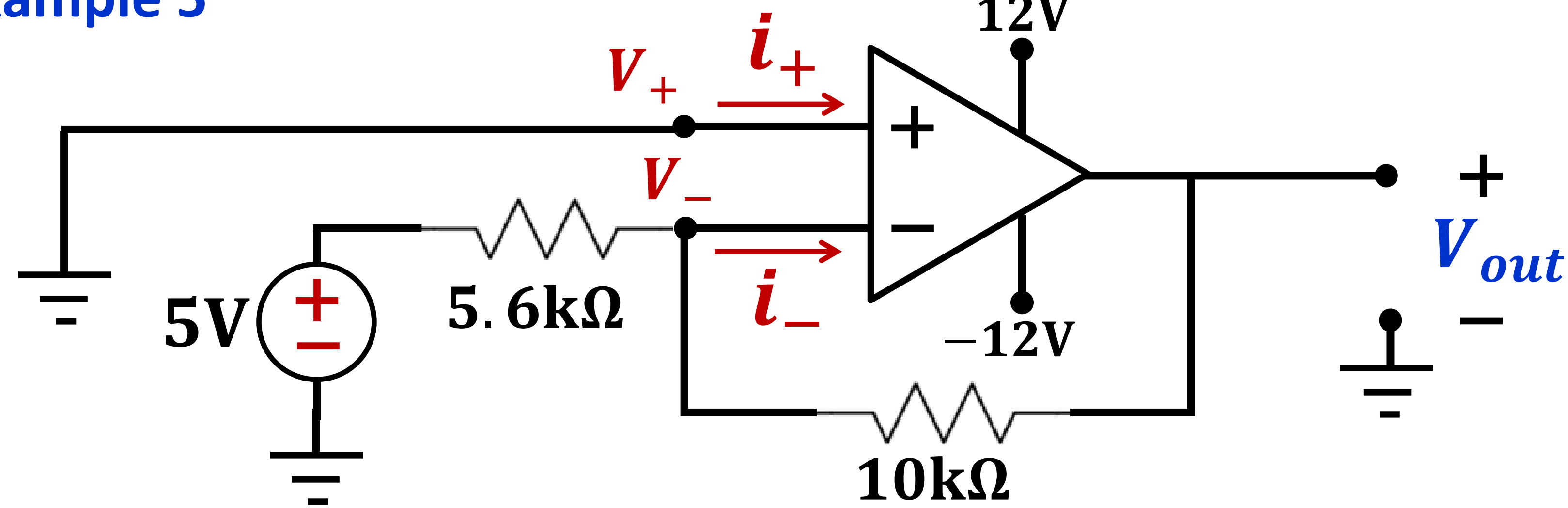


Be very careful

The configuration above is NOT a voltage follower, because the feedback loop connects to the non-inverting input

This Op Amp tends to saturate quickly to $\pm V_{CC}$ depending on polarity and magnitude of V_S (bistable circuit). This circuit DOES NOT OPERATE IN THE LINEAR RANGE AND DOES NOT DUPLICATE THE INPUT TO THE OUTPUT.

Example 5

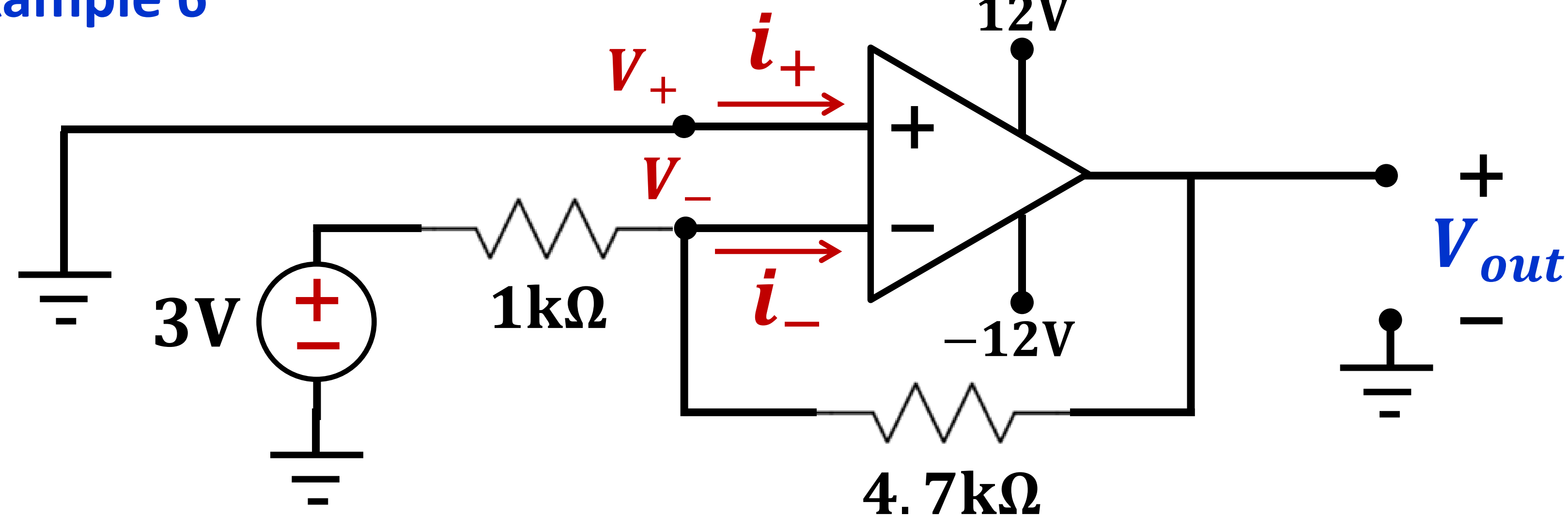


$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

$$\Rightarrow \quad \boxed{V_{out} = -5\,\frac{10\text{k}}{5.6\text{k}} = -8.93\text{V}}$$

Since $|V_{out}| < |V_{CC}|$, the result obtained is valid.

Example 6

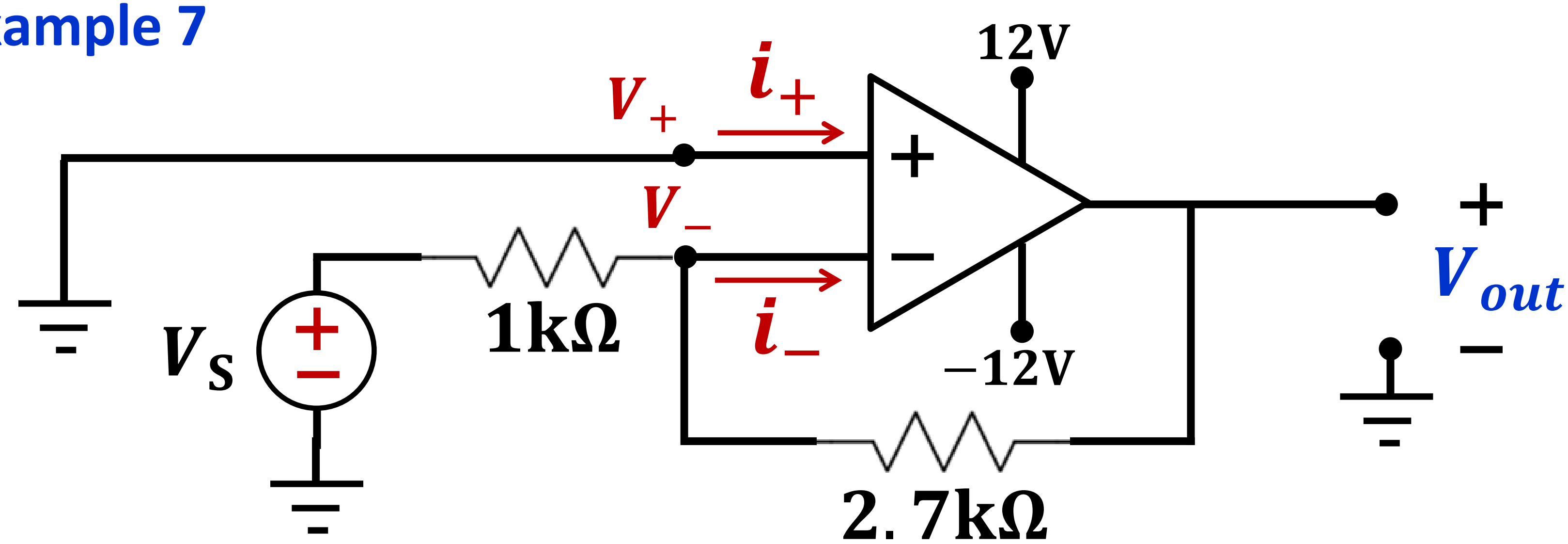


$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

$$\Rightarrow \quad \boxed{V_{out} = -3\,\frac{4.7\text{k}}{1\text{k}} = -14.1\text{V} < -V_{\text{b}}}$$

Since $|V_{out}| > |V_{CC}|$, the output voltage saturates at -12V.

Example 7

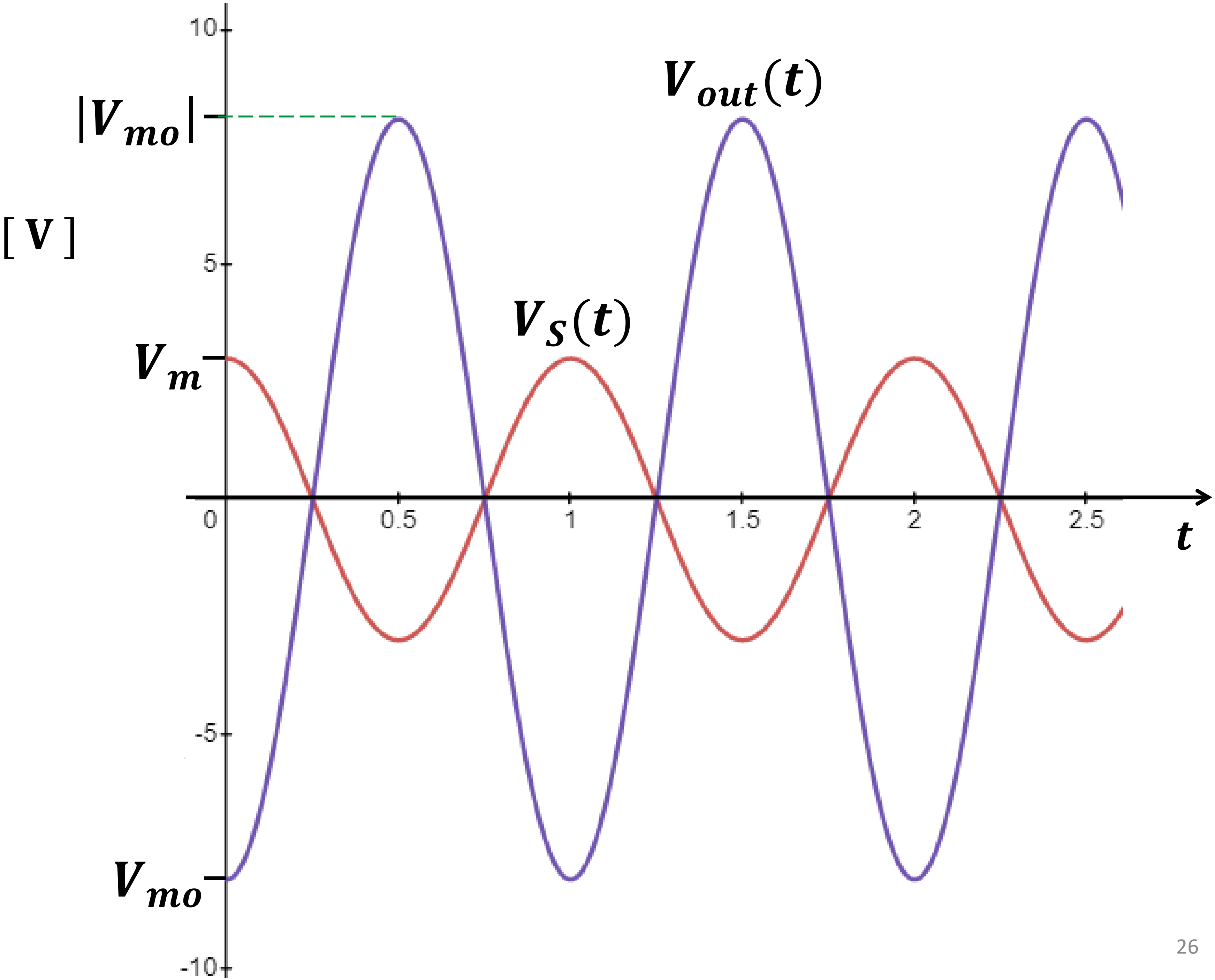


$$V_S = \mathrm{V}_m \cos(\omega t) \qquad \mathrm{V}_m = 3\mathrm{V}$$

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S} \qquad V_{out} = \mathrm{V}_{mo} \cos(\omega t)$$

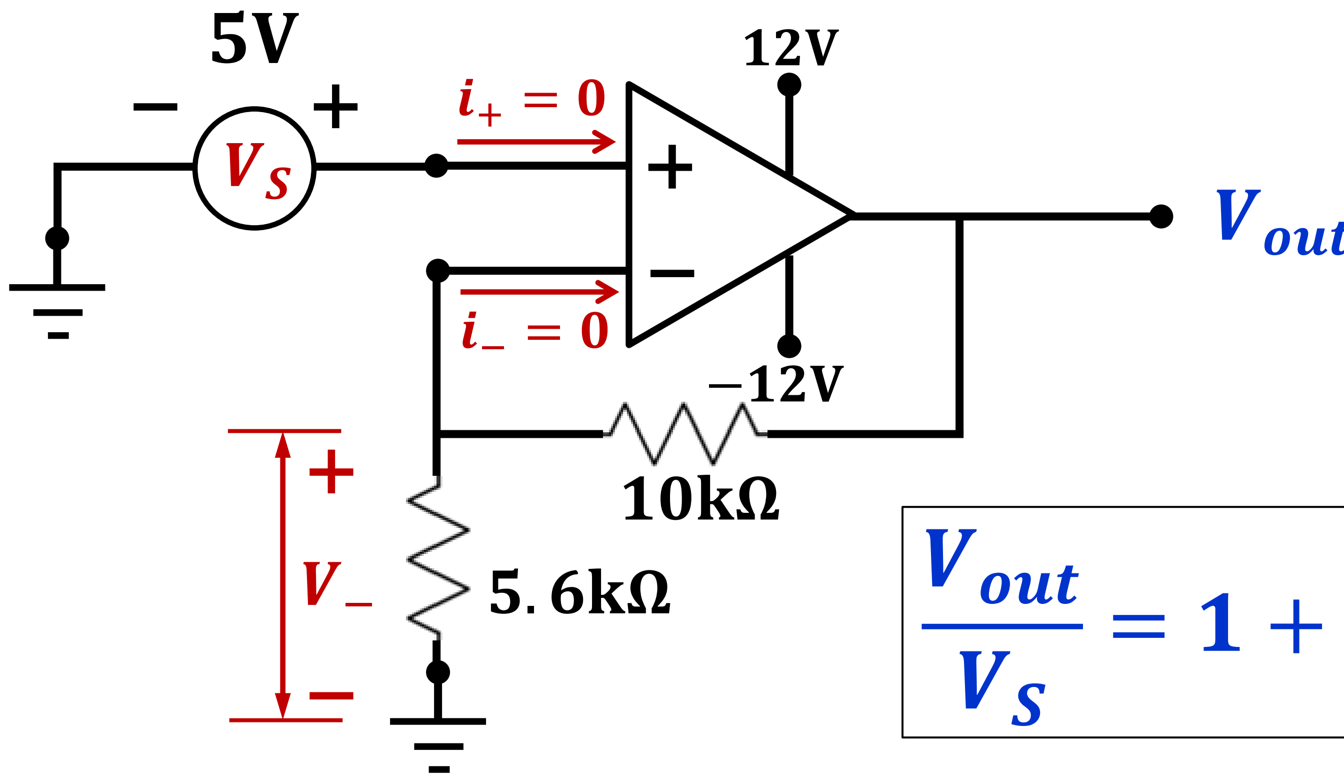
$$\boxed{\mathrm{V}_{mo} = -3\frac{2.7\mathrm{k}}{1\mathrm{k}} = -8.1\mathrm{V} > -V_{\mathrm{b}}}$$

Since $|V_{mo}| < |V_{CC}|$, the result obtained is valid.



10
$|V_{mo}|$
$V_{out}(t)$
[V]
5
$V_S(t)$
V_m
0
0.5
1
1.5
2
2.5
t
-5
V_{mo}
-10

Example 8

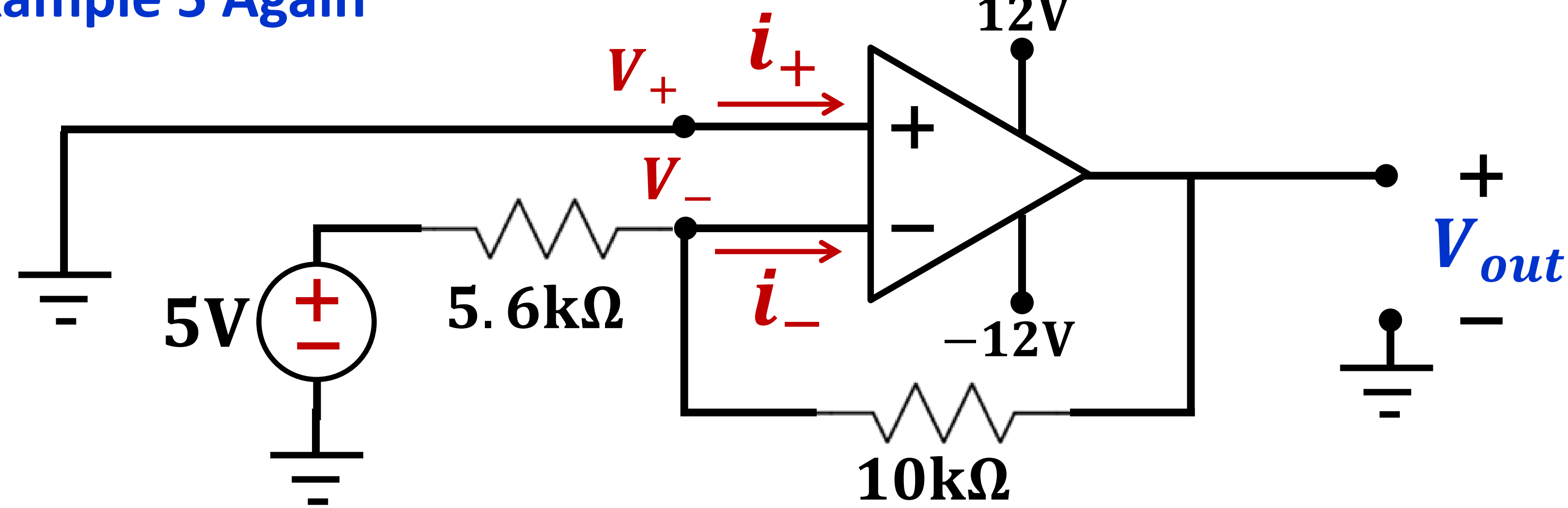


$$\frac{V_{out}}{V_S} = 1 + \frac{R_F}{R_S}$$

$$V_{out} = 5\left(1 + \frac{10\text{k}}{5.6\text{k}}\right) = 13.93\text{V}$$

Since $|V_{mo}| > |V_{CC}|$, the op-amp saturates at 12V. In Example 5, with the same resistors the inverting op-amp did not saturate.

Example 5 Again



$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

$$\Rightarrow \quad \boxed{V_{out} = -5\,\frac{10\mathrm{k}}{5.6\mathrm{k}} = -8.93\mathrm{V}}$$

Since $|V_{out}| < |V_{CC}|$, the result obtained is valid.

We have seen earlier

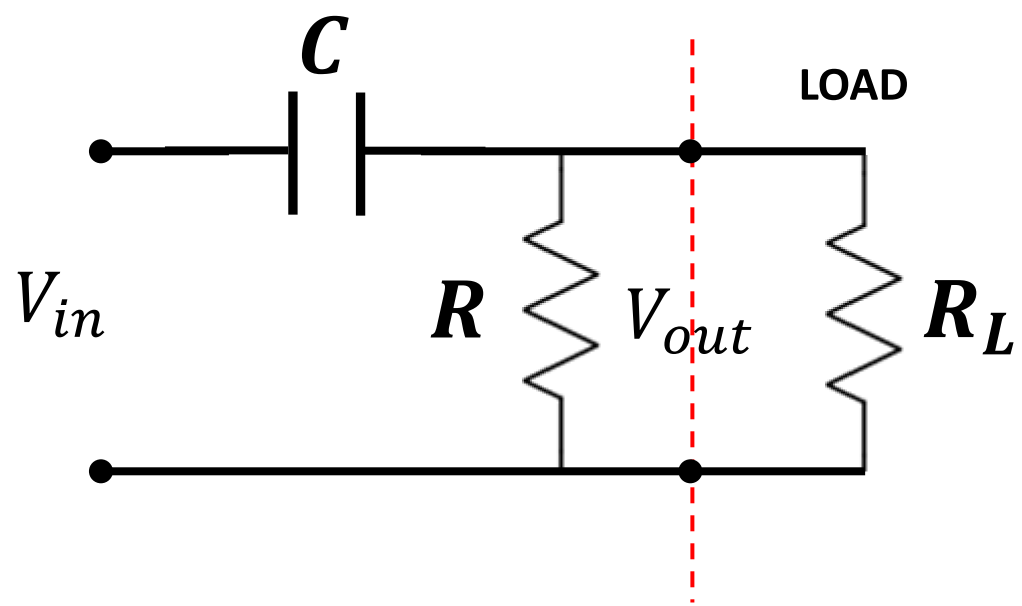
The response of a passive filter is affected by the load connected directly to it. For example, consider a high-pass RC filter:

RC constant

$$\boldsymbol{\tau = R\ C}$$

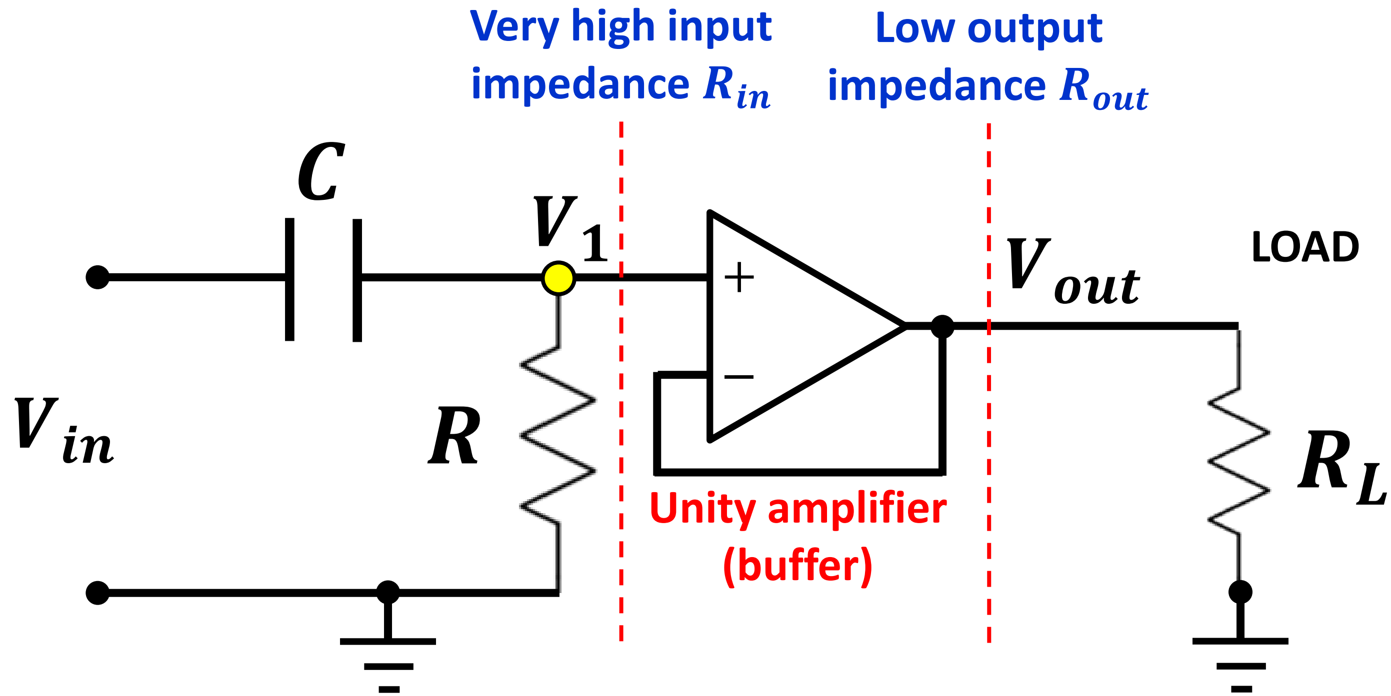
with load

$$\boldsymbol{\tau' = (R//R_L)\ C}$$



The parallel $\mathrm{R_{eff}} = R//R_L$ yields an equivalent resistance lower than either R or R_L. In particular, if connected to a small resistor R_L, the resulting cutoff frequency $\omega'_C = (R_{\mathrm{eff}}C)^{-1}$ may change considerably with respect to the original $\omega_C = (RC)^{-1}$.

Isolation of filter from load



$$V_{out} = V_1$$

$$\tau' = (R//R_{in})\,C \rightarrow (R//\infty)\,C = RC = \tau$$