ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 37

MWF – 12:00pm

Prof. Umberto Ravaioli

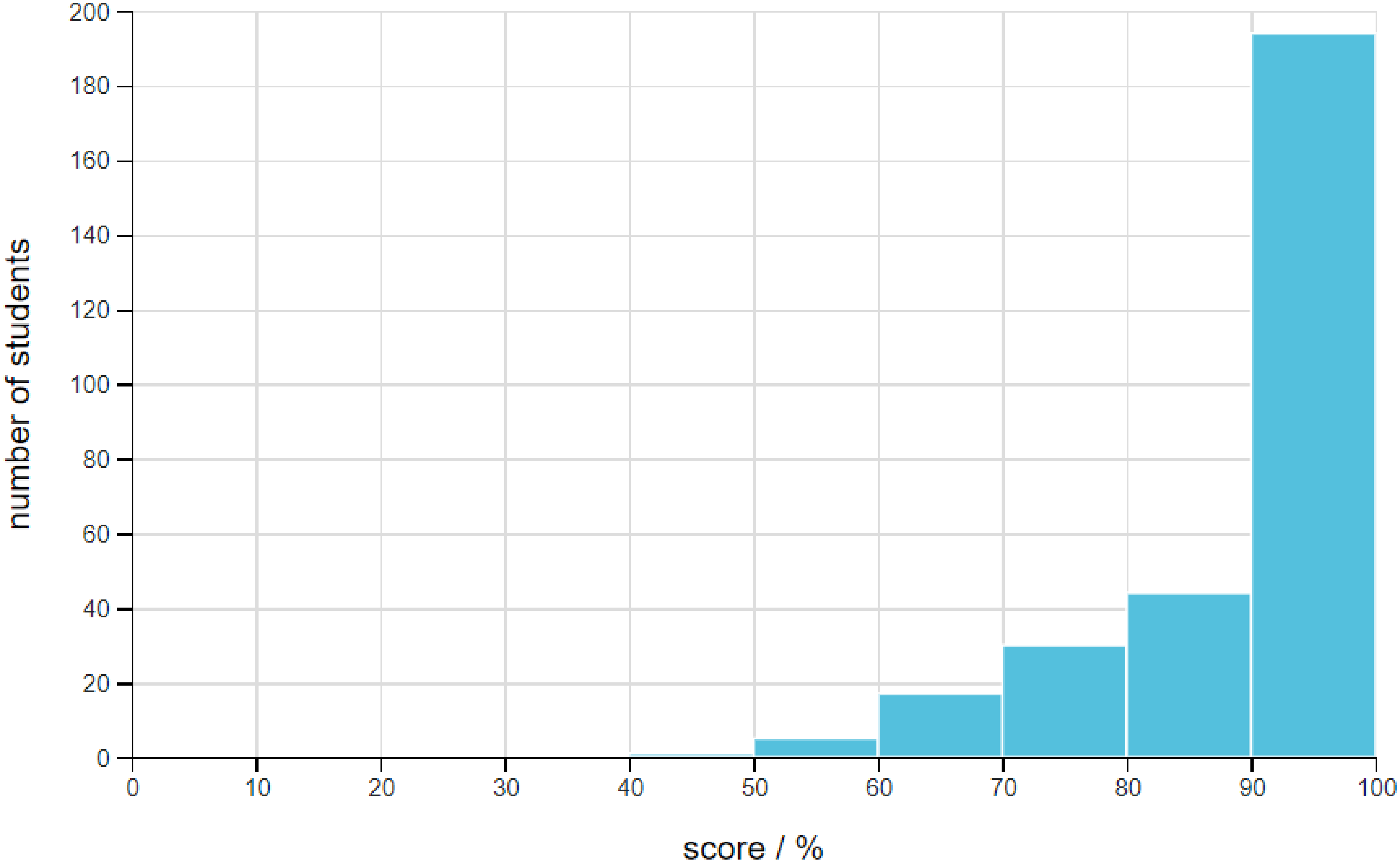
2062 ECE Building

Lecture 37 – Summary

Learning Objectives

1. Applications of Operational Amplifiers

Quiz 4 – Grade distribution

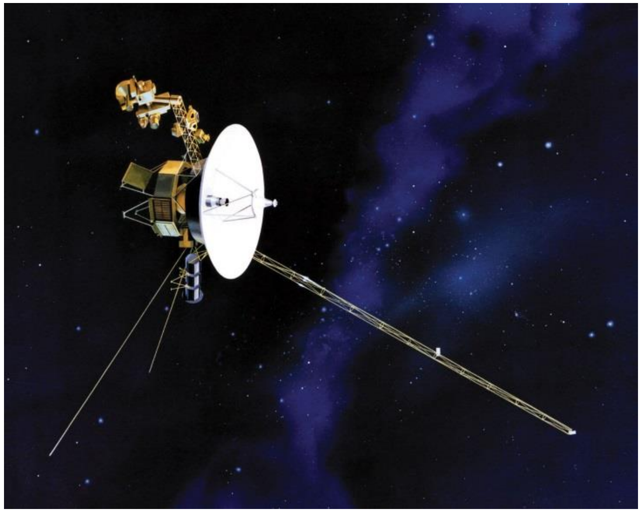


Quiz 4 – Statistics

Number of students	291
Mean score	90%
Standard deviation	11%
Median score	94%
Minimum score	45%
Maximum score	100%
Number of 0%	0 (0% of class)
Number of 100%	31 (11% of class)

Voyager 1 probe

← $\approx$ 10 km/s



April 2024

$\approx$ 15 billion miles

radio wave

≈ 22.5 hours

September 5, 1977

Went silent in November 2023 due to failure of one memory chip

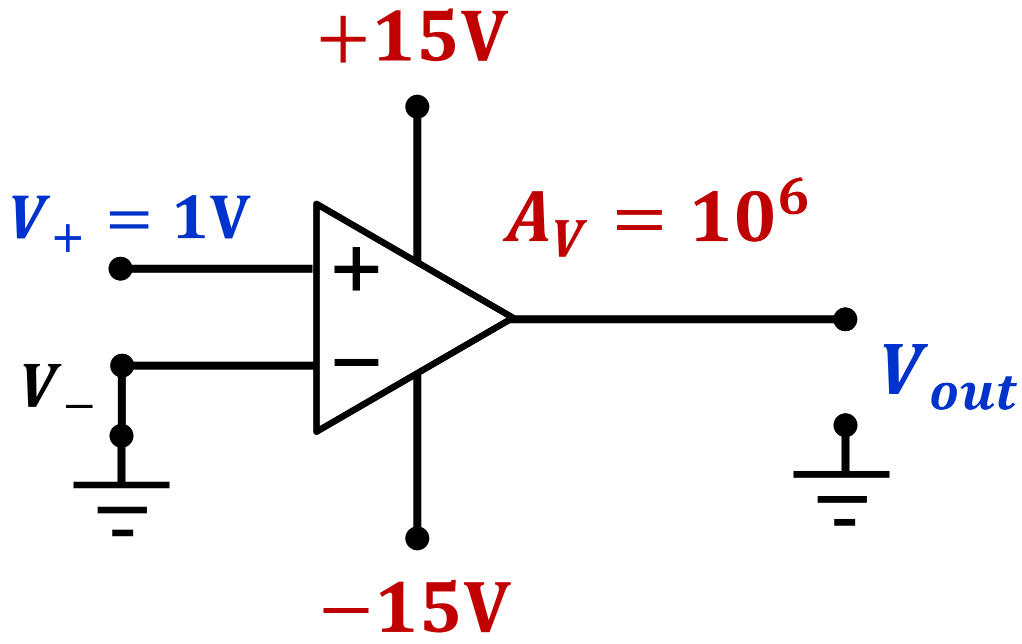
NASA engineers have rerouted the code to another memory chips

This week communications have been reestablished

The onboard computer memory is 70 Kbytes

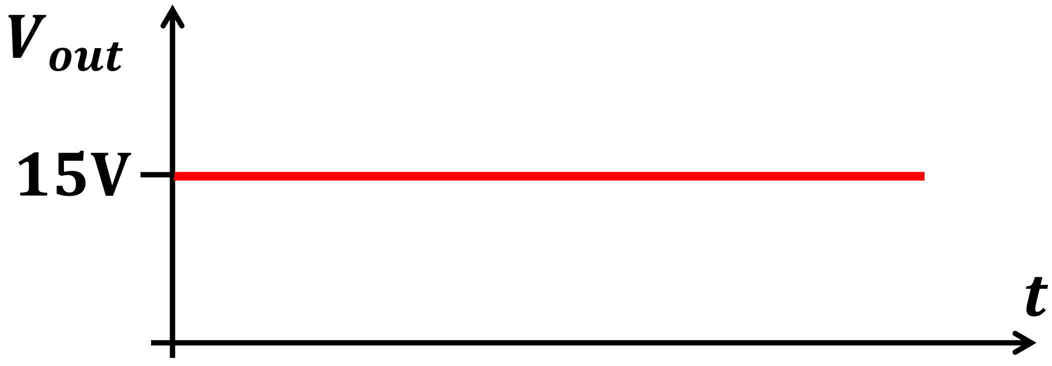
The digital tape recorder can store 67 Mbytes of data

Example 1

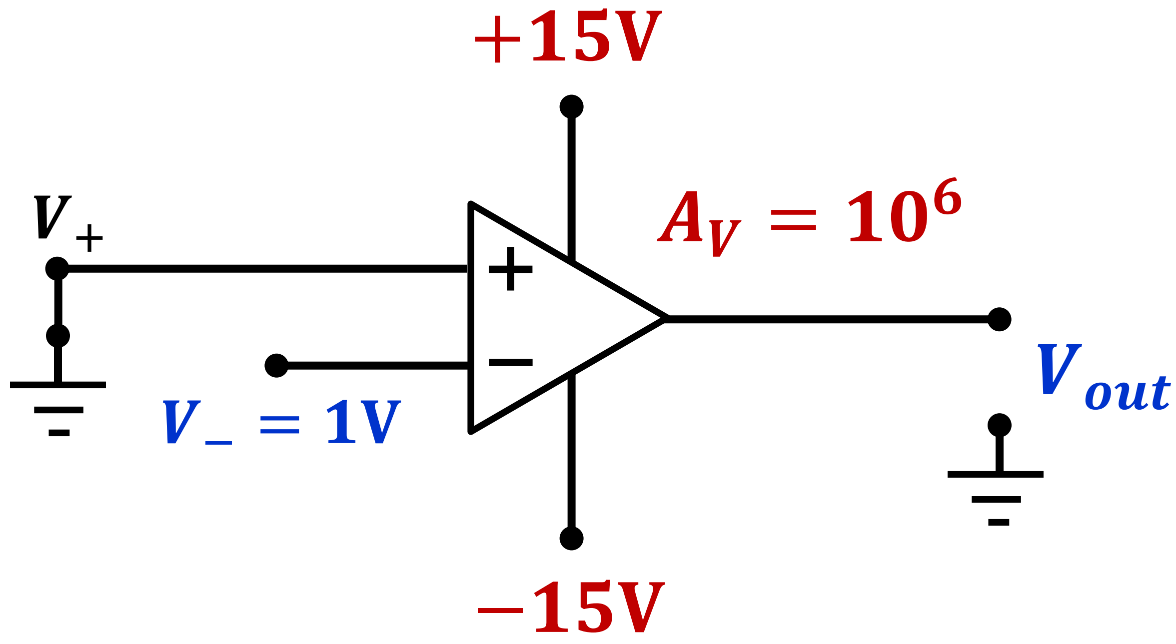


$$V_{out} = A_V \times (V_+ - V_-) = 10^6(1-0) = 10^6\text{V}$$

Saturation occurs at +15 Volts. ⇨ $$V_{out} = 15\text{V}$$

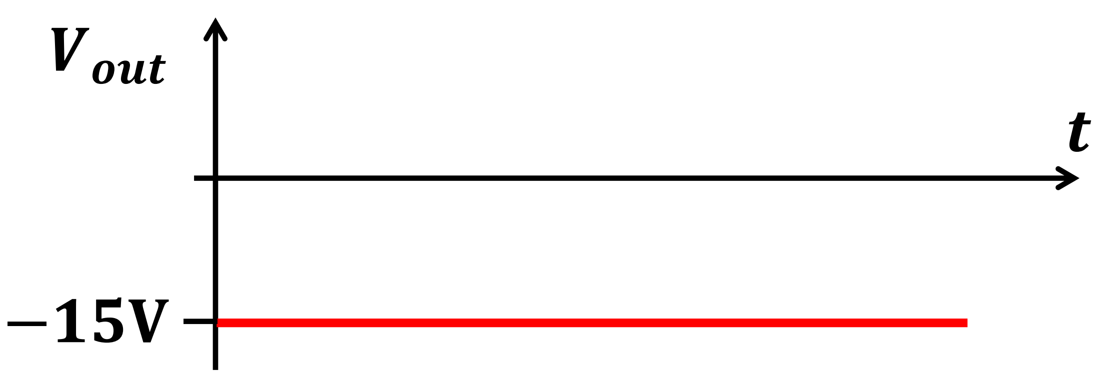


Example 2

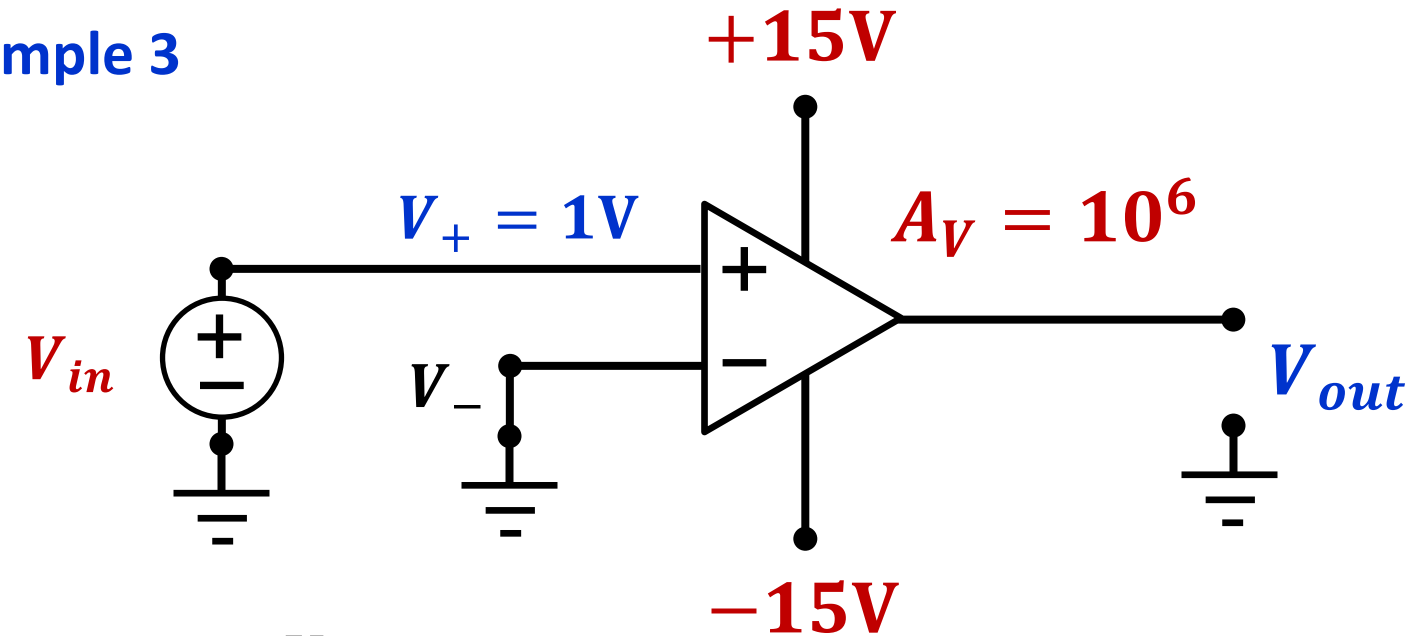


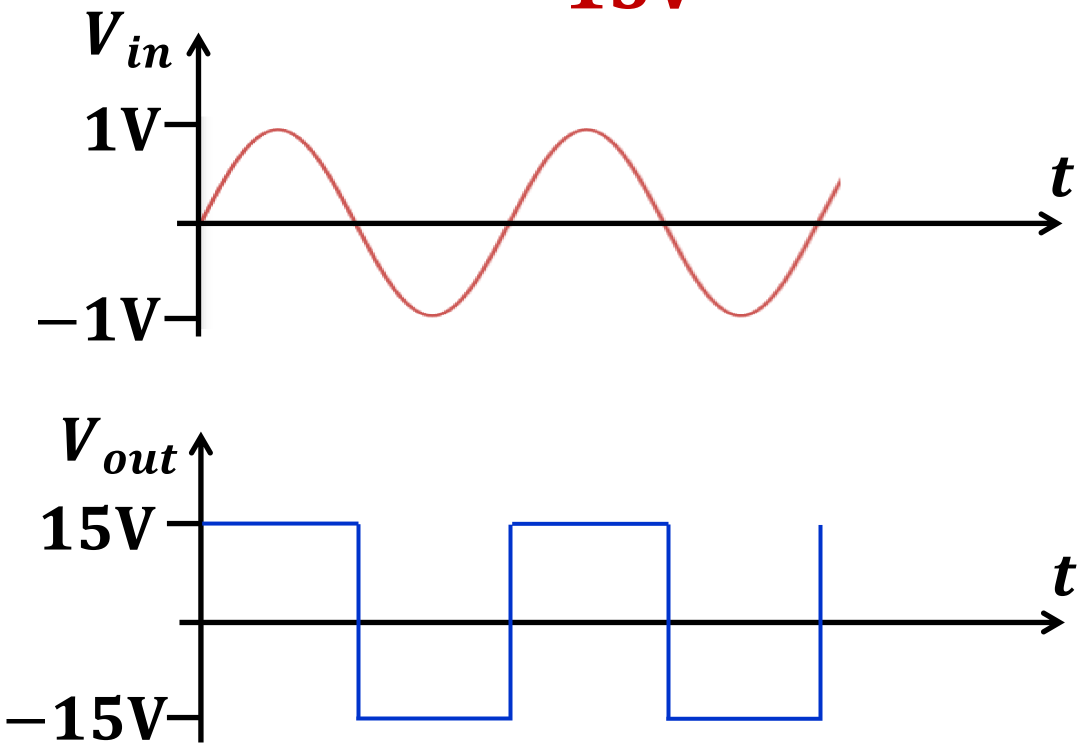
$$V_{out} = A_V \times (V_+ - V_-) = 10^6(0-1) = -10^6 V$$

Saturation occurs at -15 Volts. ⇨ $V_{out} = -15V$

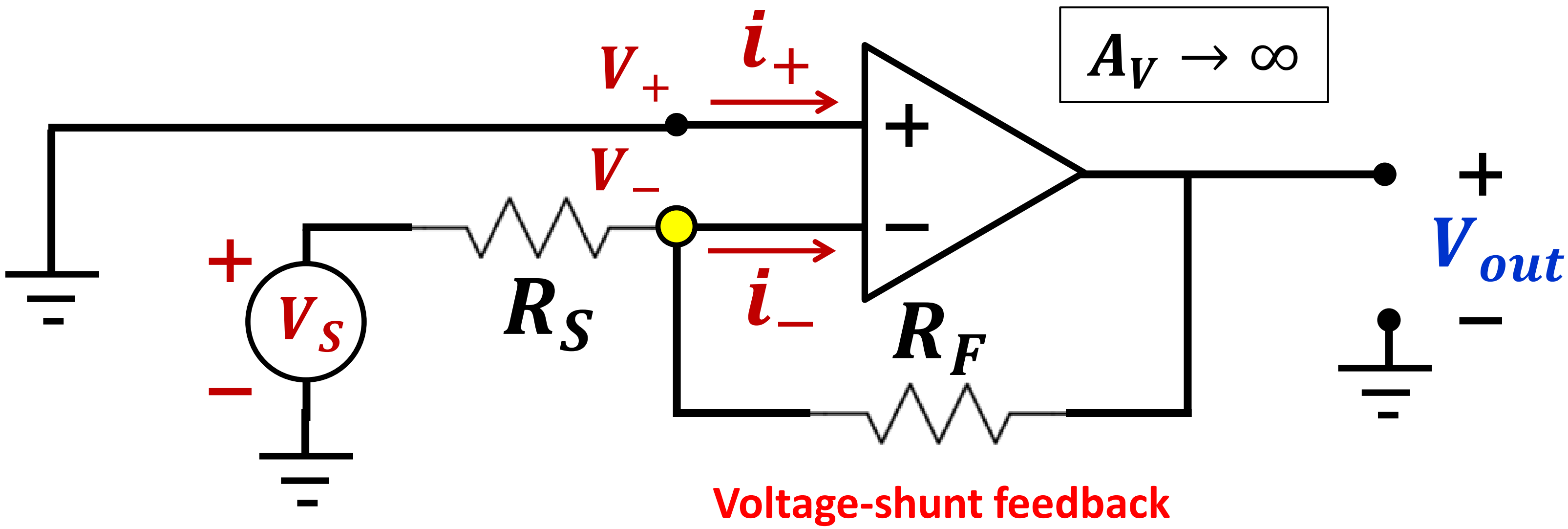


Example 3





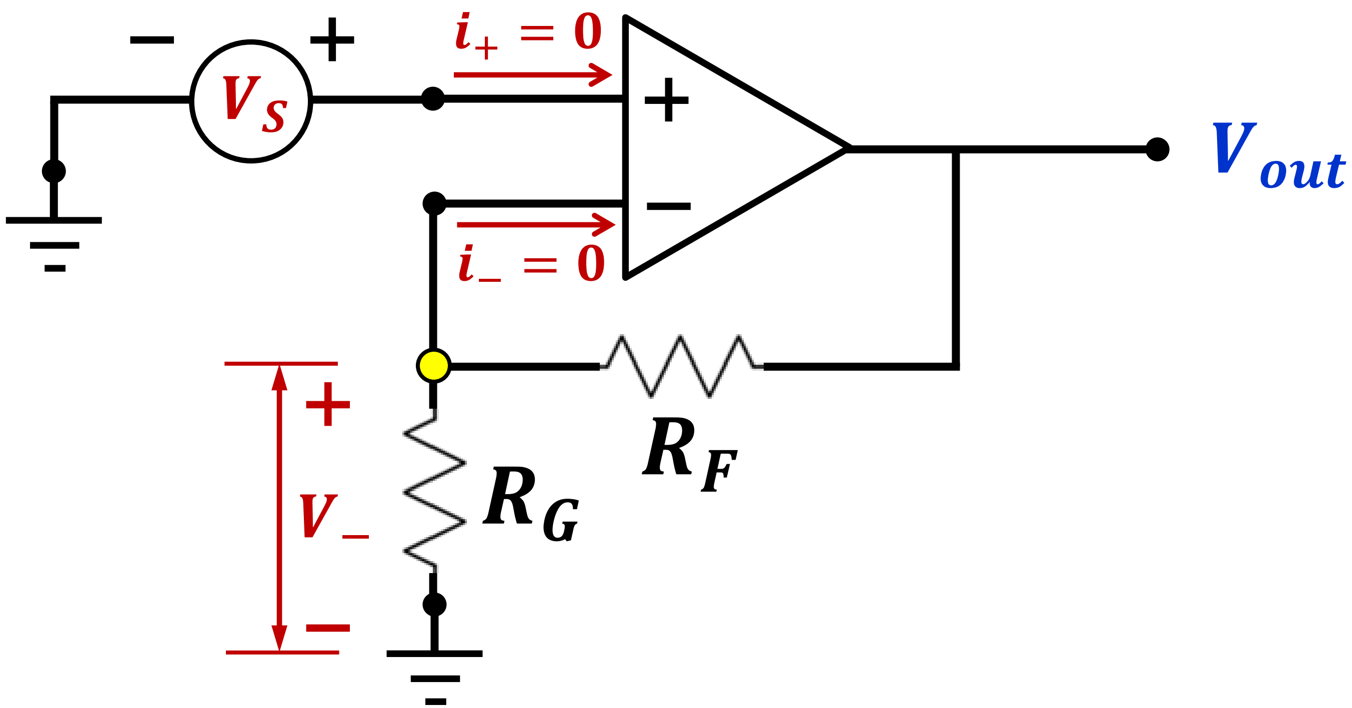
Ideal Inverting Amplifier



The **voltage gain with feedback** is

$$A_{VF} = \frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

Ideal Non-inverting Amplifier



Gain is positive

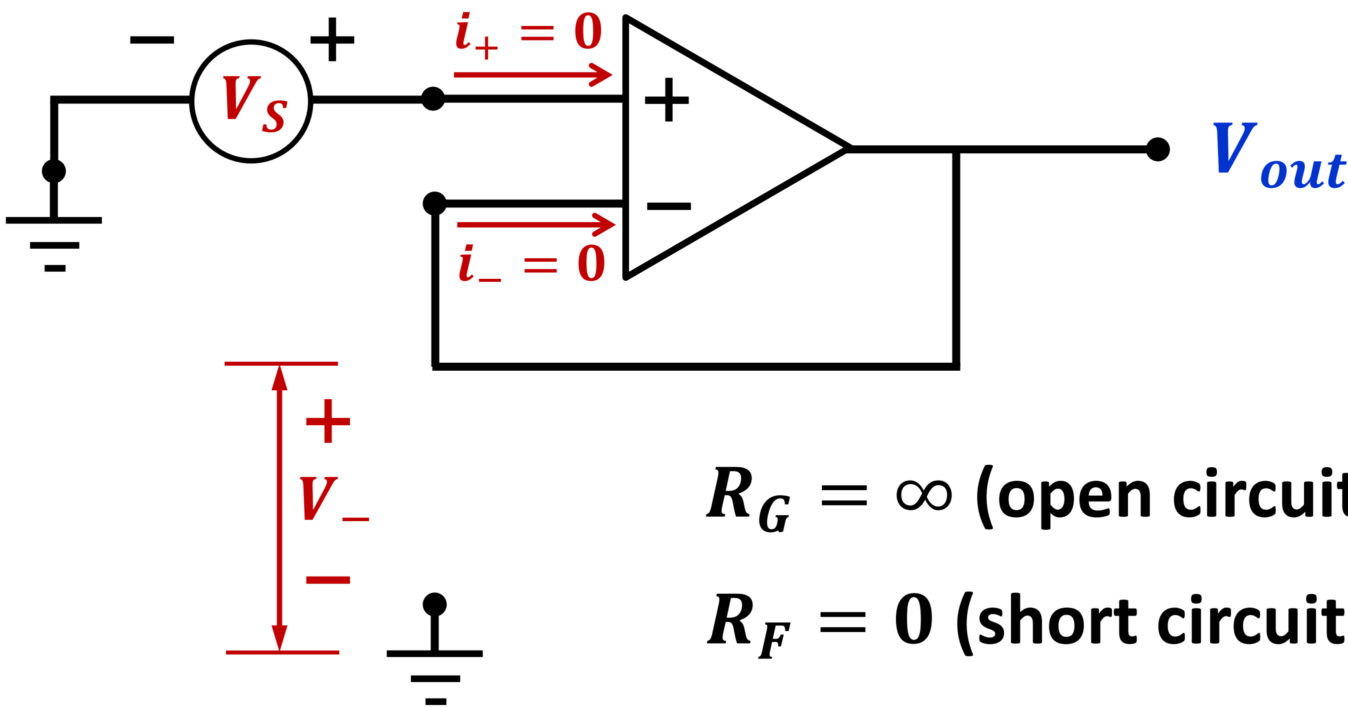
$$A_{VF} = 1 + \frac{R_F}{R_G}$$

Input and output in phase

Amplifier operates in linear region when

$$1 + \frac{R_F}{R_G} < \left|\frac{V_{CC}}{V_S}\right|$$

Example 4



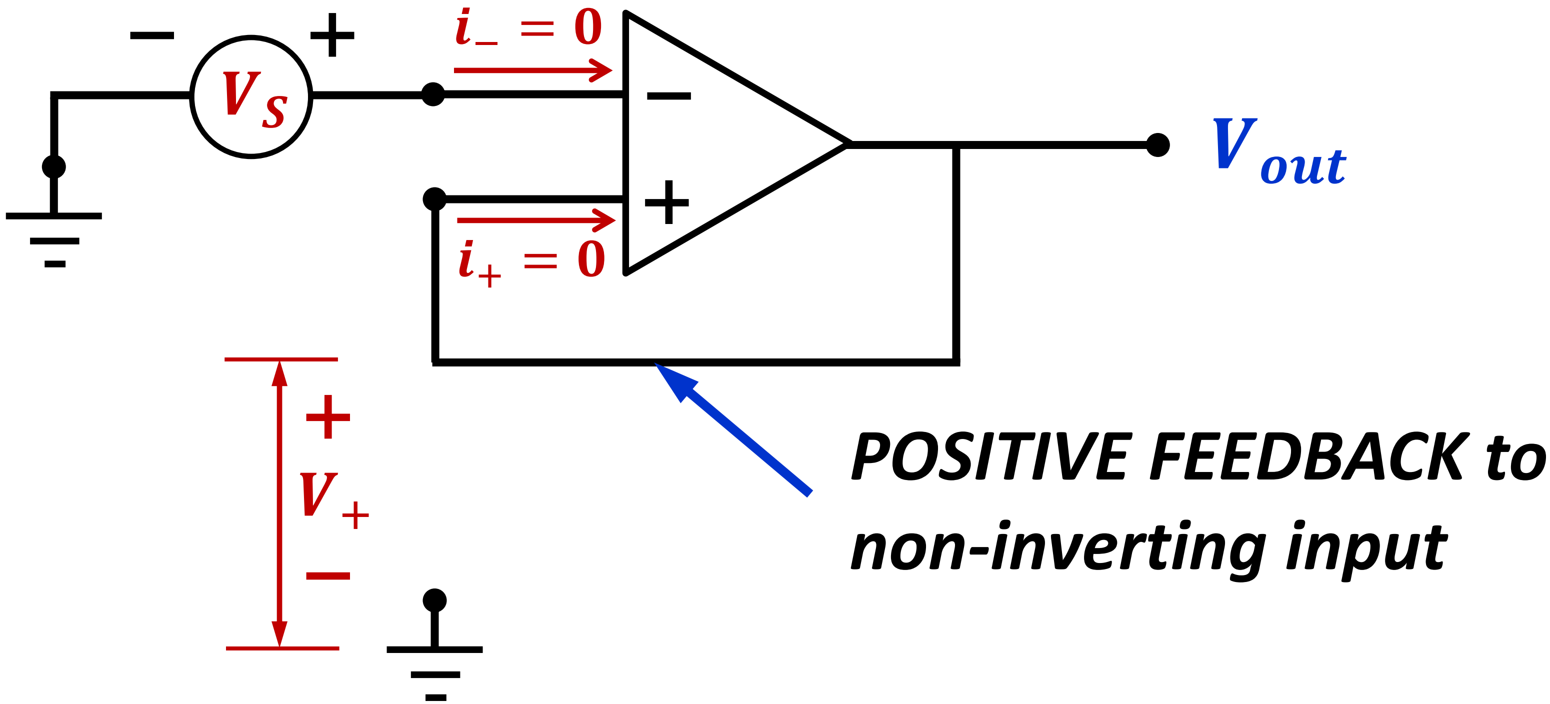
$R_G = \infty$ (open circuit)

$R_F = 0$ (short circuit)

$$A_{VF} = \frac{V_{out}}{V_S} = \frac{V_{out}}{V_-} = \frac{R_G + R_F}{R_G} = 1 + \frac{R_F}{R_G}$$

⇨ $V_{out} = V_S$

Unity amplifier (buffer) also called *"Voltage Follower"*

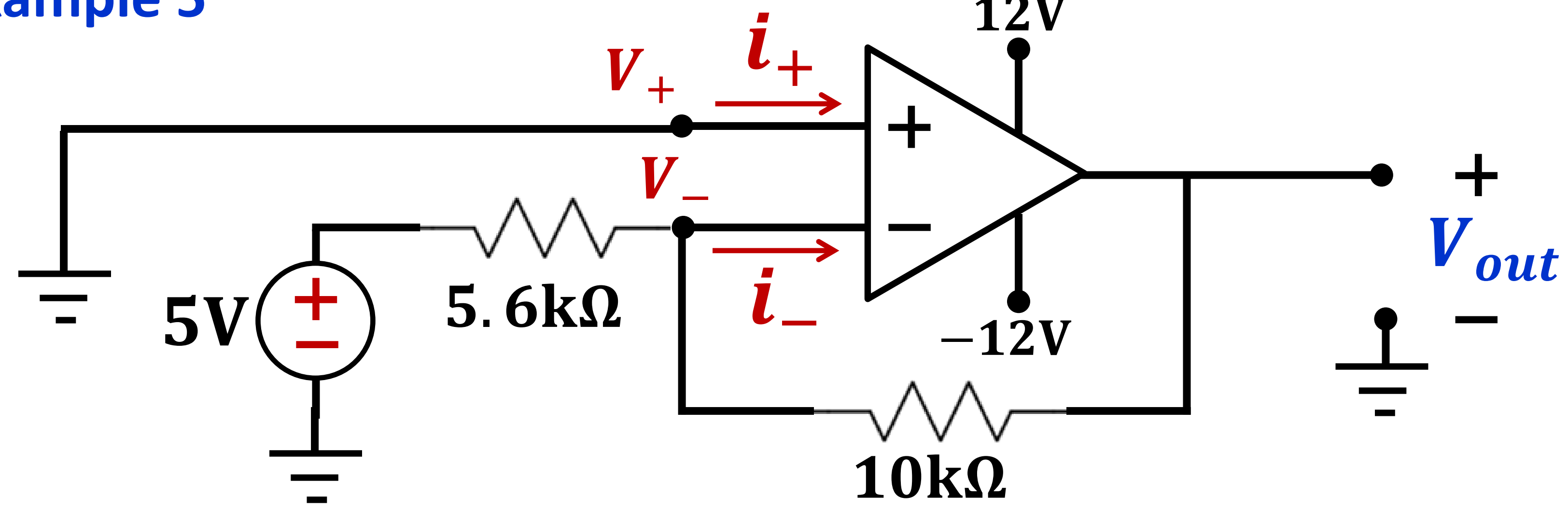


Be very careful

The configuration above is NOT a voltage follower, because the feedback loop connects to the non-inverting input

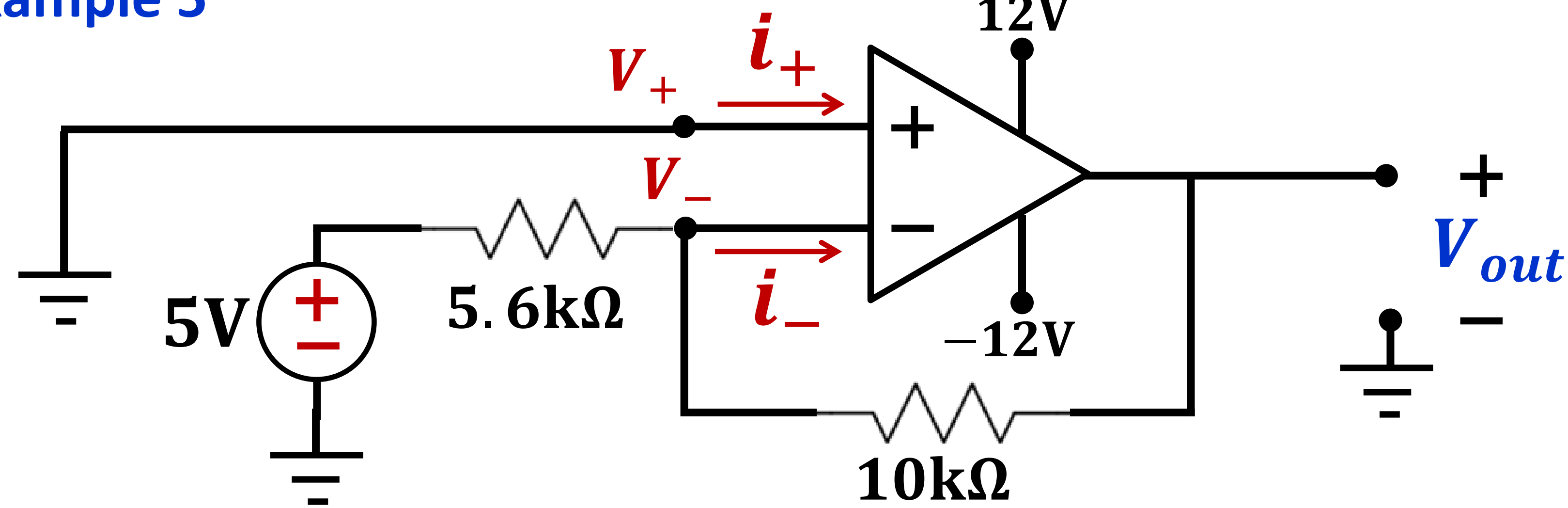
This Op Amp tends to saturate quickly to $\pm V_{CC}$ depending on polarity and magnitude of V_S (bistable circuit). This circuit DOES NOT OPERATE IN THE LINEAR RANGE AND DOES NOT DUPLICATE THE INPUT TO THE OUTPUT.

Example 5



$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

Example 5

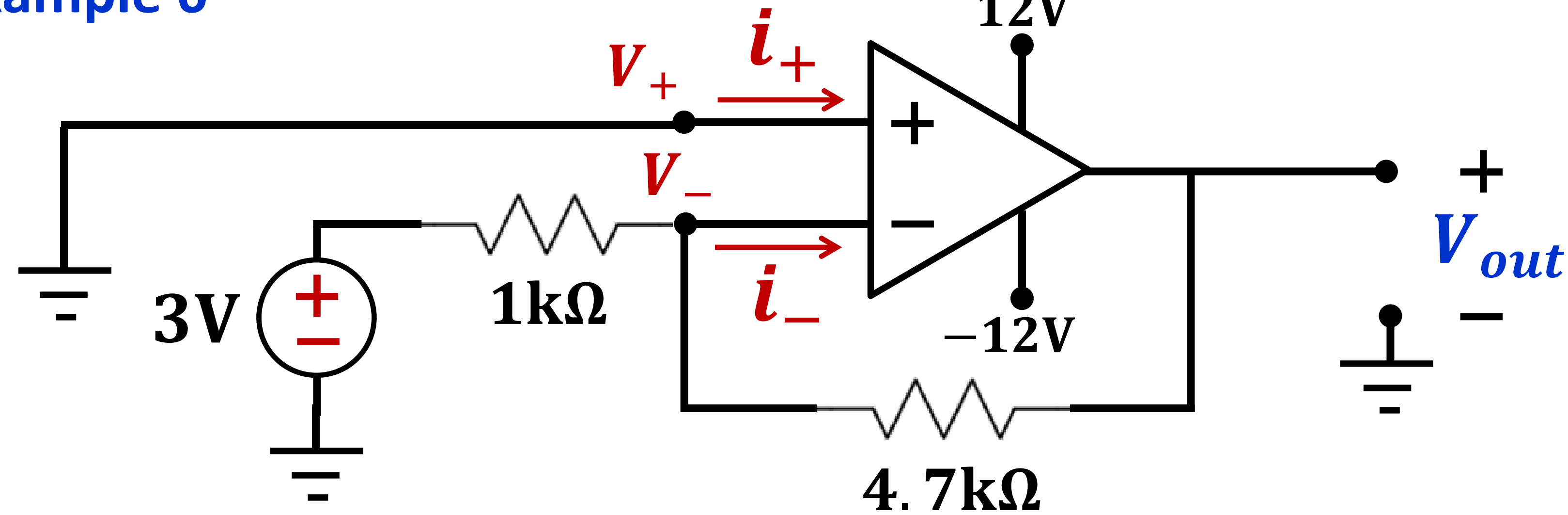


$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

$$\Rightarrow \quad \boxed{V_{out} = -5\,\frac{10\text{k}}{5.6\text{k}} = -8.93\text{V}}$$

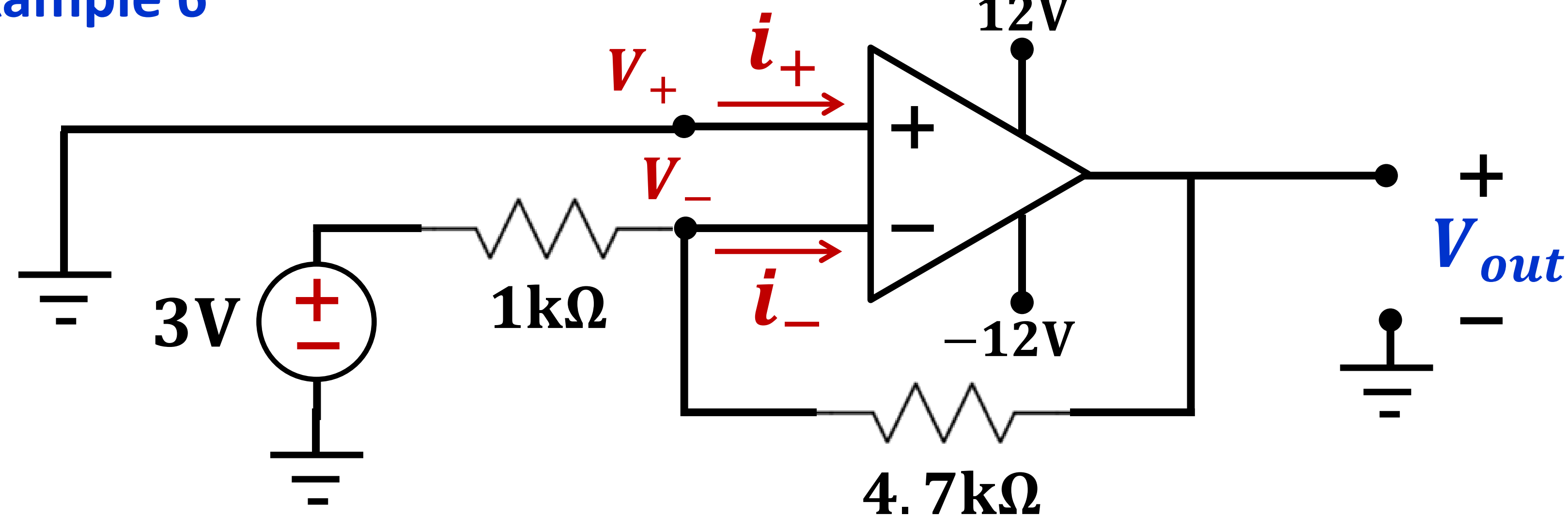
Since $|V_{out}| < |V_{CC}|$, the result obtained is valid.

Example 6



$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

Example 6

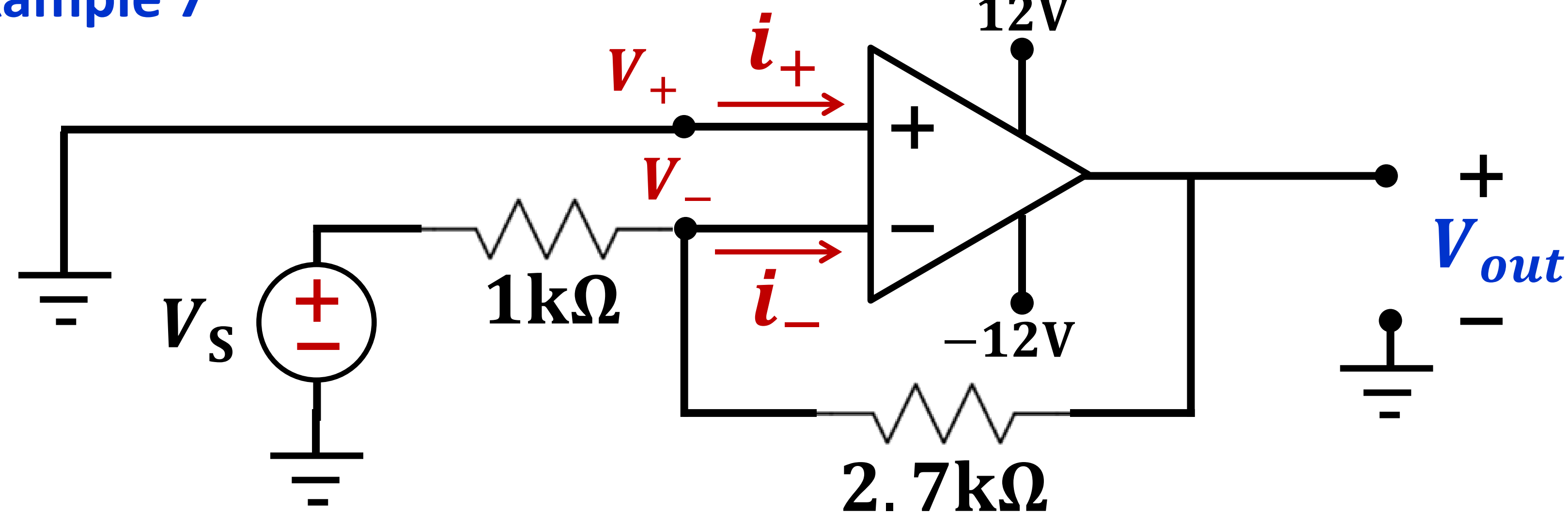


$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

$$\Rightarrow \quad \boxed{V_{out} = -3\frac{4.7\text{k}}{1\text{k}} = -14.1\text{V} < -V_{\text{b}}}$$

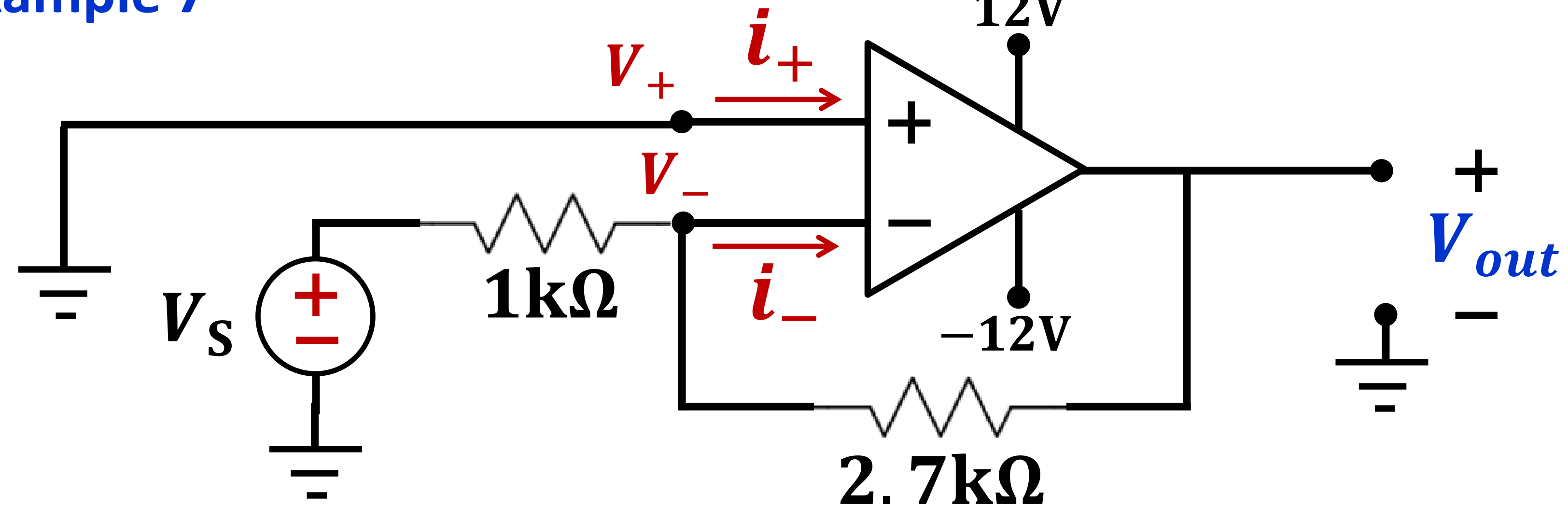
Since $|V_{out}| > |V_{CC}|$, the output voltage saturates at -12V.

Example 7



$$V_S = \mathrm{V}_m \cos(\omega t) \qquad \mathrm{V}_m = 3\mathrm{V}$$

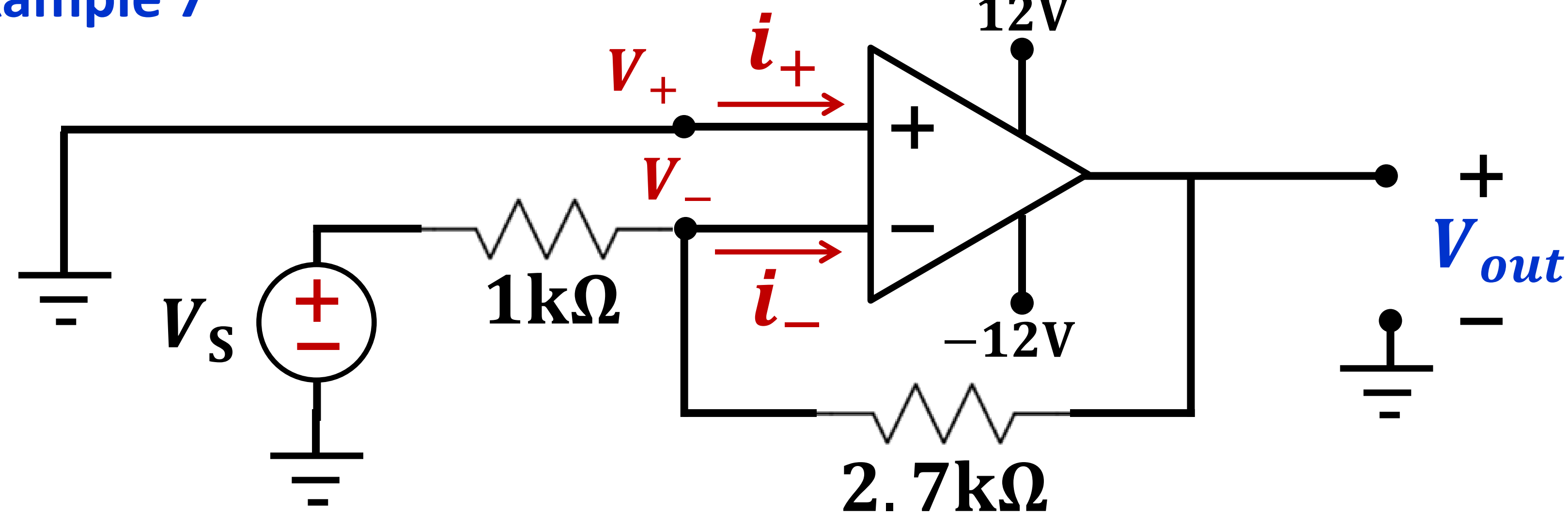
Example 7



$$V_S = \mathrm{V}_m \cos(\omega t) \qquad \mathrm{V}_m = 3\mathrm{V}$$

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S} \qquad V_{out} = \mathrm{V}_{mo} \cos(\omega t)$$

Example 7

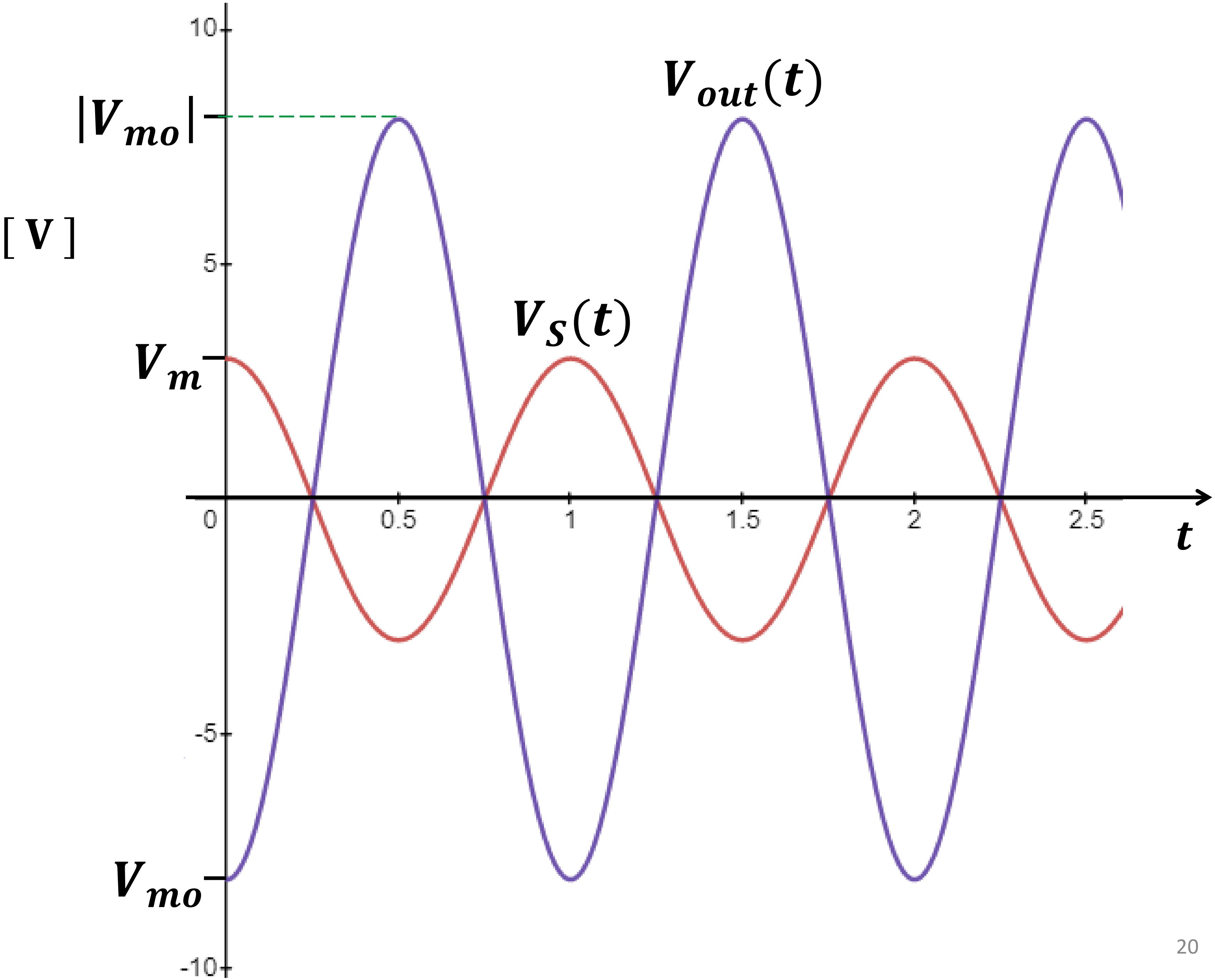


$$V_S = V_m \cos(\omega t) \qquad V_m = 3\text{V}$$

$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S} \qquad V_{out} = V_{mo}\cos(\omega t)$$

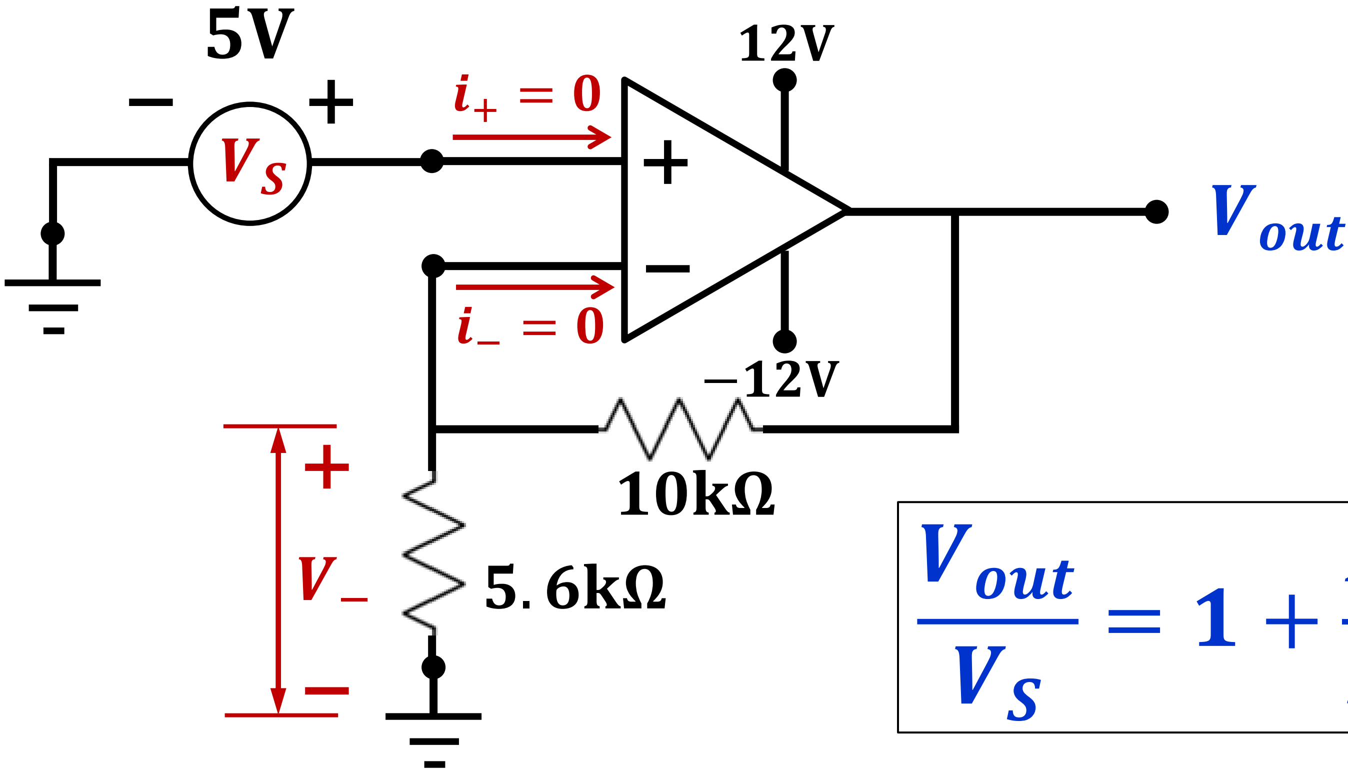
$$\Rightarrow \quad \boxed{V_{mo} = -3\,\frac{2.7\text{k}}{1\text{k}} = -8.1\text{V} > -V_b}$$

Since $|V_{mo}| < |V_{CC}|$, the result obtained is valid.



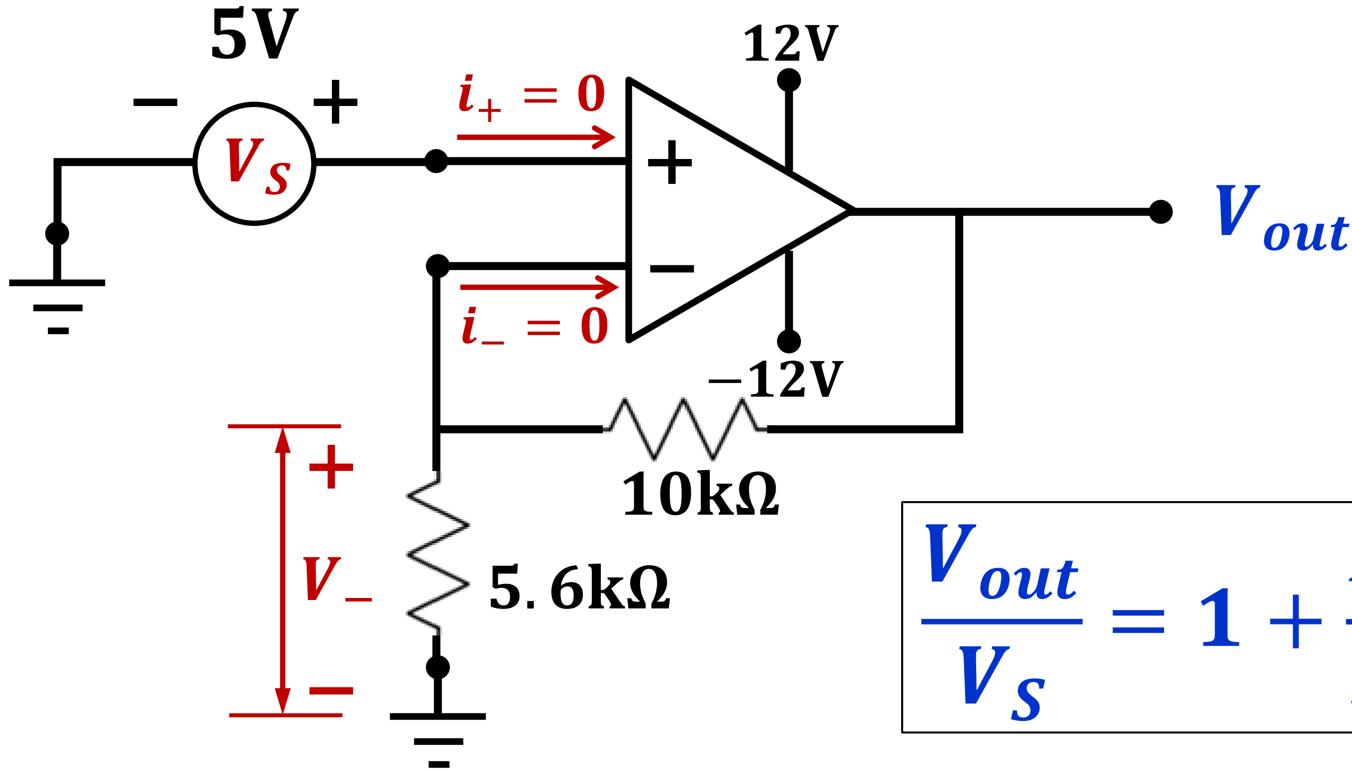
10
$|V_{mo}|$
[V]
5
$V_{out}(t)$
$V_S(t)$
V_m
0
0.5
1
1.5
2
2.5
t
-5
V_{mo}
-10

Example 8



$$\frac{V_{out}}{V_S} = 1 + \frac{R_F}{R_S}$$

Example 8

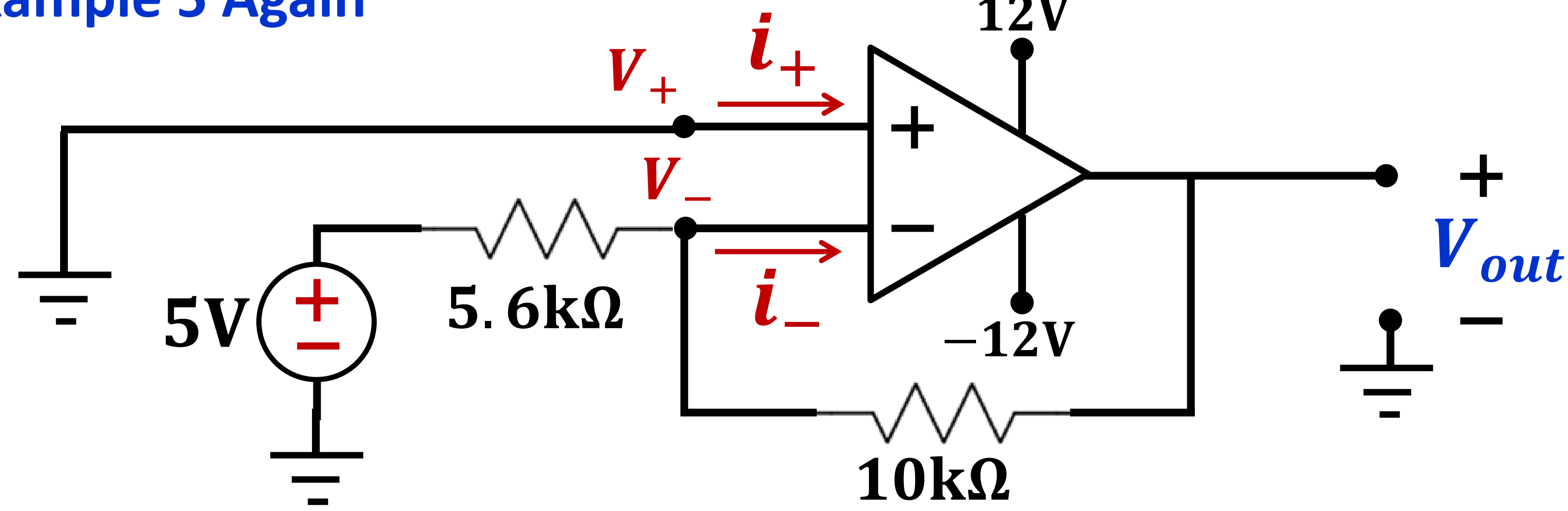


$$\frac{V_{out}}{V_S} = 1 + \frac{R_F}{R_S}$$

$$V_{out} = 5\left(1 + \frac{10k}{5.6k}\right) = 13.93V$$

Since $|V_{out}| > |V_{CC}|$, the op-amp saturates at 12V. In Example 5, with the same resistors the inverting op-amp did not saturate.

Example 5 Again



$$\frac{V_{out}}{V_S} = -\frac{R_F}{R_S}$$

$$\Rightarrow \quad \boxed{V_{out} = -5\,\frac{10\text{k}}{5.6\text{k}} = -8.93\text{V}}$$

Since $|V_{out}| < |V_{CC}|$**, the result obtained is valid.**

We have seen earlier

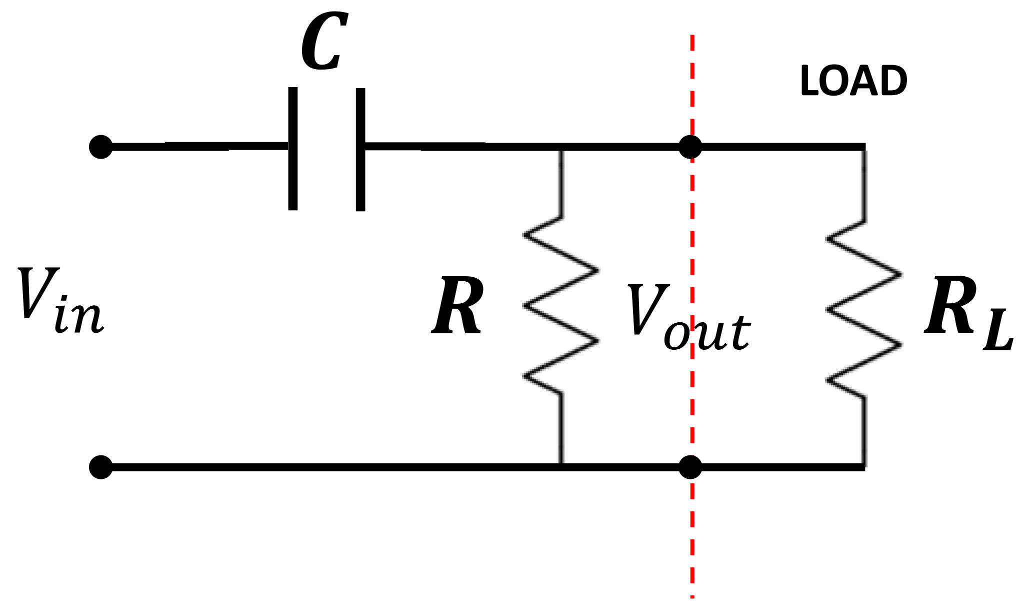
The response of a passive filter is affected by the load connected directly to it. For example, consider a high-pass RC filter:

RC constant

$\boldsymbol{\tau = R\,C}$

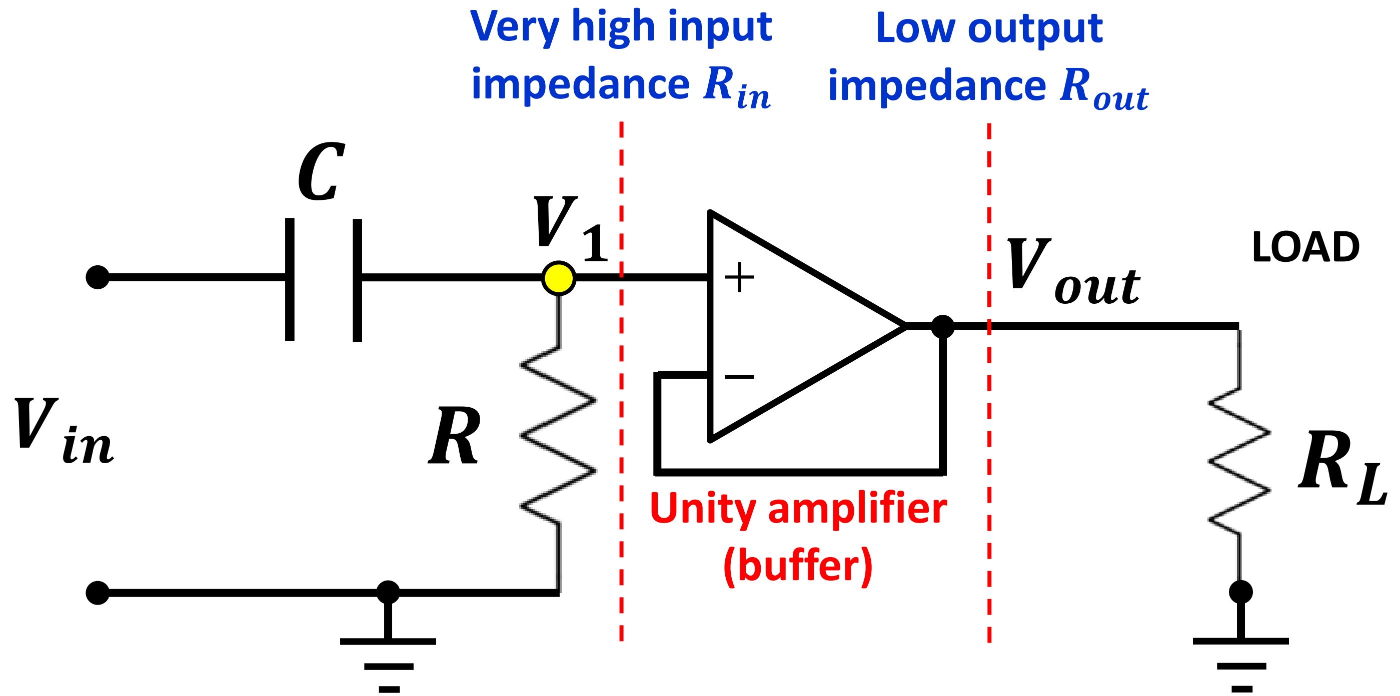
with load

$\boldsymbol{\tau' = (R//R_L)\,C}$



The parallel $\mathrm{R_{eff}} = R//R_L$ yields an equivalent resistance lower than either R or R_L. In particular, if connected to a small resistor R_L, the resulting cutoff frequency $\omega_C' = (R_{\mathrm{eff}}C)^{-1}$ may change considerably with respect to the original $\omega_C = (RC)^{-1}$.

Isolation of filter from load



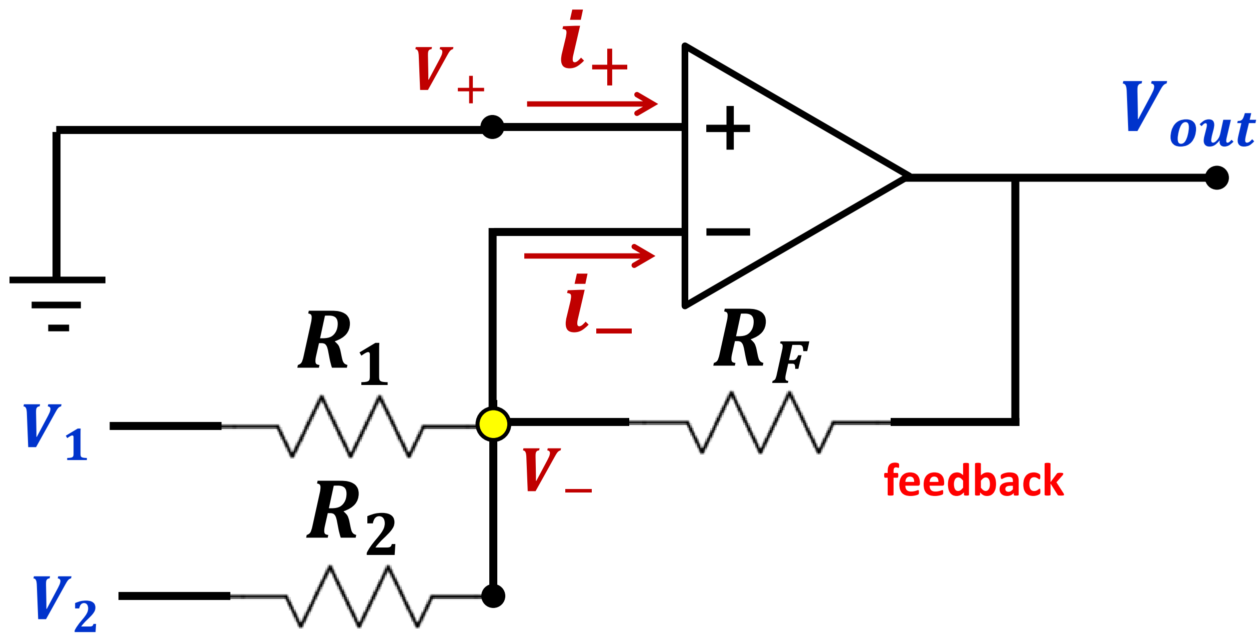
$$V_{out} = V_1$$

$$\tau' = (R//R_{in})\,C \rightarrow (R//\infty)\,C = RC = \tau$$

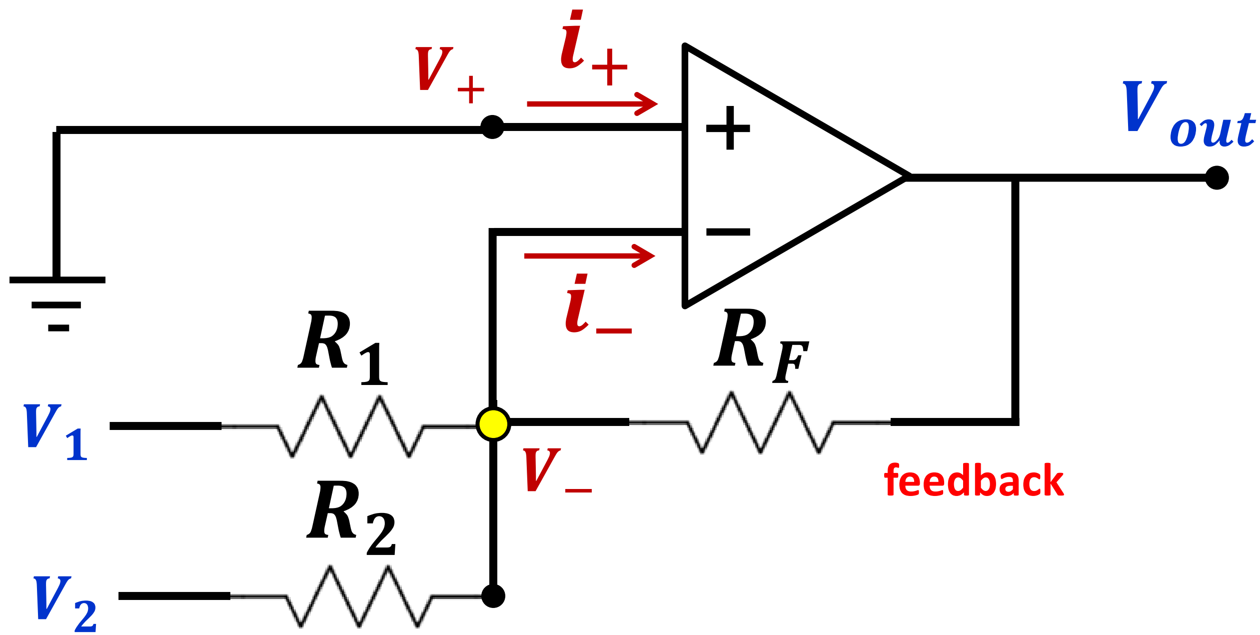
Adder OP AMP

$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$



Adder OP AMP



$V_+ = V_- = 0$

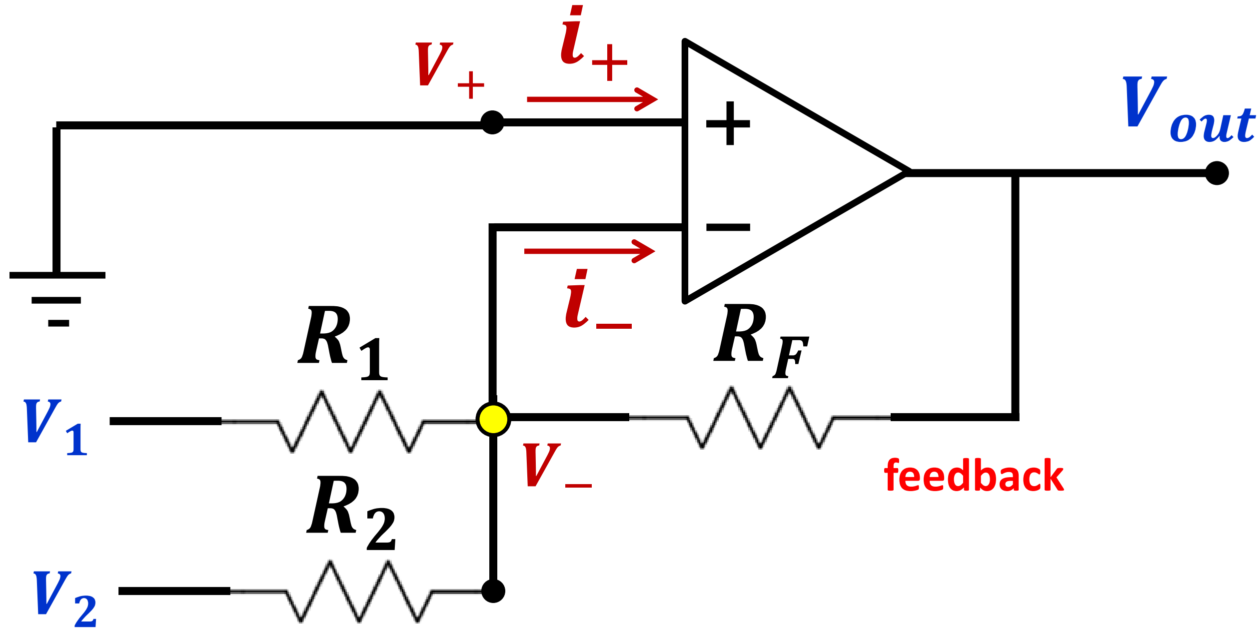
$i_+ = i_- = 0$

Node V_-

$$\frac{V_1 - V_-}{R_1} + \frac{V_2 - V_-}{R_2} + \frac{V_{out} - V_-}{R_F} = 0$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_{out}}{R_F}$$

Adder OP AMP



$$V_+ = V_- = 0$$

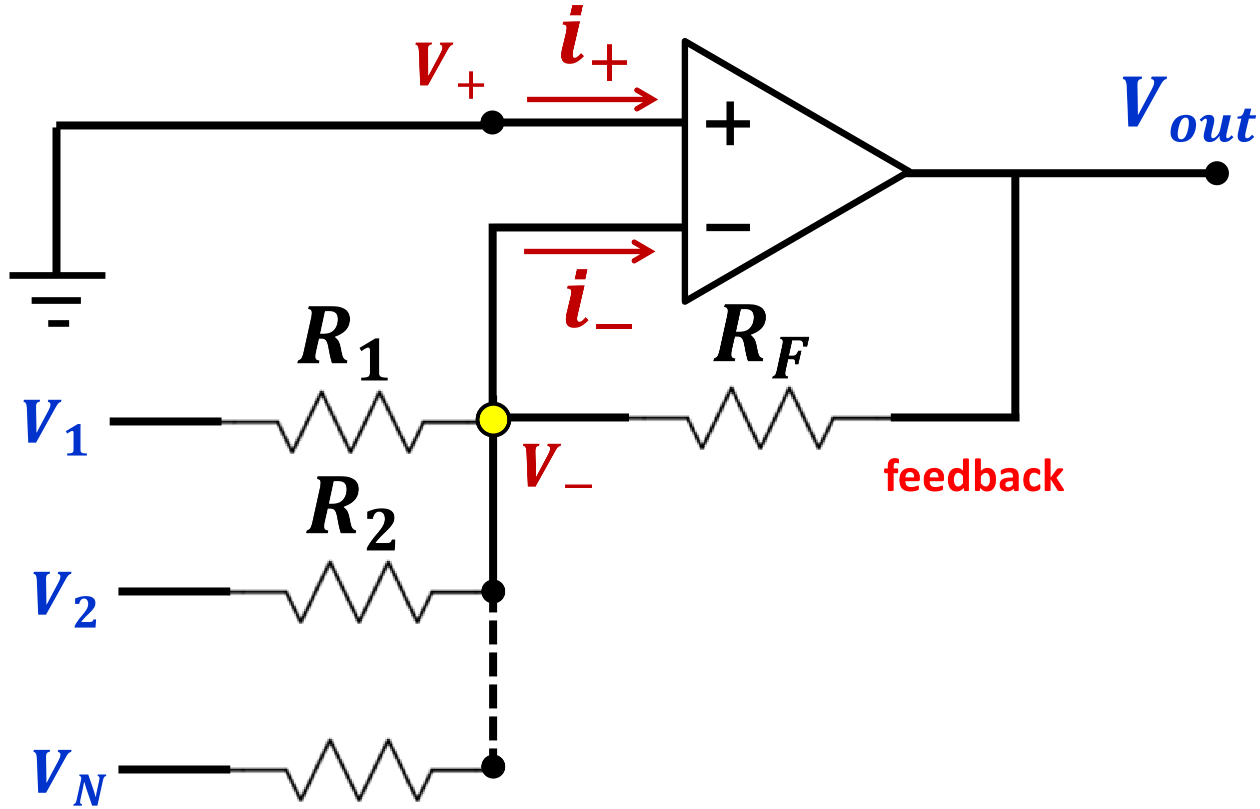
$$i_+ = i_- = 0$$

$$V_{out} = -R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

If $R_1 = R_2 = R_F$

$$\Rightarrow \boxed{V_{out} = -[V_1 + V_2]}$$

Adder OP AMP



$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$

For N inputs

$$V_{out} = -R_F \sum_{k=1}^{N} \frac{V_k}{R_k}$$

If $R_k = R_F$

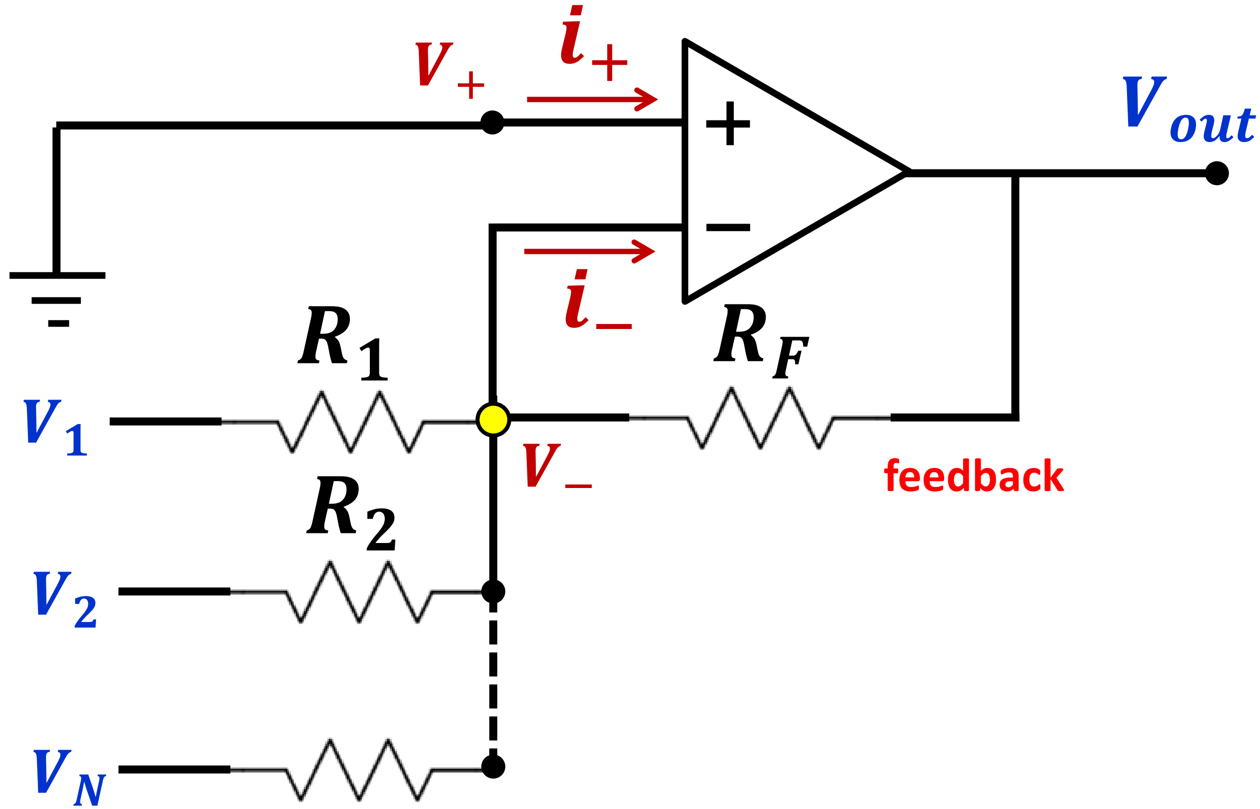
resistors all equal

$$\Rightarrow \quad V_{out} = -\sum_{k=1}^{N} V_k$$

Adder OP AMP

$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$



But one needs to verify that the output voltage does not exceed the rail bias

$$|V_{out}| \leq |V_{CC}|$$

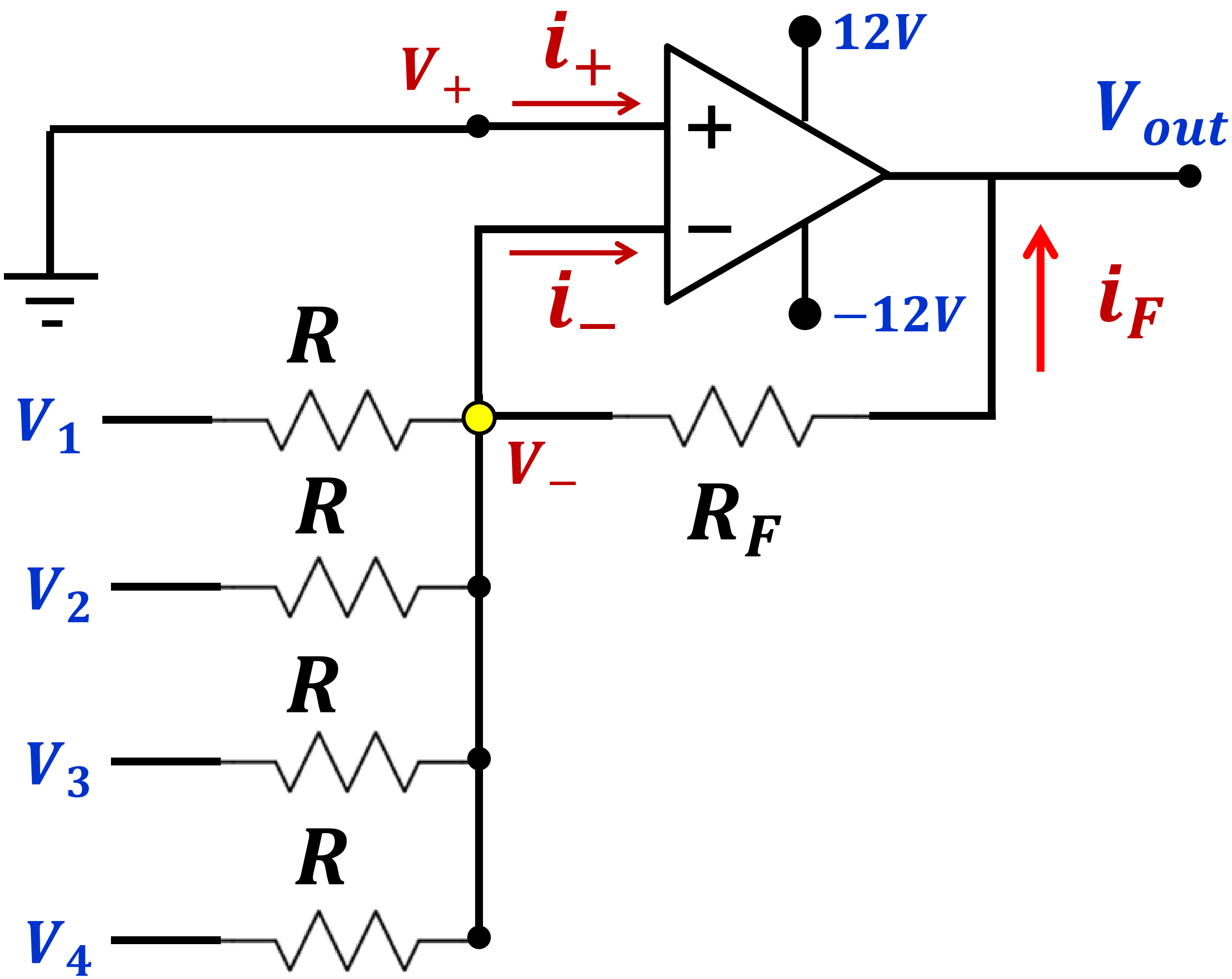
Example – Four equal resistors in input

$V_+ = V_- = 0$

$i_+ = i_- = 0$

$R = 1\text{k}\Omega$

$V_i \Big|_{\max} = 1\text{V}$



Example – Four equal resistors in input

$V_+ = V_- = 0$

$i_+ = i_- = 0$

$R = 1\text{k}\Omega$

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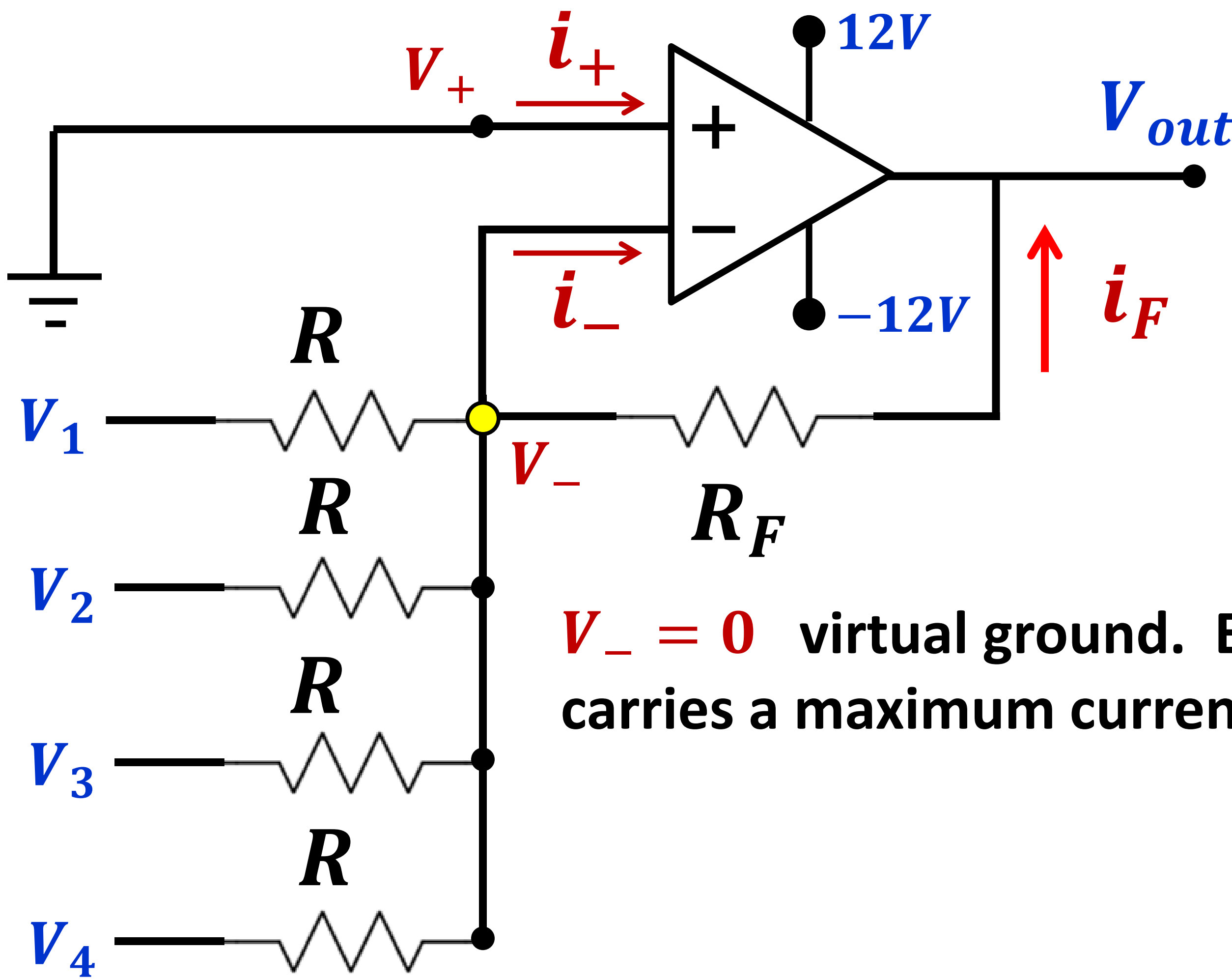


$V_- = 0$ virtual ground. Each input resistor carries a maximum current $i_k|_{\max} = 1\text{mA}$

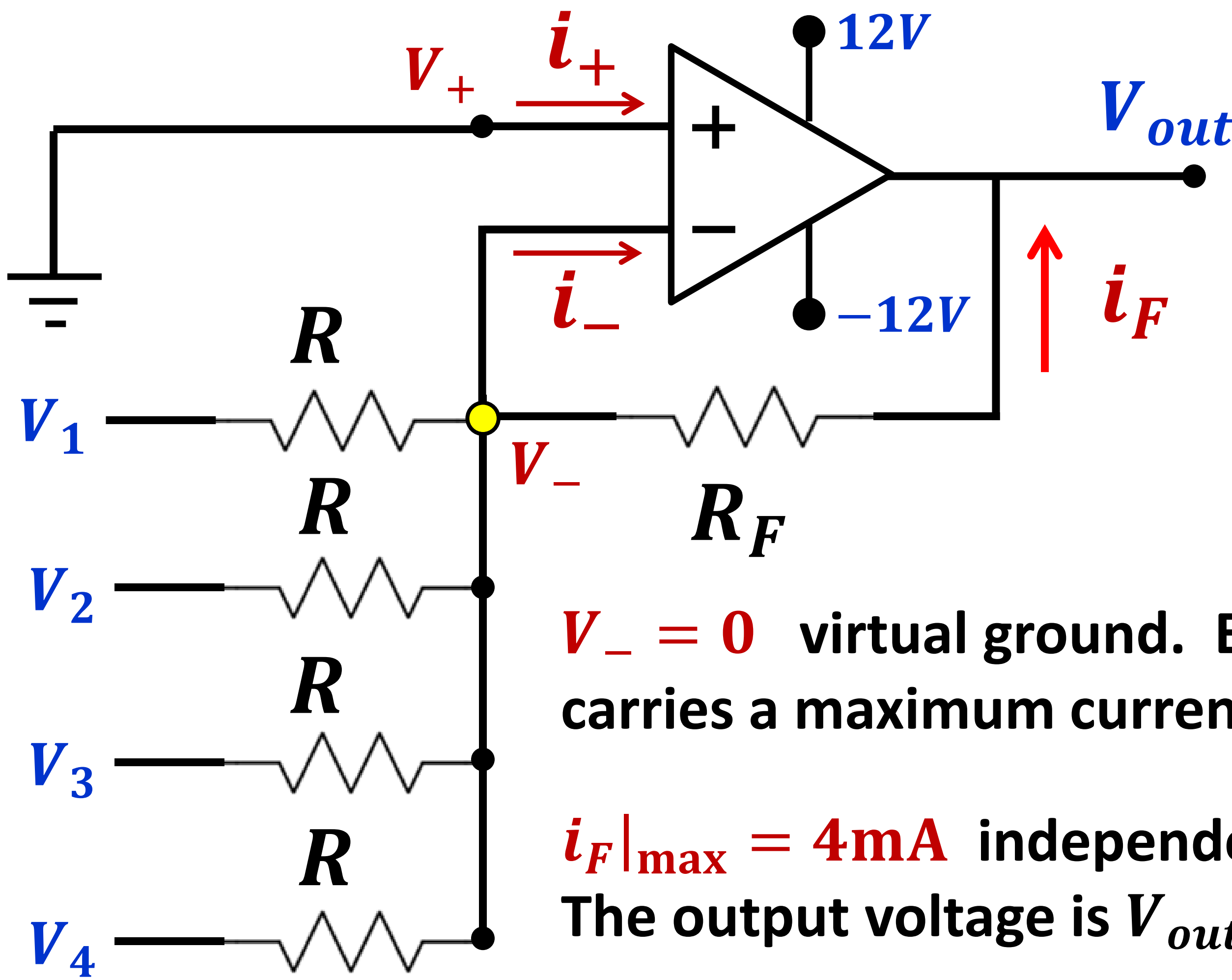
Example – Four equal resistors in input

$V_+ = V_- = 0$

$i_+ = i_- = 0$

$R = 1\text{k}\Omega$

$V_i\big|_{\max} = 1\text{V}$



$V_- = 0$ virtual ground. Each input resistor carries a maximum current $i_k\big|_{\max} = 1\text{mA}$

$i_F\big|_{\max} = 4\text{mA}$ independent of R_F.
The output voltage is $V_{out} = R_F \times i_F\big|_{\max}$

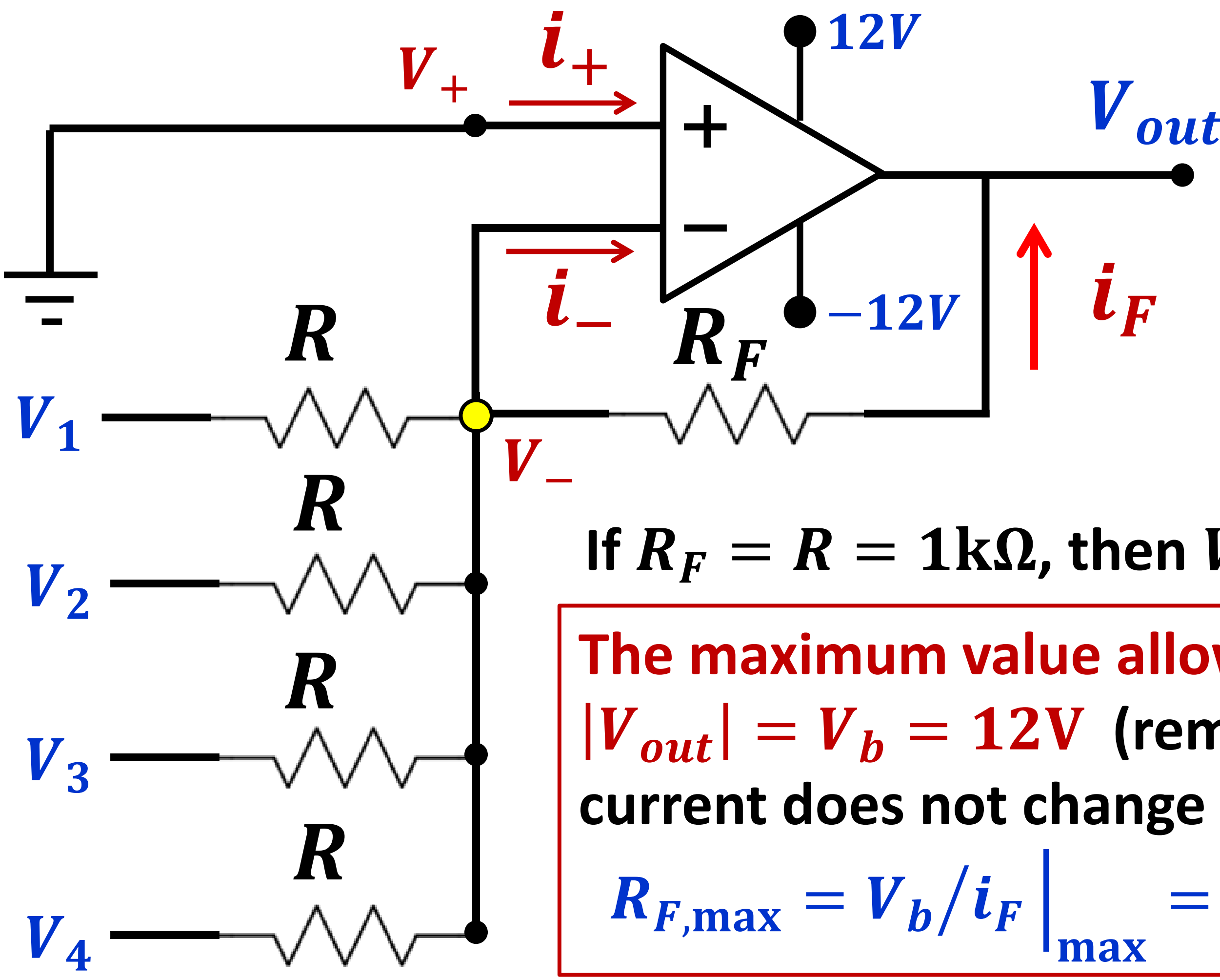
Example – Equal input resistors

$V_+ = V_- = 0$

$i_+ = i_- = 0$

$R = 1\text{k}\Omega$

$V_i\big|_{\max} = 1\text{V}$

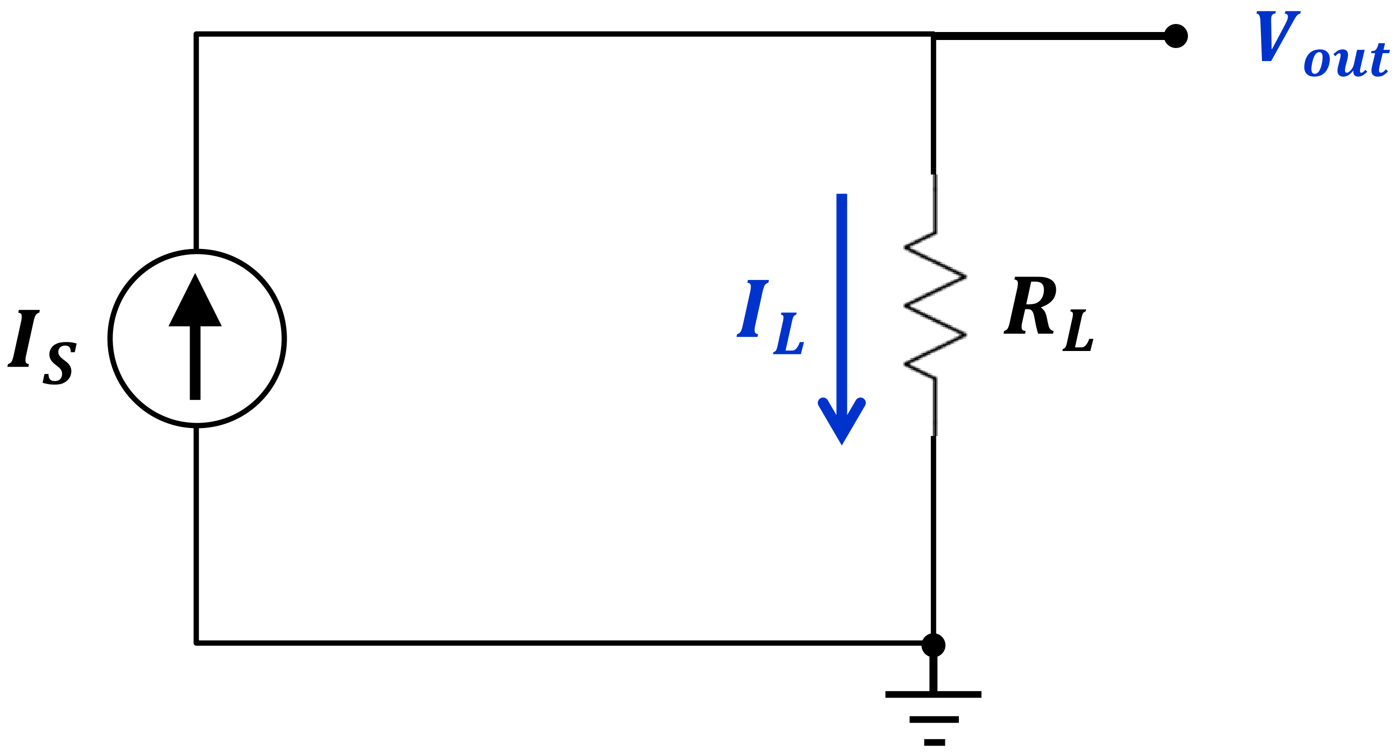


If $R_F = R = 1\text{k}\Omega$, then $V_{out} = 4\text{V}$ at most.

The maximum value allowed for R_F is when $|V_{out}| = V_b = 12\text{V}$ (remember, the input current does not change with R_F)

$$R_{F,\max} = V_b/i_F\big|_{\max} = 12\text{V}/4\text{mA} = 3\text{k}\Omega$$

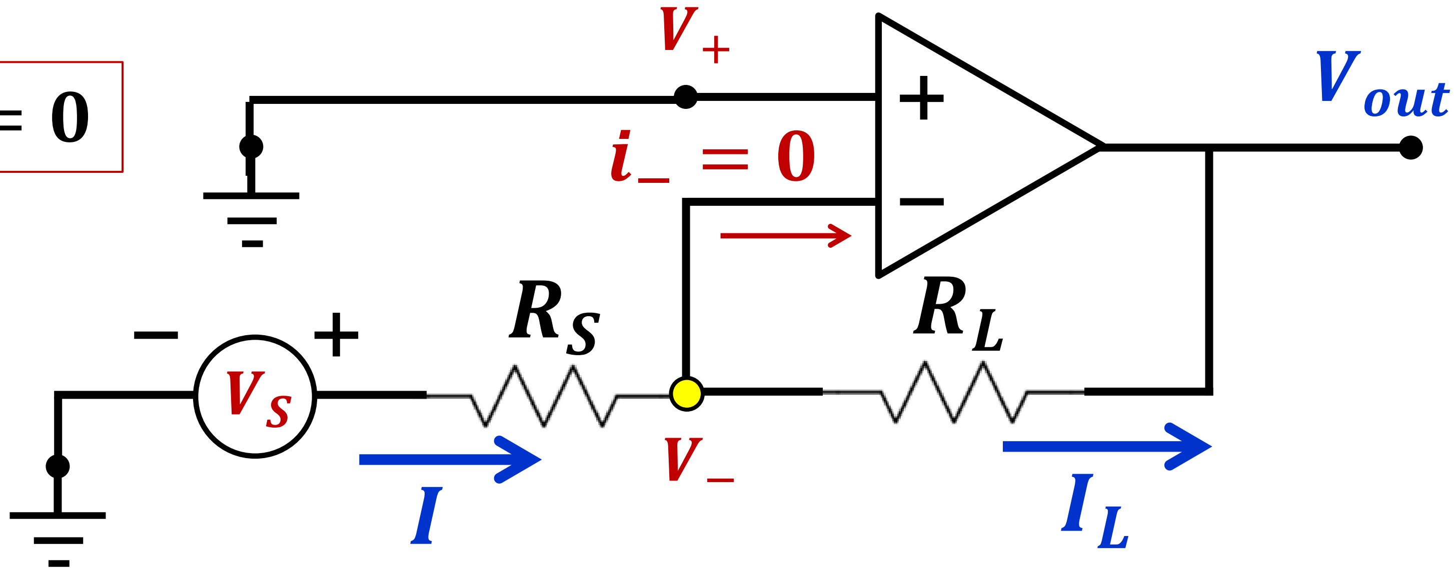
Ideal Current Source



The current generated should be constant, independently of the load R_L

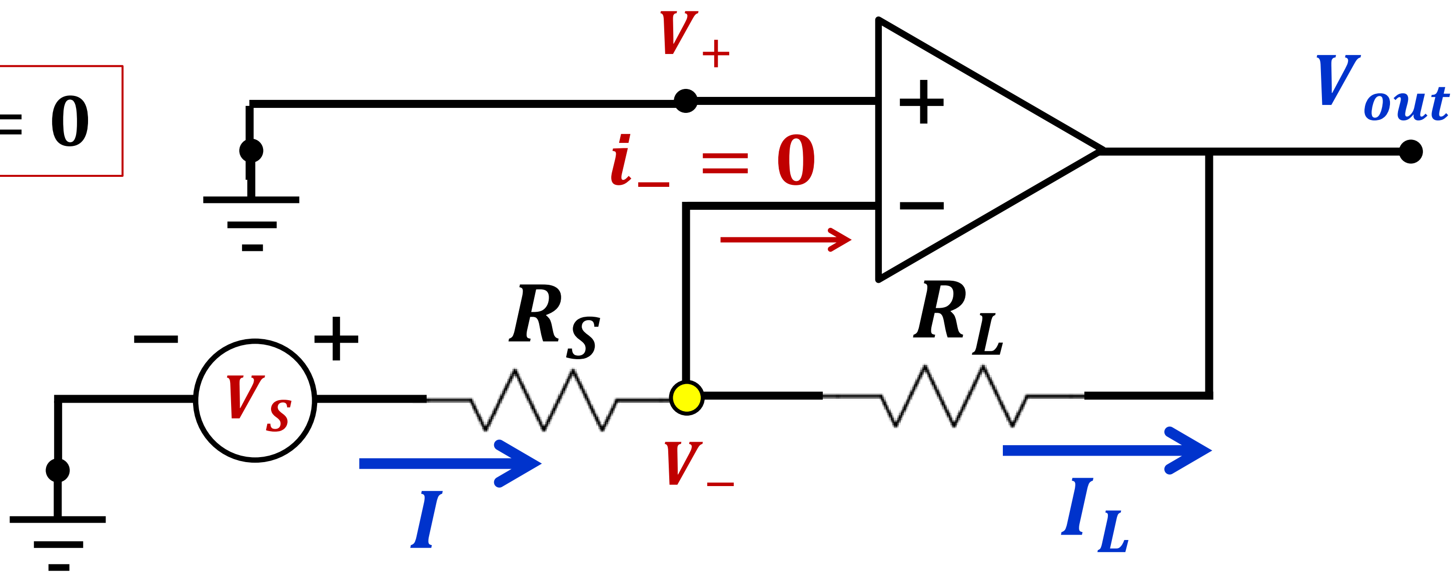
OP AMP Current Source

$$V_+ = V_- = 0$$



OP AMP Current Source

$$V_+ = V_- = 0$$

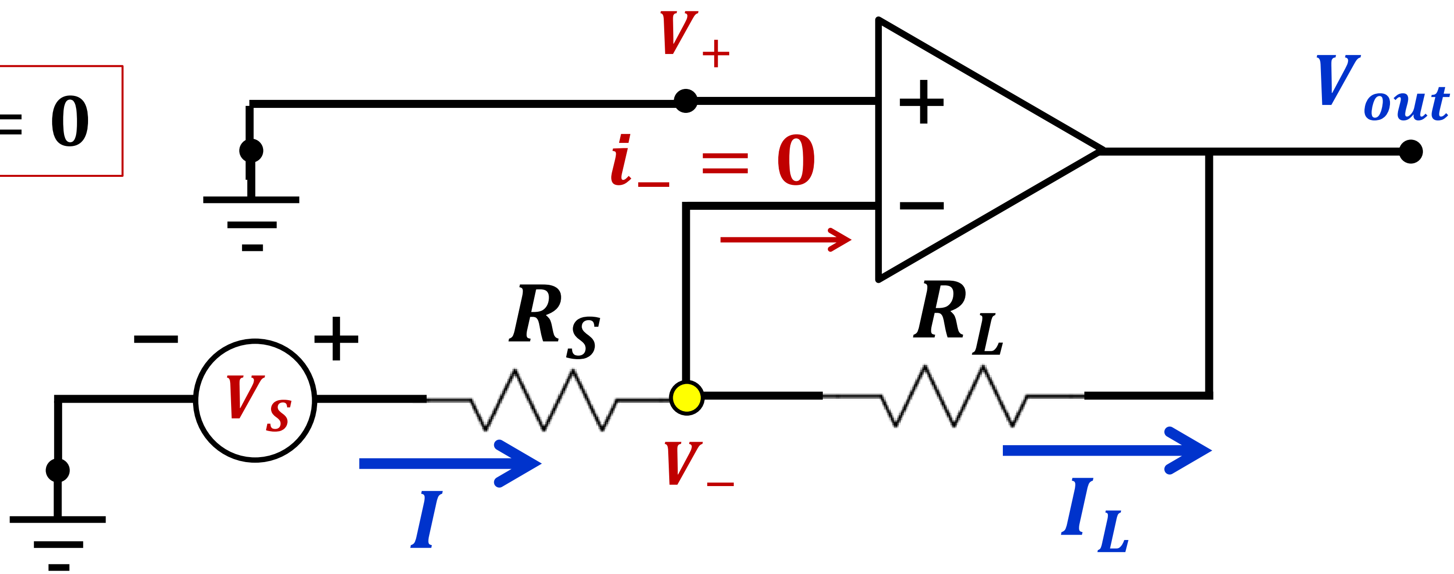


$$I = \overbrace{i_-}^{=0} + I_L = I_L$$

$$I_L = \frac{V_S - V_-}{R_S} = \frac{V_S}{R_S}$$

OP AMP Current Source

$$V_+ = V_- = 0$$



$$I = \overbrace{i_-}^{=0} + I_L = I_L$$

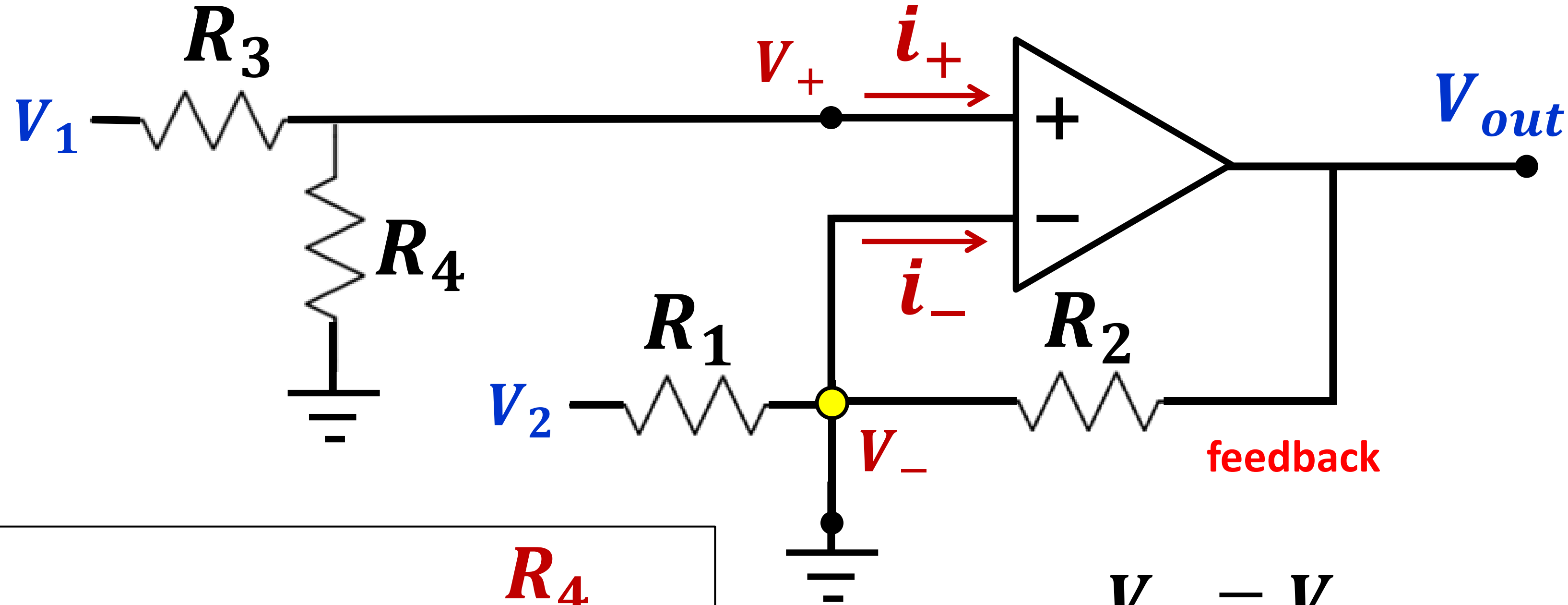
$$I_L = \frac{V_S - V_-}{R_S} = \frac{V_S}{R_S}$$

Example: $V_S = 1\text{V};\ \ R_S = 1\text{k}\Omega$

$$\Rightarrow \quad I_L = \frac{1-0}{1\text{k}\Omega} = 1\text{mA}$$

Independent of R_L

Differential OP AMP

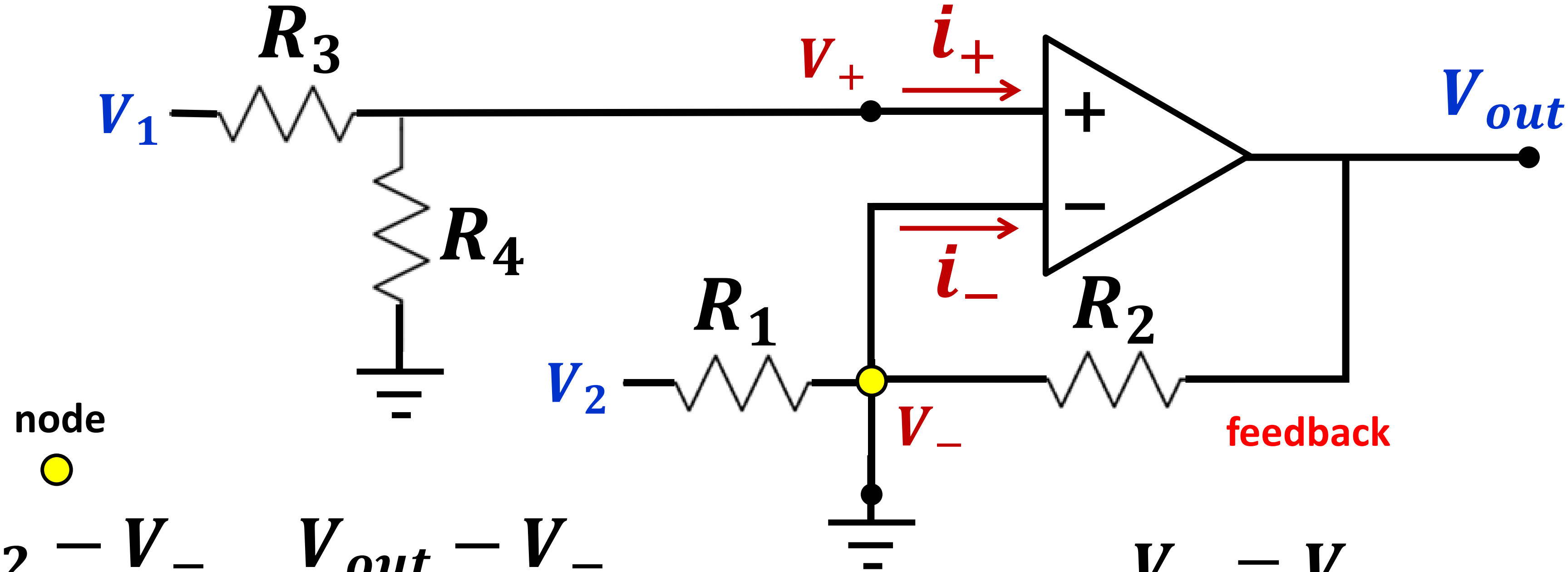


$$V_+ = V_- = V_1 \frac{R_4}{R_3 + R_4}$$

$$V_+ = V_-$$

$$i_+ = i_- = 0$$

Differential OP AMP



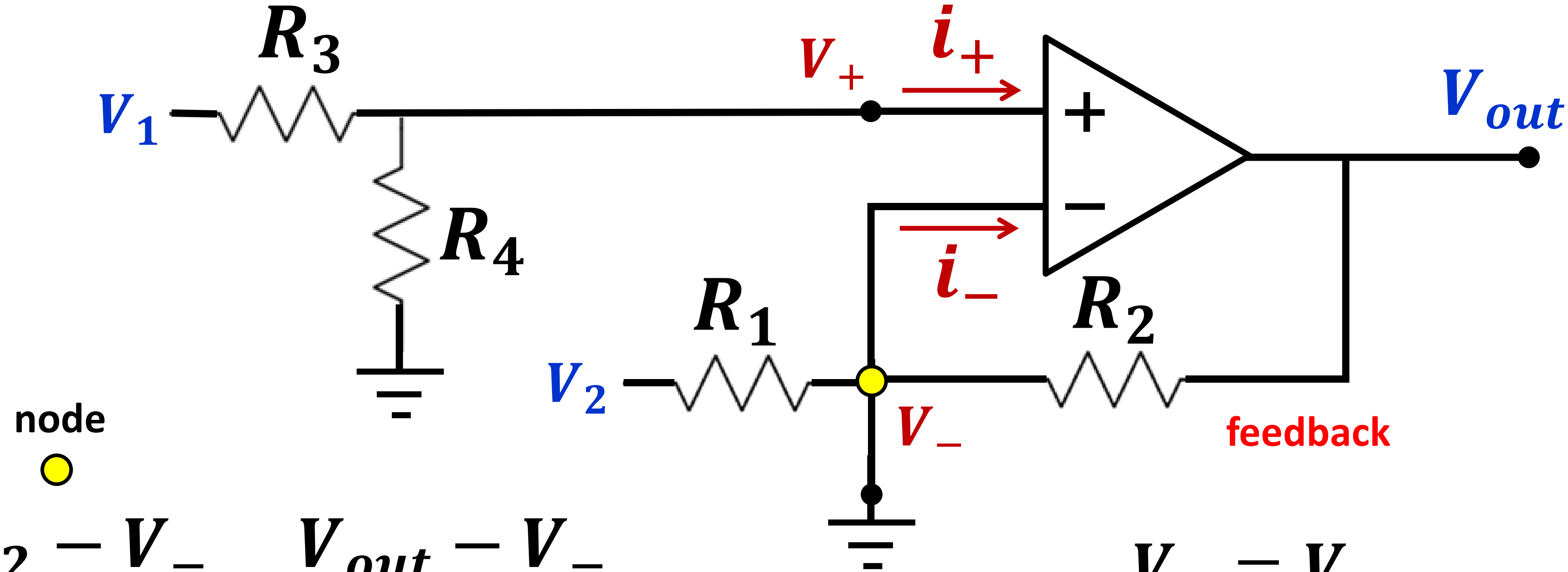
$$\frac{V_2 - V_-}{R_1} + \frac{V_{out} - V_-}{R_2} = 0$$

$$V_+ = V_-$$

$$i_+ = i_- = 0$$

$$\frac{V_{out}}{R_2} = -\frac{V_2 - V_-}{R_1} - \frac{-V_-}{R_2}$$

Differential OP AMP



$$\frac{V_2 - V_-}{R_1} + \frac{V_{out} - V_-}{R_2} = 0$$

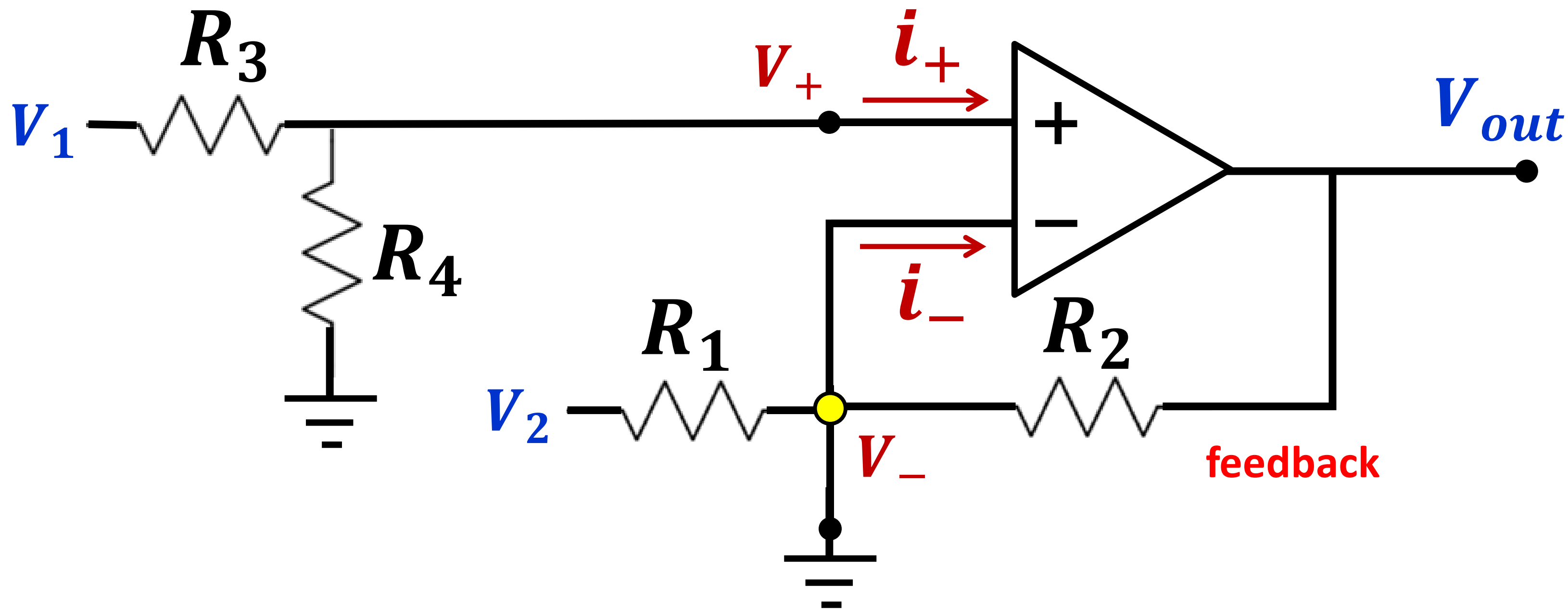
$$V_+ = V_-$$

$$i_+ = i_- = 0$$

$$\frac{V_{out}}{R_2} = -\frac{V_2 - V_-}{R_1} - \frac{-V_-}{R_2}$$

$$\Rightarrow \quad V_{out} = \frac{R_2}{R_1}(V_- - V_2) + V_-$$

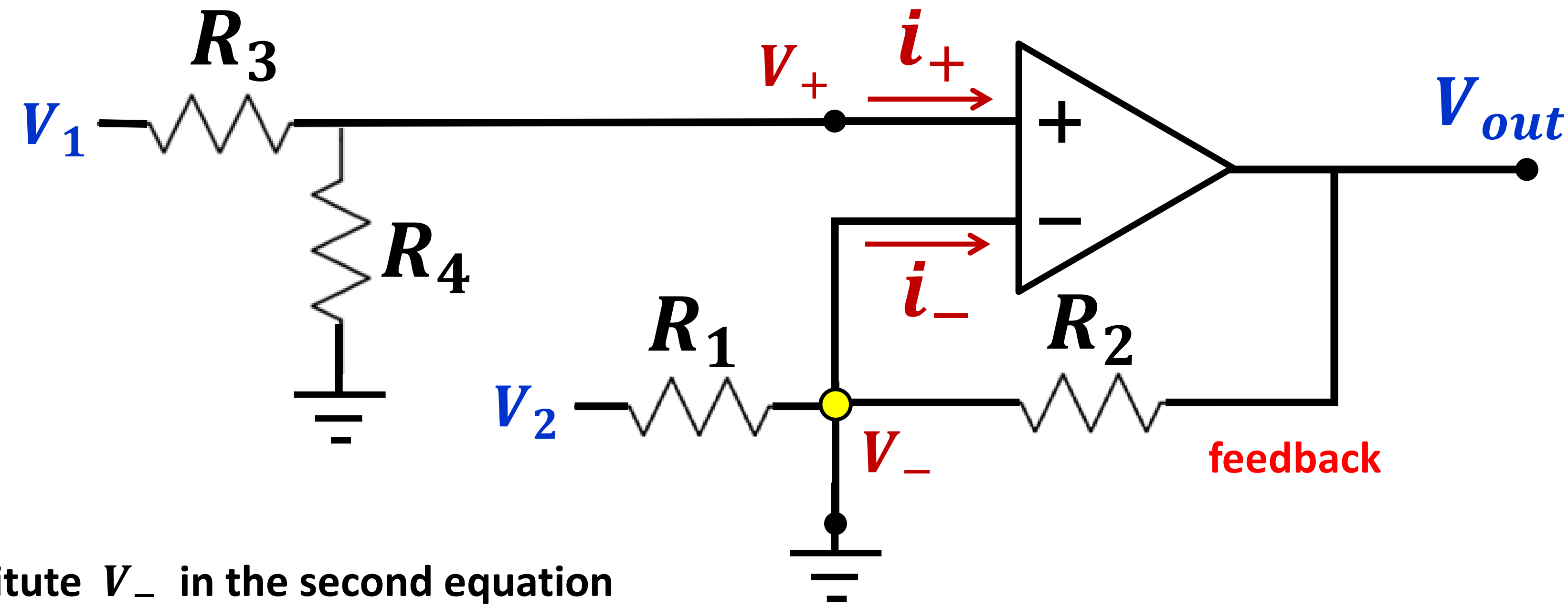
Differential OP AMP



$$V_+ = V_- = V_1 \frac{R_4}{R_3 + R_4}$$

$$V_{out} = \frac{R_2}{R_1}(V_- - V_2) + V_-$$

Differential OP AMP



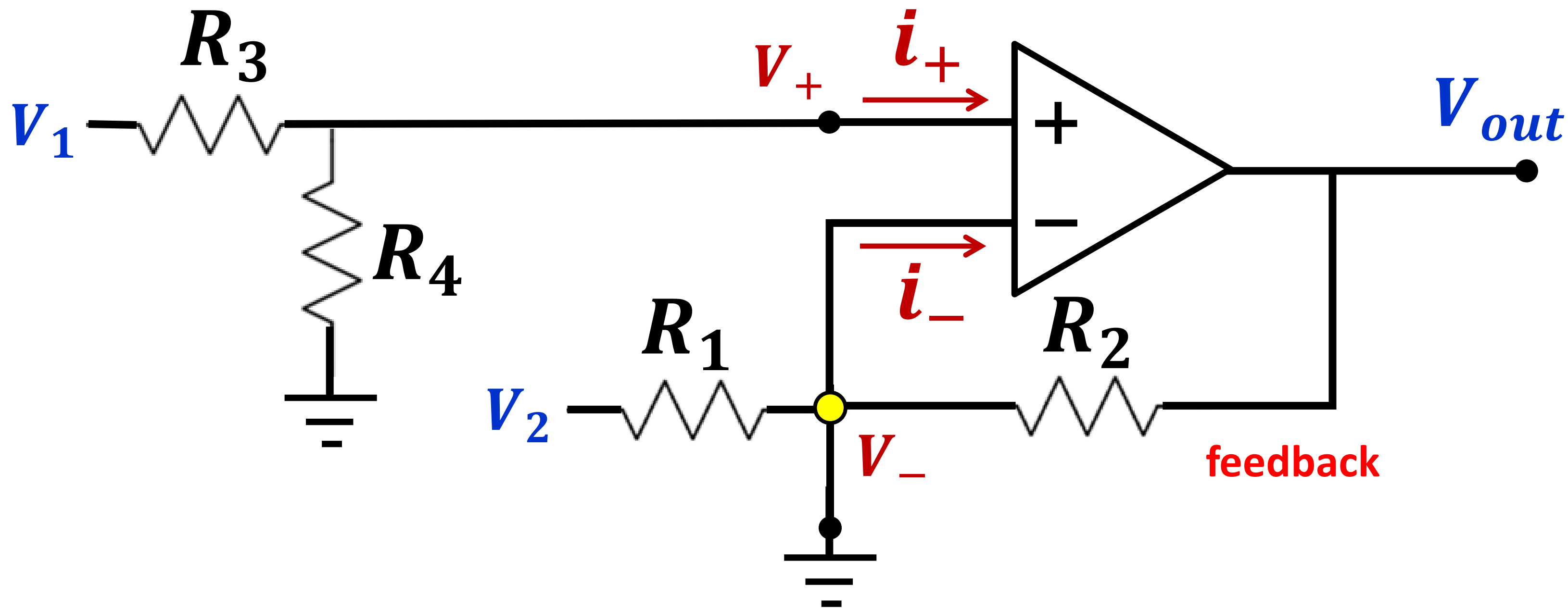
Substitute V_- in the second equation

$$V_+ = V_- = V_1 \frac{R_4}{R_3 + R_4}$$

$$V_{out} = \frac{R_2}{R_1}(V_- - V_2) + V_-$$

$$V_{out} = \frac{R_2}{R_1}\left(V_1 \frac{R_4}{R_3 + R_4} - V_2\right) + V_1 \frac{R_4}{R_3 + R_4}$$

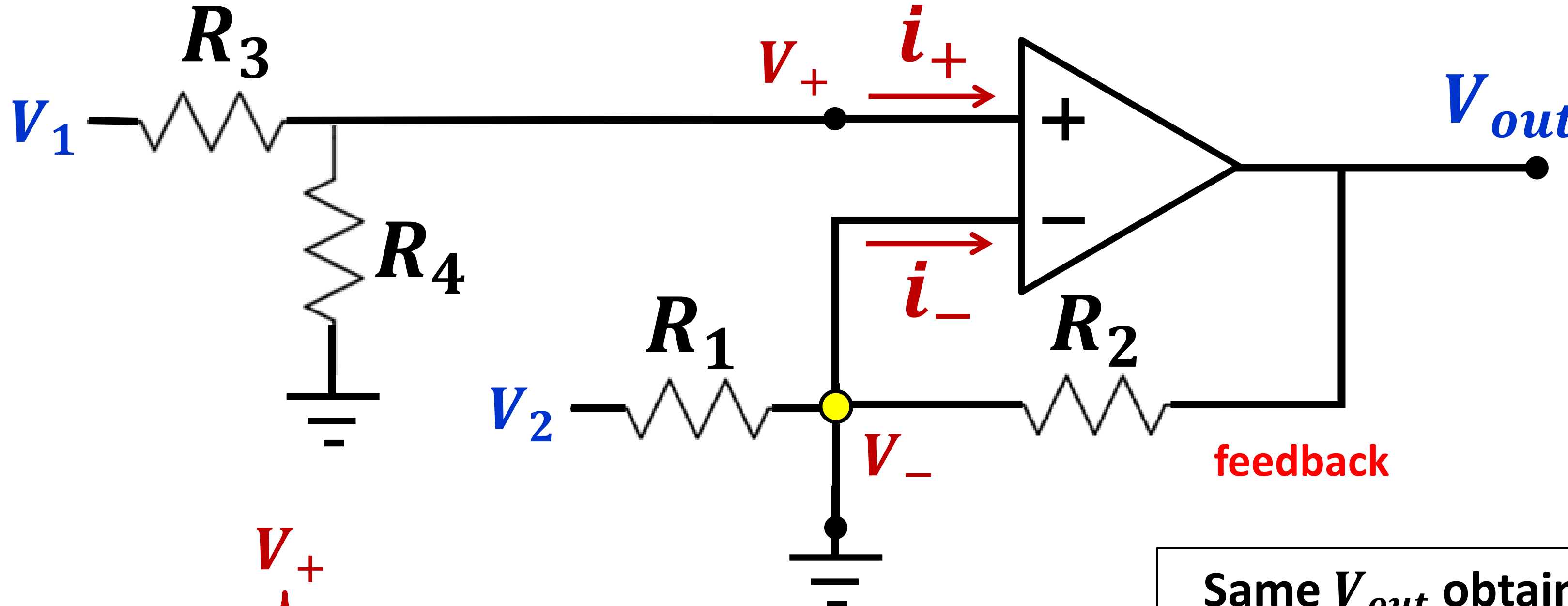
Differential OP AMP



$$V_{out} = \frac{R_2}{R_1}\left(V_1 \frac{R_4}{R_3 + R_4} - V_2\right) + V_1 \frac{R_4}{R_3 + R_4}$$

$$V_{out} = V_1 \frac{R_4}{R_3 + R_4}\left(\frac{R_2}{R_1} + 1\right) - \frac{R_2}{R_1} V_2$$

Differential OP AMP



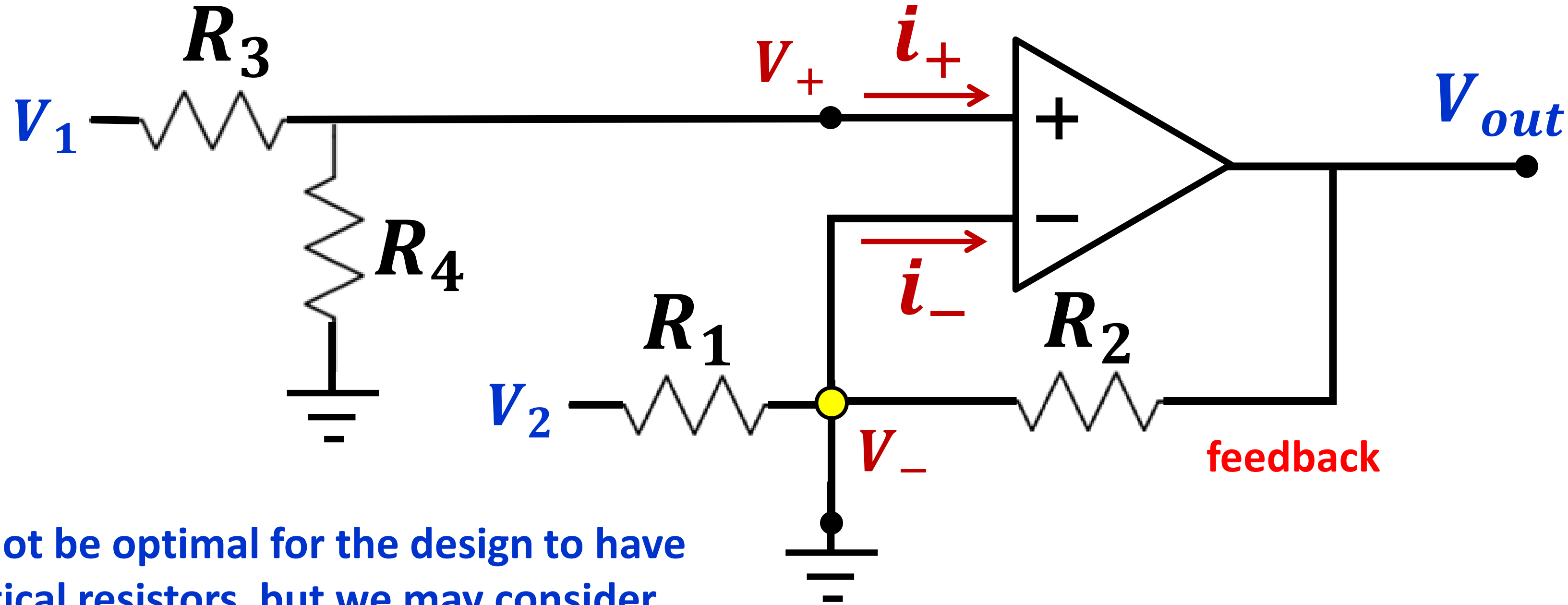
$$V_{out} = \overbrace{V_1 \frac{R_4}{R_3 + R_4}}^{V_+} \underbrace{\left(1 + \frac{R_2}{R_1}\right)}_{} - V_2 \frac{R_2}{R_1}$$

$$\underbrace{V_1 \frac{R_4}{R_3 + R_4}\left(1 + \frac{R_2}{R_1}\right)}_{V_1 \text{ contribution}} \underbrace{- V_2 \frac{R_2}{R_1}}_{V_2 \text{ contribution}}$$

Same V_{out} obtained by superposition of results with inverting and non-inverting amplifier formulas

If $R_1 = R_2 = R_3 = R_4 = R$ ⇨ $V_{out} = [V_1 - V_2]$

Differential OP AMP



It may not be optimal for the design to have all identical resistors, but we may consider pairs of resistors with identical ratio

If: $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

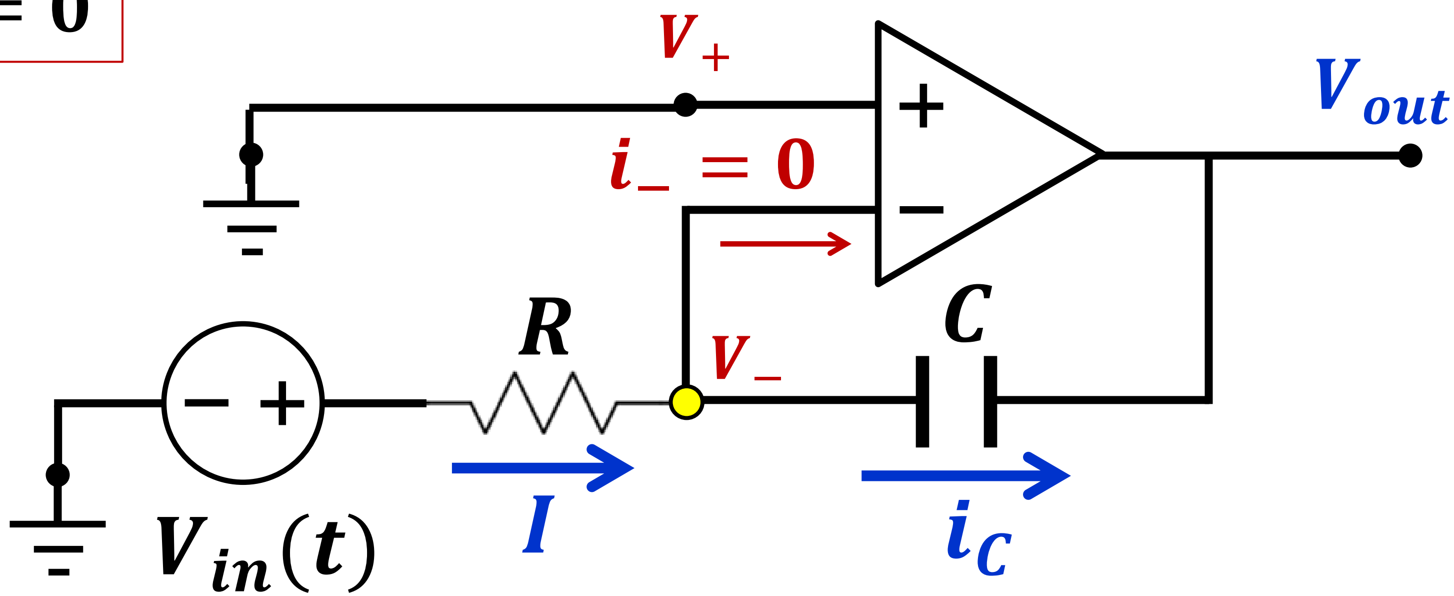
$$V_0 = \frac{R_4}{R_3 + R_4}\left(\frac{R_2}{R_1} + 1\right)V_1 - \frac{R_2}{R_1}V_2$$

$$V_0 = \frac{R_4}{\cancel{R_3 + R_4}}\left(\frac{\cancel{R_4 + R_3}}{R_3}\right)V_1 - \frac{R_2}{R_1}V_2 \Rightarrow V_0 = \frac{R_2}{R_1}(V_1 - V_2)$$

OP AMP Integrator

$V_+ = V_- = 0$

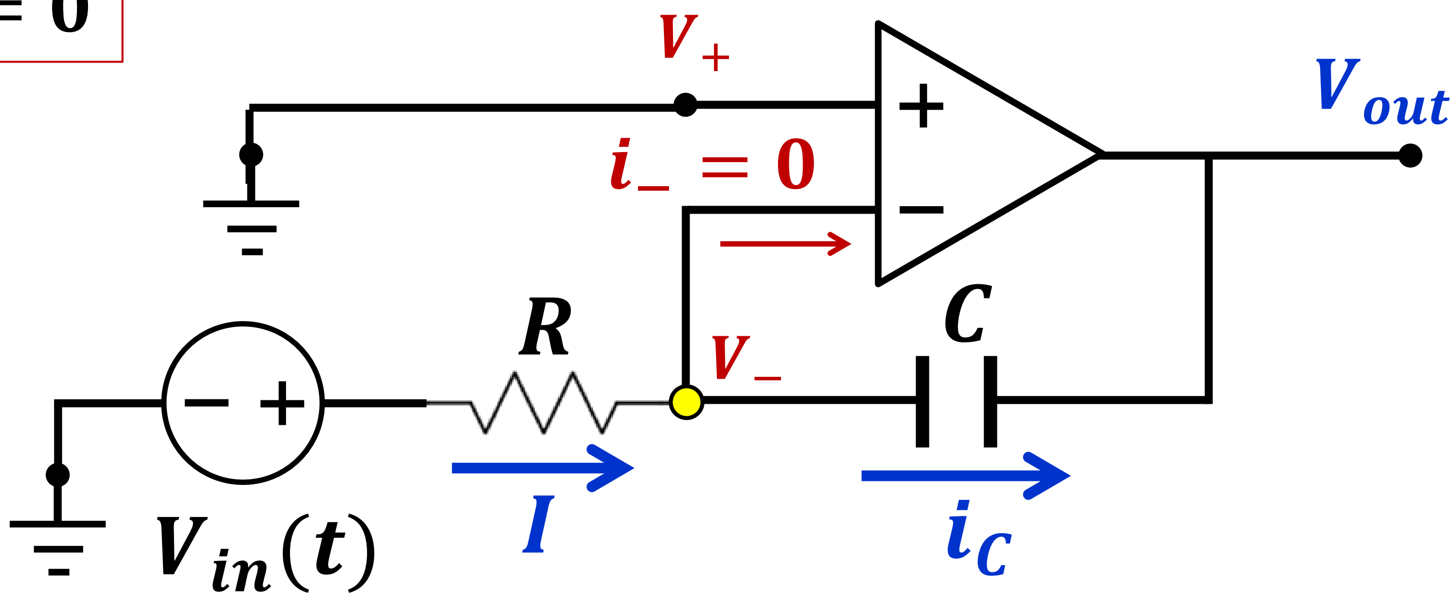
$I = i_C$



OP AMP Integrator

$$V_+ = V_- = 0$$

$$I = i_C$$



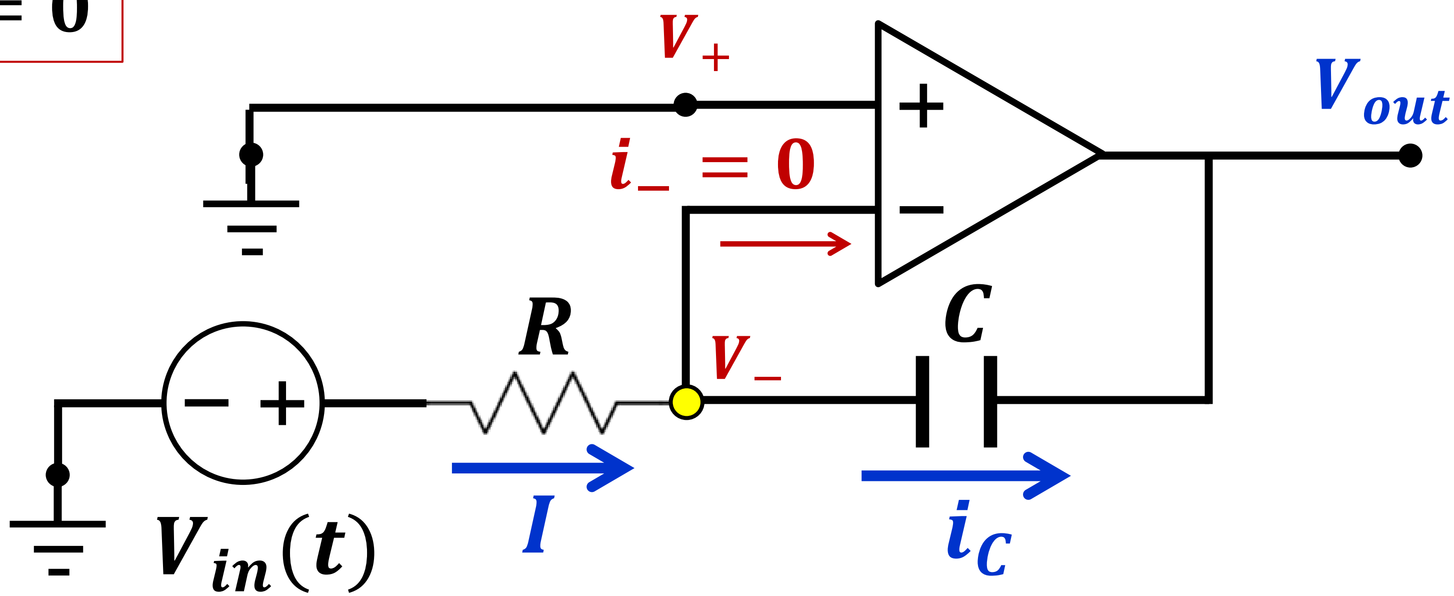
$$V_{out}(t) = -\frac{1}{C}\int_0^t i_C\,dt + V_{out}(0)$$

$$i_C = \frac{V_{in}(t)}{R}$$

OP AMP Integrator

$$V_+ = V_- = 0$$

$$I = i_C$$



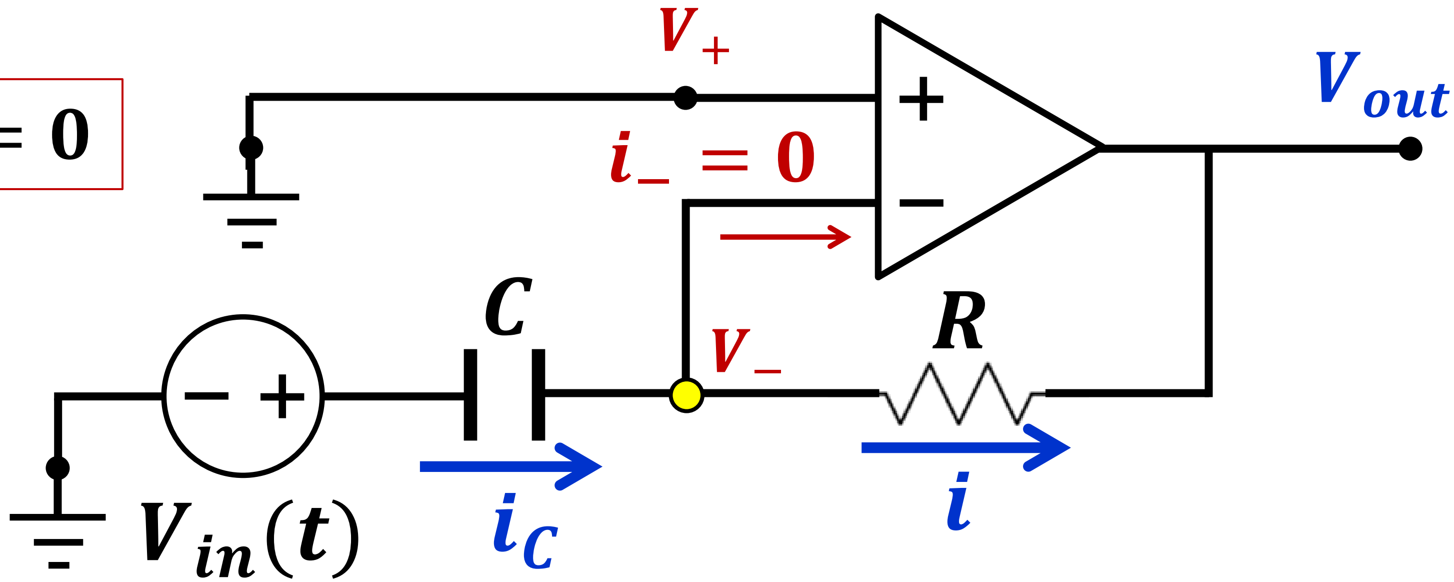
$$V_{out}(t) = -\frac{1}{C}\int_0^t i_C\, dt + V_{out}(0)$$

$$i_C = \frac{V_{in}(t)}{R}$$

$$\Rightarrow \quad V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(t)\, dt + V_{out}(0)$$

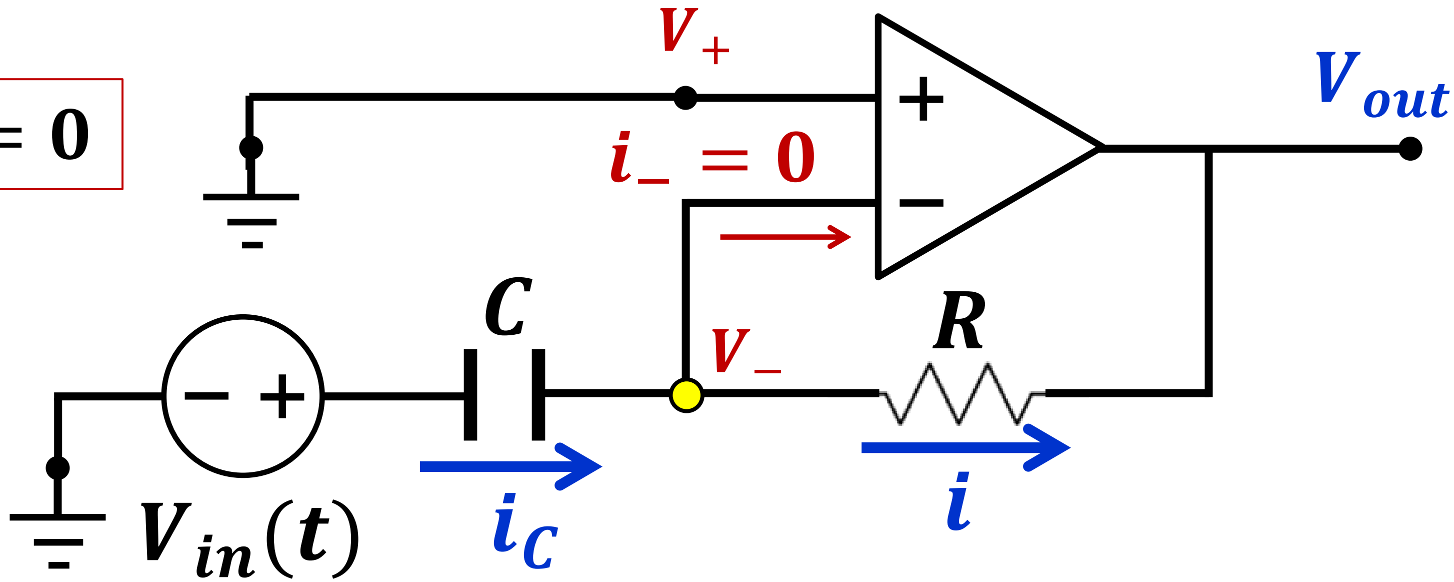
OP AMP differentiator

$$V_+ = V_- = 0$$



OP AMP differentiator

$$V_+ = V_- = 0$$

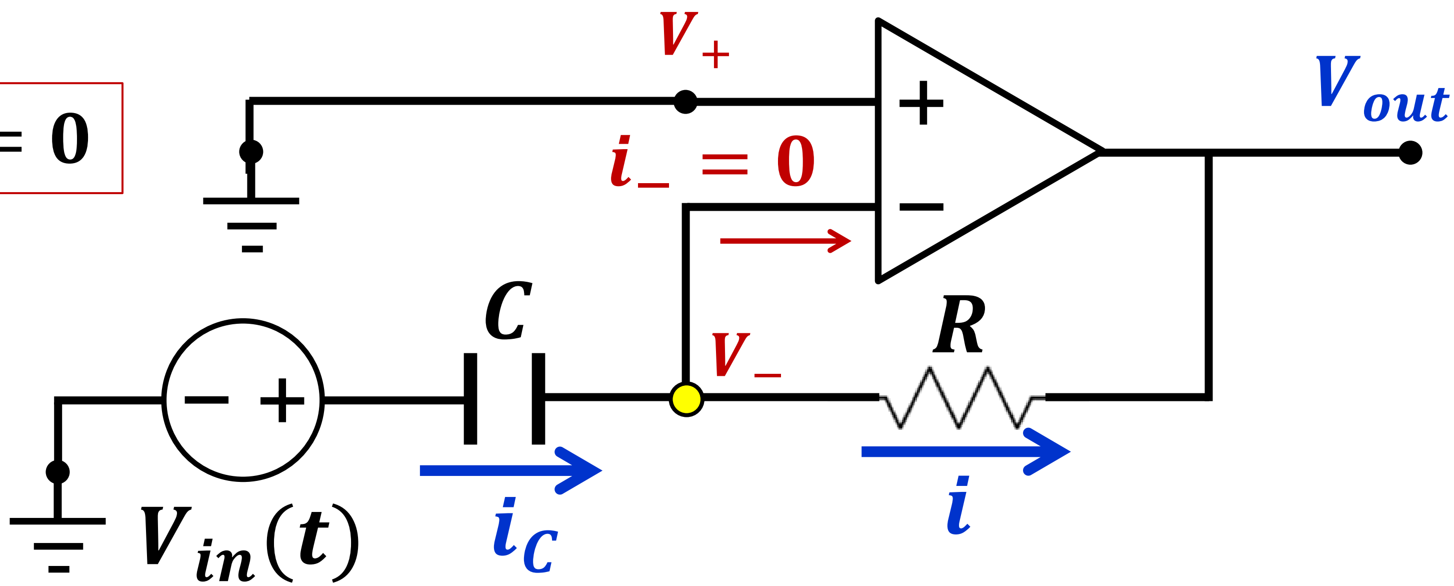


$$i_C = i = (V_{out} - V_-)/R$$

$$i_C = C\frac{d}{dt}V_{in}(t)$$

OP AMP differentiator

$$V_+ = V_- = 0$$



$$i_C = i = (V_{out} - V_-)/R$$

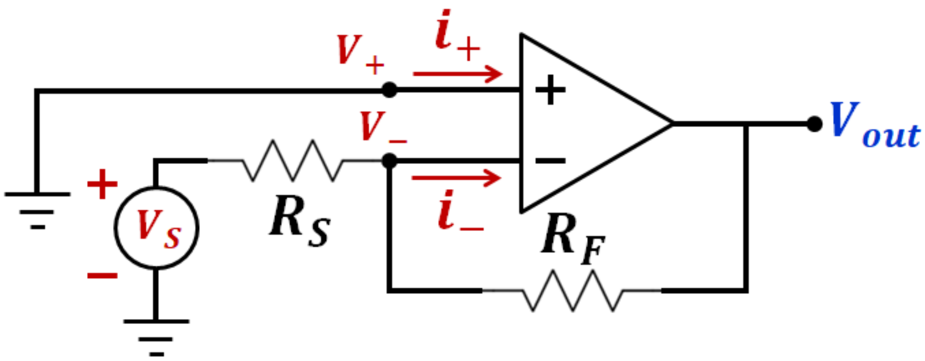
$$i_C = C\frac{d}{dt}V_{in}(t)$$

$$V_{out} = -Ri_C = -RC\frac{d}{dt}V_{in}(t)$$

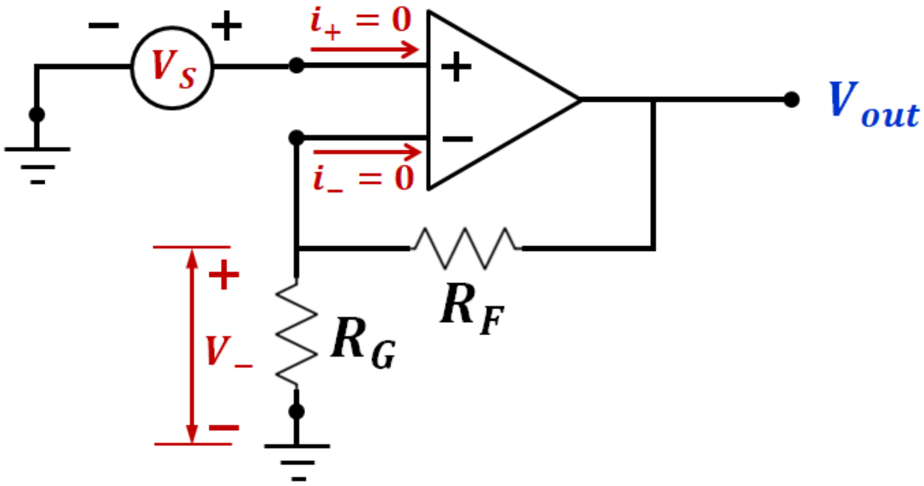
This basic circuit is typically sensitive to fluctuations and may not be very stable due to noise amplification.

Recall the basic Op Amp results

Inverting Amplifier



Non-inverting Amplifier



$$A_{VF} = -\frac{R_F}{R_S}$$

$$A_{VF} = 1 + \frac{R_F}{R_G}$$

Similar relationships are established in the OP AMP when simple resistors are replaced with impedances

- ***Inverting amplifier***

$$A_{VF} = \frac{V_{out}}{V_S} = -\frac{Z_F}{Z_S}$$

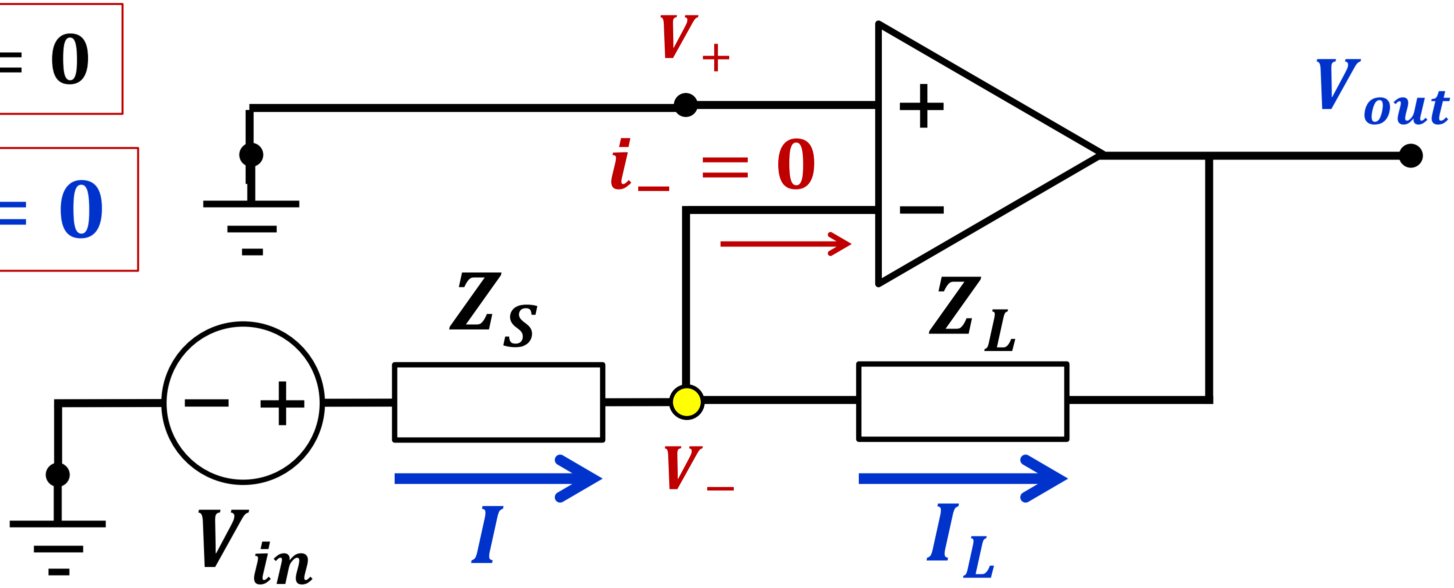
- ***Non-inverting amplifier***

$$A_{VF} = \frac{V_{out}}{V_S} = \frac{Z_G + Z_F}{Z_G} = 1 + \frac{Z_F}{Z_G}$$

OP AMP with impedances

$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$

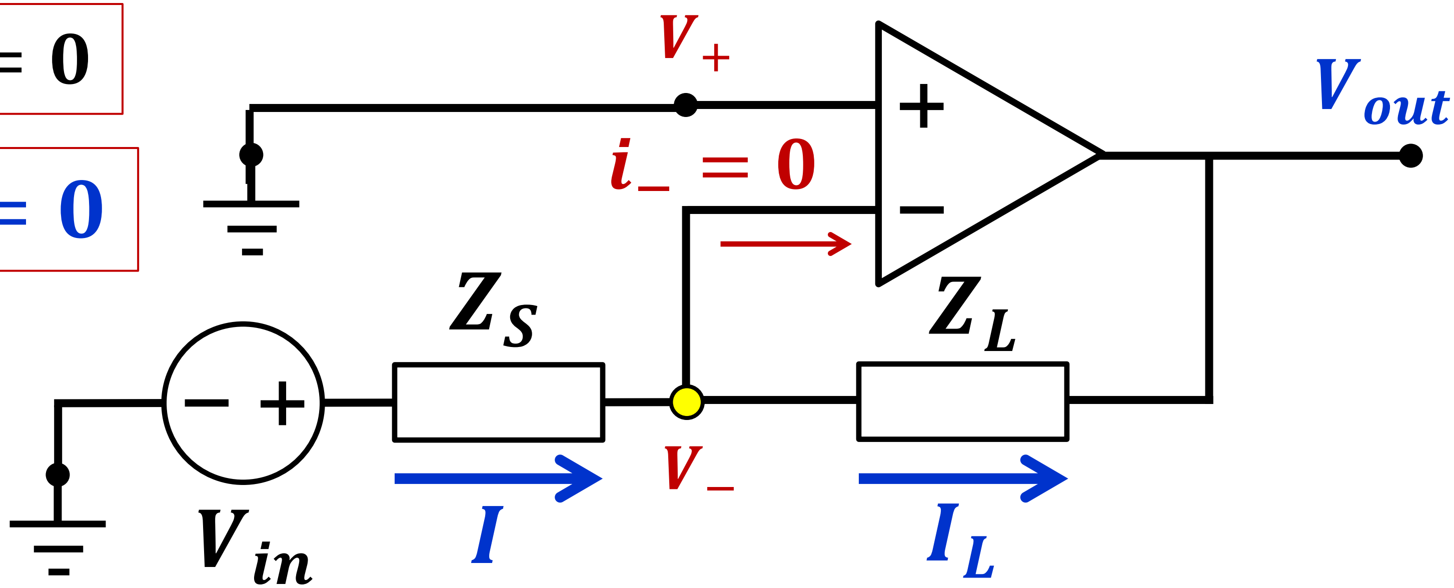


Now, voltages and currents are phasors

OP AMP with impedances

$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$



Now, voltages and currents are phasors

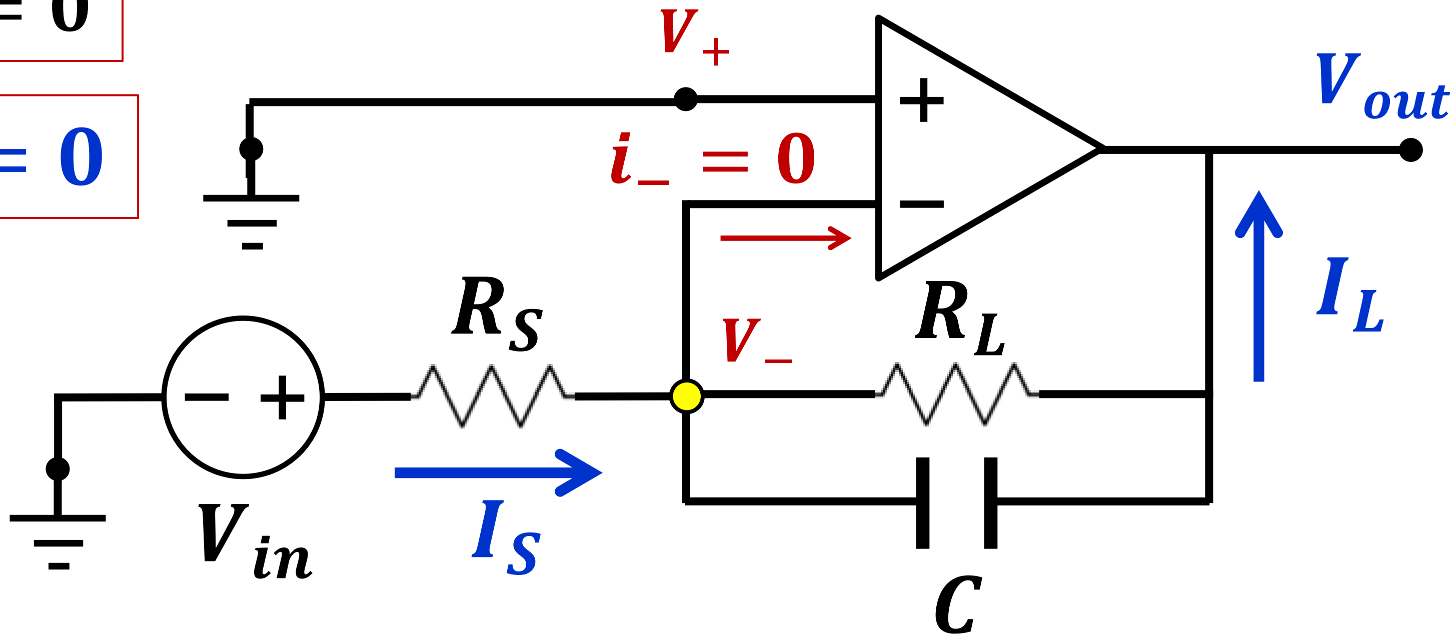
$$\frac{V_{in} - V_-}{Z_S} = \frac{V_- - V_{out}}{Z_L}$$

$$\mathrm{H}(\omega) = \frac{V_{out}}{V_{in}} = -\frac{Z_L}{Z_S}$$

RC circuit – Inverting Op Amp

$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$



$$Z_S = R_S$$

$$Z_L = R_L // \frac{1}{j\omega C} = \left[\frac{1}{R_L} + j\omega C\right]^{-1} = \frac{R_L}{1 + j\omega R_L C}$$

V_{in} = sinusoidal

$$Z_S = R_S$$

$$Z_L = R_L // \frac{1}{j\omega C} = \left[\frac{1}{R_L} + j\omega C\right]^{-1} = \frac{R_L}{1 + j\omega R_L C}$$

$$\mathrm{H}(\omega) = -\frac{Z_L}{Z_S} = -\frac{R_L/R_S}{1 + j\omega R_L C}$$

$$|\mathrm{H}(\omega)| = \frac{R_L/R_S}{\sqrt{1 + \omega^2 R_L^2 C^2}}$$

$$|\mathrm{H}(\omega)| = \frac{R_L/R_S}{\sqrt{1 + \omega^2 R_L^2 C^2}}$$