ECE 205 “Electrical and Electronics Circuits”

Spring 2024 – LECTURE 38

MWF – 12:00pm

Prof. Umberto Ravaioli

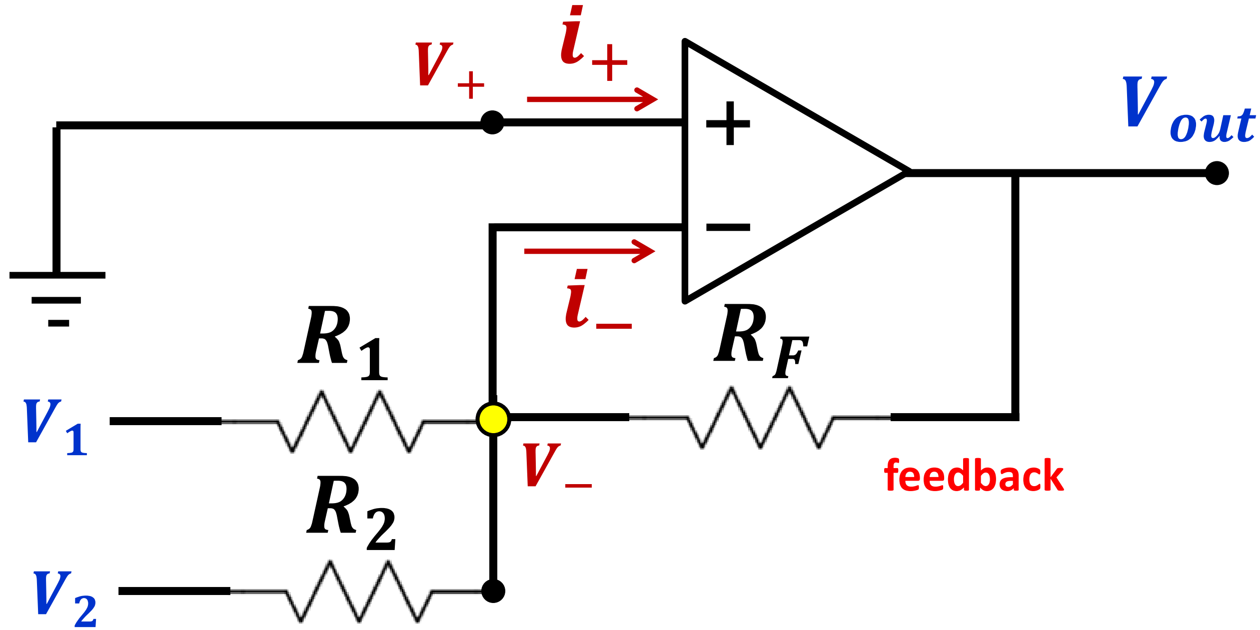
2062 ECE Building

Lecture 38 – Summary

Learning Objectives

1. More Operational Amplifiers examples

Adder OP AMP



$V_+ = V_- = 0$

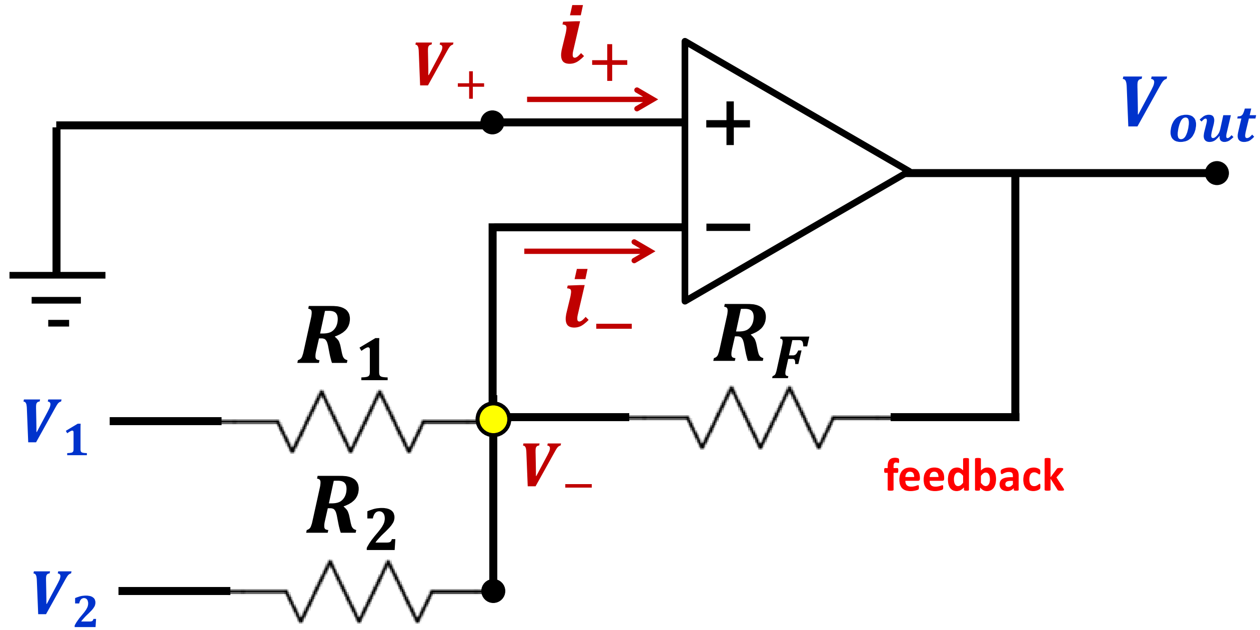
$i_+ = i_- = 0$

Node V_- ●

$$\frac{V_1 - V_-}{R_1} + \frac{V_2 - V_-}{R_2} + \frac{V_{out} - V_-}{R_F} = 0$$

$$\frac{V_1}{R_1} + \frac{V_2}{R_2} = -\frac{V_{out}}{R_F}$$

Adder OP AMP



$$V_+ = V_- = 0$$

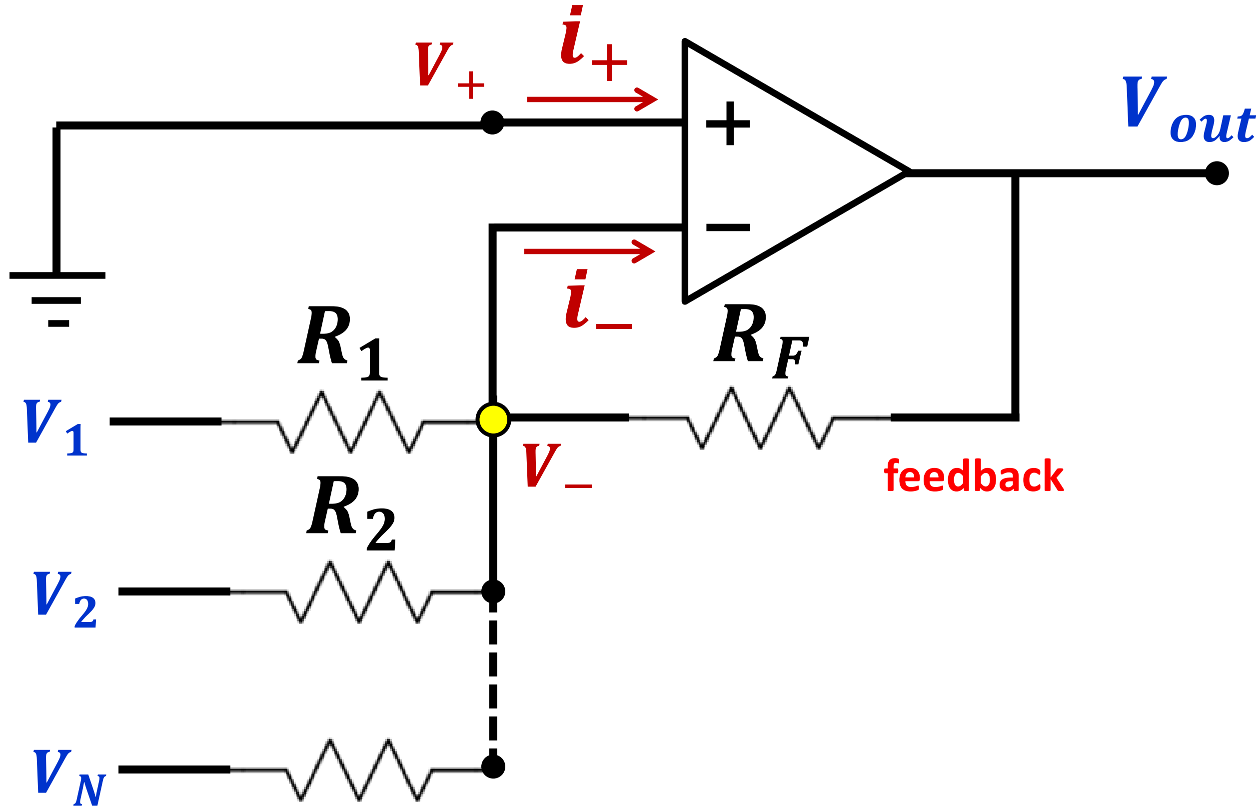
$$i_+ = i_- = 0$$

$$V_{out} = -R_F\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

If $R_1 = R_2 = R_F$

$$\Rightarrow \boxed{V_{out} = -[V_1 + V_2]}$$

Adder OP AMP



$V_+ = V_- = 0$

$i_+ = i_- = 0$

For N inputs

$$V_{out} = -R_F \sum_{k=1}^{N} \frac{V_k}{R_k}$$

If $R_k = R_F$

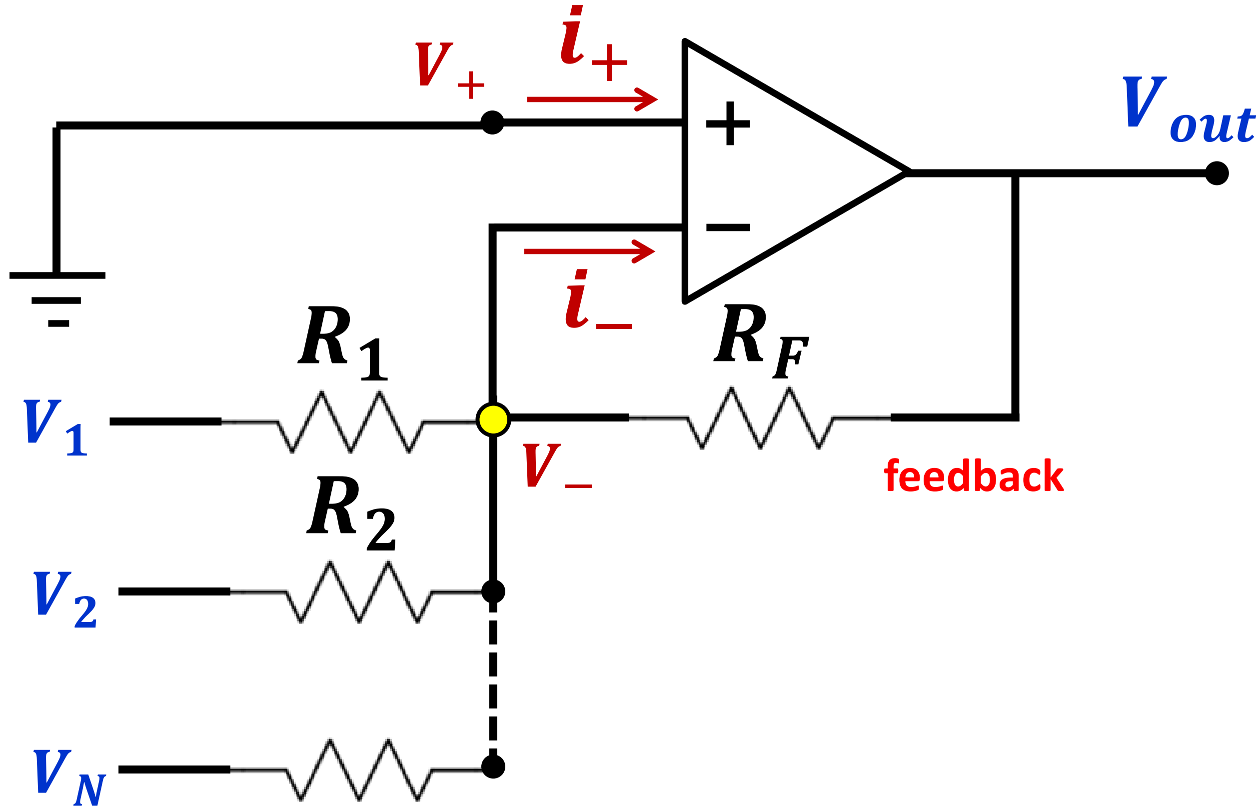
resistors all equal

$$V_{out} = -\sum_{k=1}^{N} V_k$$

Adder OP AMP

$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$



But one needs to verify that the output voltage does not exceed the rail bias

$$|V_{out}| \leq |V_{CC}|$$

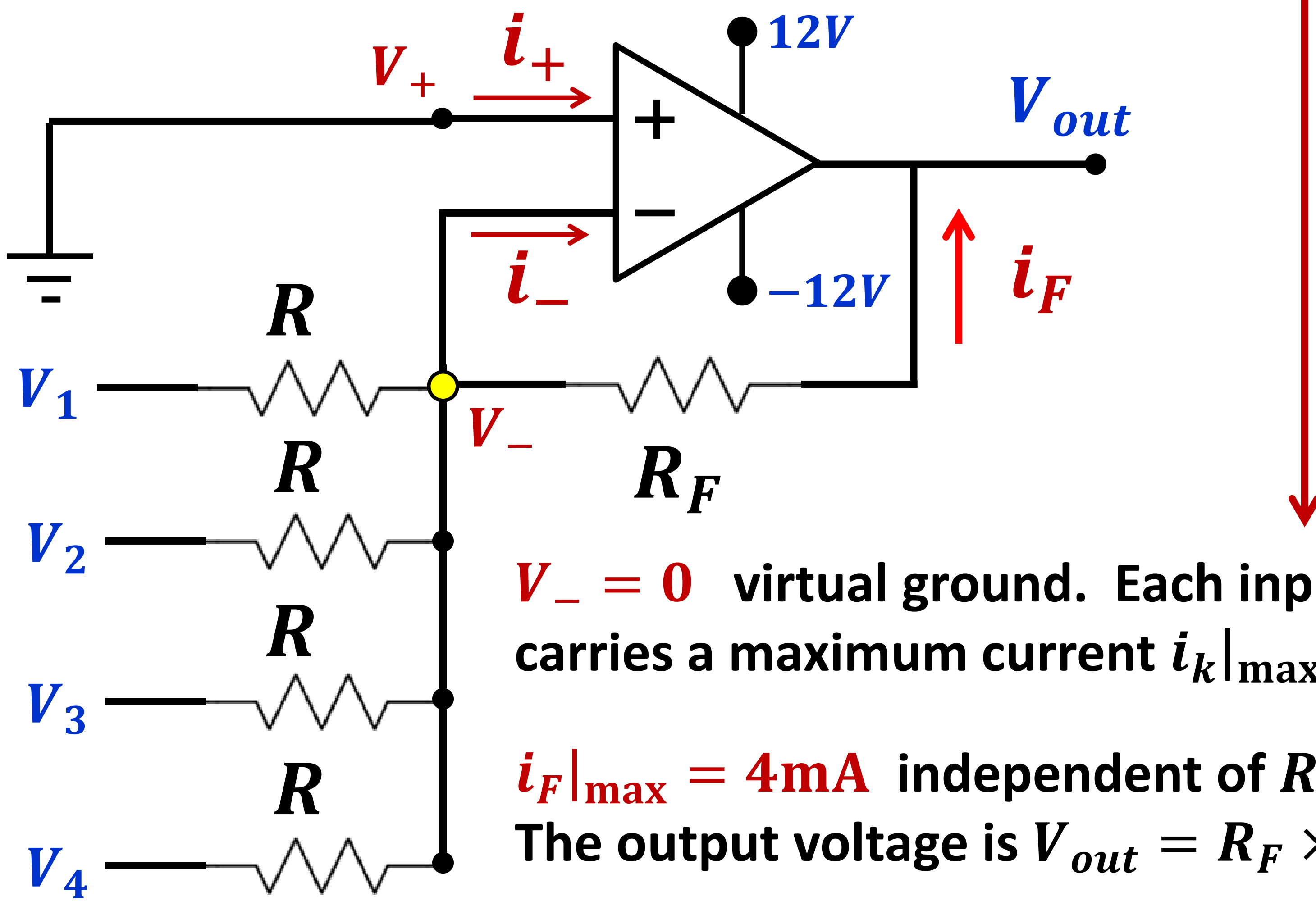
Example – Four equal resistors in input

$V_+ = V_- = 0$

$i_+ = i_- = 0$

$$R = 1\text{k}\Omega$$

$$V_i\Big|_{\max} = 1\text{V}$$



$V_- = 0$ virtual ground. Each input resistor carries a maximum current $i_k|_{\max} = 1\text{mA}$

$i_F|_{\max} = 4\text{mA}$ independent of R_F.
The output voltage is $V_{out} = R_F \times i_F|_{\max}$

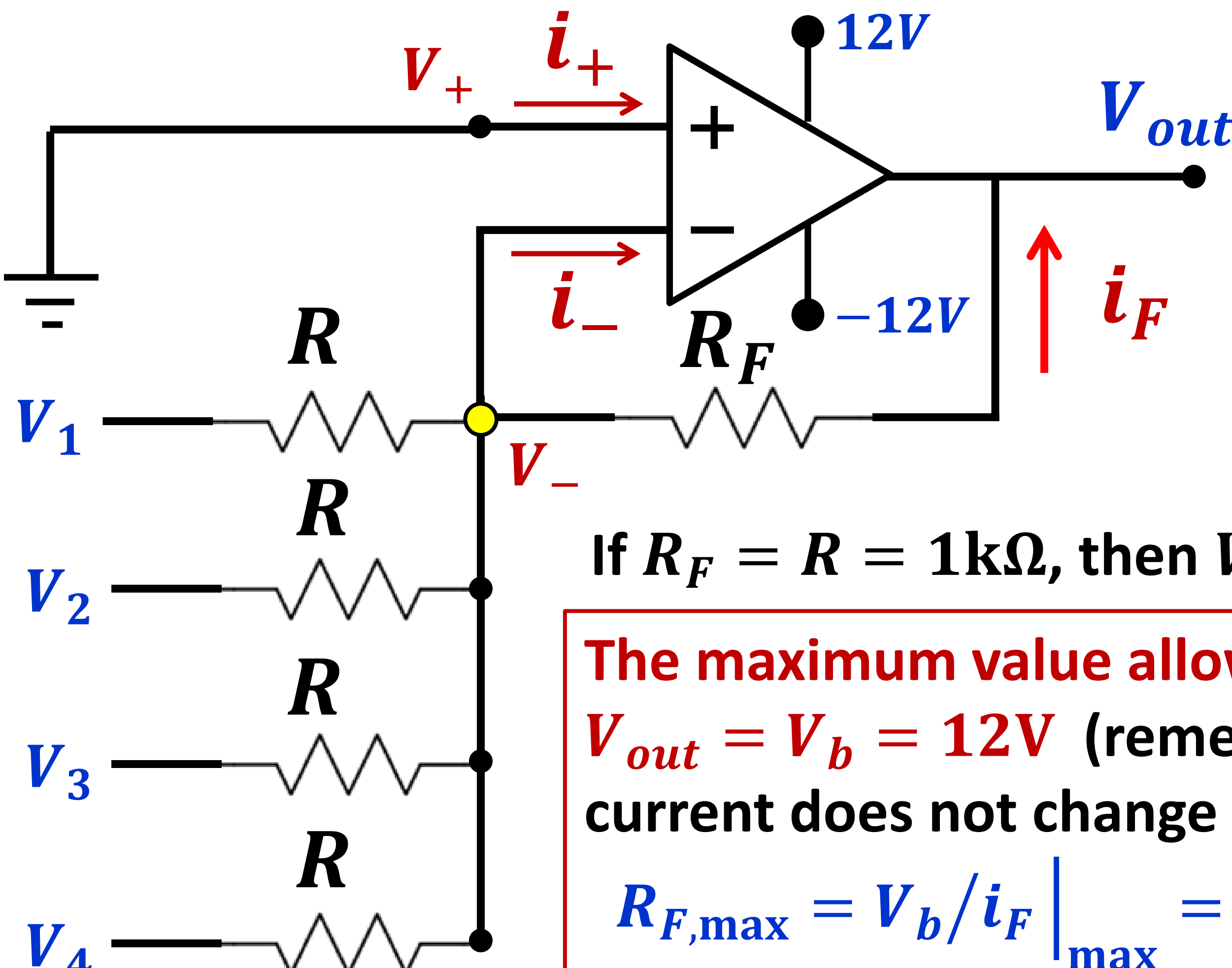
Example – Equal input resistors

$V_+ = V_- = 0$

$i_+ = i_- = 0$

$R = 1\text{k}\Omega$

$V_i\big|_{\max} = 1\text{V}$

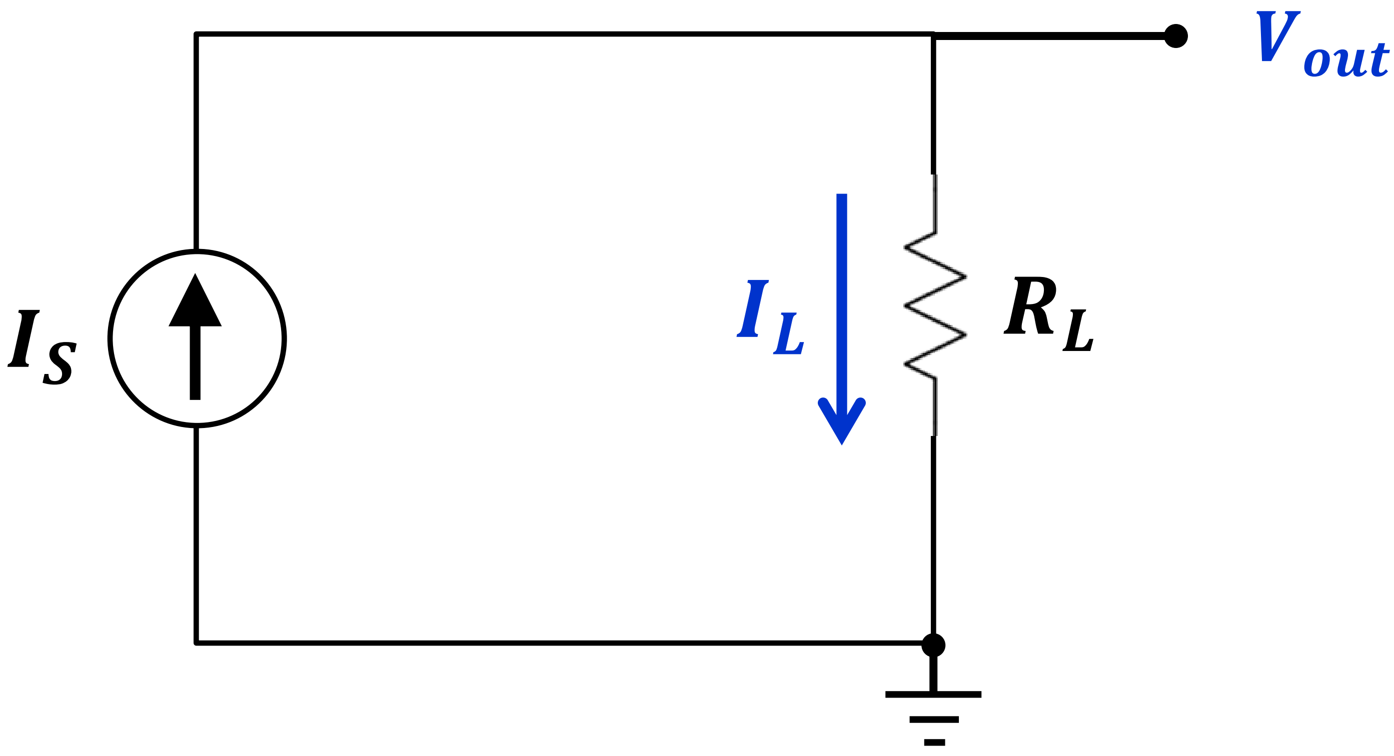


If $R_F = R = 1\text{k}\Omega$, then $V_{out} = 4\text{V}$ at most.

The maximum value allowed for R_F is when $V_{out} = V_b = 12\text{V}$ (remember, the input current does not change with R_F)

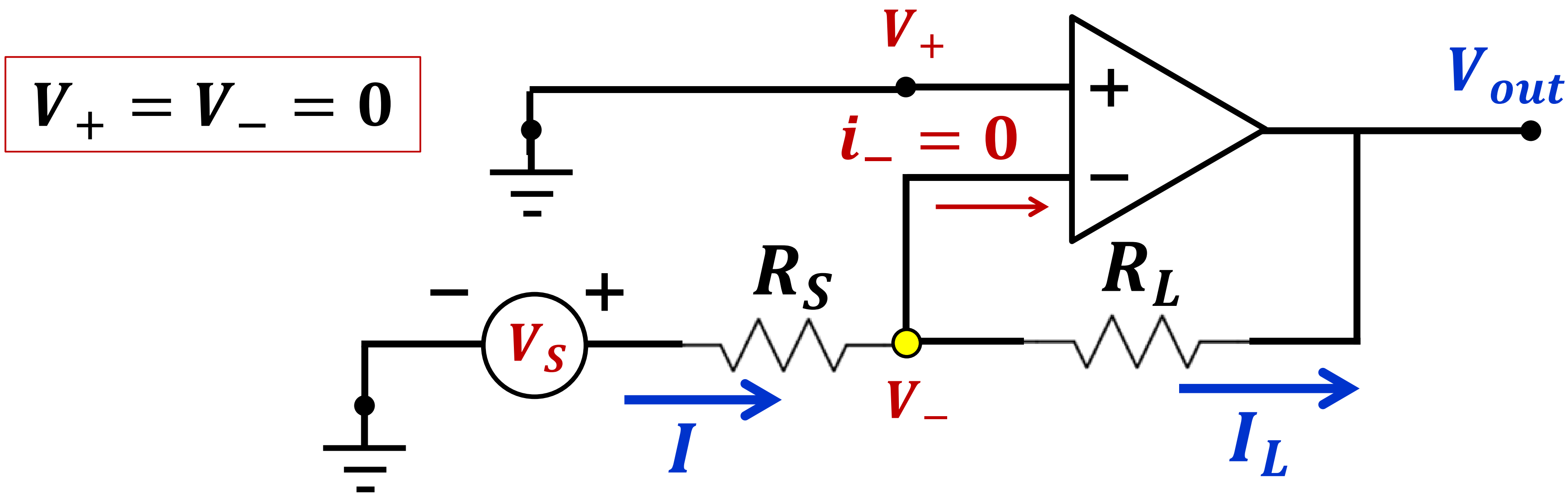
$$R_{F,\max} = V_b / i_F\big|_{\max} = 12\text{V}/4\text{mA} = 3\text{k}\Omega$$

Ideal Current Source



The current generated should be constant, independently of the load R_L

OP AMP Current Source



$$I = \overbrace{i_-}^{=0} + I_L = I_L$$

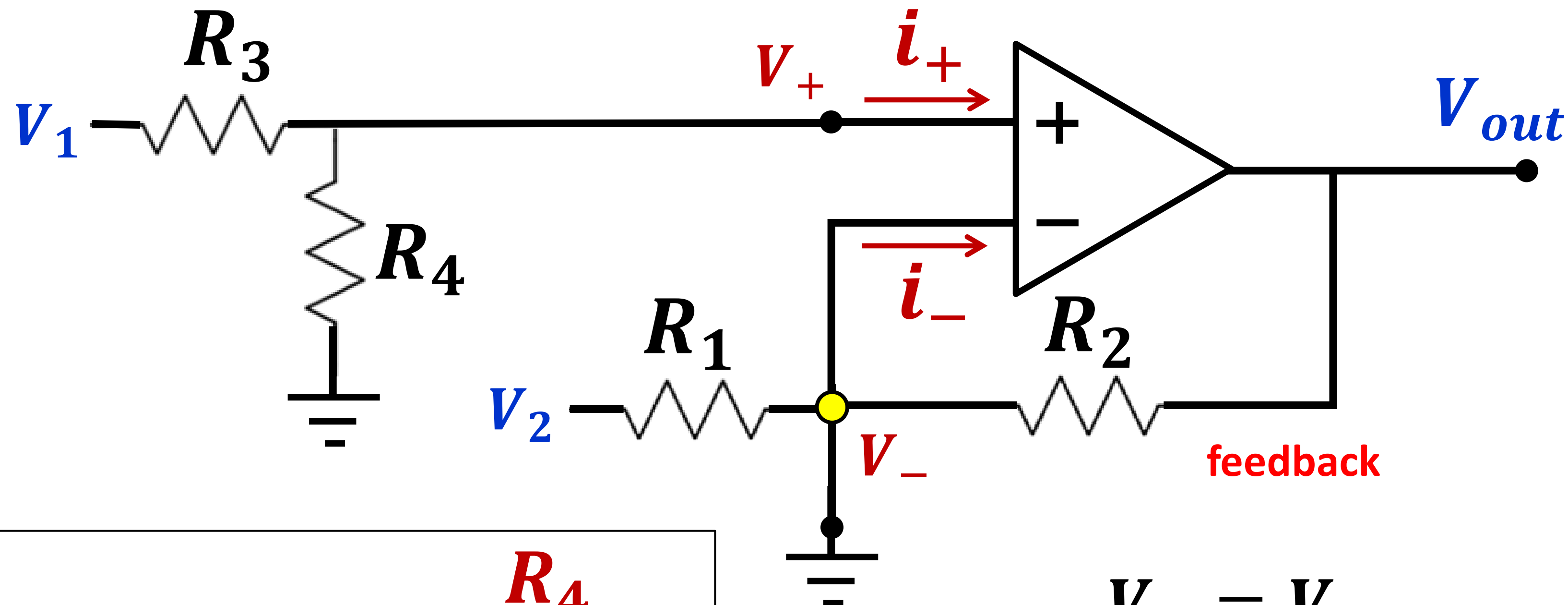
$$I_L = \frac{V_S - V_-}{R_S} = \frac{V_S}{R_S}$$

Example: $V_S = 1\text{V};\ R_S = 1\text{k}\Omega$

⇨ $$I_L = \frac{1 - 0}{1\text{k}\Omega} = 1\text{mA}$$

Independent of R_L

Differential OP AMP



$$V_+ = V_- = V_1 \frac{R_4}{R_3 + R_4}$$

$$V_+ = V_-$$

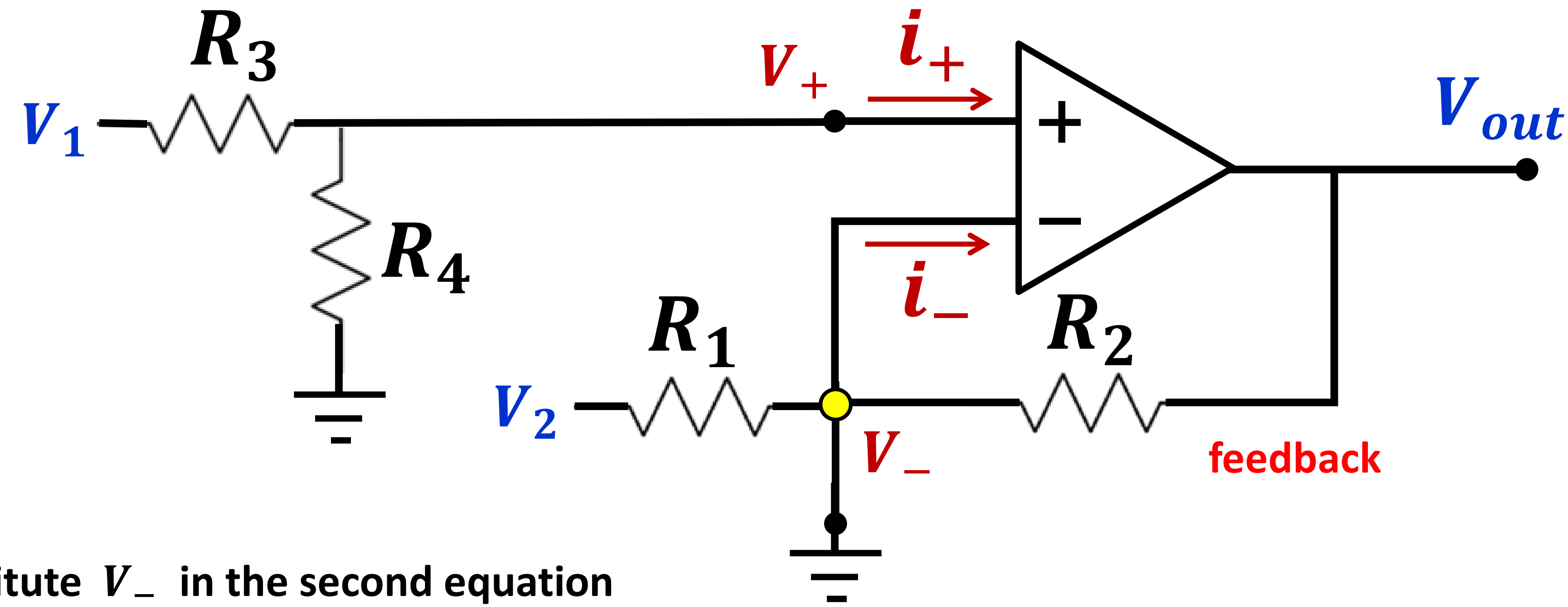
$$i_+ = i_- = 0$$

node

$$\frac{V_- - V_2}{R_1} + \frac{V_- - V_{out}}{R_2} = 0$$

$$\Rightarrow \quad V_{out} = \frac{R_2}{R_1}(V_- - V_2) + V_-$$

Differential OP AMP



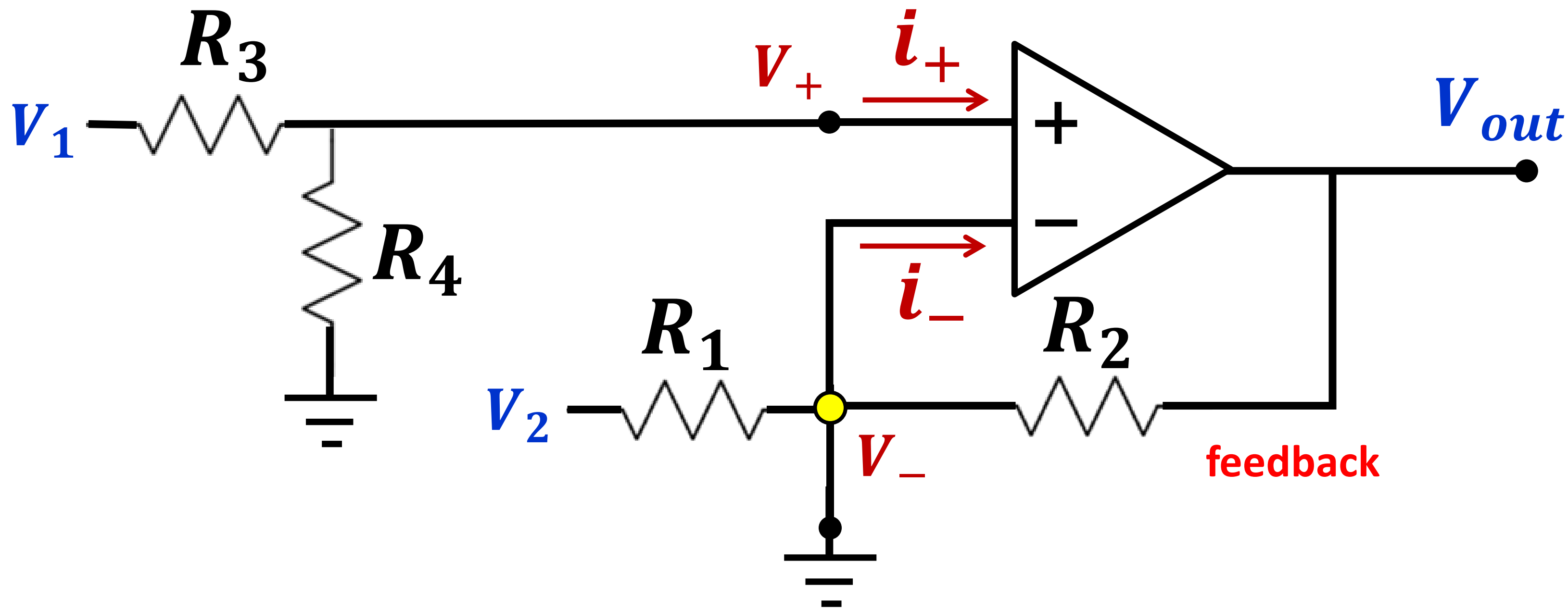
Substitute V_- in the second equation

$$V_+ = V_- = V_1 \frac{R_4}{R_3 + R_4}$$

$$V_{out} = \frac{R_2}{R_1}(V_- - V_2) + V_-$$

$$V_{out} = \frac{R_2}{R_1}\left(V_1 \frac{R_4}{R_3 + R_4} - V_2\right) + V_1 \frac{R_4}{R_3 + R_4}$$

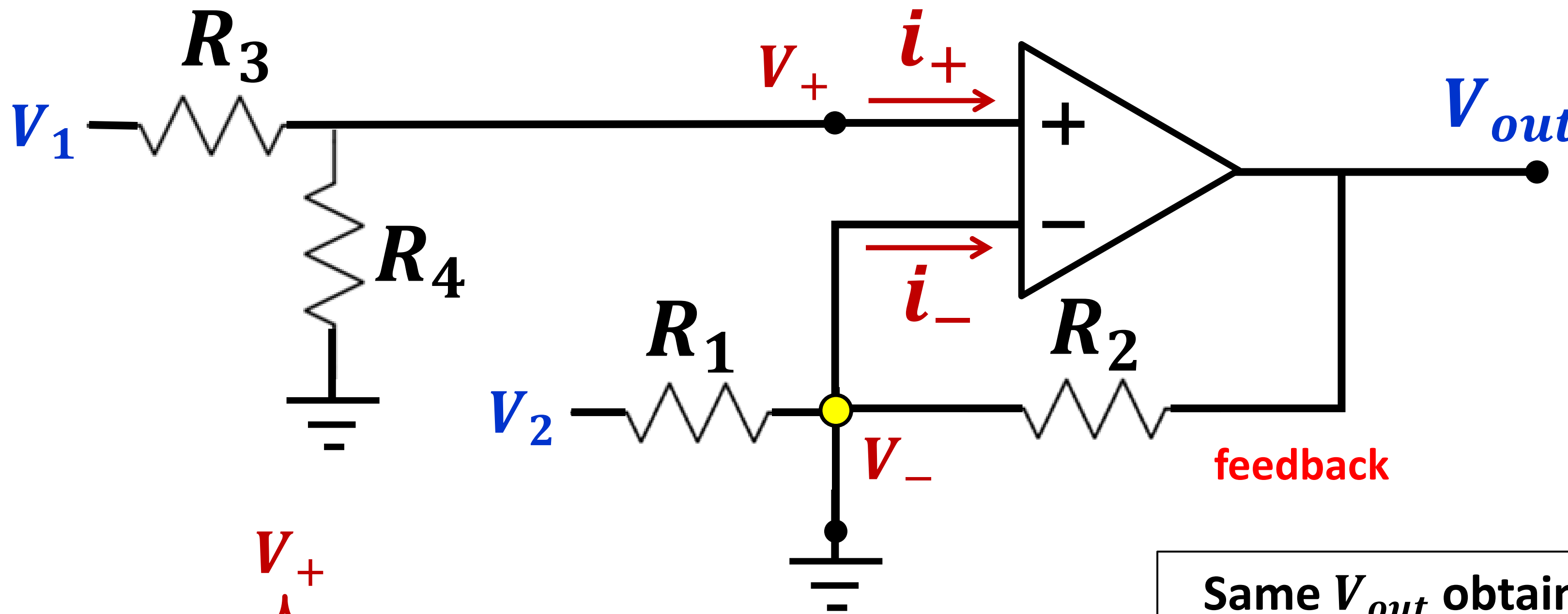
Differential OP AMP



$$V_{out} = \frac{R_2}{R_1}\left(V_1 \frac{R_4}{R_3 + R_4} - V_2\right) + V_1 \frac{R_4}{R_3 + R_4}$$

$$V_{out} = \frac{R_4}{R_3 + R_4}\left(\frac{R_2}{R_1} + 1\right) V_1 - \frac{R_2}{R_1} V_2$$

Differential OP AMP

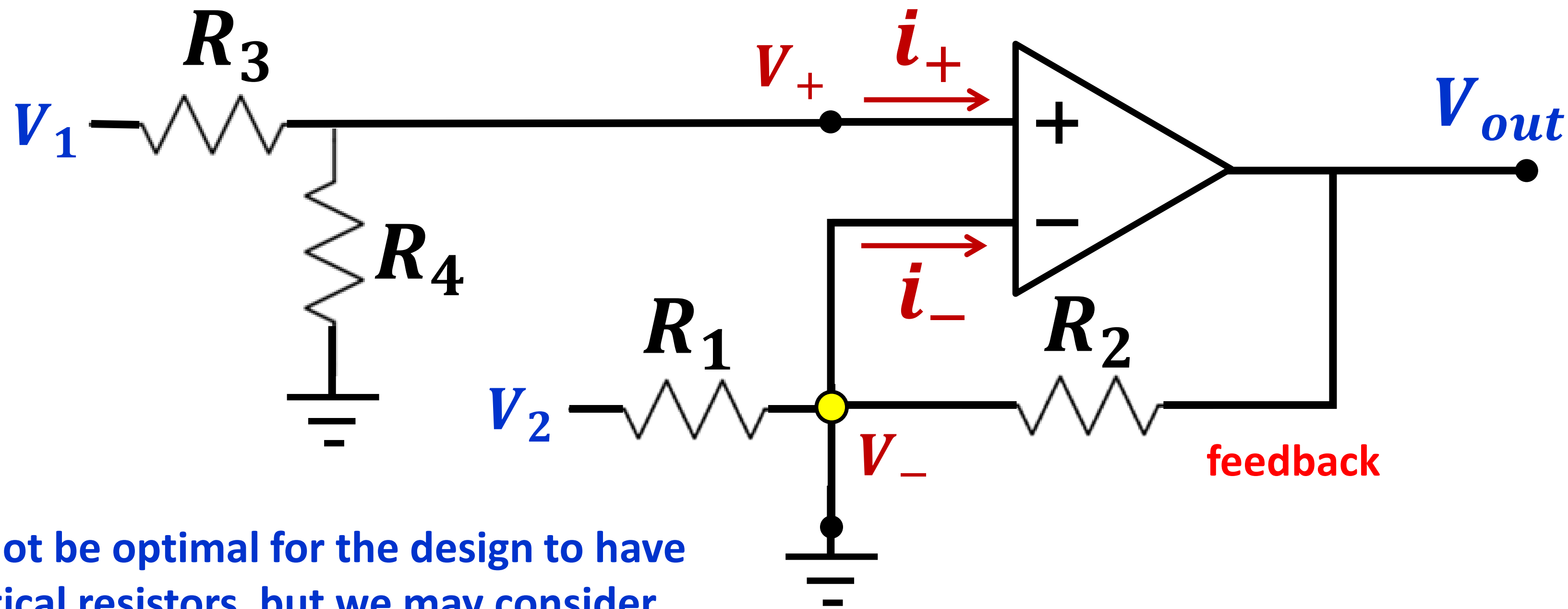


$$V_{out} = \underbrace{\overbrace{V_1 \frac{R_4}{R_3 + R_4}}^{V_+}\left(1 + \frac{R_2}{R_1}\right)}_{V_1 \text{ contribution}} \underbrace{- V_2 \frac{R_2}{R_1}}_{V_2 \text{ contribution}}$$

Same V_{out} obtained by superposition of results with inverting and non-inverting amplifier formulas

If $R_1 = R_2 = R_3 = R_4 = R$ ⇨ $V_{out} = [V_1 - V_2]$

Differential OP AMP



It may not be optimal for the design to have all identical resistors, but we may consider pairs of resistors with identical ratio

If: $\dfrac{R_1}{R_2} = \dfrac{R_3}{R_4}$

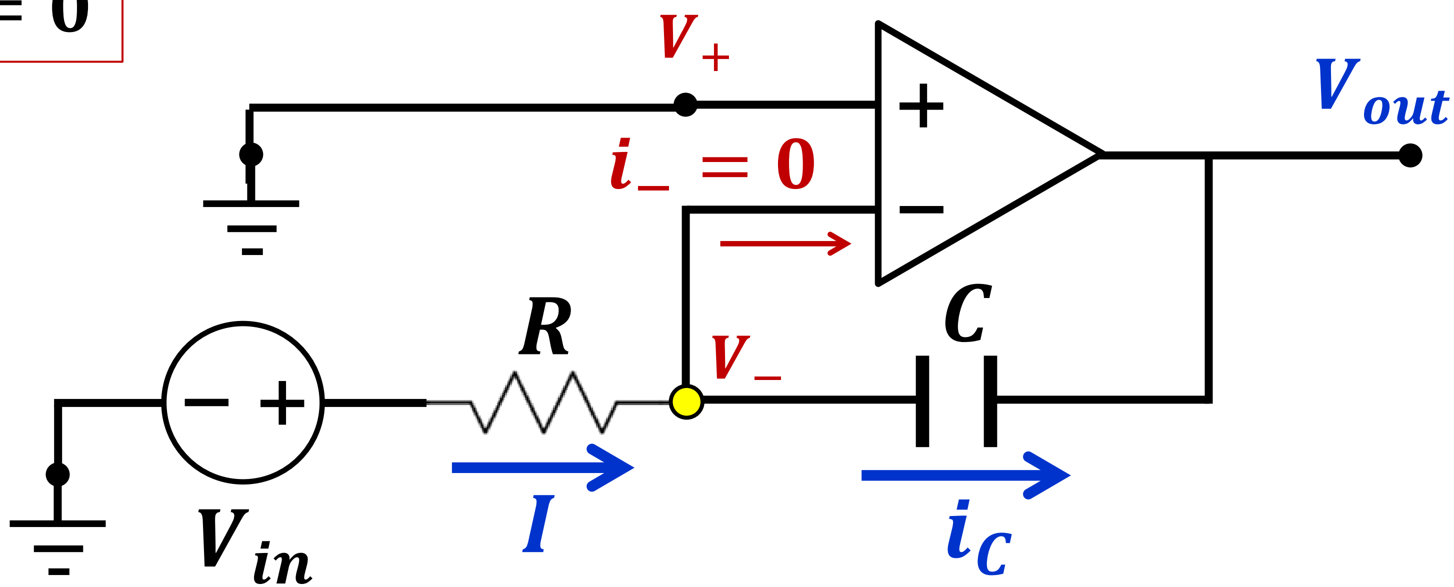
$$V_0 = \frac{R_4}{R_3 + R_4}\left(\frac{R_2}{R_1} + 1\right)V_1 - \frac{R_2}{R_1}V_2$$

$$V_0 = \frac{R_4}{R_3 + R_4}\left(\frac{R_4 + R_3}{R_3}\right)V_1 - \frac{R_2}{R_1}V_2 \Rightarrow V_0 = \frac{R_2}{R_1}(V_1 - V_2)$$

OP AMP Integrator

$$V_+ = V_- = 0$$

$$I = i_C$$



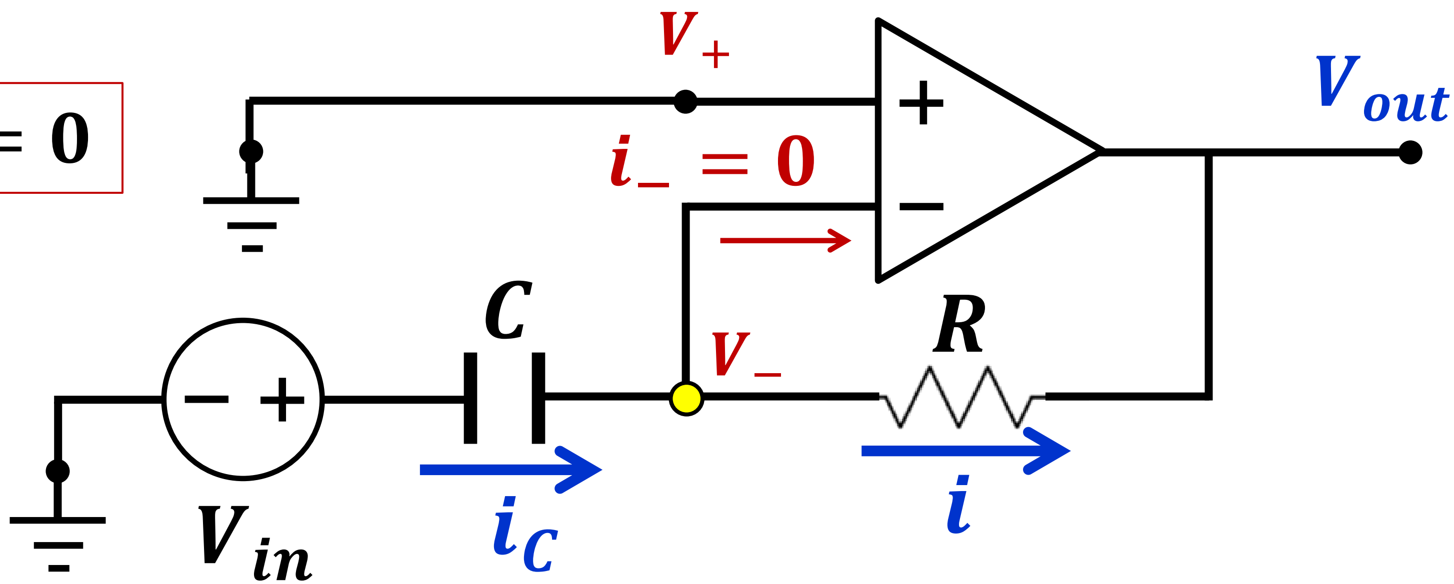
$$V_{out}(t) = -\frac{1}{C}\int_0^t i_C \, dt + V_{out}(0)$$

$$i_C = \frac{V_{in}}{R}$$

$$\Rightarrow \quad V_{out}(t) = -\frac{1}{RC}\int_0^t V_{in}(t) \, dt + V_{out}(0)$$

OP AMP differentiator

$$V_+ = V_- = 0$$



$$i_C = i = (V_{out} - V_-)/R$$

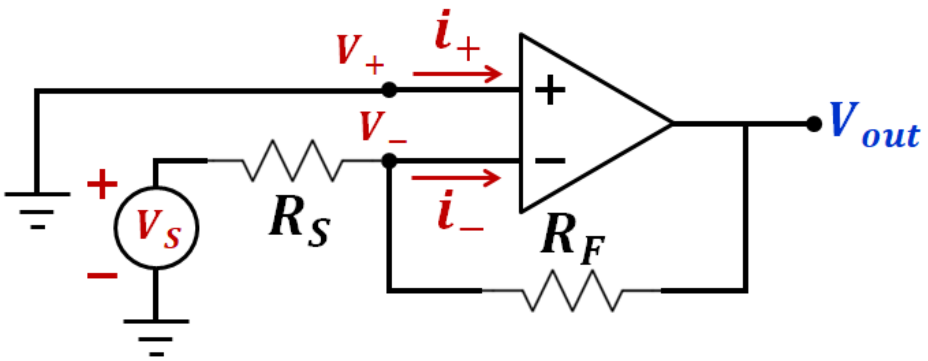
$$i_C = C\frac{d}{dt}V_{in}(t)$$

$$V_{out} = -Ri_C = -RC\frac{d}{dt}V_{in}(t)$$

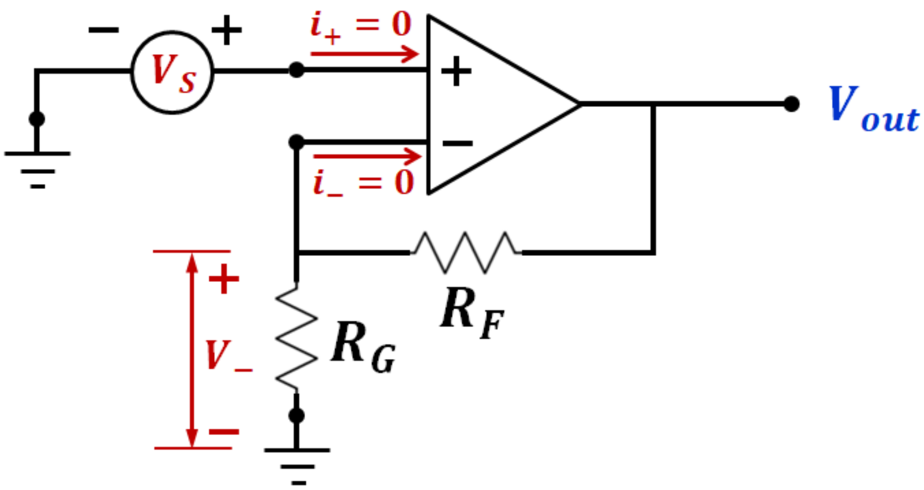
⇨ **This basic circuit is typically sensitive to fluctuations and may not be very stable due to noise amplification.**

Recall the basic Op Amp results

Inverting Amplifier



Non-inverting Amplifier



$$A_{VF} = -\frac{R_F}{R_S}$$

$$A_{VF} = 1 + \frac{R_F}{R_G}$$

Similar relationships are established in the OP AMP when simple resistors are replaced with impedances

- ***Inverting amplifier***

$$A_{VF} = \frac{V_{out}}{V_S} = -\frac{Z_F}{Z_S}$$

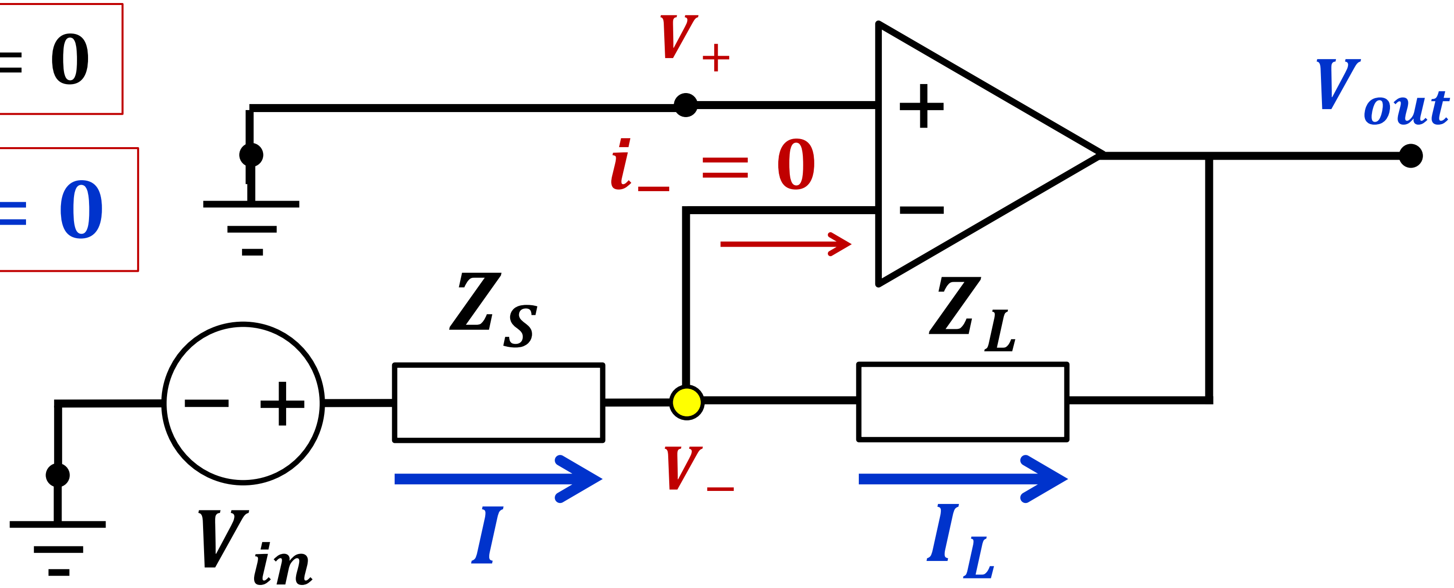
- ***Non-inverting amplifier***

$$A_{VF} = \frac{V_{out}}{V_S} = \frac{Z_G + Z_F}{Z_G} = 1 + \frac{Z_F}{Z_G}$$

OP AMP with impedances

$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$



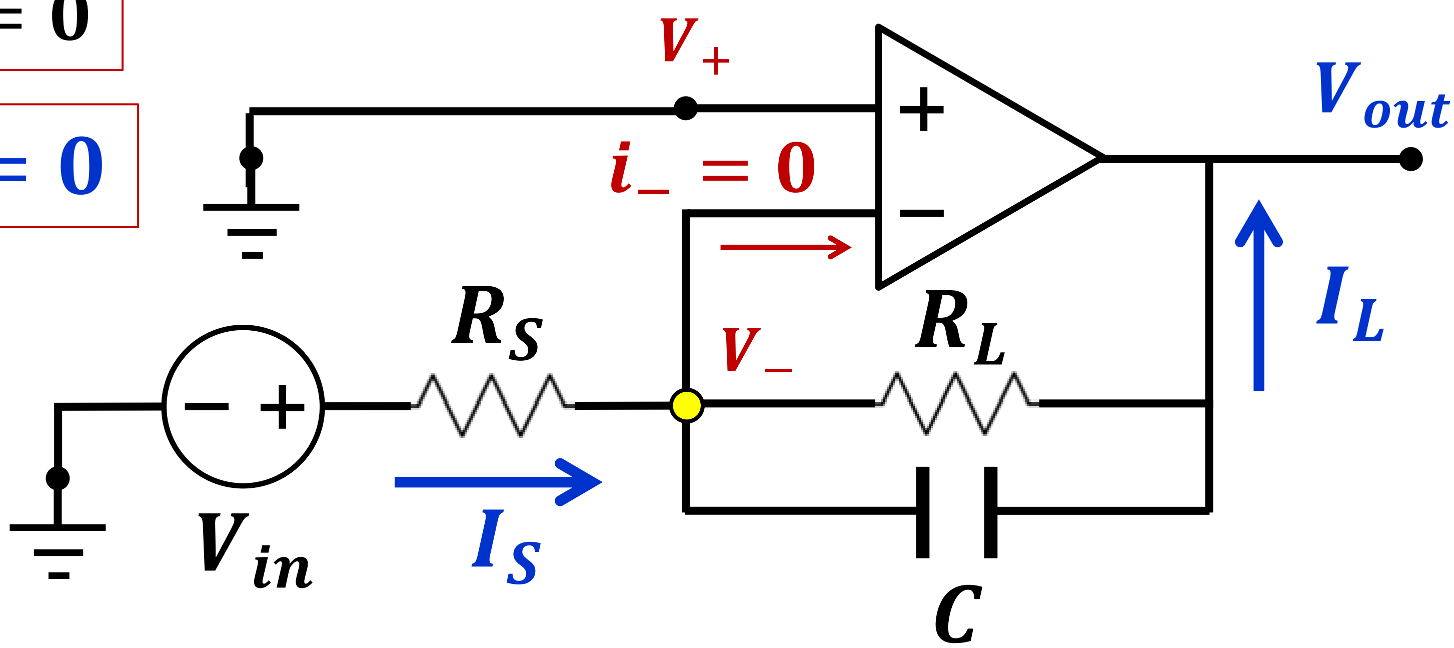
$$\frac{V_{in} - V_-}{Z_S} = \frac{V_- - V_{out}}{Z_L}$$

$$\mathrm{H}(\omega) = \frac{V_{out}}{V_{in}} = -\frac{Z_L}{Z_S}$$

RC circuit – Inverting Op Amp

$$V_+ = V_- = 0$$

$$i_+ = i_- = 0$$



$$Z_S = R_S$$

$$Z_L = R_L // \frac{1}{j\omega C} = \left[\frac{1}{R_L} + j\omega C\right]^{-1} = \frac{R_L}{1 + j\omega R_L C}$$

$$V_{in} = \text{sinusoidal}$$

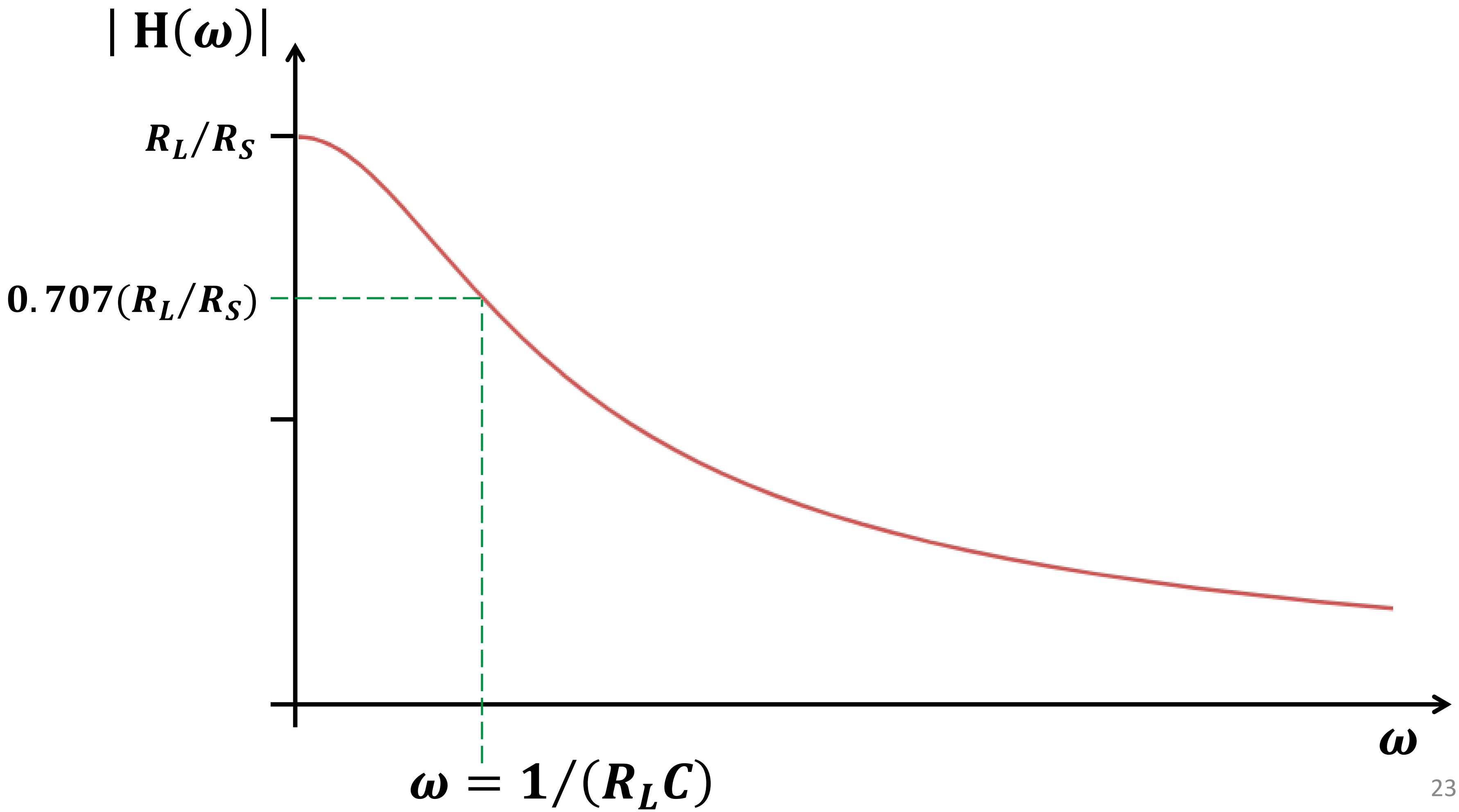
$$Z_S = R_S$$

$$Z_L = R_L // \frac{1}{\mathrm{j}\omega C} = \left[\frac{1}{R_L} + \mathrm{j}\omega C\right]^{-1} = \frac{R_L}{1 + j\omega R_L C}$$

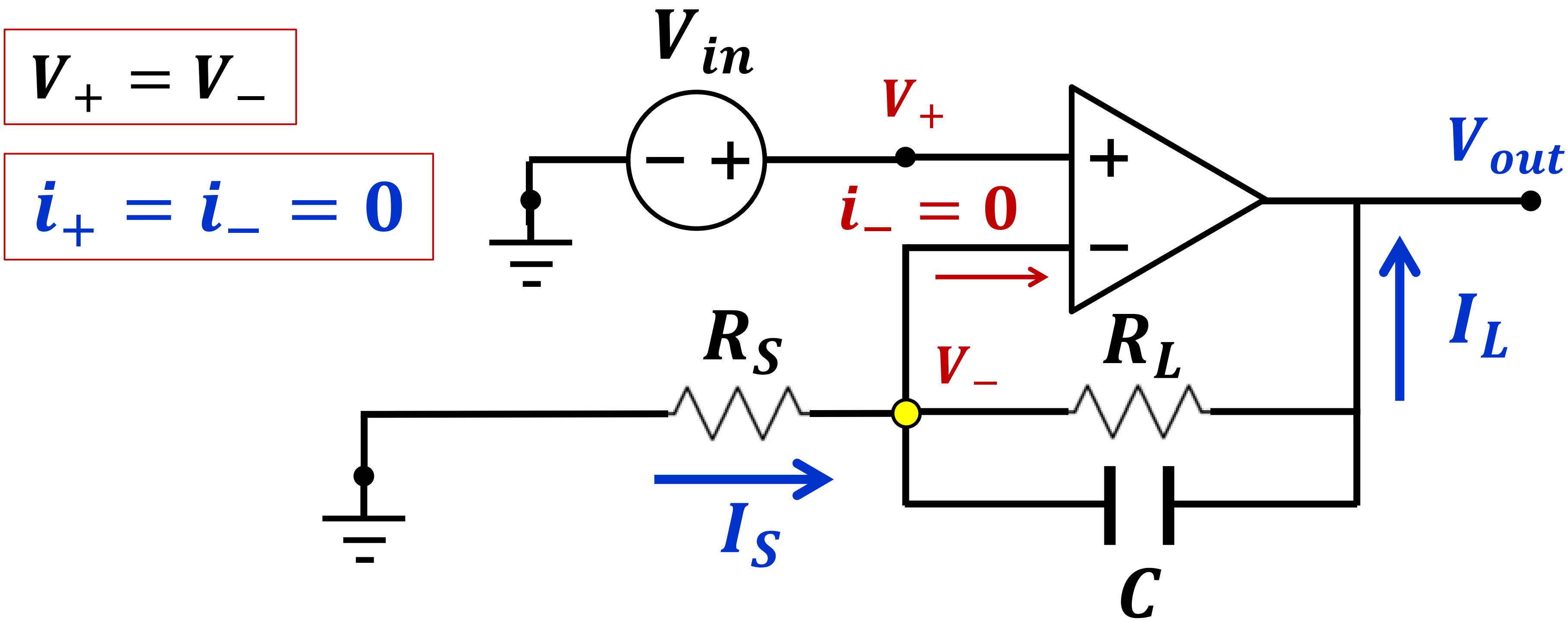
$$\mathrm{H}(\omega) = -\frac{Z_L}{Z_S} = -\frac{R_L/R_S}{1 + j\omega R_L C}$$

$$|\mathrm{H}(\omega)| = \frac{R_L/R_S}{\sqrt{1 + \omega^2 R_L^2 C^2}}$$

$$|\mathbf{H}(\omega)| = \frac{R_L/R_S}{\sqrt{1+\omega^2 R_L^2 C^2}}$$



RC circuit – Non-Inverting Op Amp



$$Z_S = R_S$$

$$Z_L = R_L // \frac{1}{\mathrm{j}\omega C} = \left[\frac{1}{R_L} + \mathrm{j}\omega C\right]^{-1} = \frac{R_L}{1 + j\omega R_L C}$$

V_{in} = sinusoidal

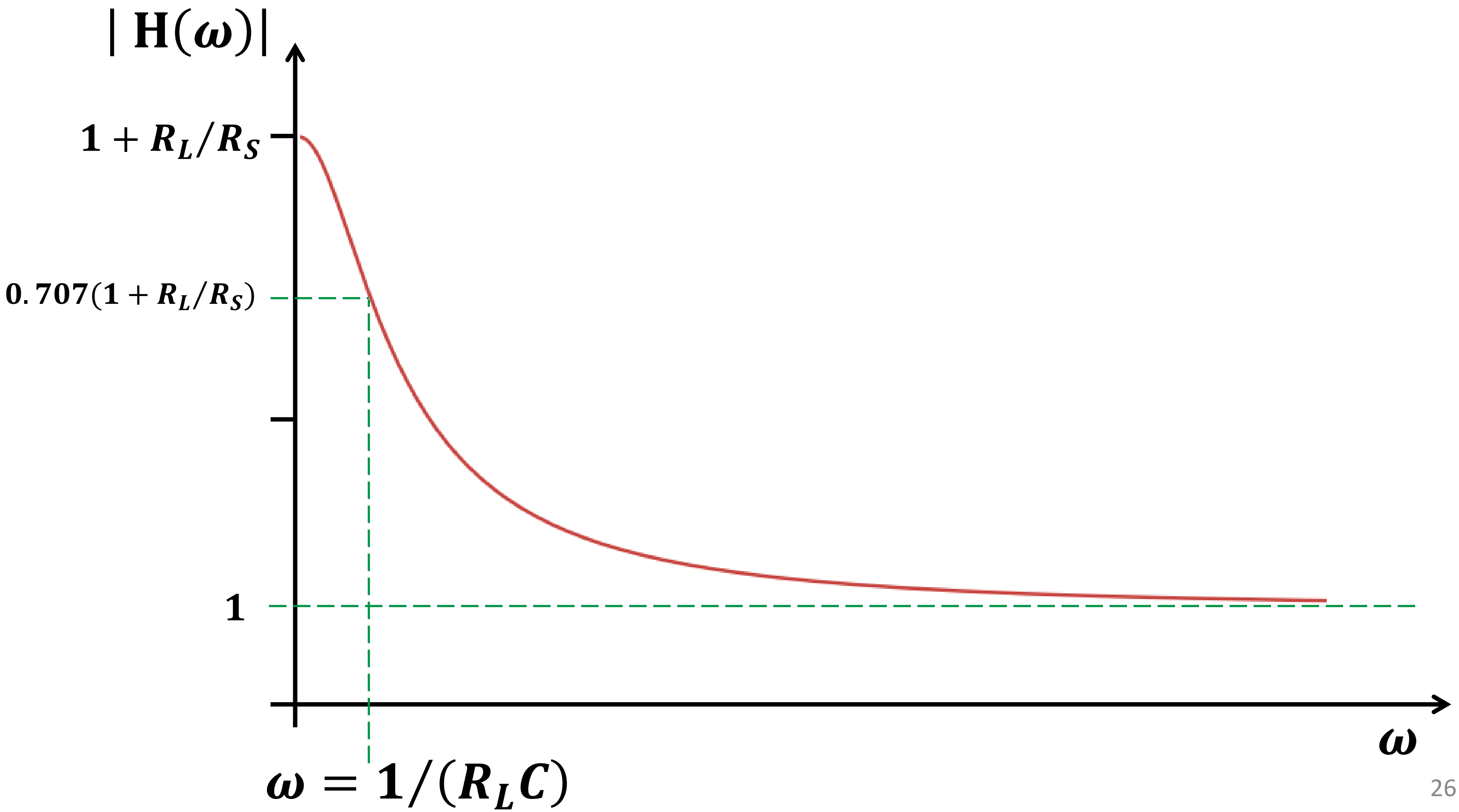
$$Z_S = R_S$$

$$Z_L = R_L // \frac{1}{j\omega C} = \left[\frac{1}{R_L} + j\omega C\right]^{-1} = \frac{R_L}{1 + j\omega R_L C}$$

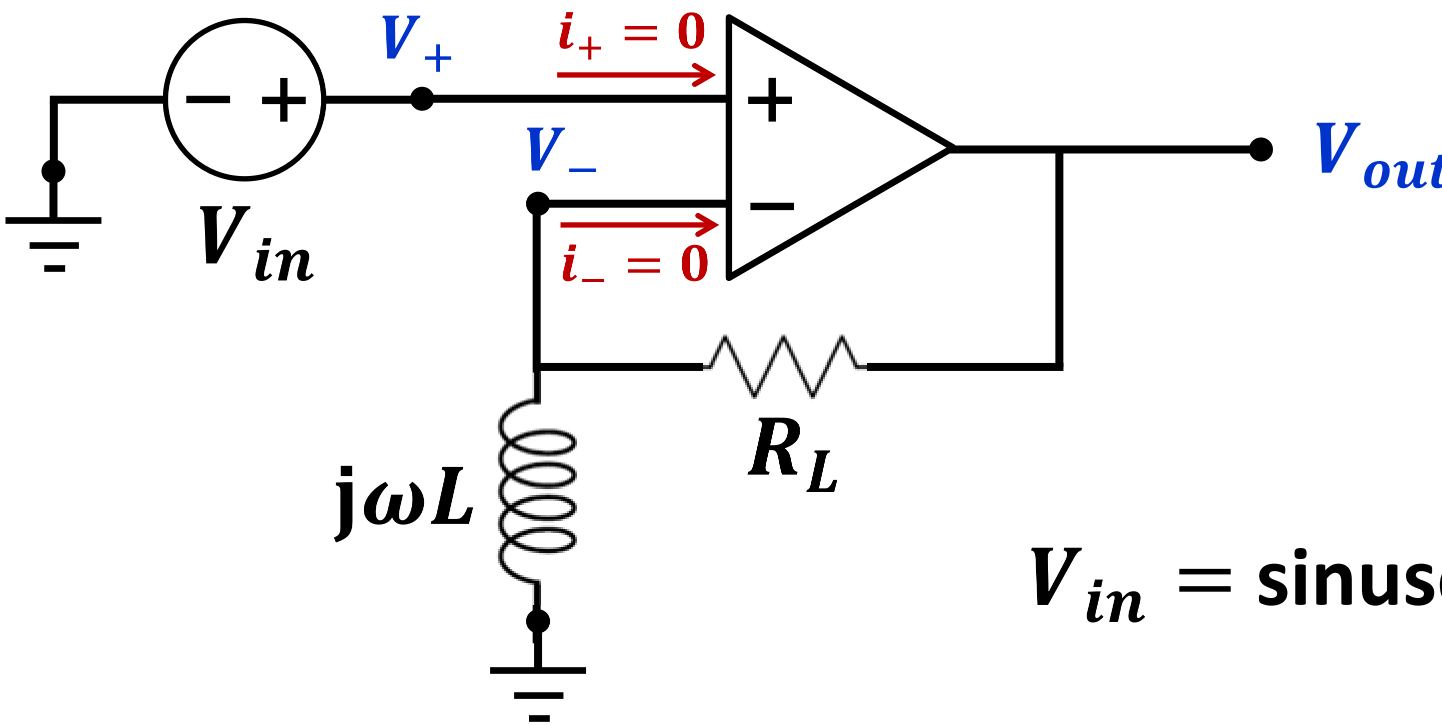
$$H(\omega) = 1 + \frac{Z_L}{Z_S} = 1 + \frac{R_L/R_S}{1 + j\omega R_L C} = \frac{1 + R_L/R_S + j\omega R_L C}{1 + j\omega R_L C}$$

$$|H(\omega)| = \frac{\sqrt{(1 + R_L/R_S)^2 + \omega^2 R_L^2 C^2}}{\sqrt{1 + \omega^2 R_L^2 C^2}}$$

$$|\mathbf{H}(\omega)| = \frac{\sqrt{(1 + R_L/R_S)^2 + \omega^2 R_L^2 C^2}}{\sqrt{1 + \omega^2 R_L^2 C^2}}$$



RL circuit – Non-inverting Op Amp



V_{in} = sinusoidal

$$V_+ = V_-$$

$$i_+ = i_- = 0$$

$$V_- = V_{in} = V_{out}\frac{j\omega L}{R_L + j\omega L}$$

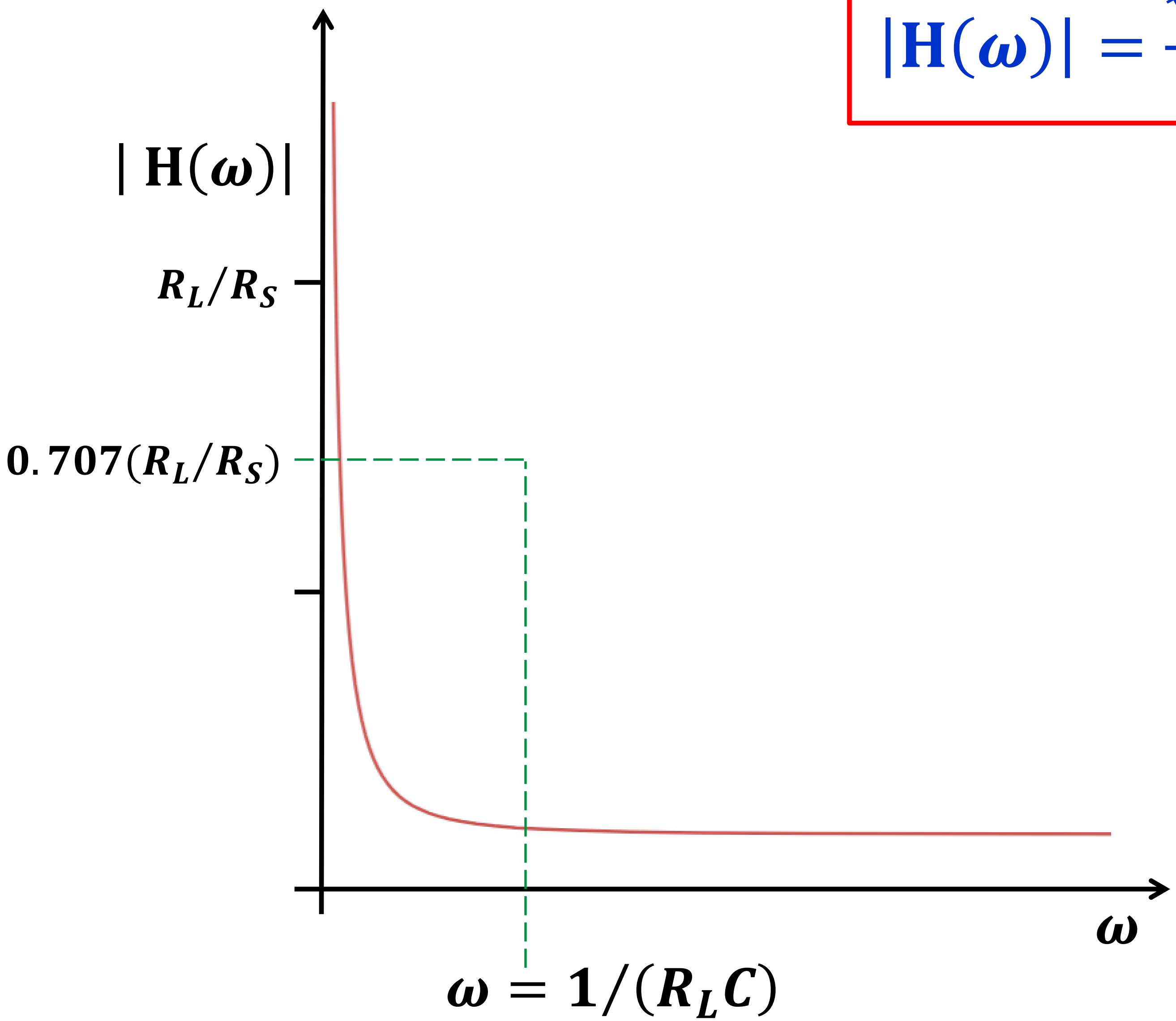
$$V_{-} = V_{in} = V_{out} \frac{j\omega L}{R_L + j\omega L}$$

$$H(\omega) = \frac{V_{out}}{V_{in}} = \frac{R + j\omega L}{j\omega L} = 1 - j\frac{R_L}{\omega L}$$

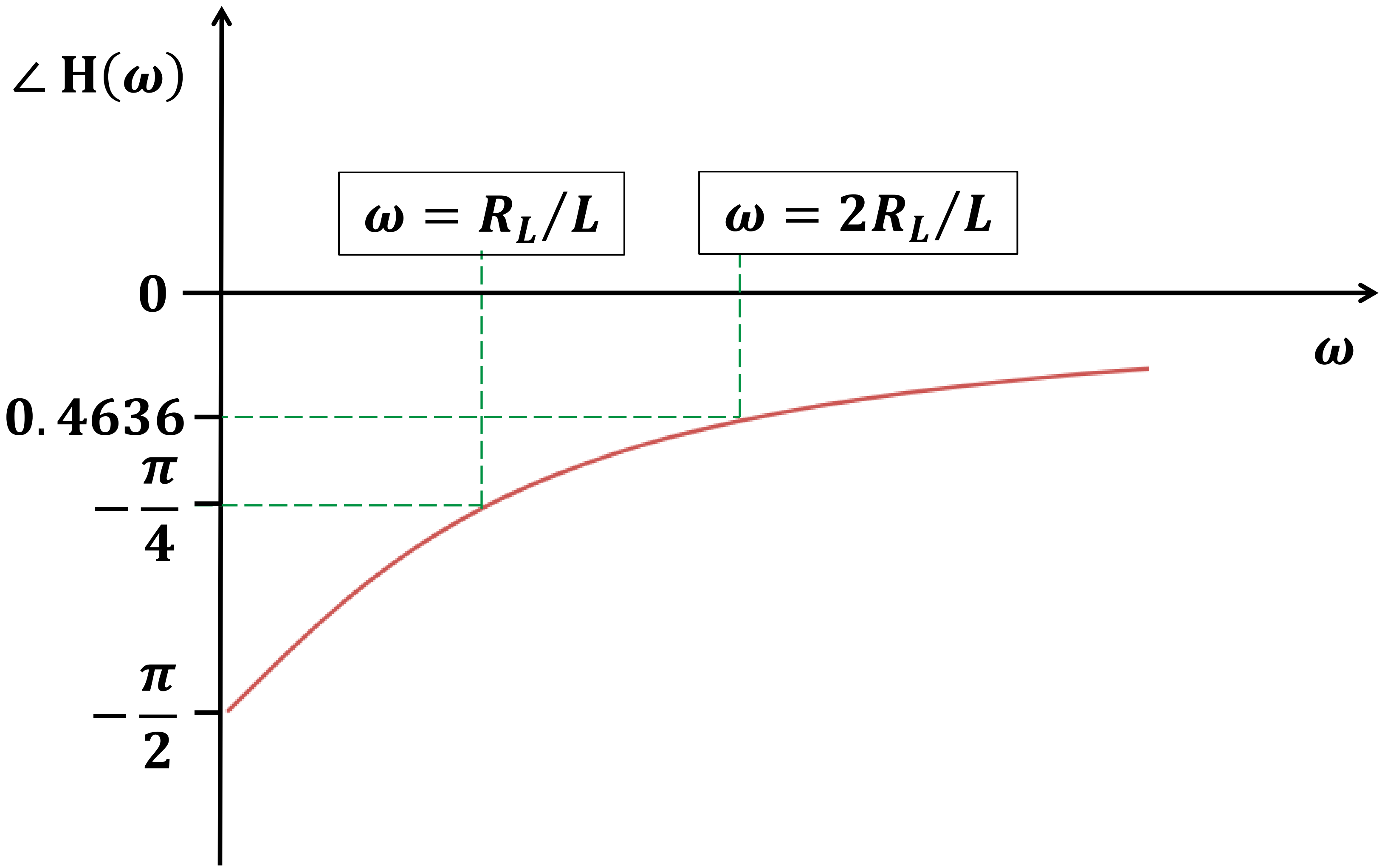
$$|H(\omega)| = \frac{\sqrt{R_L^2 + \omega^2 L^2}}{\omega L}$$

$$\angle H(\omega) = -\tan^{-1}\left(\frac{R_L}{\omega L}\right)$$

$$|\mathbf{H}(\omega)| = \frac{\sqrt{R_L^2 + \omega^2 L^2}}{\omega L}$$



$$\angle \mathbf{H}(\omega) = -\tan^{-1}\left(\frac{R}{\omega L}\right)$$



The phase of $\mathrm{H}(\omega)$ causes a time shift. Example:

$$V_S = \sin(2t) \qquad \omega = 2\ \mathrm{rad/sec}$$

$$R_L = 1.0\Omega$$

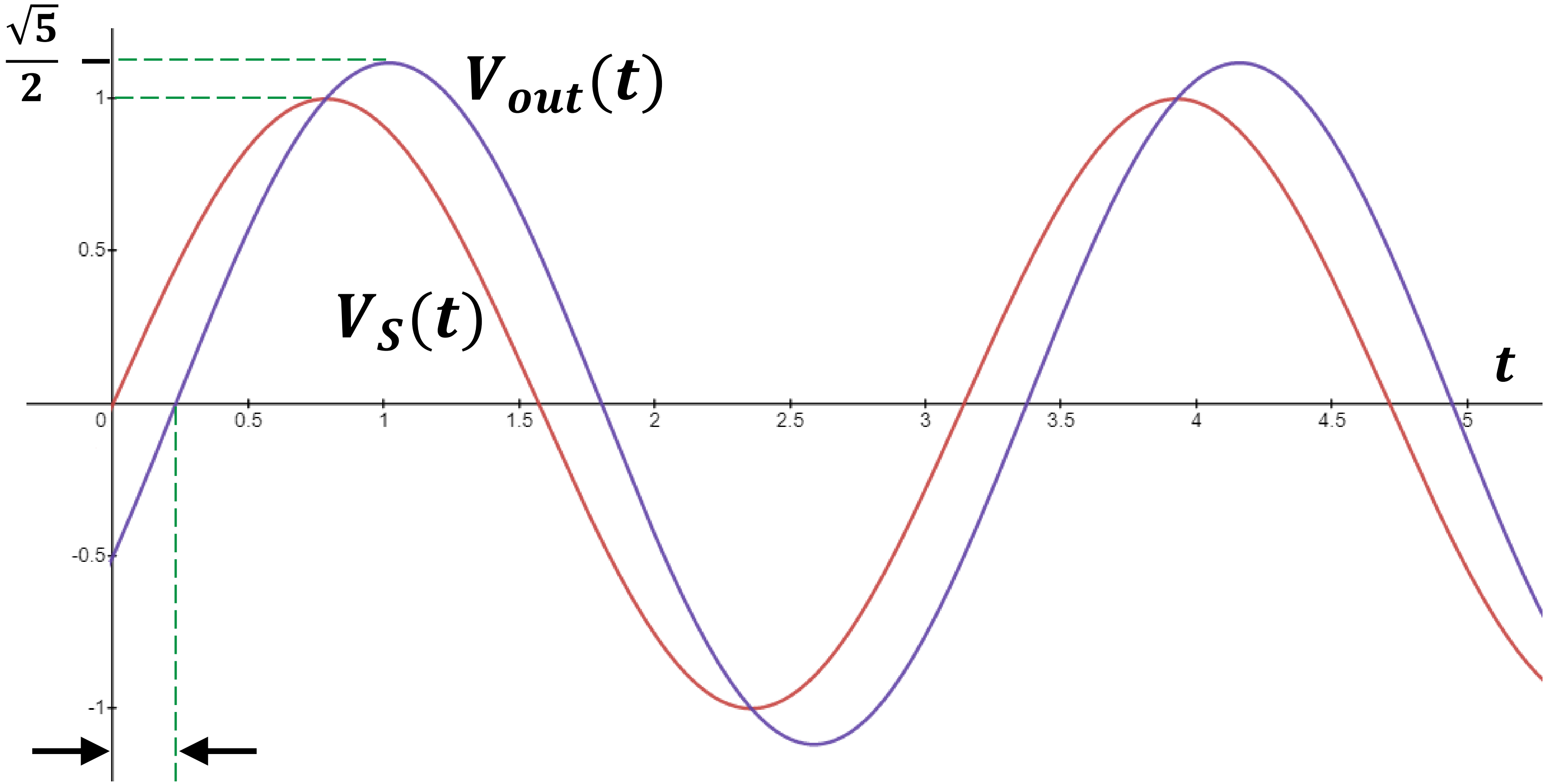
$$L = 1.0\mathrm{H}$$

$$|\mathrm{H}(\omega)| = \frac{\sqrt{R_L^2 + \omega^2 L^2}}{\omega L} = \frac{\sqrt{1^2 + 4 \times 1^2}}{2 \times 1} = \frac{\sqrt{5}}{2}$$

$$\angle\, \mathrm{H}(\omega) = -\tan^{-1}\left(\frac{R_L}{\omega L}\right) = -\tan^{-1}\left(\frac{1}{2}\right) = -0.4636$$

$$V_{out} = \frac{\sqrt{5}}{2} \sin(2t - 0.4636)$$

$$\omega = 2 \text{ rad/sec}$$



$$\Delta t = -0.2318 \text{ [s]} \quad \text{(a time delay)}$$

$$\phi = 2\pi f \, \Delta \text{t} = \omega \, \Delta t$$

$$\phi = -0.4636 \text{ [rad]} = 2\Delta t$$

Additional (more advanced) problems posted on Canvas

Module Week 15
with Lecture 38